CSCI 3104 Fall 2022
Instructors: Prof. Grochow and Chandra Kanth Nagesh

Midterm S27SOLUTION

Due Date .... Saturday Nov 19, 2022 4pm MT
Name .... **Your Name**
Student ID .... **Your Student ID**
Quiz Code (enter in Canvas to get access to the LaTeX template) **e3BfXTWKCU**

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Instructions

- You may either type your work using this template, or you may handwrite your work and embed it as an image in this template. **If you choose to handwrite your work, the image must be legible, and oriented so that we do not have to rotate our screens to grade your work.** We have included some helpful LaTeX commands for including and rotating images commented out near the end of the LaTeX template.
- You should submit your work through the **class Gradescope page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code. Failure to do so will result in your assignment not being graded.

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Problem HC.

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

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27 Standard 27: Amortized Analysis

Problem 27. Consider a data structure that is like a doubling list, except when it's full, instead of allocation twice as much memory and copying over the old data, it only allocates 1.5x as much memory and copies over the old data. The data structure is otherwise like the one discussed in class.

Given a sequence of N `add` operations, let $T(N)$ denote the total cost of those operations on the above data structure. Do the following:

1. Write down a recurrence relation for $T(N)$.
2. Solve the recurrence.
3. What is the amortized run-time of the `add` operation on this data structure?

Answer. Suppose that N is a time when we've just extended the list. Extending this list cost $O(N)$. The last time the list was extended, it had $2N/3$ elements, so the `add`s in between N and $2N/3$ each cost $O(1)$, adding a $N/3 \cdot O(1)$ to our recurrence. And then we add in the time of all the `add`s up to $2N/3$, which is $T(2N/3)$, giving us:

$$\begin{aligned} T(N) &\leq O(N) + N/3 \cdot O(1) + T(2N/3) \\ &= T(2N/3) + O(N) \end{aligned}$$

We can solve this recurrence in a variety of ways. Here we solve it by unrolling.

$$\begin{aligned} T(N) &\leq T(2N/3) + cN \\ &= (T((2/3)^2 N) + c2N/3) + cN \\ &= T((2/3)^2 N) + cN(1 + 2/3) \\ &= (T((2/3)^3 N) + c(2/3)^2 N) + cN(1 + 2/3) \\ &= T((2/3)^3 N) + cN(1 + 2/3 + (2/3)^2) \\ &= T((2/3)^k N) + cN \sum_{i=0}^{k-1} (2/3)^i \end{aligned}$$

Solving for the base case we get $k = \log_{3/2} N$, resulting in

$$T(N) = \Theta(1) + cN \cdot \Theta(1) = \Theta(N),$$

where the second $\Theta(1)$ comes from the geometric series $\sum_{i=0}^{\log_{3/2N}} (2/3)^i \leq \sum_{i=0}^{\infty} (2/3)^i = 1/(1-2/3) = 3 = \Theta(1)$.

The amortized runtime of `add` is thus $T(N)/N = \Theta(N)/N = \Theta(1)$. □