CSCI 3104 Fall 2022
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Quiz 7 S20 SOLUTION

Due Date ..... Thursday Nov 3, 2022 8pm MT
Name ..... **Your Name**
Student ID ..... **Your Student ID**
Quiz Code (enter in Canvas to get access to the LaTeX template) ..... **MQRYY**

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Instructions

- You may either type your work using this template, or you may handwrite your work and embed it as an image in this template. **If you choose to handwrite your work, the image must be legible, and oriented so that we do not have to rotate our screens to grade your work.** We have included some helpful LaTeX commands for including and rotating images commented out near the end of the LaTeX template.
- You should submit your work through the **class Gradescope page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
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Problem HC.

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

Agreed (signature here). □

20 Standard 20: QuickSort

Problem 20. Consider a modified version of QuickSort, in which the Partition subroutine *always chooses the 10-th largest element*. You may assume Partition still takes $\Theta(n)$ time on lists of length n.

Do the following **three** parts of the question (note the third part on the next page).

1. **Write down** the recurrence relation for the runtime of this modified version of QuickSort. **Justify your recurrence relation in 1–2 sentences.**

   Answer.

   $$T(n) = \begin{cases} \Theta(1) & n \leq 1 \\ T(n-10) + T(9) + \Theta(n) & n > 1 \end{cases}$$

   The base case is for lists of length 1, which only takes a constant number of steps—the same as the base case for ordinary QuickSort.

   The recursive case takes $\Theta(n)$ time for Partition (by the assumption given in the problem statement), and then recursively calls on lists of length $n-10$ and 9, since the pivot is not included in the recursive calls.

   Note that you can simplify the recursive case to $T(n-10) + \Theta(n)$, because $T(9)$, whatever it is, is just a constant, so it is $\Theta(1)$, and $\Theta(1) + \Theta(n) = \Theta(n)$.

   □

2. Solve your recurrence relation from part (1), by any method you choose. **Show your work.** Find a function $f(n)$ such that the runtime $T(n)$ is $T(n) = \Theta(f(n))$.

   Answer. We will use unrolling. Using the simplification mentioned above, we have

   $$\begin{aligned} T(n) &= T(n-10) + cn \\ &= (T(n-20) + c(n-10)) + cn \qquad \text{(unroll)} = (T(n-30) + c(n-20)) + c(n-10) + cn \qquad \text{(unroll)} \\ &= T(n-10k) + c\sum_{i=0}^{k}(n-10i) \qquad \text{(pattern)} \end{aligned}$$

   The base case is reached when $n - 10k \leq 1$, which is as soon as $k \geq (n-1)/10$. Setting $k = (n-1)/10$ and plugging back into the pattern, we get

   $$\begin{aligned} T(n) &= T(1) + c\sum_{i=0}^{(n-1)/10}(n-10i) \\ &= \Theta(1) + c\sum_{j=0}^{n/10} 10j \\ &= \Theta(1) + 10c\sum_{j=0}^{n/10} j \\ &= \Theta(1) + 10c\frac{(n/10)(n/10+1)}{2} = \Theta(n^2). \end{aligned}$$

   □

3. Write down the following three functions and put them in order from smallest to largest. If two are asymptotically equal (Θ of one another), you should clearly indicate this.

   - the best-case running time of QuickSort (when it always chooses the best possible pivot),
   - the worst-case running time of QuickSort (when it always chooses the worst possible pivot),
   - your answer from part (2) above.

   You do not need to recalculate the best- and worst-case runtimes, those you can get from class notes.

Answer. The best-case running time is $\Theta(n \log n)$.

The worst-case running time is $\Theta(n^2)$.

The running time for the modified version of QuickSort in this question is $\Theta(n^2)$, asymptotically the same as the worst-case running time. So we have

Best-case << Worst-case = Modified version

□