CSCI 3104 Fall 2022
Instructors: Prof. Grochow and Chandra Kanth Nagesh

Midterm S20

Due Date ..... Saturday Nov 19, 2022 4pm MT
Name ..... **Your Name**
Student ID ..... **Your Student ID**
Quiz Code (enter in Canvas to get access to the LaTeX template) **7X0vUY9sMh**

Contents

Instructions

- You may either type your work using this template, or you may handwrite your work and embed it as an image in this template. **If you choose to handwrite your work, the image must be legible, and oriented so that we do not have to rotate our screens to grade your work.** We have included some helpful LaTeX commands for including and rotating images commented out near the end of the LaTeX template.
- You should submit your work through the **class Gradescope page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code. Failure to do so will result in your assignment not being graded.

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Problem HC.

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

Agreed (signature here). □

20 Standard 20: QuickSort

Problem 20. Consider a modified version of QuickSort, in which the Partition subroutine *always chooses the* $(2/3)n$*-th largest element of the list.*You may assume Partition still takes $\Theta(n)$ time on lists of length n.

Do the following **three** parts of the question (note the third part on the next page).

1. **Write down** the recurrence relation for the runtime of this modified version of QuickSort. **Justify your recurrence relation in 1–2 sentences.**

   Answer.

   $$T(n) = \begin{cases} \Theta(1) & n \leq 1 \\ T(2n/3) + T(n/3) + \Theta(n) & n > 1. \end{cases}$$

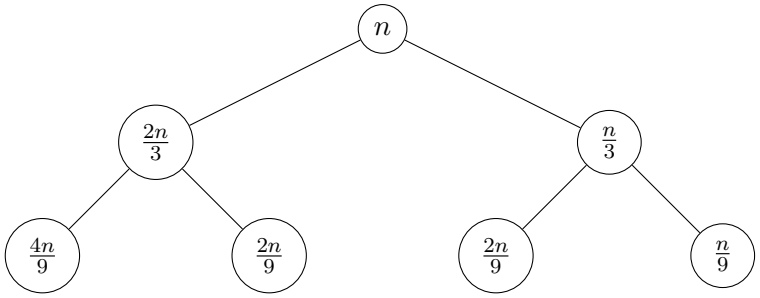
   The base case, for constant-size input, always takes constant time.

   In the recursive case, Partition costs $\Theta(n)$, and we recurse on two lists, one of size $2n/3$ and the other of size $n/3$. □

2. Solve your recurrence relation from part (1), by the tree method. **Show your work.** Find a function $f(n)$ such that the runtime $T(n)$ is $T(n) = \Theta(f(n))$.

   Answer. Note: Because partition is guaranteed to split off a constant *fraction* of the list, we should expect the final answer to be asymptotically the same as the best case, which is $\Theta(n \log n)$.

   Each node has 2 children, one of which has size $2/3$ and one of which has size $1/3$:

   At a node with imput size m, the work is $\Theta(m)$, say cm. Notice that the sum of the inputs of the nodes on every level is still n, so the sum of the total non-recursive work done at each level is cn.

   The slowest-shrinking branch is the left-most one, which decreases like $(2/3)^k n$ at the k-th level. So we reach the base case after at most $\log_{3/2} n$ levels. The fastest-shrinking branch is the right-most one, which decreases like $(1/3)^k n$ at the k-th level, so that branch ends after only $\log_3 n$ levels. Thus, the total number of levels is $\Theta(\log n)$.

   As each level sums up to cn work, the total work is $cn\Theta(\log n) = \Theta(n \log n)$ (as expected from the heuristic mentioned at the start of the solution). □

3. Write down the following three functions and put them in order from smallest to largest. If two are asymptotically equal (Θ of one another), you should clearly indicate this.

   - the best-case running time of QuickSort (when it always chooses the best possible pivot),
   - the worst-case running time of QuickSort (when it always chooses the worst possible pivot),
   - your answer from part (2) above.

   You do not need to recalculate the best- and worst-case runtimes, those you can get from class notes.

Answer. Best case is $\Theta(n \log n)$, the same (asymptotically) as the answer from part 2. The worst-case is $\Theta(n^2)$, which is much larger than the best case. □