

# Midterm S21SOLUTION

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Due Date .....Saturday Nov 19, 2022 4pm MT  
Name ..... **Your Name**  
Student ID ..... **Your Student ID**  
Quiz Code (enter in Canvas to get access to the LaTeX template) ..... **SMhbkYPyZN**

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## Instructions

- You may either type your work using this template, or you may handwrite your work and embed it as an image in this template. **If you choose to handwrite your work, the image must be legible, and oriented so that we do not have to rotate our screens to grade your work.** We have included some helpful LaTeX commands for including and rotating images commented out near the end of the LaTeX template.
- You should submit your work through the **class Gradescope page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students. Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code. Failure to do so will result in your assignment not being graded.

## Honor Code (Make Sure to Virtually Sign)

**Problem HC.**     • My submission is in my own words and reflects my understanding of the material.

- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

*Agreed (signature here).*



## 21 Standard 21: Dynamic Programming: Identify Precise Subproblems

**Problem 21.** The Counting Subset-Sum problem is defined as follows.

- **Input:** We are given  $n$  items with positive weights  $w_1, \dots, w_n > 0$ , as well as a target threshold  $W > 0$ .
- **Output:** The number of subsequences  $w_{i_1}, \dots, w_{i_k}$  such that:

$$\sum_{j=1}^k w_{i_j} = W.$$

For example, consider the input array  $A = [4, 15, 8, 16, 23, 42]$ . If  $W = 31$  then the output is 2 since there are two subsequences that add to 31:  $8 + 23 = 15 + 16 = 31$ . If  $W = 13$ , the output is 0 since no subsequence of  $A$  has sum equal to 13. If  $W = 4$ , the output is 1, since the only subsequence that adds up to 4 is  $[4]$  itself.

Suppose you are going to solve this problem by dynamic programming; this can be done with a two-dimensional table  $T$ . **Clearly define** what subproblems  $T[i, j]$  corresponds to, and which other cells  $(i', j')$  need to be considered when determining the value of  $T[i, j]$ .

*Answer.* THERE MAY BE SEVERAL WAYS TO SOLVE THIS. HERE IS ONE.

We can define  $T[i, j]$  to be the number of subsequences of  $[w_1, \dots, w_i]$  (the first  $i$  weights) that add up to  $j$ .

Then  $T[i + 1, j]$  can be determined by considering  $T[i, j - w_{i+1}]$  (covering the cases when  $w_{i+1}$  is used) and  $T[i, j]$  (covering the cases when  $w_{i+1}$  is not used).

THE QUESTION DID NOT ASK ABOUT THE FOLLOWING, BUT FOR YOUR REFERENCE, WE GIVE MORE INFO ABOUT THE DYNAMIC PROGRAMMING SOLUTION HERE.

THE FINAL ANSWER WOULD BE IN  $T[n, W]$ , AND THE BASE CASES ARE WHEN  $i$  OR  $j$  IS ZERO. THE RECURRENCE IS:

$$T[i, j] = \begin{cases} 1 & j = 0 \\ 0 & i = 0 \text{ and } j \neq 0 \\ T[i - 1, j] & w_i > j \\ T[i - 1, j] + T[i - 1, j - w_i] & w_i \leq j \end{cases}$$

NOTE THAT  $T[i, 0] = 1$  BECAUSE THERE IS A UNIQUE SUBSEQUENCE THAT ADDS UP TO 0, NAMELY THE EMPTY SUBSEQUENCE OF LENGTH 0. IF YOU CONSIDER WHAT HAPPENS AT  $T[1, w_1]$ , YOU'LL SEE WHY THIS BASE CASE IS NEEDED.  $\square$