CSCI 3104 Fall 2022
Instructors: Prof. Grochow and Chandra Kanth Nagesh

Problem Set 12

Due Date .... **Tuesday** December 6, 2022
Name .... **Your Name**
Student ID .... **Your Student ID**
Collaborators .... **List Your Collaborators Here**

Contents

Instructions

- The solutions **must be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Useful links and references on LaTeXcan be found here on Canvas.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You are welcome and encouraged to collaborate with your classmates, as well as consult outside resources. You must **cite your sources in this document. Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section Honor Code). Failure to do so will result in your assignment not being graded.

Honor Code (Make Sure to Virtually Sign the Honor Pledge)

Problem HC. On my honor, my submission reflects the following:

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.

- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

In the specified region below, clearly indicate that you have upheld the Honor Code. Then type your name.

Honor Code. □

28 Standard 28: Computational Complexity: Formulating Decision Problems

Problem 28. Recall the Maximum Flow problem from class:

> *Input:* A flow network, which consists of a directed graph $G = (V, E)$, two vertices $s, t \in V$, and a non-negative capacity function $c: E \to \mathbb{R}_{\geq 0}$.
> *Output:* The maximum value of an $s - t$ flow in G

Formulate the decision variant of this problem using the Input/Decide format below. **Hint:** See Example 172 of Levet's Lecture Notes.

Answer. Input: YOUR ANSWER HERE
Decide: YOUR ANSWER HERE □

Input: A flow network, which consist of directed graph $G = (V, E)$, two vertices $s, t \in V$, and a non-negative capacity function $c: E \to R \geq 0$. Let $k \in N$

Decide: Does there exist a flow having a value at least k?

29 Standard 29: Computational Complexity: Problems in P

Problem 29. Consider the decision variant of the Interval Scheduling problem.

- Input: Let $\mathcal{I} = \{[s_1, f_1], \ldots, [s_n, f_n]\}$ be our set of intervals, and let $k \in \mathbb{N}$.
- Decide: Does there exist a set $S \subseteq \mathcal{I}$ of at least k pairwise-disjoint intervals?

Show that the decision variant of the Interval Scheduling problem belongs to P. You are welcome and encouraged to cite algorithms we have previously covered in class, including known facts about their runtime. [**Note:** To gauge the level of detail, we expect your solutions to this problem will be 2-4 sentences. We are not asking you to come up with a new algorithm nor to analyze an algorithm in great detail.]

Answer. □

1. Apply the greedy alogorithm.

$\Theta(|\mathcal{I}|)$ a. Pick the one with the last ending time. from the set

$\Theta(|\mathcal{I}|)$ b. Check whether it is overlapped with picked ones

$\Theta(1)$ c. Remove it from the set.

d. repeat executing. Step (a), (b), (c). until we cannot pick anyone.

$\Rightarrow$ Takes. $\Theta(|\mathcal{I}|^2)$

$\Rightarrow$ It belongs to P

30 Standard 30: Computational Complexity: Problems in NP

Problem 30. Consider the Vertex Cover problem. A *vertex cover* in a graph $G = (V, E)$ is a subset of vertices, $C \subseteq V$, such that every edge touches at least one vertex in C.

> *Input:* An undirected graph $G = (V, E)$ and a positive integer k
> *Decide:* Does G have a vertex cover of $\leq k$ vertices?

verify

Show that this problem belongs to NP.

Answer. □

A viable certificate is a set of vertices S

Then. we need to verify

1. whether $S \subseteq V$. $\Rightarrow$ $\Theta(|V|^2)$
2. whether $|S| \leq k$ $\Rightarrow$ $\Theta(|V|)$
3. whether all edges in G touches any vertex in S. $\Rightarrow$ $\Theta(|V| \cdot |E|)$

Verification takes polynomial time.

$\Rightarrow$ It belongs to NP.

31 Standard 31: Structure and Consequences of P vs. NP

Problem 31. A student has a decision problem L which they know is in the class NP. This student wishes to show that L is NP-complete. They attempt to do so by constructing a polynomial time reduction from L to SAT, a known NP-complete problem. That is, the student attempts to show that $L \leq_p$ SAT. Determine if this student's approach is correct and justify your answer.

Answer. □

It is incorrect. In addition to show $L \in NP$, we also need to show that a NP-complete problem, SAT can be reduced to L. $\Rightarrow$ Then we know that

$L \geq_p SAT$

$IS. \subseteq NP$

$IS \leq_p VC$

$SAT \leq_p$ Other problems : NP-hard.

at least as hard as all problems in NP.

All NP problems $\leq_p SAT$