CSCI 3104 Fall 2022
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Quiz 5 S15 SOLUTION

Due Date .... Thursday Oct 20, 2022 8pm MT
Name .... **Your Name**
Student ID .... **Your Student ID**
Quiz Code (enter in Canvas to get access to the LaTeX template) .... **DSGZH**

Contents

Instructions

- You may either type your work using this template, or you may handwrite your work and embed it as an image in this template. **If you choose to handwrite your work, the image must be legible, and oriented so that we do not have to rotate our screens to grade your work.** We have included some helpful LaTeX commands for including and rotating images commented out near the end of the LaTeX template.
- You should submit your work through the **class Gradescope page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
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Problem HC.

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

Agreed (signature here). □

$$1 + \sum_{j=i}^{n-i} 4 = 1 + 4(n-i-i) = 1 + 4(n-2i).$$

15 Standard 15: Analyzing Code—Nested Dependent Loops

Problem 15. Analyze the *worst-case* runtime of the following algorithm. Clearly derive the runtime complexity function $T(n)$ for this algorithm, and then find a tight asymptotic bound for $T(n)$ (that is, find a function $f(n)$ such that $T(n) = \Theta(f(n))$). Avoid heuristic arguments from 2270/2824 such as multiplying the complexities of nested loops.

Notice (because they may not be what you think!):

- the lower bound on j in the inner loop, and
- the upper bound on j in the inner loop

$i = 3$ $n \leq 2 \cdot 3$

Algorithm 1 Nested Algorithm 2

```
procedure Foo7(Integer n)
    for i ← 1; i ≤ n; i ← i + 1 do
        for j ← i; j ≤ n − i; j ← j + 1 do
            print "Hi"
```

for $j \leftarrow 3$; $j \leq n-3$; $j \leftarrow j+1$ do

1 , i

Answer. We analyze from the innermost loop first.

The inner loop has:

- 1 step for the initialization $j \leftarrow i$
- 4 steps per iteration: $n - i$, comparing j to $n - i$, incrementing j by 1, and printing.
- Runs from $j = i$ to $j = n - i$, for a total of $n - 2i$ iterations.
- So the total for the inner loop is $\boxed{1 + 4(n - 2i)}$
- **Important note:** The above is only valid for $i \leq n/2$. When $i > n/2$, at the initial iteration of this loop we have $j = i > n - i$ (because $i > n/2$, so $n - i < n/2$), so the loop doesn't iterate at all, but it still incurs the cost for initialization and comparison (3 steps)

The outer loop has:

- 1 step for the initialization
- 2 steps per iteration for the comparison and incrementing i
- At the i-th iteration, the inner loop costs $1 + 4(n - 2i)$ for $i \leq n/2$, and costs 3 for $i > n/2$ (because of the important note above).
- The total cost is thus:

$$1 + \sum_{i=1}^{n/2}(1 + 2 + 1 + 4(n - 2i)) + \sum_{i=n/2+1}^{n}(1 + 2 + 3) = 1 + 4\frac{n}{2} + 4\sum_{i=1}^{n/2}(n - 2i) + 6(n/2 - 1)$$

$$= 5n - 7 + 4\sum_{i=0}^{(n-2)/2} 2i$$

$$= -7 + 5n + 8\frac{(n-2)/2 \cdot n/2}{2}$$

$$= -7 + 5n + (n - 2)n = \Theta(n^2)$$

□