ICU Bed Allocation Simulation

Game Theory Project – Spring 2020

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Project Overview

This project simulates the allocation of ICU beds among three hospitals:

- Hospital A (Social Security) with $n_A = 3$ beds
- Hospital B (University) with $n_B = 4$ beds
- Hospital C (Private) with $n_C = 2$ beds

A centralized screening center assigns critically ill COVID-19 patients to these hospitals in discrete time slots. Patients arrive from two classes (Class 1 and Class 2) with different probabilities. The duration of ICU usage follows an exponential probability density function:

$$f_i(t) = 0.5i \cdot e^{-0.5it}$$

where $i \in \{1, 2\}$ denotes the patient class.

Allocation Mechanisms

Optimal Mechanism

In each time slot, two random numbers between 0 and 1 are generated. By comparing these values with p_1 and p_2, we determine whether patients of type 1 or 2 arrive. If a patient arrives, their value is determined based on the exponential distribution corresponding to their type.

For each patient, the cost function $C_i(v_i)$ is computed, and patients are added to a waiting list sorted in descending order of $C_i(v_i)$. Beds are then allocated to patients with the highest values.

The payment for each patient is determined by the auction mechanism:

- If the number of patients with positive $C_i(v_i)$ exceeds the number of available beds plus one, the payment is equal to the $C_i(v_i)$ of the highest-valued patient who does not receive a bed.
- Otherwise, the payment is equal to the patient's own $C_i(v_i)$.

At the end of each time slot, new values are assigned to hospitalized patients based on their type's distribution, and the waiting list is updated accordingly.

Stable Matching Mechanism

In this mechanism, hospitals have equal priority toward patients. Patients are assigned to available beds based on descending order of their value. Each patient is matched to the first hospital with an available bed that aligns with their preferences.

Payments are determined solely by the patient's value at the time of allocation and are unaffected by their duration of stay.

Mortality Modeling

In both mechanisms, at the end of each time slot, a random variable is generated for each hospitalized patient to determine discharge. If mortality is considered, a similar random process is used to determine whether unassigned patients die. The probability of death is modeled as:

$$P_{\text{death}}(t) = 1 - 2 \cdot e^{-0.5it}$$

Simulation Metrics

The following metrics are computed for each mechanism:

- Average income per hospital and overall
- Average payment per patient type
- Average payment across all patients
- Number of deceased patients (if mortality is considered)

Simulation Results

Optimal Mechanism – Without Mortality

- Hospital A Average Income: up to 8 units
- Hospital B Average Income: up to 15 units
- Hospital C Average Income: up to 15 units
- Patients Type 1 Average Payment: up to 30 units
- Patients Type 2 Average Payment: up to 30 units
- Patients Overall Average Payment: up to 30 units

Optimal Mechanism – With Mortality

- Hospital A Average Income: up to 3 units
- Hospital B Average Income: up to 6 units
- Hospital C Average Income: up to 5 units
- Patients Type 1 Average Payment: up to 12 units
- Patients Type 2 Average Payment: up to 10 units
- Patients Overall Average Payment: up to 30 units
- Number of Deceased Patients: up to 400

Stable Matching – Without Mortality

- Hospital A Average Income: up to 15 units
- Hospital B Average Income: up to 30 units
- Hospital C Average Income: up to 25 units
- Patients Type 1 Average Payment: up to 25 units
- Patients Type 2 Average Payment: up to 25 units
- Patients Overall Average Payment: up to 25 units

Stable Matching – With Mortality

- Hospital A Average Income: up to 6 units
- Hospital B Average Income: up to 12 units
- Hospital C Average Income: up to 10 units
- Patients Type 1 Average Payment: up to 8 units
- Patients Type 2 Average Payment: up to 10 units
- Patients Overall Average Payment: up to 10 units
- Number of Deceased Patients: up to 800

Analysis and Conclusion

The average income of hospitals under the optimal mechanism is lower than under stable matching, because patients pay less than their full value. In contrast, under stable matching, patients pay their full value.

The average payment per patient is lower under stable matching due to insurance-based discounts. Although the optimal mechanism reduces payments below patient value, the insurance discounts in stable matching result in even lower payments on average.

Mortality rates are higher under the optimal mechanism, since only patients with positive $C_i(v_i)$ are allocated beds. In stable matching, any patient may be assigned a bed regardless of value, reducing wait times and increasing access.