Math 4800
Spring 2022

Here is the list of the suggested questions - you don't have to answer all the questions but try to think about all the questions.

Please submit your solutions/ideas on Wednesday February 23: Please submit your codes by e-mail, and pdf file of your results (and the discussion of the results) online via Canvas.

- Show that the condition number $\kappa(A)$ of the invertible matrix A satisfies $\kappa(A) \geq 1$.

- Give examples of a) strictly diagonally dominant matrices by rows; b) symmetric positive definite marices. Justify your examples.

- Show that if matrix $A, n \times n$ is strictly diagonally dominanat by rows, then Jacobi and Gauss-Seidel methods converge for the solution of $Au = f$ for any intial guess u_0.

- Implement Jacobi; SOR and (*optional*: "underrelaxed Jacobi" with $\omega = 2/3$) to solve linear system $Au = f$. Test your methods. Did the methods converge for any matrix A?

  For matrices for which SOR method converges can you think about the idea or propose method (numerical or theoretical) how to select "optimal" ω or near "optimal" ω ("optimal" ω is the ω that will produce the least number of iterations in SOR to achieve the desired tolerance/accuracy)? *You can try to research/investigate some specific class of matrices A. This question can make a very good final project if some of you would like to keep working on it afterwards as well.*

- Consider again second order centered FD method for solving

$$\begin{aligned} u'' &= f(x), \text{ in } \Omega = [0,1] \\ u(0) &= \alpha, \quad u(1) = \beta \end{aligned} \tag{1}$$

1. Use Jacobi; SOR (and *optional*: "underrelaxed Jacobi" with $\omega = 2/3$) to solve linear system $A^h u^h = f^h$ in your centered FD code

2. Apply your code to the following problem:
   True solution $u(x) = e^x$ with $\alpha = 1$, $\beta = e$ and $f(x) = e^x$. Compare the performance of your method when you use Jacobi; SOR (and *optional*: "underrelaxed Jacobi" with $\omega = 2/3$) to solve linear system. Discuss the results.

3. Apply your code now to the following problem:
   True solution $u(x) = 1 + 12x - 10x^2 + 0.5\sin(20\pi x^3)$ with $\alpha = 1$, $\beta = 3$ and $f(x) = -20 + 0.5\phi''(x)\cos(\phi(x)) - 0.5(\phi'(x))^2\sin(\phi(x))$, with $\phi(x) = 20\pi x^3$.
   First, check that this is correct right-hand side for the given true solution $u(x)$. Then, again, compare the performance of your method when you use Jacobi; SOR (and *optional*: "underrelaxed Jacobi" with $\omega = 2/3$) to solve the linear system. What do you observe?

Please write the discussion of the observed results.