MATH 4800
Spring 2022

Here is the list of the suggested questions.
Please submit your solutions/ideas on Wednesday February 2:
Please submit your codes by e-mail, and pdf file of your results (and the discussion of the results) online via Canvas.

- Implement second order centered FD (discussed in class) method for solving

$$\begin{aligned} u'' &= f(x), \text{ in } \Omega = [0,1] \\ u(0) &= \alpha, \quad u(1) = \beta \end{aligned} \tag{1}$$

- Apply your code to the following problem:
  True solution $u(x) = \sin(\pi x)$ with $\alpha = 0$, $\beta = 0$ and $f(x) = -\pi^2 \sin(\pi x)$.
  Remark: $f(x), \alpha$ and β are computed using the true solution $u(x)$ in the equation (1)

- Estimate numerically the convergence of your method to the true solution by computing the maximum error:

$$E = \max_{x_j} |u(x_j) - u_j|$$

  Remark: To do this you need to compute your numerical solution on several levels of the refinement of the grid, for example, consider $h = \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$....
  Please make the error table for the errors and compute the ratio between the errors. What do you observe?

- Investigate the dependence of the largest and smallest eigenvalues of your finite difference matrix A^h on the grid size h.
  For example, you can use Matlab command $eig(A^h)$ to compute the eigenvalues or you can try to find the analytical expression for the eigenvalues.
  Plot your numerical solution for $h = \frac{1}{4}$ and $h = \frac{1}{32}$ and graph on the same plot the exact solution $u(x)$.

- Test your code on the true solution $u(x) = x$. What do you observe?

- Implement second order centered FD method for solving

$$\begin{aligned} u'' &= f(x), \text{ in } \Omega = [0,1] \\ u_x(0) &= \gamma, \quad u(1) = \beta \end{aligned} \tag{2}$$

- How will you approximate the boundary condition at the left endpoint $x = 0$?

  You can start with the same code as for the previous problem (1) but you have to modify it appropriately to incorporate different boundary conditions.

- Suggest your own test problem and test the numerical method. Please make the error table again for the several choices of the grid, for example, $h = \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$... What do you observe?
- *Please write the discussion of the observed results.*