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To cite this article: Zhiqiang Chao *et al* 2017 *J. Phys.: Conf. Ser.* **916** 012022

View the article online for updates and enhancements.

Inverse Kinematics Solution and Verification of 4-DOF Hydraulic Manipulator

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Abstract. Aimed at four degree of freedom (DOF) hydraulic manipulator, an inverse kinematics solution is proposed from Cartesian space to drive space based on geometrical method. To the structural and diver characteristics of the manipulator, a forward kinematics is conducted by using D-H method. The position and orientation of manipulator's end-effector can be obtained under the kinematics constraint. By analyzing the structure, the solution of inverse kinematics of manipulator can be obtained, and the conversion between drive space and joint space can be got through the sport's mechanism kinematics. In order to meet the need of motion planning and control of the manipulator, the inverse kinematics and conversion are validated based on simulation.

1. Introduction

Hydraulic manipulator plays an important role in industry, agriculture and military because it has small size, high power, durability and large operation range. With hydraulic motor rotating and hydraulic cylinder expanding, the links can be moved. Further, hydraulic manipulator realize the position. Thus, in order to make sure the position of manipulator's end-effector in the Cartesian coordinate system, the conversion between Cartesian space and joint space and between joint space and drive space should be determined firstly.

Inverse kinematics of manipulator is solving the joint variables according to the known position and orientation of manipulator's end-effector. The result perhaps is no solution, unique solution or multiple solutions. The kinematics is used for analyzing, and the inverse kinematics is used for control of manipulator. At present, there are some common methods for inverse kinematics [1]: analytic method, geometric method and iterative method. Analytic method applies to multiple DOF manipulator, and its real time ability is poor because of a lot of inverse matrix multiplication in the calculation process. We always can't converge to correct solution by iterative method. Geometric method keeps the advantage of calculation being simple, and it applies to few DOF manipulator. Besides, there are some other method for inverse kinematics, such as dual matrix method, screw algebraic method [2], dual quaternion method, genetic algorithm [3] and intelligent hill climbing algorithm [4]. According to the characteristics of manipulator designed in this paper, the geometric method for inverse kinematics is adopted. In the control of hydraulic manipulator, the motion of every joint of manipulator can be acquired by inverse kinematics. The controller converts motion to control signal, and the servo valves are driven. So the hydraulic motor and cylinders can be controlled.

2. Model and forward kinematics of manipulator

Targeting 4 DOF hydraulic manipulator, the structure and coordinate system are shown in the figure 1. The four joint are all rotational. The $X_0Y_0Z_0$ is base coordinate system, the $X_1Y_1Z_1$, $X_2Y_2Z_2$, $X_3Y_3Z_3$ are link coordinate systems respectively, and $X_4Y_4Z_4$ is end-effector coordinate system. According to D-H parameters, the unique position and orientation of manipulator's end-effector in the base coordinate system can be acquired.
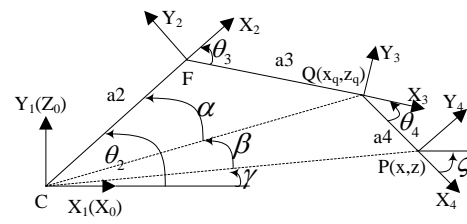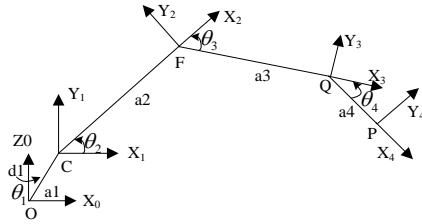

Figure 1. The simplified structure of manipulator   **Figure 2.** The simplification for inverse kinematics

Table 1. D-H parameters of manipulator

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	90	187.5mm	300mm	θ_1
2	0	L_2	0	θ_2
3	0	L_3	0	θ_3
4	0	L_4	0	θ_4

L1=515mm，L2=1100mm，L3=1350mm，L4=280mm

The mission is usually described in the Cartesian space, so the unique position and orientation can be determined in the Cartesian coordinate system. According to conversion matrix, the position and orientation of 4 DOF manipulator's end-effector can be described as

$$\begin{cases} X = c_1(a_4c_{234} + a_3c_{23} + a_2c_2 + a_1) \\ Y = s_1(a_4c_{234} + a_3c_{23} + a_2c_2 + a_1) \\ Z = a_4s_{234} + a_3s_{23} + a_2s_2 + d_1 \\ \xi = \theta_2 + \theta_3 + \theta_4 \end{cases} \qquad (1)$$

3. Inverse kinematics

Compared to forward kinematics, inverse kinematics is more complicated. Its result perhaps is no solution or multiple solutions. Duffy proved that the result of inverse kinematics exists if three successive joints of manipulator are parallel [6]. According to this proof, the manipulator designed in this paper can acquired at most 16 groups solutions. The solution that rotates a minimum angle will be chosen. In this paper, the geometric method is chosen for inverse kinematics.

As shown in the figure 2, at first the rotation from joint 1 is not considered. So $a_1 = \theta_1 = d_1 = 0$, and point C is considered as point O. Taking the hinge point C of link 2 as the origin, the base coordinate system is built. And the position and orientation of manipulator's end-effector are $[x, 0, z, \varsigma]^T$, and $\varsigma = \theta_1 + \theta_2 + \theta_3$.

The angle γ shown in the figure 2 is solved firstly.

$$\gamma = \arctan \frac{z}{x} \tag{2}$$

The angle β shown in the figure 2 is solved secondly. According to coordinate value of point P, the coordinate value of point Q can be acquired by coordinate conversion. The equation ${}^{i}p = {}^{i}A_{i+1}{}^{i+1}p$ [7] has been known, so

$$\begin{bmatrix} x_q \\ y_q \\ z_q \\ 1 \end{bmatrix} = {}^{0}A_3 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = {}^{0}A_4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{3}$$

Because the rotation of joint 1 is not considered, ${}^{0}A_1$ become identity matrix. ${}^{0}A_3$ 和 ${}^{0}A_4$ are substituted to ${}^{0}p = {}^{0}A_4{}^{4}p$, so

$$\begin{bmatrix} x_q \\ y_q \\ z_q \\ 1 \end{bmatrix} = \begin{bmatrix} a_3 c_{23} + a_2 c_2 \\ 0 \\ a_3 s_{23} + a_2 s_2 \\ 1 \end{bmatrix} \quad \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} a_4 c_{234} + a_3 c_{23} + a_2 c_2 \\ 0 \\ a_4 s_{234} + a_3 s_{23} + a_2 s_2 \\ 1 \end{bmatrix} \tag{4}$$

The two equation are compared

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x_q \\ y_q \\ z_q \\ 1 \end{bmatrix} \begin{bmatrix} a_4 c_{234} \\ 0 \\ a_4 s_{234} \\ 0 \end{bmatrix} \tag{5}$$

We can know that

$$\begin{aligned} x_q &= x - a_4 \cos \varsigma \\ y_q &= y \qquad\qquad\qquad \varsigma = \theta_1 + \theta_3 + \theta_4 \\ z_q &= z - a_4 \sin \varsigma \end{aligned} \tag{6}$$

So far, the point Q can be acquired. In triangle PQC, according to cosine theorem, we can know

$$\beta = \arccos \frac{CQ^2 + CP^2 - a_4{}^2}{2CQ \cdot CP} \tag{7}$$

In triangle FQC, according to cosine theorem, we can know

$$\alpha = \arccos \frac{CQ^2 + a_2{}^2 - a_3{}^2}{2CQ \cdot a_2} \tag{8}$$

Based on equation (2)(7)(8)

$$\theta_2 = \alpha + \beta + \gamma$$

$$\theta_3 = \pi - \arccos \frac{a_2^2 + a_3^2 - CQ^2}{2a_2 a_3} \tag{9}$$

$$\theta_4 = \varsigma - \theta_2 - \theta_3$$

Now the rotation of the first joint is considered, the orientation of manipulator' end-effector doesn't change, and the polygon CPQF consisting of link2, Link3, and link4 maintains the same shape. So $x = \sqrt{(x - a_1)^2 + y^2} \; and \; z = z - d_1$ are substituted to equation from (6) to (9). It means that the joint 1 take a counter rotating θ_1, and the origin Q is move back to O. So the line CP will project the X-axis just right. Finally

$$\theta_1 = \arctan \frac{y}{x}, \theta_2 = \alpha + \beta + \gamma, \theta_3 = \pi - \arccos \frac{a_2^2 + a_3^2 - CQ^2}{2a_2 a_3}, \theta_4 = \varsigma - \theta_2 - \theta_3 \tag{10}$$

In this equation, the coordinate value is in the base coordinate system of manipulator. The CQ and CP can be expressed by equation(6) and point P coordinate.

$$CQ = \sqrt{(x_q - a_1)^2 + y^2 + (z_q - d_1)^2}, CP = \sqrt{(x - a_1)^2 + y^2 + (z - d_1)^2}$$

And α, β, γ respectively is

$$\alpha = \arccos \frac{CQ^2 + a_2{}^2 - a_3{}^2}{2CQ \cdot a_2}, \beta = \arccos \frac{CQ^2 + CP^2 - a_4{}^2}{2CQ \cdot CP}, \gamma = \arctan \frac{z - d_1}{\sqrt{(x - a_1)^2 + y^2}} \tag{11}$$
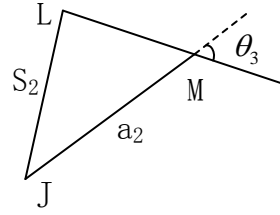
4. Conversion between joint space and drive space

Like the relationship between Cartesian space and joint space, the extension length of hydraulic cylinder can be acquired by manipulator joint angle. On the contrary, the manipulator joint angle can be acquired by extension length of hydraulic cylinder [8]. The space conversion depends on geometric structure of manipulator. Like inverse kinematics, geometric method can be used for solutions because of no coupling in these solution for each other.

In this paper, under the premise of ignoring joint 1, the relationship between joint angle and expansion length of hydraulic cylinder is built. The specific steps are:

4.1. The relationship between joint angle of link 2 and expansion length of hydraulic cylinder 1

The link 2 is manipulator's upper arm. It is driven by hydraulic cylinder 1. The relationship between joint θ_2 of link 2 and length S_1 of hydraulic cylinder can be acquired by simplified figure 3.
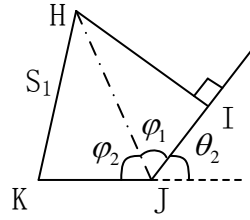
Figure 3. The mechanism sketch of link 2   **Figure 4.** The mechanism sketch of link 3

S_1 is hydraulic cylinder length to be solved. JI is a part of link 2. HI $\perp$ IJ. The lengths of IJ,KJ and HJ are known. Based on Cosine theorem,

$$\varphi_1 = \arccos \frac{JI^2 + JH^2 - HI^2}{2JI \cdot JH} \tag{12}$$

$$\varphi_2 = \arccos \frac{JK^2 + JH^2 - S_1^2}{2JK \cdot JH} \tag{13}$$

Because $\theta_2 = \pi - \varphi_1 - \varphi_2$, the relationship between θ_2 and S_1 is

$$\theta_2 = \pi - \arccos \frac{JI^2 + JH^2 - HI^2}{2JI \cdot JH} - \arccos \frac{JH^2 - JK^2 - S_1}{2JH \cdot JK} \tag{14}$$

*4.2. The relationship between joint angle of link 3 and expansion length of hydraulic cylinder 2*
The link 3 is manipulator's lower arm. It is driven by hydraulic cylinder 2. The relationship between joint θ_3 of link 3 and length S_2 of hydraulic cylinder can be acquired by simplified figure 4.

S_2 is hydraulic cylinder length to be solved. JM is the length a_2 of link 2. The length of LM is known. Based on Cosine theorem,

$$\theta_3 = \arccos \frac{LM^2 + a_2^2 - S_2}{2LM \cdot a_2} \tag{15}$$

4.3 The relationship between joint angle of link 4 and expansion length of hydraulic cylinder 3
The link 4 is the manipulator's wrist. It is driven by hydraulic cylinder 3. The relationship between joint θ_4 of link 4 and length S_3 of hydraulic cylinder can be acquired by simplified figure 5.
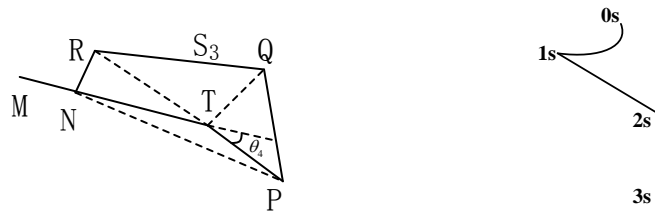
Figure 5. The mechanism sketch of link4 **Figure 6.** Desirable trajectory of manipulator's end-effector

S_3 is hydraulic cylinder length to be solved. The lengths of NR, QP, TP, NP, RT, TQ and NT are known. Because $\angle PTN + \angle NTR + \angle RTQ + \angle QTP = 2\pi$, we can acquire that

$$(\pi - \theta_4) + \angle NTR + \angle RTQ + \angle QTP = 2\pi$$

Based on Cosine theorem,

$$\angle NTR = \arccos \frac{TR^2 + TN^2 - NR^2}{2TR \cdot TN} \tag{16}$$

$$\angle RTQ = \arccos \frac{TR^2 + TQ^2 - S_3^{\ 2}}{2TR \cdot TQ} \tag{17}$$

$$\angle PTQ = \arccos \frac{TP^2 + TQ^2 - QP^2}{2TP \cdot TQ} \tag{18}$$

According to equation(16)(17)(18), we can acquire

$$\theta_4 = \arccos \frac{TR^2 + TN^2 - NR^2}{2TR \cdot TN} + \arccos \frac{TR^2 + TQ^2 - S_3^{\ 2}}{2TR \cdot TQ} + \arccos \frac{TP^2 + TQ^2 - QP^2}{2TP \cdot TQ} - \pi \tag{19}$$

5. Simulation and verification

The trajectory of manipulator's end-effector is known, the variable of joint and drive can be analyzed by MATLAB. If the trajectory has been given, the specific motion of manipulator's end-effector can be described as:

Begin-the first second: the only joint 1 rotates 25° counterclockwise;

The first second-the second seconds: uniform moves down in the straight line at a certain inclination angle;

The second second-the third second: uniform moves down in the straight line perpendicular to the ground.

Based on the inverse kinematics, the angle of joint space and length of hydraulic cylinder can be acquired in the figure 7 and figure 8.
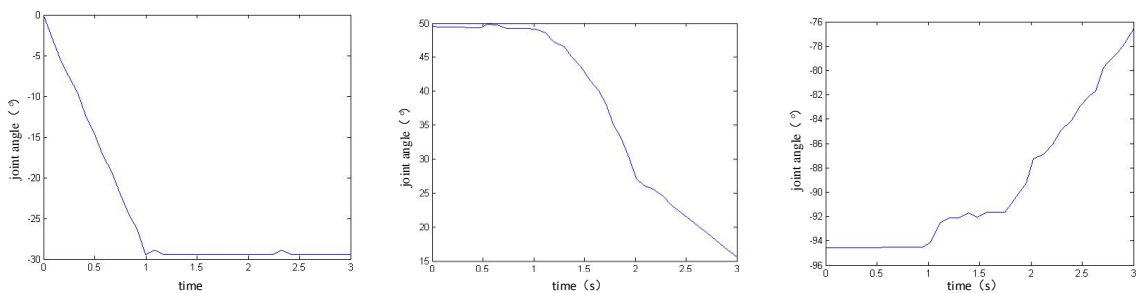

Figure 7(a). Angle curve of joint 1**Figure 7(b).** Angle curve of joint 2 **Figure 7(c).** Angle curve of joint 3
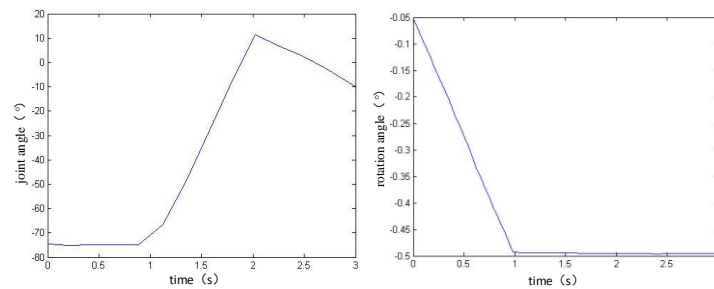
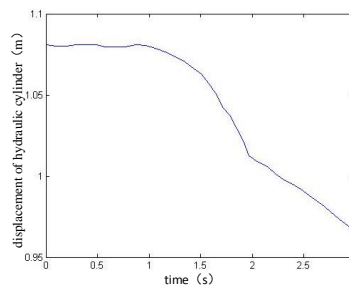**Figure 7(d).** Angle curve of joint 4   **Figure 8(a).** Angle curve of motor

Figure 8(b). Displacement curve of cylinder 1
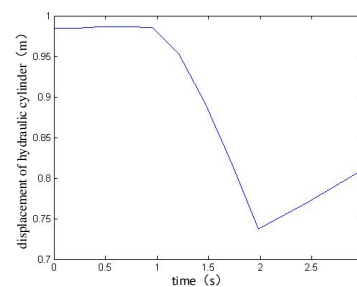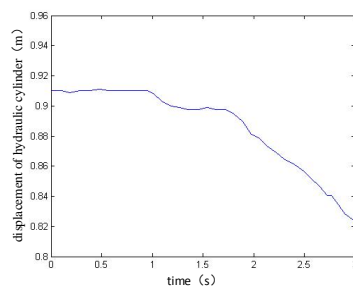

Figure 8(c). Displacement curve of cylinder 2   **Figure 8(d).** Displacement curve of cylinder 3

The simulation shows that the only joint 1 moves and joint 2,3,4 is stationary from begin to the first second. Then the manipulator's joints make a compound motion from the first second to the third second. It meets the given motion requirement.

6. Conclusion

The kinematics analysis is the basis on motion planning and control of manipulator. Taking 4 DOF manipulator as a study object in this paper, the forward kinematics can be acquired by D-H method. Based on the structure of the manipulator, the geometric method is chosen for the inverse kinematics. On this basis, the relationship between joint angle and expansion length of hydraulic cylinder can be expressed. Finally MATLAB validates the correctness. This paper provides a solving method for reverse conversion from Cartesian space to joint space to drive space. It provides theoretical basis for motion planning and control study of manipulator.

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