

PDE Project2 Report

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0.1 Procudure

We can know that $f(x, y) = \sin(\pi * x) * (\pi^2 * y * (\exp(y) - \exp(1)) - (2 * \exp(y) + y * \exp(y)))$. Given the weights matrices for u and f, we can get the corresponding coefficient matrices A and C easily:

- As for the coefficient matrix A of u takes the form:

$$A = \frac{1}{6h^2} \begin{pmatrix} B_1 & B_2 & & & \\ B_2 & B_1 & B_2 & & \\ & \ddots & \ddots & \ddots & \\ & & B_2 & B_1 & B_2 \\ & & & B_2 & B_1 \end{pmatrix}_{n^2 \times n^2}$$

where

$$B_1 = \begin{pmatrix} -20 & 4 & & & \\ 4 & -20 & 4 & & \\ & \ddots & \ddots & \ddots & \\ & & 4 & -20 & 4 \\ & & & 4 & -20 \end{pmatrix}_{n \times n} \quad B_2 = \begin{pmatrix} 4 & 1 & & & \\ 1 & 4 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & 4 & 1 \\ & & & 1 & 4 \end{pmatrix}_{n \times n}$$

- As for the coefficient matrix C of f takes the form:

$$C = \frac{1}{12} \begin{pmatrix} B_3 & I & & & \\ I & B_3 & I & & \\ & \ddots & \ddots & \ddots & \\ & & I & B_3 & I \\ & & & I & B_3 \end{pmatrix}_{n^2 \times n^2}$$

where

$$B_3 = \begin{pmatrix} 8 & 1 & & & \\ 1 & 8 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & 8 & 1 \\ & & 8 & 1 & \end{pmatrix}_{n \times n}$$

So the equation becomes $Au = Cf$. Then we can use $u = A^{-1}Cf$ to calculate numerical solution.

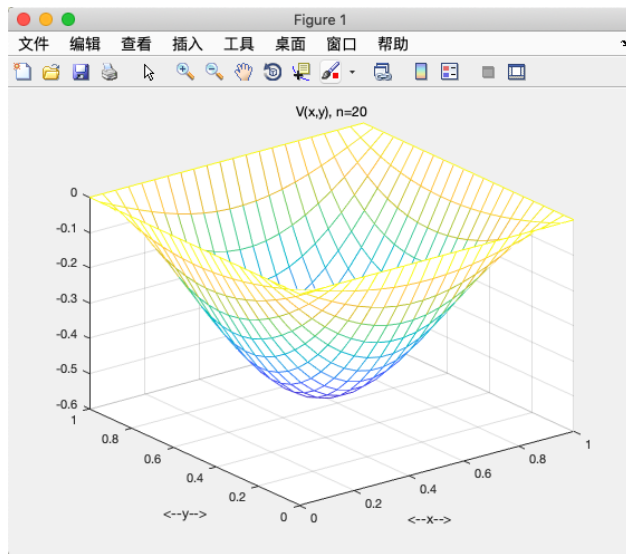
Table 1: $Eh - h$

n	h	Eh	Rate of Convergence
3	$\frac{1}{4}$	$1.5 * 10^{-3}$	
10	$\frac{1}{11}$	$2.527 * 10^{-6}$	4
20	$\frac{1}{21}$	$1.920 * 10^{-6}$	4
40	$\frac{1}{41}$	$1.324 * 10^{-7}$	4

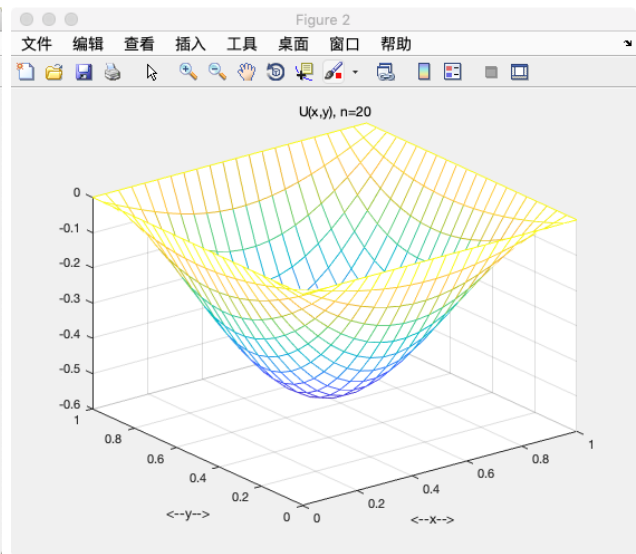
0.2 Result

0.2.1 Grid Refinement Analysis

0.2.2 Plot

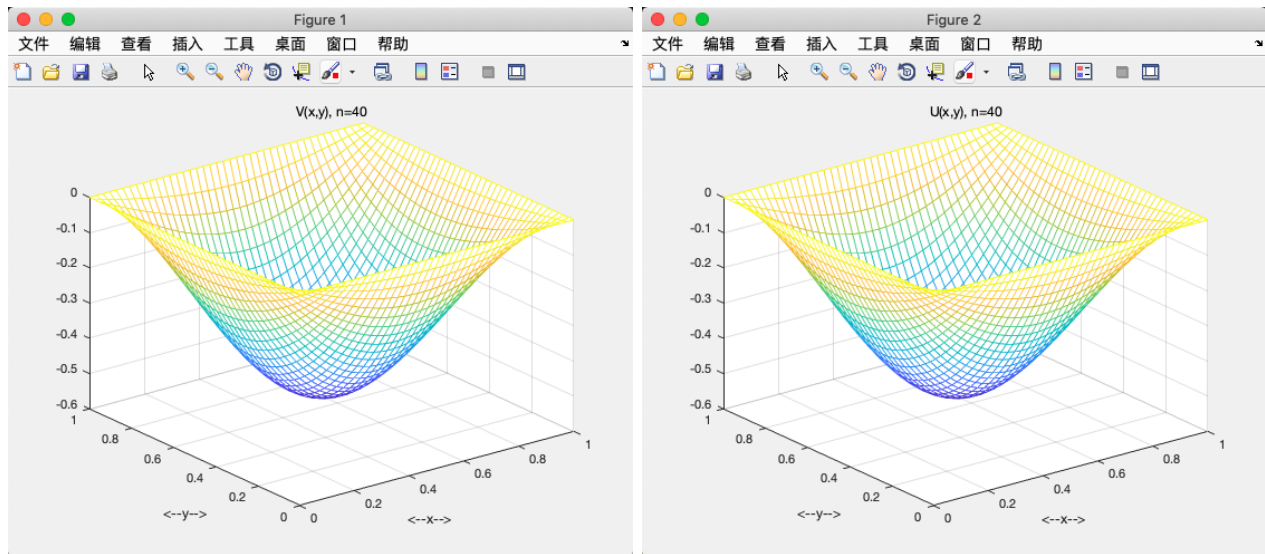


(a) $v, n=20$



(b) $u, n=20$

Figure 1: $n=20$



(a) $v, n=40$

(b) $u, n=40$

Figure 2: $n=40$

0.3 Code

```

1  clc;
2  clear all;
3  close all;
4  % format long e
5  %%
6  n=3;
7  h=1./(n+1);
8  h4=h.^4;
9  x1 = linspace ( 0, 1.0, n+2);
10 y1 = linspace ( 0, 1.0, n+2);
11
12 x = x1(2:(n+1));
13 y = y1(2:(n+1));
14
15 v = zeros(n.^2,1);
16 u = v;
17 f = v;
18 Fxx=f;
19 Fyy=f;
20
21 A = zeros(n.^2,n.^2);
22 C = A;
23

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24 syms a b;
25 F(a,b)= sin(pi.*a) * (pi.^2 * b * (exp(b) - exp(1)) - (2*exp(b) + b * exp(b)));
26 Faa(a,b)=diff(F(a,b),a,2);
27 Fbb(a,b)=diff(F(a,b),b,2);
28
29 for j=1:n
30     for i=1:n
31         u(i+(j-1)*n) = sin(pi * x(i)) * y(j) * (exp(y(j)) - exp(1));
32         f(i+(j-1)*n) = sin(pi.*x(i)) * (pi.^2 * y(j) * (exp(y(j)) - exp(1)) -
33 %         f(i+(j-1)*n) = -((-pi.*pi.*sin(pi.*x(i)).*y(j).*(exp(y(j))-exp(1)))
34     end
35 end
36
37 for j=1:n
38     for i=1:n
39         Fxx(i+(j-1)*n)=Faa(x(i),y(j));
40         Fyy(i+(j-1)*n)=Fbb(x(i),y(j));
41     end
42 end
43
44 B2 = diag(8*ones(n,1))+ diag(ones(n-1,1),1)+ diag(ones(n-1,1),-1);
45 I = eye(n,n);
46
47 for i=1:n
48     for j=1:n
49         for k=1:n
50             C(n*(i-1)+j,n*(i-1)+k)=B2(j,k);
51         end
52     end
53 end
54
55 for i=1:(n-1)
56     for j=1:n
57         for k=1:n
58             C(n*(i-1)+j,n*i+k)=I(j,k);
59         end
60     end
61 end
62
63 for i=1:(n-1)
64     for j=1:n
65         for k=1:n
66             C(n*i+j,n*(i-1)+k)=I(j,k);
67         end
68     end

```

```

69 end
70
71
72 B1 = diag((-20)*ones(n,1))+ diag(4*ones(n-1,1),1)+ diag(4*ones(n-1,1),-1);
73 B3 = diag(4*ones(n,1))+ diag(ones(n-1,1),1)+ diag(ones(n-1,1),-1);
74 for i=1:n
75     for j=1:n
76         for k=1:n
77             A(n*(i-1)+j , n*(i-1)+k)=B1(j , k);
78         end
79     end
80 end
81
82 for i=1:(n-1)
83     for j=1:n
84         for k=1:n
85             A(n*(i-1)+j , n*i+k)=B3(j , k);
86         end
87     end
88 end
89
90 for i=1:(n-1)
91     for j=1:n
92         for k=1:n
93             A(n*i+j , n*(i-1)+k)=B3(j , k);
94         end
95     end
96 end
97
98
99
100
101 A1=A/(6*(h^2));
102 C1=C/12;
103
104 A1_inv=inv(A1);
105
106 d = (h^2/12)*(Fxx+Fyy)+f;
107 v=-A1_inv*d;
108 v;
109
110 Eh=(max(abs(u-v)));
111
112 error1 = -A1*u;
113 error2 =d;

```

```

114 error=max(abs(error1−error2 ));
115
116 V=zeros (n+2,n+2);
117 U=zeros (n+2,n+2);
118 for i=1:n
119     for j=1:n
120         V(i+1,j+1)=v((i−1)*n+j );
121         U(i+1,j+1)=u((i−1)*n+j );
122     end
123 end
124
125 s=sprintf( 'V(x,y) , n=%d' ,n );
126
127 figure ;
128 mesh ( x1 , y1 , V);
129 xlabel ( '←x→' );
130 ylabel ( '←y→' );
131 title (s);
132
133 s=sprintf( 'U(x,y) , n=%d' ,n );
134 figure ;
135 mesh ( x1 , y1 , U);
136 xlabel ( '←x→' );
137 ylabel ( '←y→' );
138 title (s);

```