PDE Project2 Report

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0.1 Procudure

We can know that $f(x,y) = sin(pi*x)*(pi^2*y*(exp(y)-exp(1))-(2*exp(y)+y*exp(y)))$. Given the weights matrices for u and f, we can get the corresponding coefficient matrices A and C easily:

- As for the coefficient matrix A of u takes the form:

$$A = \frac{1}{6h^2}\begin{pmatrix} B_1 & B_2 & & & \\ B_2 & B_1 & B_2 & & \\ & \ddots & \ddots & \ddots & \\ & & B_2 & B_1 & B_2 \\ & & & B_2 & B_1 \end{pmatrix}_{n^2\times n^2}$$

  where

$$B_1 = \begin{pmatrix} -20 & 4 & & & \\ 4 & -20 & 4 & & \\ & \ddots & \ddots & \ddots & \\ & & 4 & -20 & 4 \\ & & & 4 & -20 \end{pmatrix}_{n\times n} \quad B_2 = \begin{pmatrix} 4 & 1 & & & \\ 1 & 4 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & 4 & 1 \\ & & & 1 & 4 \end{pmatrix}_{n\times n}$$

- As for the coefficient matrix C of f takes the form:

$$C = \frac{1}{12}\begin{pmatrix} B_3 & I & & & \\ I & B_3 & I & & \\ & \ddots & \ddots & \ddots & \\ & & I & B_3 & I \\ & & & I & B_3 \end{pmatrix}_{n^2\times n^2}$$

  where

$$B_3 = \begin{pmatrix} 8 & 1 & & & \\ 1 & 8 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & 8 & 1 \\ & & 8 & 1 & \end{pmatrix}_{n\times n}$$

So the equation becomes $Au = Cf$. Then we can use $u = A^{-1}Cf$ to calculate numerical solution.

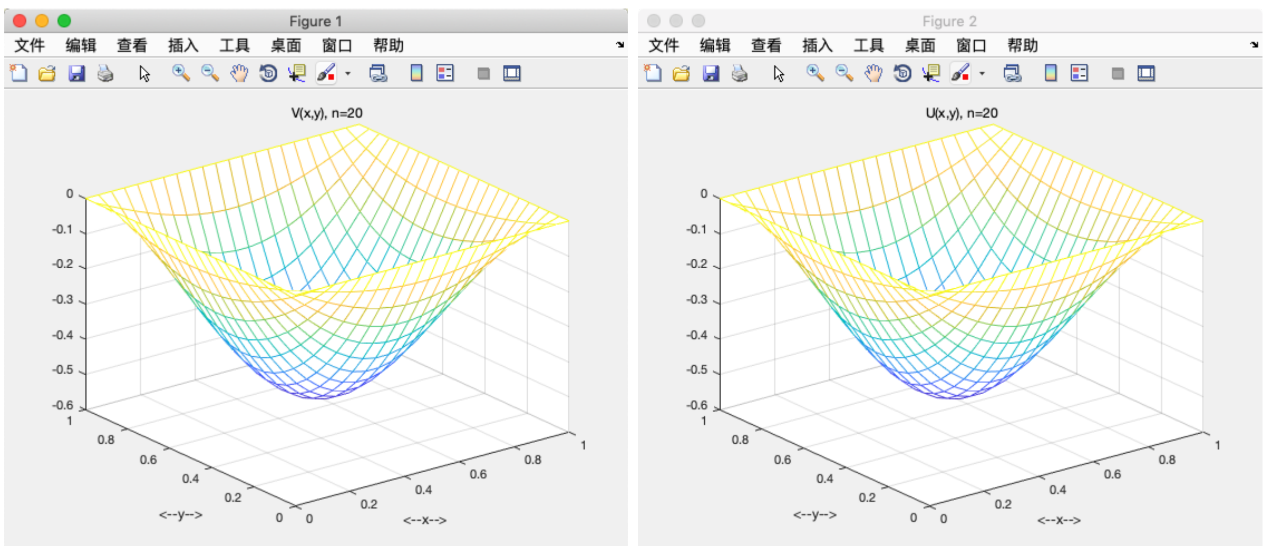
Table 1: $Eh - h$

n	h	Eh	Rate of Convergence
3	$\frac{1}{4}$	$1.5 * 10^{-3}$	
10	$\frac{1}{11}$	$2.527 * 10^{-6}$	4
20	$\frac{1}{21}$	$1.920 * 10^{-6}$	4
40	$\frac{1}{41}$	$1.324 * 10^{-7}$	4

0.2 Result

0.2.1 Grid Refinement Analysis

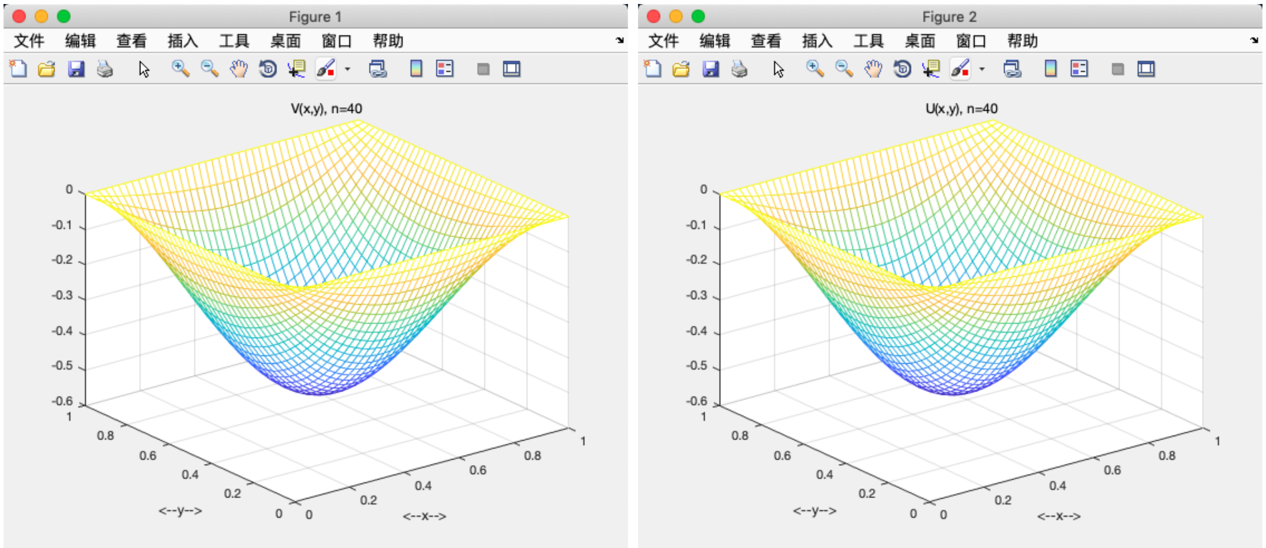
0.2.2 Plot



(a) v,n=20

(b) u,n=20

Figure 1: n=20



(a) v,n=40

(b) u,n=40

Figure 2: n=40

0.3 Code

```
clc;
clear all;
close all;
% format long e
%%
n=3;
h=1./(n+1);
h4=h.^4;
x1 = linspace ( 0, 1.0, n+2);
y1 = linspace ( 0, 1.0, n+2);

x = x1(2:(n+1));
y = y1(2:(n+1));

v = zeros(n.^2,1);
u = v;
f = v;
Fxx=f;
Fyy=f;

A = zeros(n.^2,n.^2);
C = A;

```

```
syms a b;
F(a,b)= sin(pi.*a) * (pi.^2 * b * (exp(b) - exp(1)) - (2*exp(b) + b * exp(b))
Faa(a,b)=diff(F(a,b),a,2);
Fbb(a,b)=diff(F(a,b),b,2);

for j=1:n
    for i=1:n
        u(i+(j-1)*n) = sin(pi * x(i)) * y(j) * (exp(y(j)) - exp(1));
        f(i+(j-1)*n) = sin(pi.*x(i)) * (pi.^2 * y(j) * (exp(y(j)) - exp(1)) -
%           f(i+(j-1)*n) = -((-pi.*pi.*sin(pi.*x(i)).*y(j).*(exp(y(j))-exp(1)))
    end
end

for j=1:n
    for i=1:n
        Fxx(i+(j-1)*n)=Faa(x(i),y(j));
        Fyy(i+(j-1)*n)=Fbb(x(i),y(j));
    end
end

B2 = diag(8*ones(n,1))+ diag(ones(n-1,1),1)+ diag(ones(n-1,1),-1);
I = eye(n,n);

for i=1:n
    for j=1:n
        for k=1:n
            C(n*(i-1)+j,n*(i-1)+k)=B2(j,k);
        end
    end
end

for i=1:(n-1)
    for j=1:n
        for k=1:n
            C(n*(i-1)+j,n*i+k)=I(j,k);
        end
    end
end

for i=1:(n-1)
    for j=1:n
        for k=1:n
            C(n*i+j,n*(i-1)+k)=I(j,k);
        end
    end
```

```
end


B1 = diag((-20)*ones(n,1))+ diag(4*ones(n-1,1),1)+ diag(4*ones(n-1,1),-1);
B3 = diag(4*ones(n,1))+ diag(ones(n-1,1),1)+ diag(ones(n-1,1),-1);
for i=1:n
    for j=1:n
        for k=1:n
            A(n*(i-1)+j,n*(i-1)+k)=B1(j,k);
        end
    end
end

for i=1:(n-1)
    for j=1:n
        for k=1:n
            A(n*(i-1)+j,n*i+k)=B3(j,k);
        end
    end
end

for i=1:(n-1)
    for j=1:n
        for k=1:n
            A(n*i+j,n*(i-1)+k)=B3(j,k);
        end
    end
end




A1=A/(6*(h^2));
C1=C/12;

A1_inv=inv(A1);

d = (h^2/12)*(Fxx+Fyy)+f;
v=-A1_inv*d;
v;

Eh=(max(abs(u-v)));

error1 = -A1*u;
error2 =d;
```

```
error=max(abs(error1-error2));

V=zeros(n+2,n+2);
U=zeros(n+2,n+2);
for i=1:n
    for j=1:n
        V(i+1,j+1)=v((i-1)*n+j);
        U(i+1,j+1)=u((i-1)*n+j);
    end
end

s=sprintf('V(x,y), n=%d',n);

figure;
mesh( x1, y1, V);
xlabel( '<---x--->' );
ylabel( '<---y--->');
title (s);

s=sprintf('U(x,y), n=%d',n);
figure;
mesh( x1, y1, U);
xlabel( '<---x--->' );
ylabel( '<---y--->');
title (s);
```