

THEORY OF COMPUTATION

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→ Language Processing.

→ Compiler → Converts HLL to machine language.
→ language acceptor / translator.
language → Set of strings processed by compiler.

Grammar → Set of rules to generate strings for language.

→ Production rule of grammar

$$G_1 \left\{ \begin{array}{l} S \rightarrow AB \\ A \rightarrow aA / \lambda \xrightarrow{\text{NULL}} \\ B \rightarrow bB / \lambda \end{array} \right.$$

→ String is a set of terminals (small letter) ~~and string consists of non-terminal / variable (capital letter)~~.
→ String consists of only capital letters

$$S \rightarrow A B$$

$$aA B$$

$$\downarrow aA B$$

$$a a \lambda B$$

$$a a \lambda B$$

$$a a B \rightarrow a a b B \rightarrow a a b \underline{\lambda}$$

$$a a b$$

~~Sentential Form~~

→ Generating string from grammar.

$$L_1 = \{ \lambda, a, b, ab, aab, abb, \dots \}$$

$$L_1 = \{ a^n b^m \mid n, m \geq 0 \}$$

→ Non-terminal are used to generate strings while terminal are part of string.

$$\rightarrow G_2 \left\{ S \rightarrow aSb / \lambda \right. \quad \begin{array}{l} \diagup \\ aSb \\ \diagdown \end{array} \quad \begin{array}{l} \diagup \\ \lambda \\ \diagdown \end{array}$$

$$aSb \rightarrow a\lambda b \rightarrow ab$$

$$\underline{aaSbb} \rightarrow \underline{aabbb}$$

$$L_2 = \{ \lambda, ab, aabb, \dots \} = a^n b^n \mid n \geq 0.$$

\hookrightarrow Number of a = Number of b
and a is followed by b.

\rightarrow Cardinality of $\lambda = 0$.
" of a = 1.

$$\rightarrow G_3 \Rightarrow \left\{ \text{as} \rightarrow aSb / bSb / \lambda \right.$$

$$L_3 = \{ \lambda, ab, bb, aabb, abbb, babb, bbbb, \dots \}$$

$$\begin{array}{ll} \text{as} & bSb \rightarrow bb \\ \downarrow & \rightarrow basbb \rightarrow babb \\ \downarrow & \rightarrow bbSbb \rightarrow bbbb \end{array}$$

$$\begin{array}{ccccc} & & bSb & & \\ \text{as} & & \diagup & \diagdown & \\ \text{aas} & & basbb & & bbSbb \\ \text{bb} & \diagup & \text{bab} & & \downarrow \\ a^n b^n & & b^m a^n b^m & & b^n \end{array}$$

$$\rightarrow G_4 = \{ S \rightarrow aSb / bSa / \lambda \}$$

\Downarrow
No. of a = No. of b.

\hookrightarrow Even length palindromic
are generated by this
grammar.

→ Chomsky Classification of grammar.

→ Classifies into 4 categories according to production rule.

① Type 0 (Unrestricted G)

$$\alpha \rightarrow \beta$$

α, β are set of terminal and non-terminal strings.

② Type 1 (Context Sensitive G) $|\alpha| \leq |\beta|$ (Follows type 0)

③ Type 2 (Context free G) $\downarrow \alpha \rightarrow \beta$ (Follows type 0 + type 1)

$|\alpha|=1$ and α should be non-terminal only

④ Type 3 (Regular G) → Follows type 0 + 1 + 2.

β can be $aA / Aa / a / \lambda$

can be anything

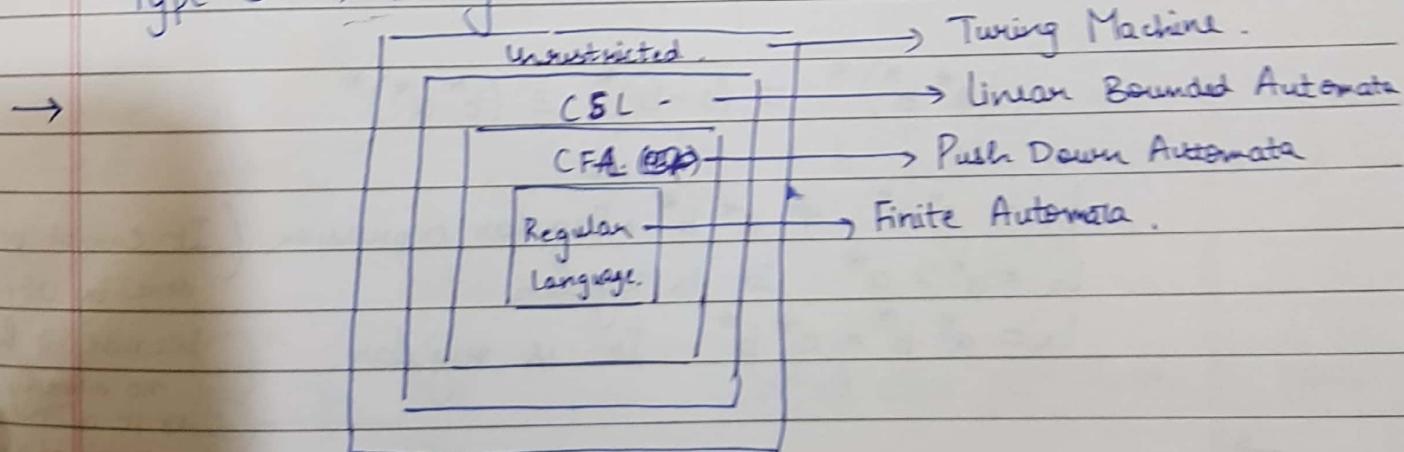
It cannot be $A \rightarrow BB$.

Type 3 $\overset{\text{Subset}}{\subseteq}$ Type 2 \subseteq Type 1 \subseteq Type 0

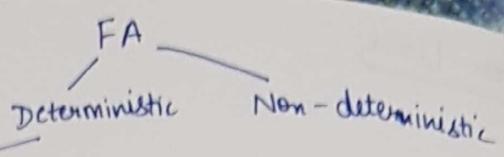
→ For every category of grammar, there will be a corresponding language and machine.

→ Type 3 → DFA

Type 0 → Turing machine.



→ DFA / Regular Expression.



→ DFA / Regular Expression → Regular expression used to represent regular language

$$a^* = \{ \lambda, a, aa, \dots a^n \}$$

$$* = \{ 0 - \infty \}$$

$$a^+ = \{ a, aa, aaa, \dots \}$$

$$+ = \{ 1 - \infty \}$$

$$a^* b^* = \{ \lambda, a, b, ab, aab, abb, \dots \}$$

$$a^+ b^+ = \{ ab, aab, abb, \dots \}$$

$$L = (a+b)^*$$

$$= \{ aa, ab, ba, bb \}$$

$$a^* b^* \neq b^* a^*$$

$$L = (a+b)^3$$

$$\sum_{n=2}^{\infty} = \{ a, b \}$$

→ Regular expression for all strings possible using a and b

$$(a+b)^{**} = \{ \lambda, a, b, ab, ba, \dots \}$$

$$(a^* b^*)^* = (a+b)^* = (b^* a^*)^*$$

$L_1 = a^n b^n \neq a^* b^*$ Not a regular expression (It cannot be done by DFA because it has no memory so it can't store count of a and b)

$$L_2 = a^n = a^*$$

It is regular.

$$L_3 = a^n b^m = a^* b^*$$

It is regular.

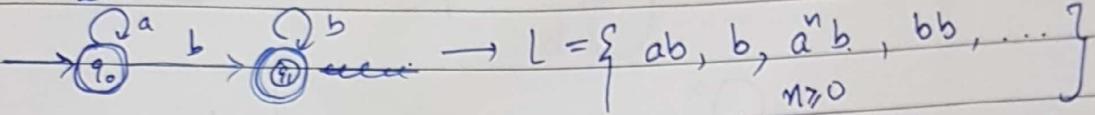
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→ Finite Automata

- If it is possible to draw FA for any language, then language is regular.
- FA - Set of states in which there is one initial state and more than one final state.

DFA - Deterministic Finite Automata.

→ DFA



Initial State
NFA

$L = \{ a^n b \mid n \geq 0 \}$ (abb is also accepted)

$$L = a^n b^m = a^* b^*$$

$n \geq 0 \quad m \geq 0$

- In DFA, there should be transition for every input.

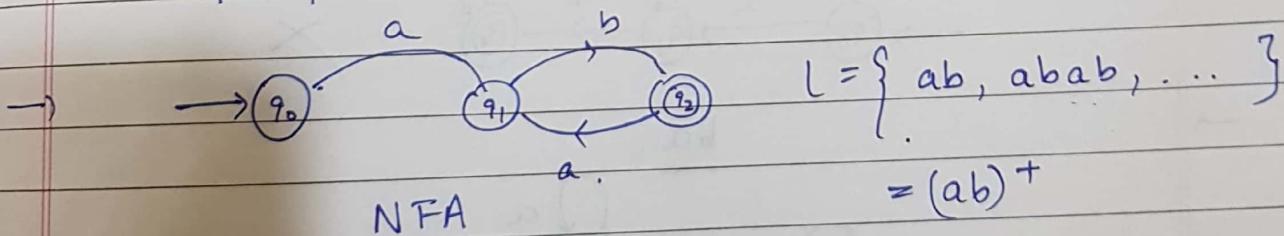
$$\rightarrow q_0 \quad L = \{ \phi \} = \{ \}$$

$$\rightarrow q_0 \quad L = \{ \lambda \}$$

$$a \cdot \lambda = a$$

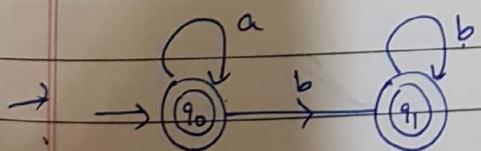
$$ab \cdot \lambda = ab$$

$$a \cdot \phi = \phi$$



$$L = \{ ab, abab, \dots \}$$

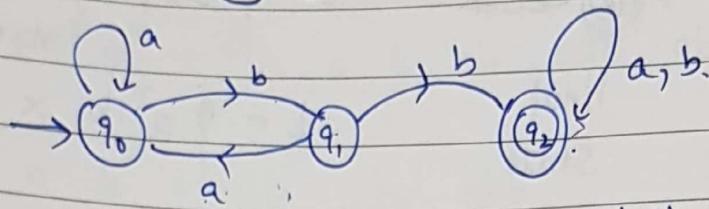
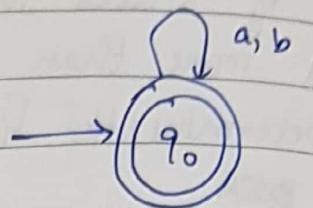
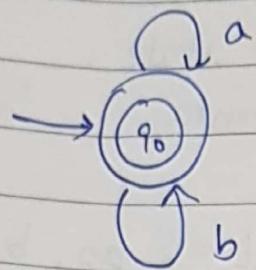
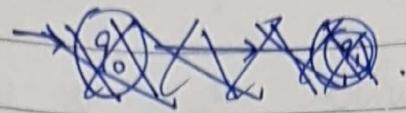
$$= (ab)^+$$



Initial state represented by arrow.

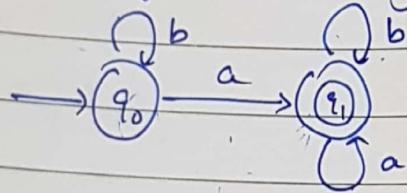
$$L = \{ \lambda, a, b, \dots \} = a^* b^*$$

$$L = \{a+b\}^*$$



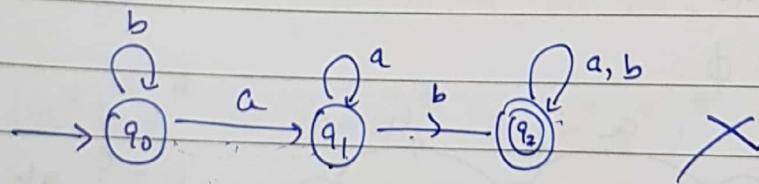
$$L = \{ abba, abbb, \cancel{babab}, bbbb, babb, bbba, \\ bbbba, aabb \}$$

→ Q Design DFA - for the language containing atleast one a.

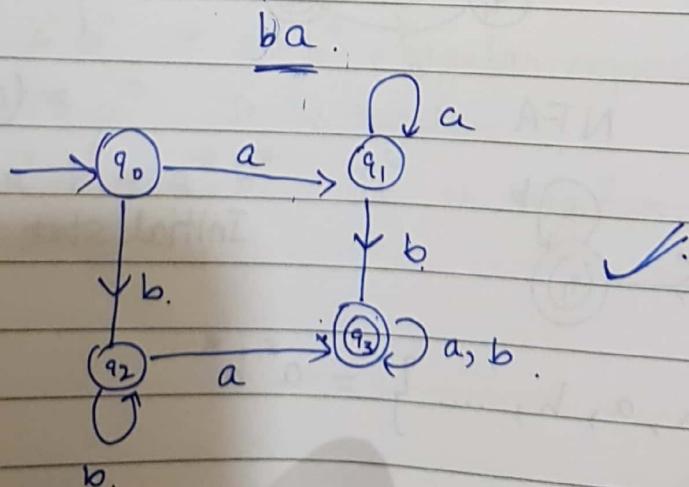


Every state should have exactly one transition for one input

→ Q Design DFA for language containing atleast one a and a b.

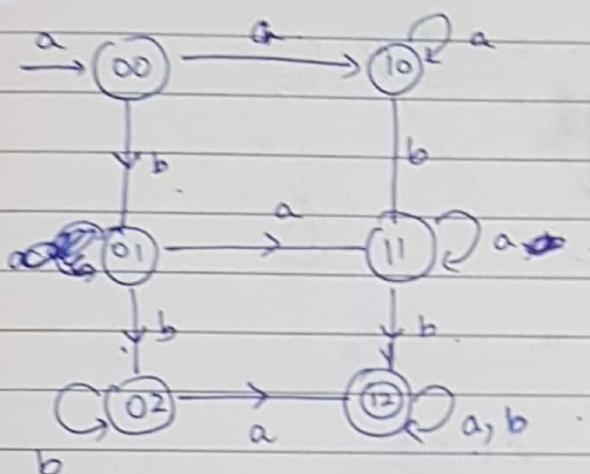


→ ~~b a.~~



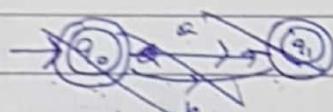
✓

Q. Design FA with atleast one a and two b.

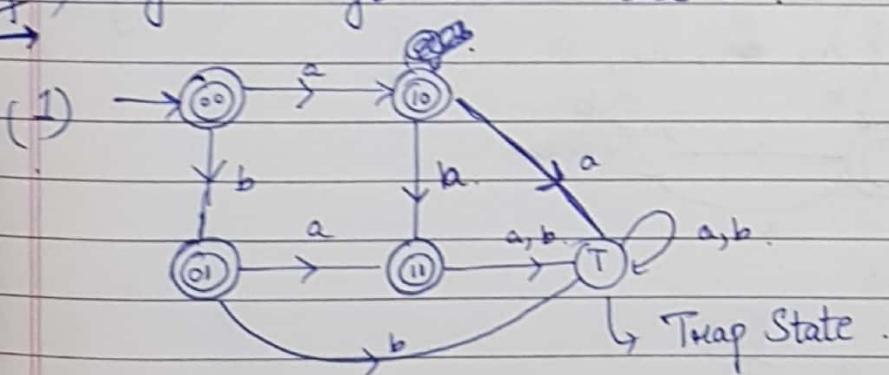


Q 1) Atmost one a and b.

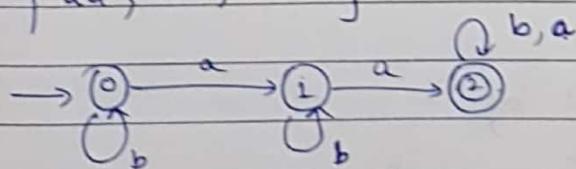
$$L = \{ \lambda, a, b, ab, ba \}$$



Q 2) Design DFA for atleast two a.

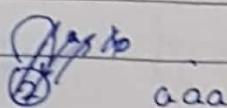
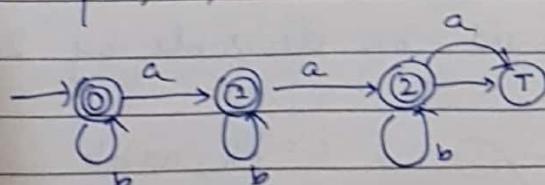


(2) $L = \{ aa, aba, \dots \}$

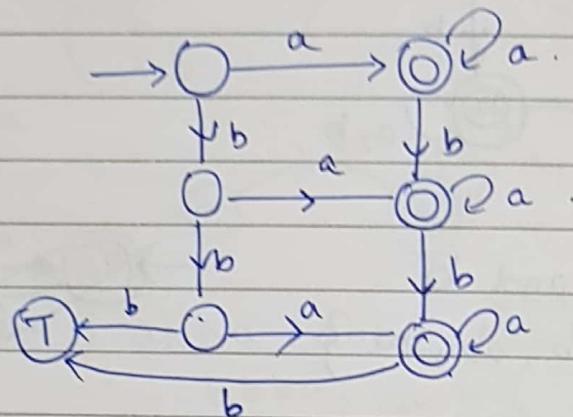
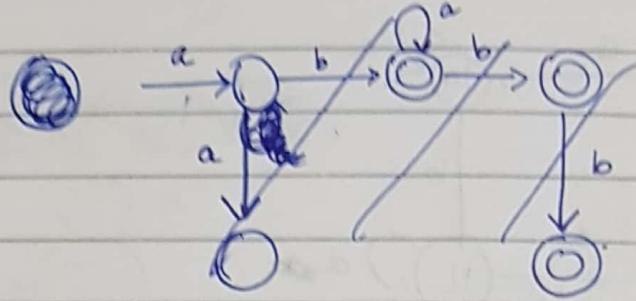


→ ③ Atmost 2 a.

$$L = \{ \lambda, a,$$

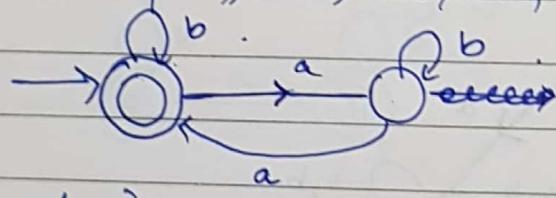


④ Design DFA with atleast one a and atmost 2 b

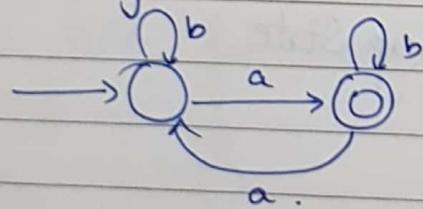


⑤ Design DFA in which number of a divisible by 2.

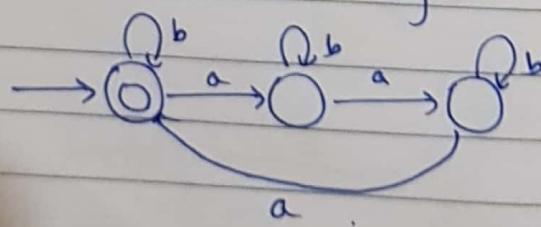
$$L = \{ \lambda, \underline{\underline{aa}}, aa, bb, \dots \}$$



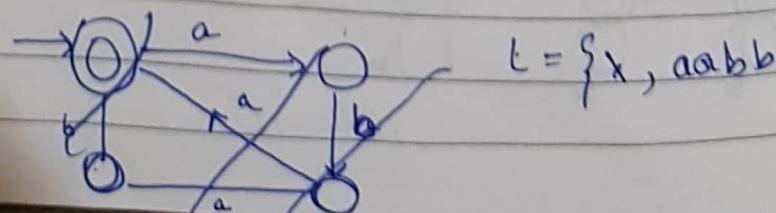
⑥ Odd no. of a's



⑦ No. of a is divisible by 3



⑧ No. of a and b should be divisible by 2.

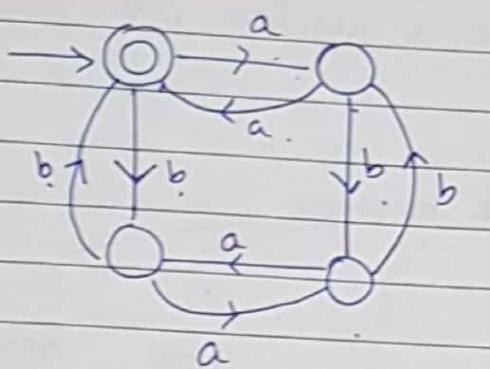


$$L = \{ \lambda, aabb \}$$

~~$aabb$~~ \times
 ~~$a^+ b^*$~~
 $c \cdot bb$

$n(a) \% m = 0$
 and $n(b) \% l = 0$.
 then no. of states $= m \times l$.

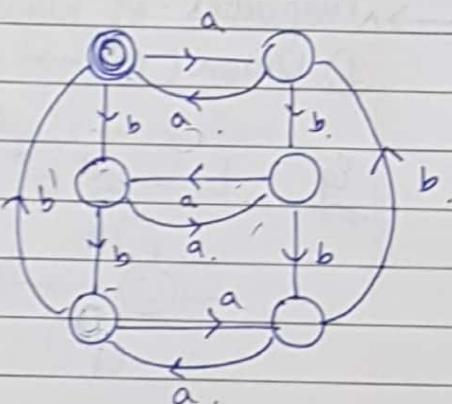
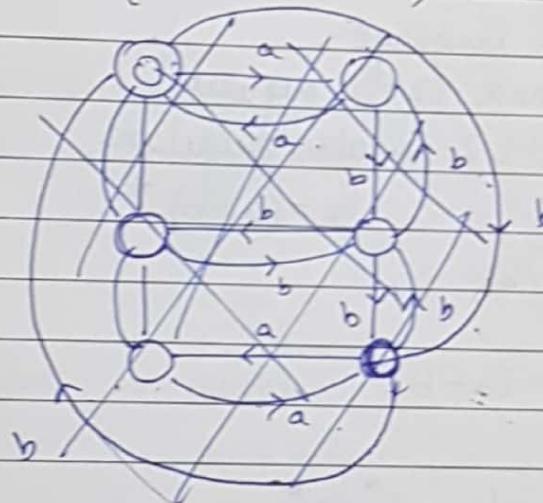
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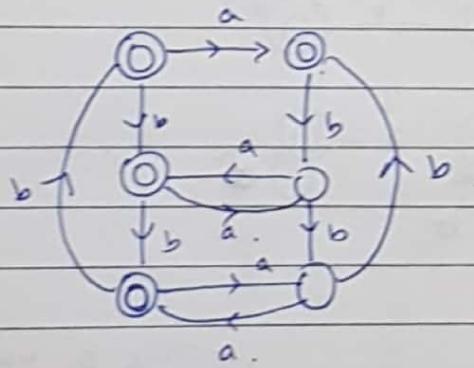
- (9) No. of a should be divisible by 2 and no. of b is divisible by 3.

$$L = \{ \lambda, aa, bbb, \dots \}$$

(10)



$$\rightarrow (10) n(a) \% 2 = 0 \text{ or } n(b) \% 3 = 0$$



$$\rightarrow L = a^n b^n \quad n \geq 0 \quad \text{No. of strings} \rightarrow \text{Infinite}$$

\hookdownarrow Not regular

Theorem: $L = a^n b^n, n \leq 3$ No. of strings \rightarrow Finite

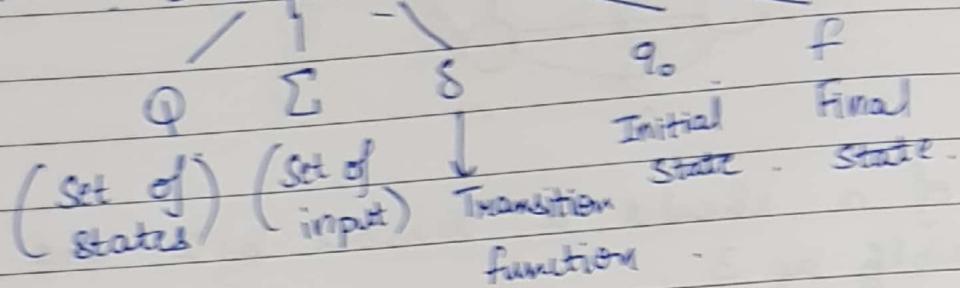
\hookdownarrow For finite no. of strings,
NFA is possible to make. Then it is regular

$$(A \cup B)^c = A^c \cap B^c$$

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→ Every finite language is regular. But it's closure is not true.

→ DFA / NFA \rightarrow 5 tuple machine

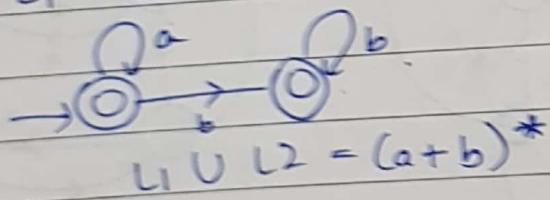


Closure

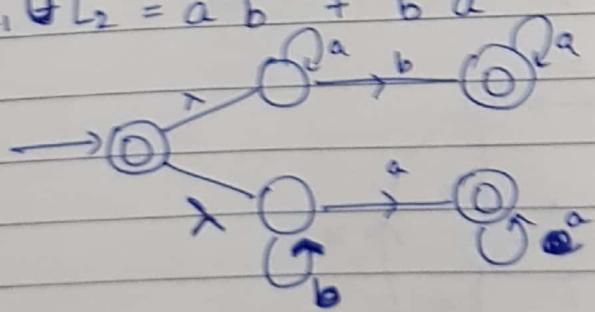
→ Properties of regular language.

① Union (\cup) $\rightarrow L_1$ and L_2 is regular
 $L_1 \cup L_2$ is also regular

Eg $L_1 = a^* b^*$ $L_2 = (a+b)^*$



Eg $L_1 = a^* b^*$ $L_2 = b^* a^*$
 $L_1 \cup L_2 = a^* b^* + b^* a^*$



→ Infinite Union

$$\{a, b\} \cup \{aa, bb\} \cup \{aaa, bbb\} \dots$$

$$= a^n b^n$$

↳ This is regular.

↳ This is not regular.

~~ASSMATE~~

~~(aⁿ) X (bⁿ)~~

~~a* b*~~
~~1 to n~~
~~aⁿ bⁿ~~

~~{a} U {ab} U {aab} U ...~~
~~{a} U {b}~~

~~and infinite~~

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$\rightarrow \{b\} \cup \{bb\} \cup \{bbb\} \cup \dots = b^n$
 It is regular.

Infinite union may or may not be regular. Therefore, regular language is not closed.

\rightarrow Regular Union Non Regular.

$$a^* b^* \cup a^n b^n = a^* b^*$$

R NR \hookrightarrow Regular.

$$\emptyset \cup a^n b^n = a^n b^n$$

R NR \hookrightarrow Non-Regular.

(2) Intersection (\cap)

$\rightarrow L_1$ and L_2 are regular $\Rightarrow L_1 \cap L_2$ is regular.

$$L_1 = a^* b^* \quad L_2 = b^* a^*$$

$$L_1 \cap L_2 = a^* + b^*$$

\rightarrow Infinite Intersection

$$\{ab\} \cap \{aabb\} \cap \{aaabbb\} \cap \dots$$

$= \emptyset$ (regular).

Assignment
 Q) Whether regular language are closed under infinite intersection?

\rightarrow Regular Intersection Non-Regular.

$$a^* b^* \cap a^n b^n = a^n b^n$$

R NR NR.

$$\emptyset \cap a^n b^n = \emptyset$$

R NR R.

(3) Subset (\subseteq)

$$L_1 = a^* b^*$$

$$L_2 = a^n b^n \subseteq L_1$$

\hookrightarrow It is not regular

Therefore, regular language are not closed under subset.

- Every non-regular language has a regular subset and regular superset $(a+b)^*$.
- Every non-regular language is ~~not~~ infinite whereas every regular language can be finite or infinite.

④ Concatenation

$$\begin{aligned} L_1 &= \{ab, bab\} \\ L_2 &= \{bb, ba\} \end{aligned}$$

$$L_1 \cdot L_2 = \{abbb, abba, babbb, babba\}$$

$L_1 \cdot L_2$ is always regular.

→ Regular language is closed under concatenation.
 $L_1 \cdot \lambda = L_1$

$$L_1 \cdot \emptyset = \emptyset$$

⑤ Commutative

$$L_1 \cdot L_2 \neq L_2 \cdot L_1$$

Concatenation is not commutative.

$$L_1 \cup L_2 = L_2 \cup L_1$$

$$L_1 \cap L_2 = L_2 \cap L_1$$

- Regular. ~~or~~ Non-Regular \Rightarrow Regular / Non-Regular
- x. $a^n b^n = \text{Non-Reg. (NR)}$
- p. $a^n b^n = \emptyset$ (NR)

⑥ Kleen's closure (Power of a set)

$$\begin{aligned} L^0 &= \{\emptyset\} \\ L^1 &= \{ab\} \\ L^2 &= \{abab\} \\ L^3 &= \{ababab\} \end{aligned}$$

$$L^* = L^0 \cup L^1 \cup L^2 \dots \cup \{ab\}^*$$

Kleen's closure of a regular language is always regular.

$$\rightarrow L = \{ \lambda \} \Rightarrow L^* = \{ \lambda \}$$

$$L = \emptyset \Rightarrow L^* = \{ \lambda \}$$

A.

$$L = \{ a^n b^n \}_{\text{closure}}$$

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Assignment

λ is always there in Kleen's closure.

Q.2) Kleen's closure of non regular lang. can be regular or non-regular. Justify with example.

Q.3) $L = a^p$ where p is prime number.

is possible.

\rightarrow Find the Kleen's closure. Also check whether it's DFA

⑥ Complement (L^c)

\rightarrow All strings that are not part of that language

$$L = a^* b^*$$

$$L^c = (a+b)^* - L$$

$$\rightarrow L = (a+b)^*$$

$$L^c = \emptyset$$

Regular language are closed under complement.

$$\rightarrow L = a^n b^n$$

$$L^c = (a+b)^* - a^n b^n \rightarrow \text{Non Regular.}$$

(Because checking for inequality also requires memory).

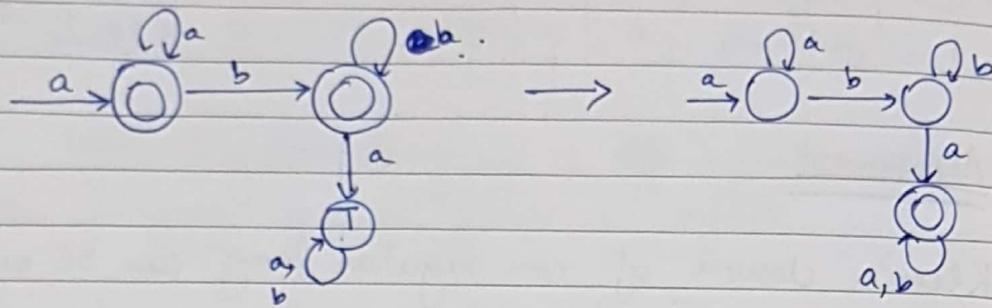
Complement of non-regular is always non-regular.

\rightarrow Steps for making DFA of complement of a language.

① Design DFA of language whose complement has to be designed including trap state.

② Make final states to non-final and non-final states to final.

$$L = a^* b^*$$



⑦ Transpose

→ Transpose of ~~non~~ regular language.

$$A \rightarrow T^* B$$

or

$$A \rightarrow T^*$$

Linear ~~Regular~~ Grammar

① Right linear

$$A \rightarrow aB$$

Non-terminal after

terminal .

② Left linear .

$$A \rightarrow BT^* / A \rightarrow T^*$$

Non-terminal

on left .

$$S \rightarrow aA \mid bB$$

$$A \rightarrow a$$

$$B \rightarrow b$$

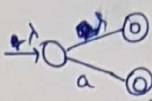
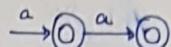
→ Language generated by left / right linear grammar is definitely regular but language generated by linear grammar may or may not be regular .

→ For finding transpose of L :

① Write grammar of L

② Reverse it's variable / non terminal

→ Regular language are closed under transpose .



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→ Non-Standard Operations.

① If L_1 and L_2 are regular

$L_1 - L_2$ returns lang which is part of L_1 but not part of L_2 .

$$L_1 - L_2 = L_1 \cap L_2'$$

→ Regular language is closed under this property.

② Reverse

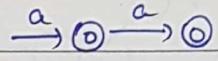
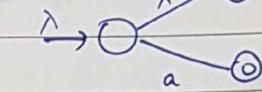
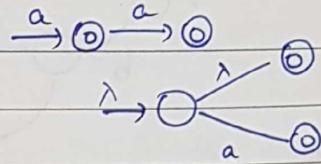
$$L_1 = \{ab\}$$

$$\text{Reverse} \rightarrow \{ba\}$$

Final becomes initial state
and vice versa.

③ Prefix of L .

$$L = \{ab, bba\}$$



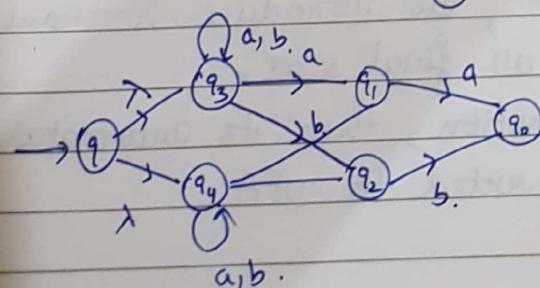
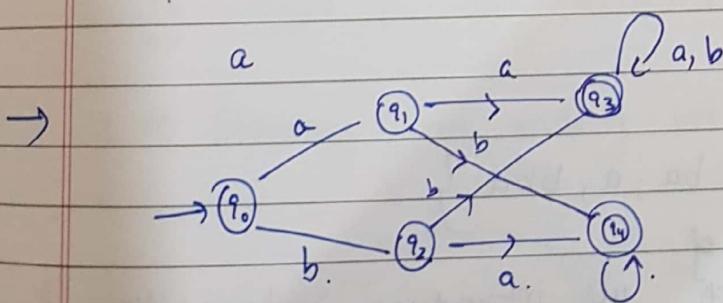
$$\text{Prefix}(L) = \{\lambda, a, b, ab, bb, bba\}$$

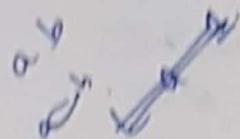
Converting non-final to final will give DFA of prefix of L .

→ Closed under prefix of L .

→ Design DFA of L and $\text{prefix}(L)$.

$$L = \{ab, bba\}$$





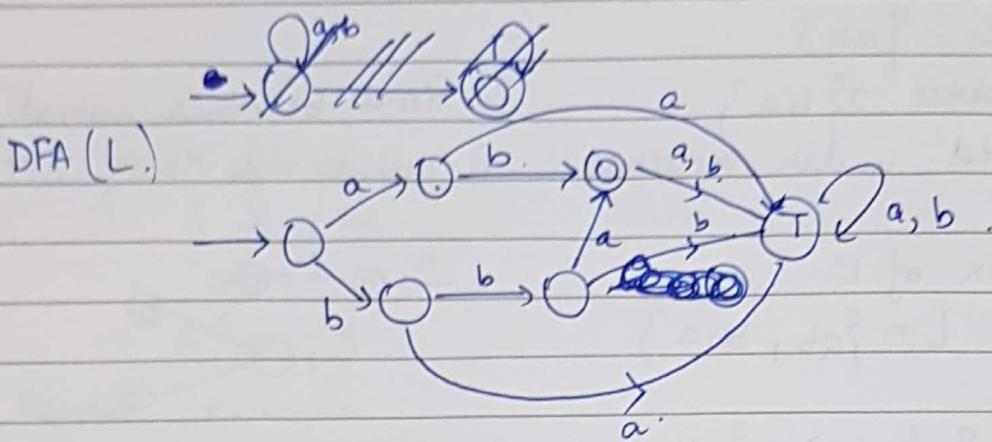
$a, b \rightarrow \textcircled{0}$ ($a + b$) *Automata*
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$$\rightarrow L = \{ \underline{ab}, \underline{bba} \}$$

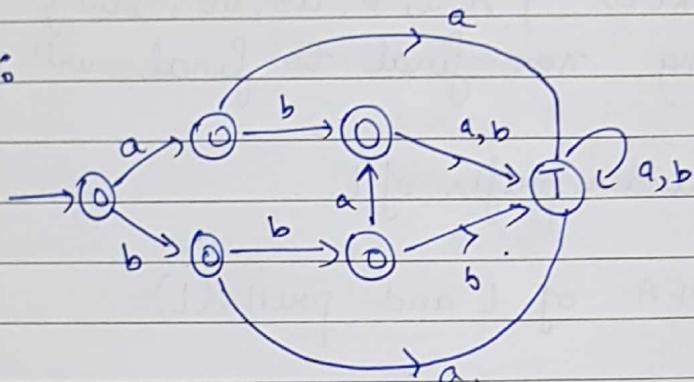
$$L_p = \{ \lambda, \underline{a}, ab, \underline{b}, bb, \underline{bba} \}$$

① Non-final states are made final for drawing
 DFA ~~of~~ *prefix*

\rightarrow



DFA(L_p):



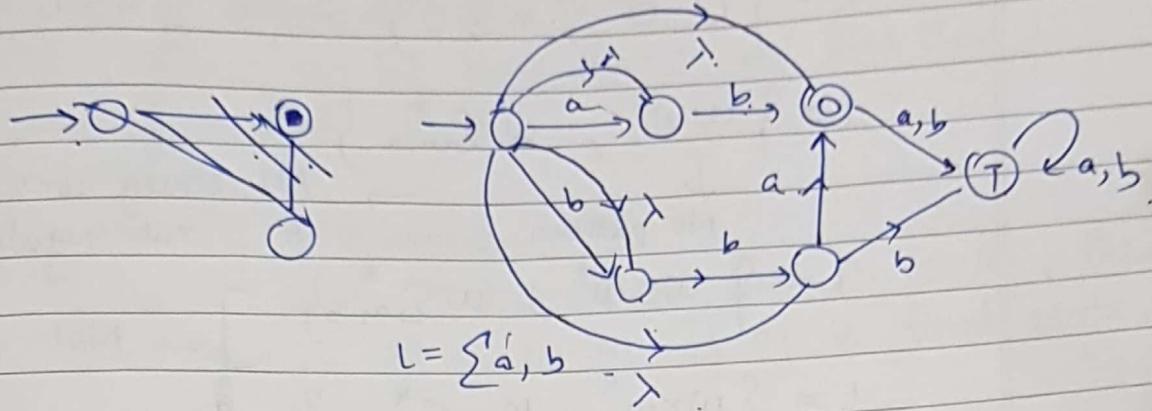
④ Suffix

$$L = \{ ab, bba \}$$

$$L_s = \{ \lambda, ab, \underline{ab}, ba, a, bba \}$$

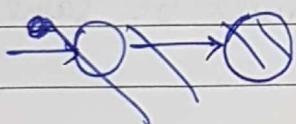
For drawing ~~of~~ DFA, we introduce λ transition from initial state to all final state.

If there is λ transition, then its automata is NFA. Every NFA can be converted to DFA



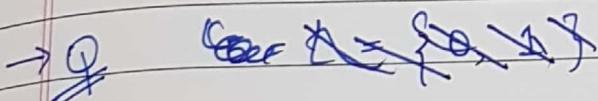
$$\rightarrow L = \{a^i ; i \geq 0\}$$

$$L = \{a, a^2, a^4, a^8, \dots\}$$



No DFA is possible
because no pattern.

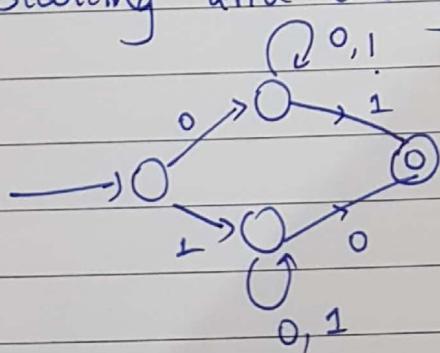
$$\rightarrow L = \{a^n b^m ; n \leq m\}$$



$$L = \{0, 1\}$$

$L = \{\text{All strings with equal number of } 01 \text{ and } 10\}$
eg $\bar{0}1\bar{0}1$ is not accepted by this L .

$L \Rightarrow$ Starting and ending bit is same.



Regular expression:
 $1(0+1)^* + 1+0 (0+1)^* 0$



$$n = (a+b)^*$$

classmate
Date _____
Page _____

~~$w = ab$~~
 ~~$a^b (a+b)^{b-1}$~~
 ~~$a b a (a+b)^{b-1}$~~

→ $L = \{0^n 0^n; n \geq 0\} = \{0^{2n}; n \geq 0\} \rightarrow \text{Regular}$

$$L = \{ww^*; w \in \Sigma_{a,b}^*\}$$

↓ All string consisting of a and b.
No pattern ∴ no DFA ∴ not regular.

$$L = \{wcw^*; w \in \Sigma_{a,b}^*\}] - \text{Not regular.}$$

$$L = \{wxw^*, w, x \in \Sigma_{a,b}^*\}$$

$$(a+b)^*$$

$$a(a+b)^*a$$

$$\boxed{b(a+b)^*b}a$$

↳ This is regular.

↳ This can be seen as X.

It's starting and ending will be same.

→ $a^n c a^n \rightarrow \text{Not regular.}$

$a^n x a^n \rightarrow \text{Regular.}$

$$a^n (a)$$

Q $L = \Sigma_{0,1}$

↳ Accepts binary string whose decimal equivalent is divisible by 5.

→ 000, ~~101~~¹⁰¹, 1010, 1111, ...

→

ab ba

a → b

classmate

Date _____

Page _____

Conversion of ~~NFA~~ to DFA

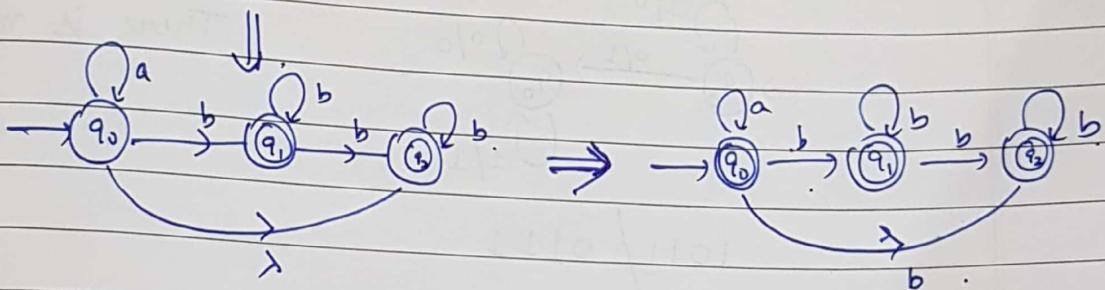
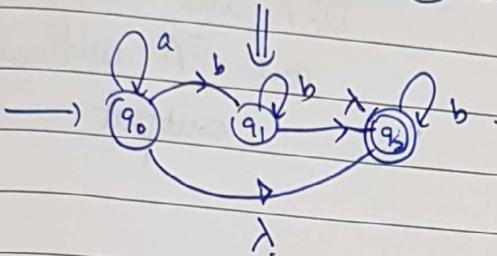
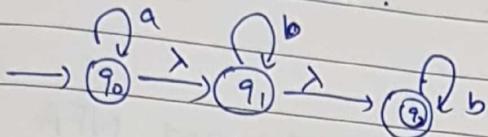
In NFA, there can be more than 1 or less than 1

transition for single input.

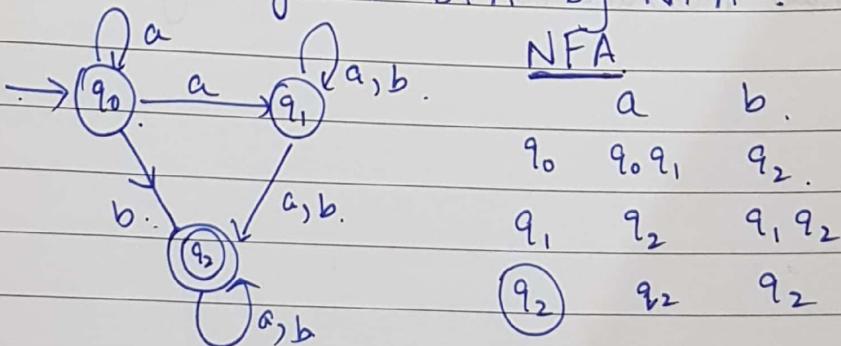
Conversion steps:

① Remove all the λ transition

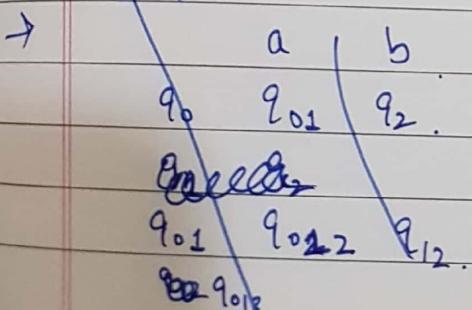
② If there exists λ transition from a_i to a_j , then copy all transition of a_j to a_i . If a_j is final state, then make a_i final state.



③ Make state table of ~~DFA~~ by NFA.



DFA

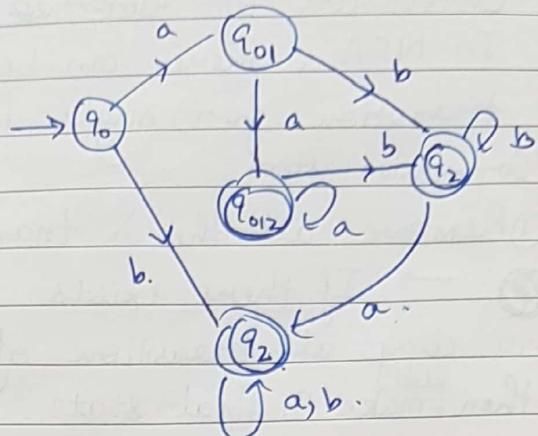


From NFA, in first row, if there are more than one trans merge them ~~using~~.

DFA .

	a	b
q_0	q_{01}	q_2
q_{01}	q_{012}	q_{12}
q_{012}	q_{012}	q_{12}
q_{12}	q_2	q_{12}
q_2	q_2	$q_{12} \times$

Final State
 $\therefore q_2$ is final



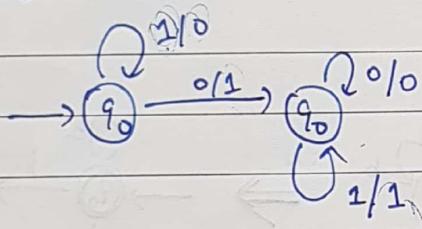
→ FA with output .

↳ Mealy machine

↳ Moore machine → Output depends only on state

DFA and NFA

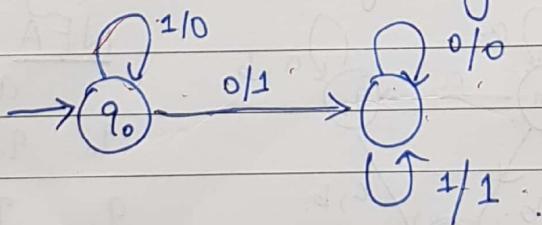
are FA without output .



There is no final state

1011 / 0111

Q → Draw machine to increment binary number by 1 .



110 011
~~111~~ 100
~~111~~ 100

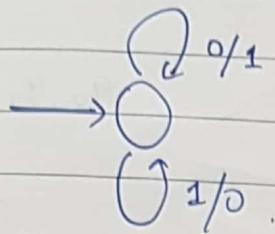
$$\begin{array}{r} 110 \\ + 1 \\ \hline 101 \end{array}$$

$$\begin{array}{r} 111 \\ + 1 \\ \hline 110 \end{array}$$

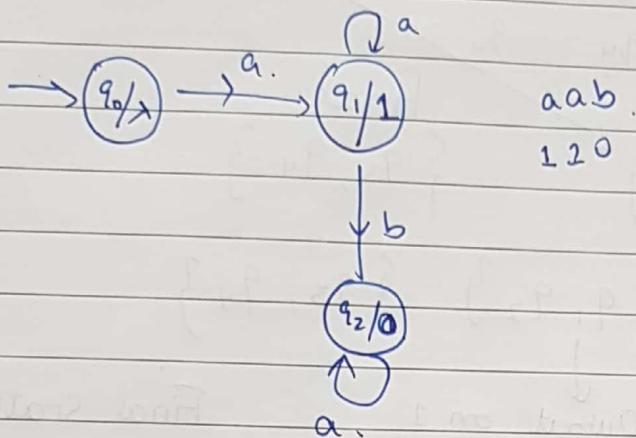
$$\begin{array}{r} 1010 \\ + 1 \\ \hline 1011 \end{array}$$

$$\begin{array}{r} 1000100 \\ + 1 \\ \hline 1 \end{array}$$

Mealy machine to complement a binary number



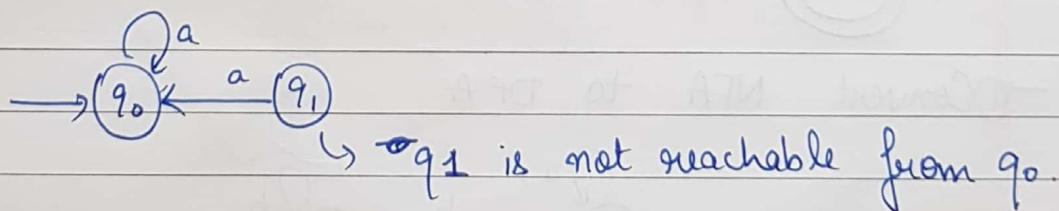
→ Moore machine.



Mealy and moore machine are equivalent in power i.e. number of strings accepted by mealy and moore machine are equal.

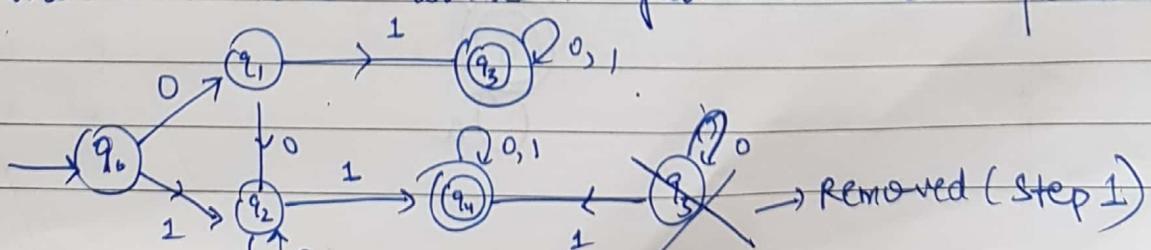
Minimization of DFA

- ① Remove all the states unreachable from initial state.

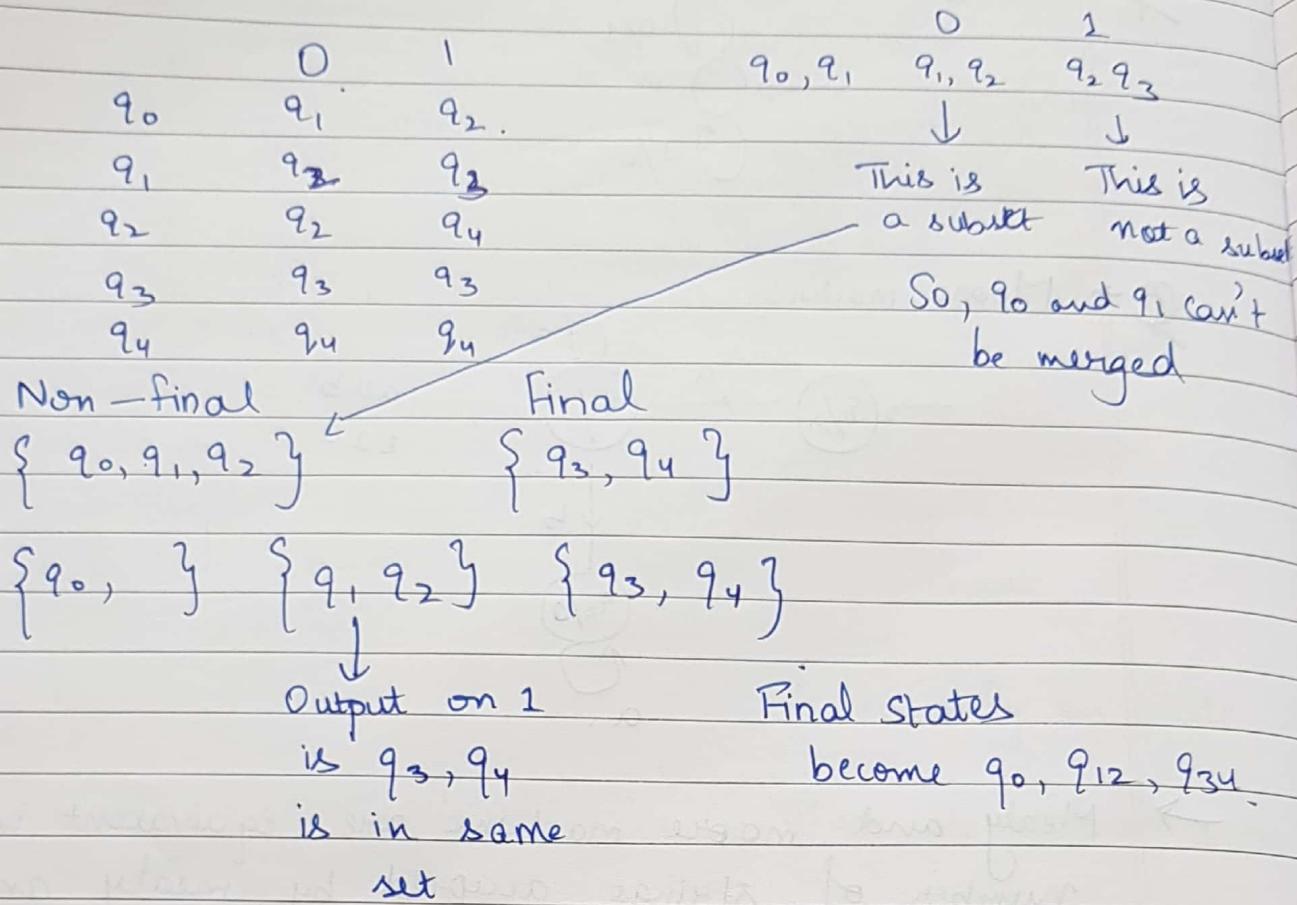


- ② Divide set of state into two subset - set of final state and sets of non-final state.

Further divide those until no further division possible

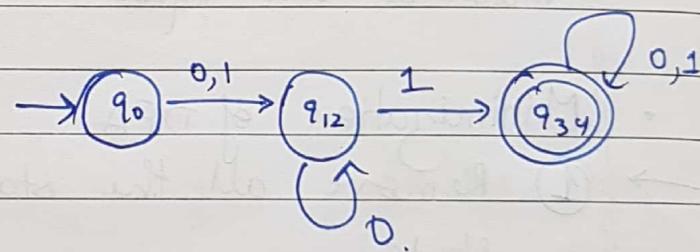


③ Draw its transition table

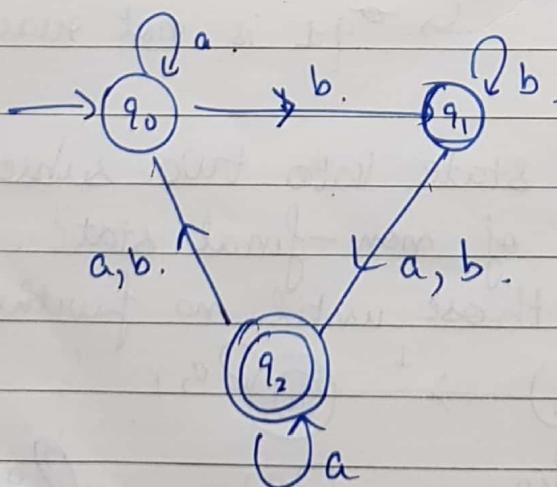


Transition Table

	0	1
q_0	q_{12}	q_{12}
q_{12}	q_{34}	q_{34}



Convert NFA to DFA.

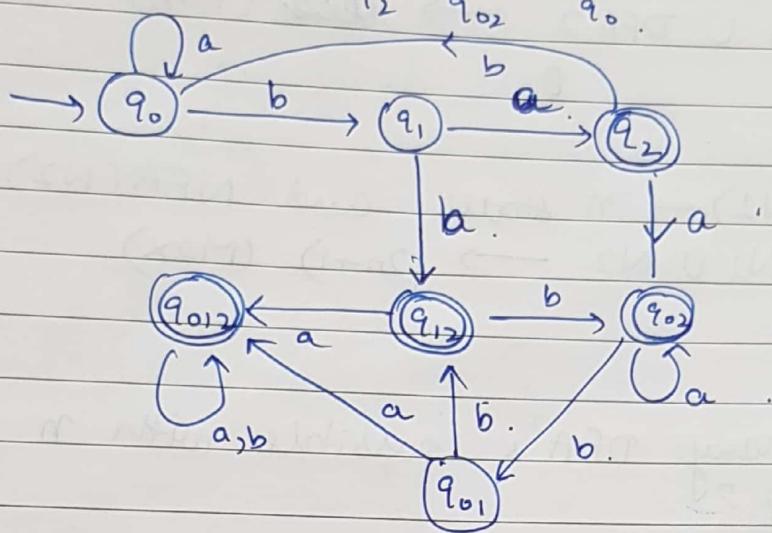


NFA

	a	b
q_0	q_0	q_1
q_1	q_2	q_{12}
q_2	q_{2, q_0}	q_0

DFA A

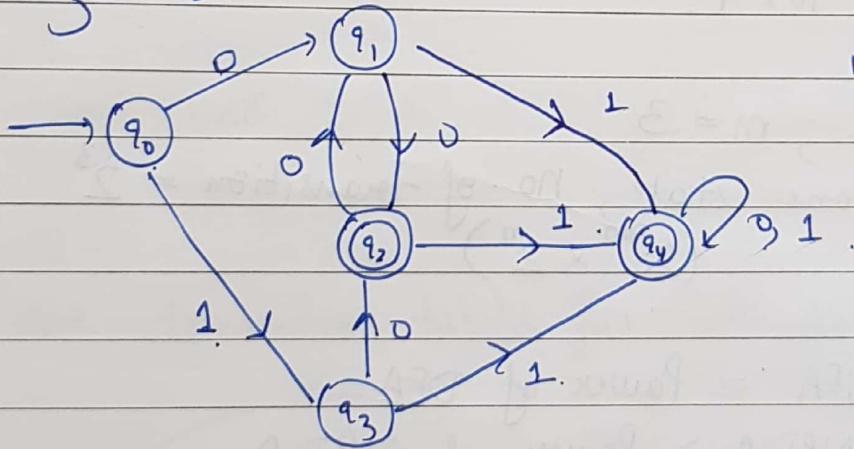
	a	b
q_0	q_0	q_1
q_1	q_2	q_{12}
q_2	q_{012}	q_{02}
q_{012}	q_{012}	q_{012}
q_{02}	q_{02}	q_{01}
q_0	q_0	q_0



If NFA contains n states

then how many max states in equivalent DFA?
 $\rightarrow 2^n$

② Minimize DFA



0 1

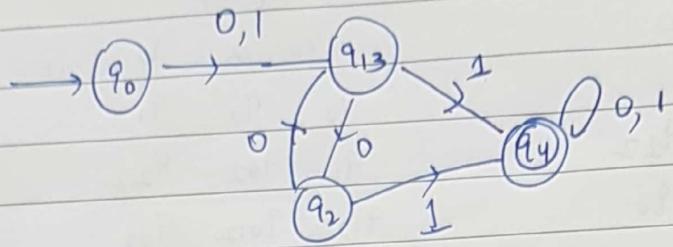
q_0	q_1	q_3	$\{q_0, q_1, q_3\}$	$\{q_2, q_4\}$
-------	-------	-------	---------------------	----------------

q_1 q_2 q_4 .

q_2	q_1	q_4 .	$\{q_0\}$	$\{q_1, q_3\}$	$\{q_2\} \{q_4\}$
-------	-------	---------	-----------	----------------	-------------------

q_3 q_2 q_4

q_4 q_4 q_4



Q DFA₁ → n states and DFA₂ → m states
 DFA₁ ∪ DFA₂ → ~~2^{n+m}~~^{2^{n+m}} (Max)

Q NFA (N₁) → n states and NFA (N₂) → m states
 N₁ ∪ N₂ → (2^{n+m}) (Max)

Q How many DFA's possible with n states and m input symbol?

→ ~~recastates~~

$$\text{e.g. } n = 2, m = 2 \\ \Rightarrow 16 \times 4$$

$$n = 2; m = 3$$

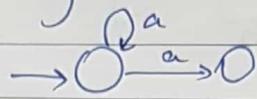
On one state, no. of transition = 2^3
 $(2^{mn} \times 2^n)$

→ Power of NFA = Power of DFA
 Power of NPDA > Power of DPDA

Push Down

Q How to decide if 2 DFA are equivalent power?

Q.2) How many NFA with 2 state and 2 i/p are possible



Possible Transitions

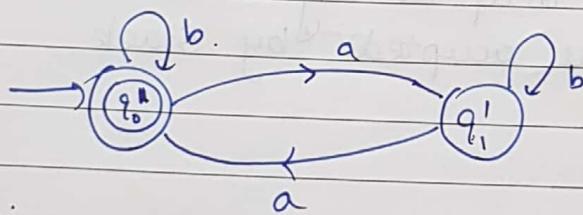
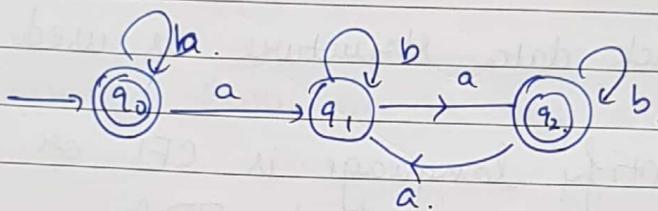
- ① Self loop
- ② \rightarrow
- ③ Self loop + \rightarrow
- ④ Null.

On State 1, total states = $4 \times 4 + 1 = 17$.

On state 2, total states = 17

4 possibilities on final state.

$$(4^n + 1)^m \times 2^n$$



Way 1:

① Minimize both DFA

Way 2:

① Make transition table for both DFA.

		a	b.	Chose either both final or both non-final state.
q_0	q_0'	q_1, q_1'	q_0, q_0'	
q_1	q_1'	q_2, q_0'	q_1, q_1'	
q_2	q_2'	q_1, q_1'	q_2, q_2'	

Check in all pairs

if there is any pair of final and non-final, then DFA are not equivalent

Not in Midsem

Auden's Theorem → For finding regular expression corresponding to given DFA.

CONTEXT FREE LANGUAGE (Push Down Automata)
↳ PDA.

Deterministic

DCFL

Non-deterministic

NCFL

CFL overcome disadvantage of regular language that it has no memory.

In CFL, stack data structure is used.

How to identify language is CFL or not?

- ① It must be accepted by PDA
- ② All languages accepted by stack.

$$S \rightarrow T^*$$

↓

Single non terminal

$$S \rightarrow S + S.$$

$$S \rightarrow S * S$$

$$S \rightarrow id$$

$$S \rightarrow S + S$$

$$S \rightarrow id + S$$

$$S \rightarrow id + S * S$$

$$S \rightarrow id + id * id.$$

}

Sentential (Intermediate forms
Form. before final form)

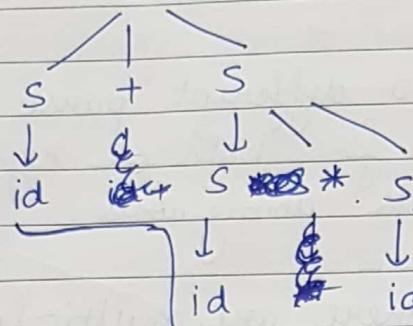
$$S \rightarrow S * S$$

$$S \rightarrow S + S * S$$

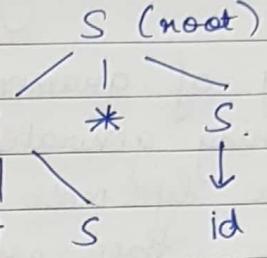
$$\Rightarrow S \rightarrow id + id * S \quad S \rightarrow id + id * id.$$

→ Derivation Tree . / Parse Tree .

$$S \rightarrow S + S$$



$$S \rightarrow S * S$$



→ We traverse from left to right and top to bottom

$\text{id} + \text{id} * \text{id}$.

For one string, there can be one or more than one sentential form.

derivation

left most

Derivation

gightmost

derivation

→ In this, ~~first~~^{always} left most variable is replaced.

Always sight most.

is supplied.

$$S \rightarrow S + S$$

$$\downarrow \text{id} + \downarrow S$$

$$\text{id} + \frac{1}{s} * s$$

\downarrow
 $id + id \neq s$

$$id + id * id$$

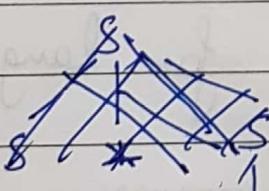
\downarrow
e-s*id

$S \rightarrow S \neq u.$

$$S \rightarrow S + S * id$$

$S \rightarrow S^+ id * id$

\downarrow
 $s \rightarrow id * id * id$



There is no change in parse tree for leftmost and rightmost.

→ For a string, parse tree for left most and right most is same always.

→ Ambiguity of grammar:

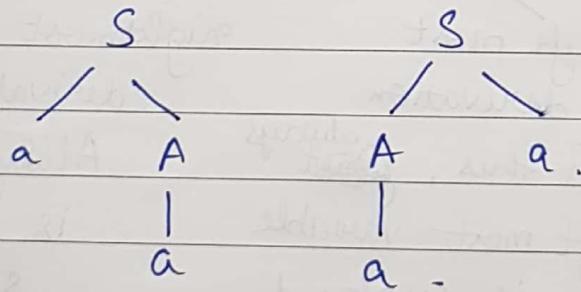
If a string generated by two different parse tree using either left most derivation for both or rightmost derivation, then grammar is ambiguous.

For a string in grammar, if there are multiple left most derivation or multiple rightmost derivation true, then grammar is ambiguous.

→ Context Free Grammar is ambiguous

→ Regular — Two non-terminal can't come together

$$S \xrightarrow{*} S * a . \quad S \rightarrow aA / Aa \\ A \rightarrow a .$$



So, regular grammar is ambiguous.

→ All programming language are designed with context free grammar.

We can ~~use~~ multiple grammar for language.

C language → Context sensitive grammar

In context free grammar, there is no concept of forward declaration.

$S \rightarrow aS$ classmate AVL
 $aS \rightarrow a$ Date DF
 $aS \rightarrow S$ page AT

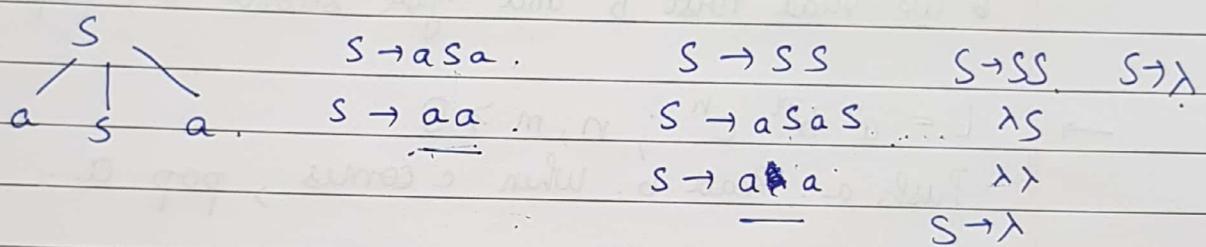
context-free

Ambiguity of grammar is undecidable because there is no algorithm.

Language cannot be ambiguous.

There exists some language for which no unambiguous grammar is available. So that language is known as inherently ambiguous language. Eg. deterministic context free language.

① $S \rightarrow aSa / bSb / SS / \lambda$



$S \rightarrow aSbS / bSaS / \lambda$

$S \rightarrow aSbS$

$S \rightarrow bSaS$

$S \rightarrow \lambda$

ab

ba

$S \rightarrow aSbS$

$S \rightarrow aSbS$

$S \rightarrow aa\underline{S}b\underline{S}$

$S \rightarrow ab\lambda$

$S \rightarrow aabb$

These are diff.

$S \rightarrow a\underline{S}bS$

$a\underline{S}bS$

$\alpha \underline{bS}aS bS$

$a\underline{b}aSbS$

$ab ab$

$ab ab$

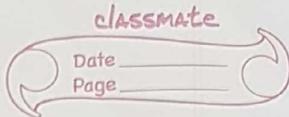
Same

So, ambiguous

Regular \rightarrow 5 tuple
CFL \rightarrow 7 tuple.



abc



$L = a^n b^n$ is context free.

$L = a^n b^n$ is context free.
PDA is FA with a single stack with infinite length.
PDA with finite length stack \rightarrow Not CFL
 \hookrightarrow Regular.

\rightarrow Stack \rightarrow LIFO.

Example of context free.

$L = a^n b^{2n}$.

We read first b and for second b, pop a.

$L = a^n b^m c^n ; n, m \geq 0$.

Push a. Read b. When c comes, pop a.

If variables have 2 equalities, then it is not context free
($a^n b^n c^n$)

$L = a^n b^m c^n d^m ; n, m \geq 0$.

\hookrightarrow Not possible (Not CFL)

$L = a^n b^n c^m d^m$.

\hookrightarrow CFL

$L = a^n b^m c^k$

$m = n$ or $m = k$.

\hookrightarrow NCFL

because we don't know when to push and when to pop.

$L = w c w^R ; w \in a, b^*$.

\hookrightarrow CFL (deterministic).

$L = w w^R$

abbbba. \rightarrow Till which point, we will push

\hookrightarrow we don't know.

\hookrightarrow It is in NCFL.

No conversion in NCFL and DCFL