Chapter 4

Block Matrix Multiplication

4.1 Objectives:

- At the end of the lecture, the learner would be able to:
  - Understand the definition of a block multiplication
  - Determine how to multiply two matrices using block multiplication .

4.2 (Block)Matrix Multiplication Procedure

- Let us consider two matrices A and B each of order nxn where n=2k.
  - First A and B are partitioned into four smaller matrices each of order kxk.

$$A = \begin{pmatrix} A11 & A12 \\ A21 & A22 \end{pmatrix}$$

$$B = \begin{pmatrix} B11 & B12 \\ B21 & B22 \end{pmatrix}$$

  - After partitioning of A and B, C can be computed as follows.

$$C = \begin{pmatrix} C11 & C12 \\ C21 & C22 \end{pmatrix}$$

implies

$$C = \begin{pmatrix} A11B11 + A12B12 & .. \\ .. & .. \end{pmatrix}$$

  - Assign a unique process to do the block matrix multiplication.
  - Let us consider the following matrices.

$$A = \begin{pmatrix} 1 & 1 & 2 & 3 \\ 3 & -1 & 1 & 5 \\ -2 & -3 & -4 & 2 \\ -5 & 2 & 0 & 2 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 1 & 2 & -3 \\ -5 & -4 & 2 & 1 \\ -3 & -3 & -4 & 2 \\ -1 & 2 & 0 & 0 \end{pmatrix}$$

- Step 1: Now first divide the two matrices into four blocks where each block contains two 2X2 matrices. The elements of A are in red and the elements of B are in green Compute the product of each block.

$$C = \begin{pmatrix} 1,1 & 1,1 & |1,1 & 2,-3 \\ 3,-1 & -5,-4 & |3,-1 & 2,1 \\ ---- & ---- & ---- & ---- \\ 2,-3 & 1,1 & |-2,3 & 2,-3 \\ -5,2 & -5,4 & |-5,2 & 2,1 \end{pmatrix}$$

=>

$$C = \begin{pmatrix} -4 & -3 & 4 & -2 \\ 8 & 7 & 4 & -10 \\ 17 & -10 & -10 & 9 \\ -15 & 3 & -6 & 17 \end{pmatrix}$$

- Now consider the next partition. Compute the product and sum up with the previous result.

$$C = \begin{pmatrix} 2,3 & 3,-3 & |2,3 & -4,2 \\ 1,5 & -1,2 & |1,5 & 0,0 \\ ---- & ---- & ---- & ---- \\ -4,2 & 3,-3 & |-4,2 & -4,2 \\ 0,2 & -1,2 & |0,2 & 0,0 \end{pmatrix}$$

=>

$$C = \begin{pmatrix} 3 & 0 & -8 & 4 \\ -2 & 7 & -4 & 2 \\ -14 & 16 & 16 & -8 \\ -2 & 4 & 0 & 0 \end{pmatrix}$$

After summing up the two C matrices we will get the final result.

4.3 Analysis

Let us assume that there are m x m sub matrices and q=n/m. Therefore q^2 sub matrices for a matrix and q^2 processes or processors.

Communication

Every slave processor will receive m^2 elements from a row and a column. Also all the slave processors will return a resultant sub matrix to the root processor.
Therefore, $T_{Communication}$=$q^2$2(startuptime+nmdatatime)+(startuptime+m^2datatime)

Computation

A slave processor will carry out q sub matrix multiplication and q sub matrix addition. A sequential sub matrix multiplication carry out m^3 multiplications and m^3 additions. Each sub matrix addition carry out m^2 additions. Therefore, $T_{Computation}=q(2m^3+m^2)=O(qm^3)$

Chapter 5

References

1. Parallel Computing, Theory and Practice, M.J.Quinn, McGraw Hill Publications,2002.

2. Introduction to Parallel Computing, Grama, Gupta, Kumar, Karypis, Addison Wesley, ISBN:0-201-64865-2, 2003.

3. Parallel Programming: Techniques and Applications Using Networked Workstations and Parallel Computers, Barry Wilkinson and Michael Allen, Prentice-Hall, 2nd edition, 2005.