Data Description

The dataset contains 435 lines. One per member of the House of Representatives. Each line contains the political party, followed by their votes on 17 selected issues.

Each vote can be *yes*, *no*, or *no-vote*.

Algorithm

Because I want to consider non-voting choices, I use a Normal Distribution with the following values for each choice.

yes = 1

no = 0

no-vote = 0.5

The MLE equation for computing the mean μ given a sample is therefore

$$\mu = \sum_{i=1}^{N} \frac{\bar{x}_i}{N} \tag{1}$$

The MLE equation for standard deviation, σ, is

$$\sigma = \sqrt{\frac{1}{N} \sum_{i-1}^{N} (x_i - \mu)} \tag{2}$$

First, a subset of the voting entries are taken and then split into a Republican group and Democrat group.

A mean and standard deviation are computed for each party, for each vote. The end result is

$$\begin{aligned} V_r &= \{(\mu_{r_1}, \sigma_{r_1}), (\mu_{r_2}, \sigma_{r_2}), \cdots, (\mu_{r_N}, \sigma_{r_N})\} \\ V_d &= \{(\mu_{d_1}, \sigma_{d_1}), (\mu_{d_2}, \sigma_{d_2}), \cdots, (\mu_{d_N}, \sigma_{d_N})\} \end{aligned} \tag{3}$$

Classification

For classification, a test set with the same dimensionality as the training set is used.

A test sample is compared against each party. For the quality metric, we use the z-score.

Given $S = (x_1, x_2, \cdots, x_N)$ where N = number of votes and x = (Y,N, no-vote).

\# Set initial total to zero
$total\_score = 0$

\# Iterate over each vote
for $i = [1 : N]$ **do**

  \# Compute the Republican Z-Score
  $Z_{r_i} = \frac{|\mu_{r_i} - x_i|}{\sigma}$

  \# Compute the Democrat Z-Score
  $Z_{d_i} = \frac{|\mu_{d_i} - x_i|}{\sigma}$

  \# Compute the Best Choice
  if $Z_{r_i} < Z_{d_i}$ **then**
    $total\_score += Z_{r_i}$
  else
    $total\_score -= Z_{d_i}$
  end if
end for

\# Compute Total Party
if $total\_score > 0$ **then**
  S is a Republican
else
  S is a Democrat
end if

Binomial Distribution

Probability Mass Function

$$f(n, k|p) = \binom{n}{k} p^k (1-p)^{n-k} = \left(\frac{n!}{k!(n-k)!}\right) p^k (1-p)^{n-k} \tag{4}$$

Likelihood

$$\mathfrak{L}(p|n, k) = \binom{n}{k} p^k (1-p)^{n-k} = \left(\frac{n!}{k!(n-k)!}\right) p^k (1-p)^{n-k} \tag{5}$$

Log Likelihood

$$\ln \mathfrak{L}(p|n, k) = \ln \left(\frac{n!}{k!(n-k)!}\right) + k \ln p + (n-k) \ln(1-p) \tag{6}$$

Derivative of Log-Likelihood

$$\frac{\partial \ln \mathfrak{L}(p|n, k)}{\partial p} = \frac{k}{p} - \frac{(n-k)}{1-p} = \frac{k-np}{p(1-p)} \tag{7}$$

Solution

$$\frac{\partial \ln \mathfrak{L}(p|n, k)}{\partial p} = 0 \tag{8}$$

$$\frac{k}{p} - \frac{(n-k)}{(1-p)} = 0 \tag{9}$$

$$\frac{k-np}{p(1-p)} = 0 \tag{10}$$

$$p = \frac{k}{n} \tag{11}$$

Normal Distribution

Probability Density Function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{12}$$

Likelihood Function

$$\mathfrak{L}(\bar{x}|\theta) = \mathfrak{L}(\bar{x}|\mu, \sigma^2) = \prod_{i=1}^{n} f(x_i, \mu, \sigma) \tag{13}$$

$$\mathfrak{L}(\bar{x}|\theta) = \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} \exp^{-\frac{(x_i-\mu)^2}{2\sigma^2}} \tag{14}$$

$$\mathfrak{L}(\bar{x}|\theta) = \left(\frac{1}{2\pi\sigma^2}\right)^{\frac{n}{2}} \exp^{\left(-\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2}\right)} \tag{15}$$

Log Likelihood

$$\ln \mathfrak{L}(\bar{x}|\theta) = \ln \left(\frac{1}{2\pi\sigma^2}\right)^{\frac{n}{2}} \exp^{\left(-\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2}\right)} \tag{16}$$

$$\ln \mathfrak{L}(\bar{x}|\theta) = \frac{n}{2} \ln \frac{1}{2\pi\sigma^2} - \frac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i - \mu)^2 \tag{17}$$

Derivative of Log Likelihood w.r. μ

$$\frac{\partial\theta}{\partial\mu} \ln \mathfrak{L}(\bar{x}|\theta) = \frac{\partial\theta}{\partial\mu} \left(\frac{n}{2} \ln \frac{1}{2\pi\sigma^2}\right) - \frac{\partial\theta}{\partial\mu} \left(\frac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i - \mu)^2\right) \tag{18}$$

$$\frac{\partial\theta}{\partial\mu} \ln \mathfrak{L}(\bar{x}|\theta) = 0 - \frac{-n\left(\bar{x} - \mu\right)}{\sigma^2} \tag{19}$$

$$\frac{\partial\theta}{\partial\mu} \ln \mathfrak{L}(\bar{x}|\theta) = \frac{n\left(\bar{x} - \mu\right)}{\sigma^2} \tag{20}$$

Maximizing

$$\frac{\partial\theta}{\partial\mu} \ln \mathfrak{L}(\bar{x}|\theta) = 0 \tag{21}$$

$$\frac{n\left(\bar{x} - \mu\right)}{\sigma^2} = 0 \tag{22}$$

$$n\bar{x} - n\mu = 0 \tag{23}$$

$$\mu = \frac{\bar{x}}{n} = \sum_{i=1}^{n} \frac{x_i}{n} \tag{24}$$

Derivative of Log Likelihood w.r. σ

$$\frac{\partial\theta}{\partial\sigma} \ln \mathfrak{L}(\bar{x}|\theta) = \frac{\partial\theta}{\partial\sigma} \left(\frac{n}{2} \ln \frac{1}{2\pi\sigma^2} \right) - \frac{\partial\theta}{\partial\mu} \left(\frac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i - \mu)^2 \right) \tag{25}$$

$$\frac{\partial\theta}{\partial\sigma} \ln \mathfrak{L}(\bar{x}|\theta) = \left(\frac{n}{2} \right) \left(-\frac{1}{\pi\sigma^3} \right) \left(\frac{2\pi\sigma^2}{1} \right) - \frac{1}{\sigma^3} \left(\sum_{i=1}^{n} (x_i - \mu)^2 \right) \tag{26}$$

$$\frac{\partial\theta}{\partial\sigma} \ln \mathfrak{L}(\bar{x}|\theta) = -\frac{n}{\sigma} - \frac{\sum_{i=1}^{n} (x_i - \mu)^2}{\sigma^3} \tag{27}$$

Maximizing

$$\frac{\partial\theta}{\partial\sigma} \ln \mathfrak{L}(\bar{x}|\theta) = 0 \tag{28}$$

$$-\frac{n}{\sigma} - \frac{\sum_{i=1}^{n} (x_i - \mu)^2}{\sigma^3} = 0 \tag{29}$$

$$-\frac{n}{\sigma} = \frac{\sum_{i=1}^{n} (x_i - \mu)^2}{\sigma^3} \tag{30}$$

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2 \tag{31}$$

$$\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2} \tag{32}$$