Complexity in Prosody: A Nonlinear Dynamical Systems Approach for Dyadic Conversations; Behavior and Outcomes in Couples Therapy

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Abstract

In this paper, we model dyadic human conversational interactions from a nonlinear dynamical systems perspective. We focus on deriving measures of the underlying system complexity using the observed dyadic behavioral signals. Specifically, we analyze different measures of complexity in prosody of speech (pitch and energy) during dyadic conversations of couples with marital conflict. We evaluate the importance of these measures as features by correlating them with different behavioral attributes of the couple codified in terms of behavioral codes. Furthermore, we investigate the relation between the computed complexity and outcomes of couples therapy. The results show that the derived complexity measures are more correlated to session level behavioral codes, and to the marital therapy outcomes, compared to traditional speech prosody features. It shows that nonlinear dynamical analysis of speech acoustic features can be a useful tool for behavioral analysis.

Index Terms: nonlinear dynamical systems, complexity, prosody, behavioral analysis, dyadic conversations

1. Introduction

Dynamical systems and chaotic analysis have been extensively studied for modeling speech waveforms [1–3] because of their ability to account for the nonlinear phenomena underlying the time-series. One characterization of non-linear systems is through the notion of complexity which provides a quantitative measure of 'degrees of freedom' or 'detail' in the minimal representation of the system. For example, a chaotic or irregular signal corresponds to higher complexity than a deterministic or periodic signal. Researchers have used complexity measures and other nonlinear dynamical features for several speech-based applications such as speaker recognition [4], phoneme classification [5], pathological speech classification [6] and speech synthesis [1].

However, little effort has been made on analyzing complexity patterns of *interacting* signal streams, such as the ones that can be found in dyadic interactions. Coordination and accommodation, also known as entrainment, are commonly occurring phenomena in such interactions [7–9], where interlocutors tend to adapt to each other's verbal and non-verbal behavior reflected in their speech patterns. It includes convergence or divergence of their patterns, as well as how they synchronize in time. Within the framework of nonlinear dynamical systems modeling of speech, the interactions can be viewed as joint (coupled) dynamical systems. One can argue that the complexity of the joint system formed by two

interlocutors depends on the extent of their mutual influence. More specifically, the complexity can be deemed lower if there is more coordination between the speakers in the form of behavioral similarity and synchrony [8].

In this paper, we explore the possible link between system coordination and complexity of conversational speech. Our analysis focuses on interactions of married couples, that were clinically assessed to have a distressed relationship. This work is motivated by several studies in the emerging domain of behavioral signal processing [10], that have shown that human interaction dynamics are influenced by underlying behavioral states of the interlocutors [11]. Lee *et al.* [12] quantified entrainment reflected in prosody and investigated its relationship with codified behavioral attributes of speakers, such as positivity and negativity. Several studies have shown importance of mutual influence in emotion during dyadic interactions [13–15]. Finally, our previous study on couples interactions used different acoustic features including turn-level information within and across speakers to predict possible relationship status change [16].

The current work also attempts to link signal-driven approaches to quantify couple's behavior and numerous theoretical and empirical research in couples therapy with dynamical systems approaches. Felmlee and Greenberg [17] modeled dyadic interaction of intimate couples as a dynamical system and argued that 'cooperation' between the couple leads to more stability in the system. Karney *et al.* [18] investigated how complexity in cognitive behavior influences marital relationships. Gottman *et al.* [19] also proposed mathematical models for behavioral constructs (such as *marital satisfaction*) in couples.

We propose a framework to analyze different complexity measures of spoken interactions using the prosody features (pitch and energy) of the observed speech signal. We associate the feature streams from each speaker with individual dynamical systems, while features from both speakers together are used to model a joint system, capturing coordination between individuals. The complexity measures computed on these systems are then investigated in relation to behavioral codes characterizing the dyad and the outcome of the couple therapy.

2. Feature Extraction

2.1. Audio Preprocessing

As the first step, we segment the raw audio stream into speaker-homogeneous regions. The segmentation quality is crucial because the dynamical analysis performed on a certain speaker's speech may be erroneous if it is corrupted by speech

segments from another speaker. Therefore, instead of relying on a fully automatic process of voice activity detection (VAD) and speaker diarization, we use a speech-text alignment algorithm, *SailAlign* [20]. It uses the transcription of the audio to obtain more accurate timestamps for each segment.

2.2. Prosodic Features: Pitch and Energy

We extract two commonly used prosodic features for all speech regions: pitch and energy. A state-of-the-art pitch tracking algorithm implemented in Praat toolbox [21] is used to extract pitch and energy. However, since pitch detection is not very robust under noisy conditions, we perform a smoothing by median filtering with a window size of 5 samples for every speaker-homogeneous segment on the raw pitch stream. We also perform linear interpolation for the instances in the speech region where pitch is detected to be zero such as for unvoiced phoneme regions. This step also attempts to rectify jumps in pitch that may happen due to doubling or halving error. We do not perform any smoothing for energy since this feature is more robust to outliers. Then pitch normalization is performed for each speaker on a logarithmic scale: $f_{0\text{norm}} = \log(f_0/f_{0\mu})$, where $f_{0\mu}$ is the mean pitch per speaker. Similarly, we normalize energy as $E_{\text{norm}} = E/E_\mu$ where E_μ denotes the mean energy per speaker.

3. Complexity of Nonlinear Dynamical Systems

According to dynamical systems theory, a time-series can be described through a mapping function starting from an initial state and evolving over a state space. In this section, we first describe a method to obtain an embedded representation of the state space of the considered time-series; then we discuss four different measures of complexity of the dynamical system. The complexity measures are calculated on reconstructed space (with the exception of the last measure in Section 3.2, obtained by Katz's algorithm) and aim to characterize how chaotically the system behaves.

3.1. Reconstructed State Space Embedding

The reconstructed state-space basically consists of a set of shifted versions of the original signal. Let us consider a scalar time series $z(n)$ sampled from $s(t)$, which is the observed signal of an nonlinear dynamical system with a finite-dimensional state space. The temporal evolution of the system is characterized by a mapping function Φ as $\mathbf{x}(t) = \Phi^t(\mathbf{x}(0))$, where $\mathbf{x}$ denotes a state in the original state space of the system. Given this formulation, one can construct a mapping F from the original state space to a reconstructed state space in $\mathbb{R}^d$ as shown in eqn. (1), where d is called the embedding dimension and Δ is the time delay. This mapping is also known as delay coordinates map.

$$\mathbf{x} \mapsto \mathbf{y} = F(\mathbf{x}) = \big(s(t), s(t+\Delta), ..., s(t+(d-1)\Delta)\big) \quad (1)$$

Although this formulation was originally justified by the celebrated Takens' theorem [22] for continuous signals $s(t)$, it was later extended to discrete-time signals or time series $z(n)$ by considering $z(n)$ as a sampled version of $s(t)$ [23–25].

As it can be seen from the eqn. (1), the embedding is dependent on parameters d and Δ. In this work, we first estimate the time delay Δ by finding the location of the first local minimum in the mutual information function of the signal with its delayed versions [26]. Next, we estimate the

optimal embedding dimension d using Cao's method [27], which requires the value of Δ. Finally we embed the time series into the reconstructed state space using eqn. (1). Once the reconstructed state space has been established, the temporal evolution of system can be examined from this space itself. Analysis of the reconstructed state space provides us with different characteristics of the chaotic patterns (*attractors*) present in the system through different complexity measures.

3.2. Different Complexity Measures

3.2.1. Lyapunov Exponents

The Lyapunov exponents (LE) describe the sensitivity of the initial conditions of a system and dynamics of neighboring trajectories in the state space embedding. Formally it is defined as the average exponential rate of convergence or divergence of two neighboring trajectories in a given direction of the embedded space. Let us assume the $\|\delta\mathbf{x}(0)\|$ is the initial separation of two trajectories and $\|\delta\mathbf{x}(t)\|$ is the separation after time t. Then the Lyapunov exponent λ_i in i^{th} direction is given by:

$$\frac{\|\delta\mathbf{x}(t)\|}{\|\delta\mathbf{x}(0)\|} = e^{\lambda_i t} \ (t \to \infty) \quad (2)$$

Largest Lyapunov exponent (LLE): The largest Lyapunov exponent λ_m is of specific interest because it is much easier to compute in a robust way and provides a measure of the complexity of the dynamical system. It is often calculated to discriminate between periodic and chaotic nature of a dynamical system. A positive value of the largest Lyapunov exponent typically indicates chaos, whereas a negative value indicates the orbits in the phase space approaching a common fixed point. The notion of Lyapunov exponents has been extended to nonlinear time series (discrete time) [28], as required in real-world applications. The present study uses the robust algorithm proposed by Sato *et al.* [29] to estimate the largest Lyapunov exponent from the reconstructed time series embedding.

3.2.2. Fractal Dimensions

The fractal dimension is a generalized term for several measures of geometric complexity of a set, or a pattern. In the case of dynamical system in the embedded space it refers to the active degrees of freedom of the system as reflected in chaotic behavior of the attractor [30]. The fractal dimension represents a lower bound on the number of equations required to model the underlying dynamical system. In this work, we used three measures of fractal dimensions [31], as described next.

Correlation Dimension (CD) [32] is probably the most widely used measure of fractal dimensions for nonlinear time series analysis. It is referred to as the second member ($q = 2$) in the infinite family of generalized dimensions D_q defined by Grassberger *et al.* [33]. The Correlation Dimension (D_2) is defined by the exponential scaling of the correlation sum $C_d(r)$ which computes the fraction of neighboring points closer than r as shown in the eqns. (3a) and (3b),

$$C_d(r) \propto r^{D_2} \quad (3a)$$

$$C_d(r) = \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=i+1}^{N} \Theta(r - \|\mathbf{y}^i - \mathbf{y}^j\|) \quad (3b)$$

where $\Theta(\cdot)$ is the Heaviside step function with $\Theta(x) = 1$ for $x \geq 0$ and zero elsewhere and $\mathbf{y}^i$, $\mathbf{y}^j$ are points in the reconstructed

state space. Takens [22] proposed a maximum likelihood estimator for the correlation dimension, which has been used in this work.

Information Dimension (ID), proposed by Radii and Politi [34], considers k nearest neighbors of a reference point and uses the distances of these points from the reference point in time-delay reconstructed space, as shown below:

$$D = -\frac{\log N}{\frac{1}{N}\sum_{n=1}^{N}\log r_n} \tag{4}$$

where r_n is the average distance of the nth reference point from its neighbors, and N is the number of samples. This is known as a fixed mass approach with k as a parameter.

Fractal Dimension by Katz's algorithm (KD) [35] treats the signal as a piecewise linearly connected set of points in the time series representation ($z(n)$ *vs.* n). If L is the total length of the curve or the sum of distances between consecutive points, one can compute the fractal dimension D with following formula:

$$D = \frac{\log n}{\log d_c - \log L + \log n} \tag{5}$$

where $n = L/\bar{a}$, $\bar{a}$ denoting the average distance between successive points, and d_c is the diameter of the curve defined as

$$d_c = \max_i \max_j \|\mathbf{z}^i - \mathbf{z}^j\| \tag{6}$$

Numerous definitions of fractal dimension have been introduced in literature to quantify the complexity of nonlinear dynamical systems. Although each of these definitions are related and similar in the sense that a higher value means a more complex system, some of them are intuitively different and capture different aspects of complexity. The above three approaches are chosen with an attempt to sample over different classes of algorithms with distinct characteristics, in order to obtain complementary information about the system.

4. Database

We use the Couples Therapy Corpus [36] in this work, which consists of recorded interactions of 134 heterosexual married couples with serious and chronic marital distress. It includes three recording sessions for each couple at different time points over a span of two years while they were undergoing therapy–the beginning of the therapy, after 26 weeks, and 2 years since the therapy started. The couple talked for 10 minutes on the wife's chosen topic and another 10 minutes on the husband's chosen topic without any therapist or research staff present. These interactions were recorded using a far-field microphone.

4.1. Behavioral Codes

In addition to the recordings of the interactions, we have manually annotated behavioral attributes, also known as *behavioral codes*, for each spouse in each session. In total the corpus has 33 behavioral codes, following two established behavioral coding systems: the Couples Interaction Rating System (CIRS, [37]) and the Social Support Interaction Rating System (SSIRS, [38]). Some examples of the codes are *blame, sadness, agreement, humor, negativity etc.* Each session was annotated by two to nine trained evaluators using these two rating systems on an integer scale of 1 to 9 and the average of their ratings are used as the reference.

4.2. Outcome Ratings

Finally, the corpus also included therapy outcome ratings of the couples based on where they stood in terms of their relationship compared to the condition before therapy. Each couple had one rating for each of the *post*-therapy sessions–26 weeks and 2 years. The ratings are provided on a 4-point scale; 1 (decline), 2 (no change), 3 (partial recovery), and 4 (complete recovery).

4.3. Preprocessing and Variables of Interest in the Study

In this work, we use 372 sessions out of total 574 sessions in the corpus, as the rest were too noisy to achieve good alignment, just as in some of the earlier studies [11, 12]. We choose two behavioral codes, *agreement* and *blame*, out of 33 codes for analysis. These two codes have respectively positive and negative correlation with the outcome; also intuitively speaking, agreement is more related to coordination of the speaker, whereas blame is not. Since annotations for these codes are at the session-level and for both husband and wife, we have 744 samples in total for the experiments with behavioral codes. On the other hand, a *pre*-therapy session and either of the corresponding *post*-therapy sessions (26 weeks or 2 years) constitute one sample for outcome experiments. As there were many couples with one or more of these sessions missing, we ended up with 64 samples for outcomes after discarding the missing sets.

5. Individual and Joint Complexity Measures

Following the steps of pre-processing and feature extraction as described in Section 2, we obtain pitch and energy streams for every session. Each feature stream is also divided into speaker homogeneous regions associated with either husband or wife. Thus we can have two speaker-specific sub-streams for each session, considering the speech of the husband and the wife to be individual systems and pitch and energy being the observed variable. Again, if we consider the original feature stream as the observed variable, we can model the speech of the husband and the wife together to form a joint system. Since the features were normalized per speaker, this construction does not include any apparent discontinuity. On each of these feature streams (husband, wife, and joint), we apply the complexity measures described in Section 3.2 (LLE, CD, ID, and KD). We denote a complexity measure as $\mathscr{C}(\cdot)$, a function of the feature stream. Finally we compute a normalized complexity of the joint feature stream of the couple (s_J) with respect to the individual feature streams of the husband (s_h) and the wife (s_w).

$$\text{normalized joint complexity} = \frac{\mathscr{C}(s_J)}{\sqrt{\mathscr{C}(s_h) \cdot \mathscr{C}(s_w)}} \tag{7}$$

6. Experiments and Results

We set up our experiments to investigate the following hypotheses in regard to the system complexity measures based on speech prosody:

- *H1*: The joint complexity measures are meaningful representations of interactions. (Section 6.1)

- *H2*: The complexity measures relate to the behavioral codes. (Section 6.2)

- *H3*: The complexity measures relate to the outcome of the couple therapy. (Section 6.3)

6.1. Verification of Joint Complexity Measures

In this experiment, we verify whether the complexity measure of the joint feature streams are meaningful. To set up the procedure to test this, we first reverse the sequence of the one of the interlocutors (husband or wife, chosen randomly) and then combine it with the intact stream of the other on a turn-by-turn basis to create a stream (s_A) corresponding to an artificial dialog. These artificial feature streams should have more complexity than original conversations, as the latter involves phenomena such as speech accommodation [7], and entrainment [12]. We test this hypothesis with all four complexity measures of pitch and energy streams and the percentages of sessions for which $\mathscr{C}(s_A) > \mathscr{C}(s_J)$ are shown in Table 1. The results show that all of these measures support this hypothesis for over 92% of the interactions.

Prosody	complexity measure used			
	LLE	CD	ID	KD
Pitch	98.92	97.31	94.35	96.51
Energy	95.16	95.43	92.74	94.09

Table 1: *Percentage of sessions with $\mathscr{C}(s_A) > \mathscr{C}(s_J)$ with different complexity measures*

6.2. Relation to Behavioral Codes

We perform a correlation analysis with different complexity measures of prosody with two behavioral codes. As the candidate feature streams, we use the individual feature streams of the specific person whose behavioral codes are being considered along with the joint feature stream. For each of these streams, the highest Spearman's ρ (in terms of absolute value) from each feature set, as correlation measures with both behavioral codes, is reported in Table 2. As baseline feature sets, we use:

- Baseline 1: mean pitch and energy,
- Baseline 2: individual and joint distribution of prosodic patterns preceeded by quantization, as used in [39].

Feature set	agreement		blame	
	pitch	*energy*	*pitch*	*energy*
Baseline 1	0.2802	0.2621	0.2445	0.2643
Baseline 2	0.2426	0.2344	0.2523	0.2712
Individual	0.2664	0.2830	0.2536	0.2719
Joint	0.2935	0.3187	0.2313	0.2787

Table 2: *Spearman's ρ (absolute) of the most correlated feature from different feature sets with agreement and blame*

Complexity measures of both pitch and energy turn out to have a generally high correlation with both of the codes, agreement and blame, when compared to baseline features. Moreover, joint complexity features seem to be more correlated with agreement than to blame. This is in accordance with the intuition that agreement is more interpersonal and dynamic as a behavior, hence can be captured well with the joint dynamical features. We also find that joint complexity measures are negatively correlated with agreement, and positively with blame (*i.e.,* in the last row of Table 2, first two values ρ values have negative sign, and the latter two are positive). We also perform a statistical significance test for the results of individual and joint complexity measures against the null hypothesis that

they are not correlated to the behavioral codes. For each of the measures (both individual and joint, corresponding to the last two rows of Table 2), $p < 0.05$ is obtained, indicating significant correlation.

6.3. Relation to Outcomes

Finally we perform another experiment to investigate importance of complexity measures, with relationship outcome as the variable of interest. Along with the two baseline feature sets as mentioned before, we use complexity measures of joint feature streams and the normalized joint complexity measures as defined in eqn. (7). The results are shown in Table 3.

Feature set	outcome	
	pitch	*energy*
Baseline 1	0.2772	0.2983
Baseline 2	0.2181	0.1878
Joint	0.3473	0.2565
Normalized Joint (7)	0.4146	0.2636

Table 3: *Spearman's ρ (absolute) of the most correlated feature from different feature sets with therapy outcome*

We find that the normalized joint complexity of pitch has the highest correlation with outcome. However, according to the results, complexity of energy seems to be less relevant to outcome. In both cases, normalized joint complexity feature has higher correlation than the unnormalized one. The correlations between outcome and complexity measures (the last two rows of Table 3) are also found to be statistically significant ($p < 0.05$) and negative in sign. The latter observation might indicate that higher complexity in interaction is related to lower value of the outcome variable, *i.e.,* decline in the relationship of the couple.

7. Conclusion

Human dyadic spoken interactions between two interlocutors can be modeled as coupled dynamical systems. These models may be useful for behavioral analysis of the interlocutors, with an emphasis on characterization of behavioral entrainment and mutual influence. In this paper, we explore such opportunities using prosodic features as observations of the dynamical systems. Then we investigate different complexity measures of these systems and evaluate their correlation with behaviors of couples during interactions and as well the therapy outcomes. The experimental results show that these complexity measures are useful for behavior analysis. We also observe that increased complexity in speech during dyadic interactions are associated with negative behavior (such as blame), and indicate decline in the couple's relationship.

In the future, we intend to use the introduced complexity measures for various tasks related to modeling interaction behavior. One can also apply more sophisticated dynamical models and computational methods introducing turn-taking behavior as another component of the system. Another direction is to extend this analysis to multimodal behavioral cues, such as language use, gestures and body language.

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9. References

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