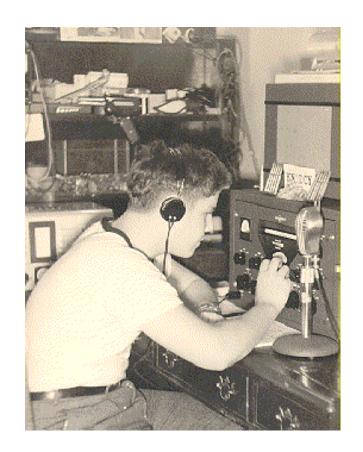
Chapter 6

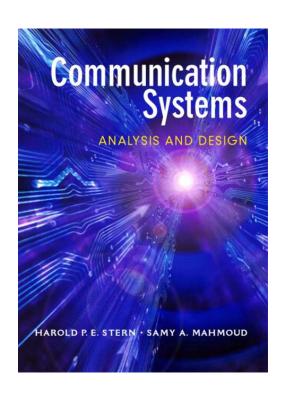
Analog Modulation and Demodulation



Chapter 6

Analog Modulation and Demodulation

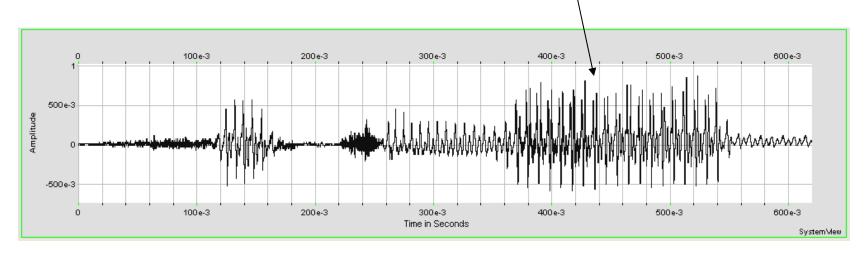
- Amplitude Modulation
- Pages 306-309



 The analytical signal for double sideband, large carrier amplitude modulation (DSB-LC AM) is:

$$s_{DSB-LC AM}(t) = A_c (c + s(t)) cos (2\pi f_C t)$$

where c is the DC bias or offset and A_C is the carrier amplitude. The continuous analog signal s(t) is a baseband signal with the information content (voice or music) to be transmitted.

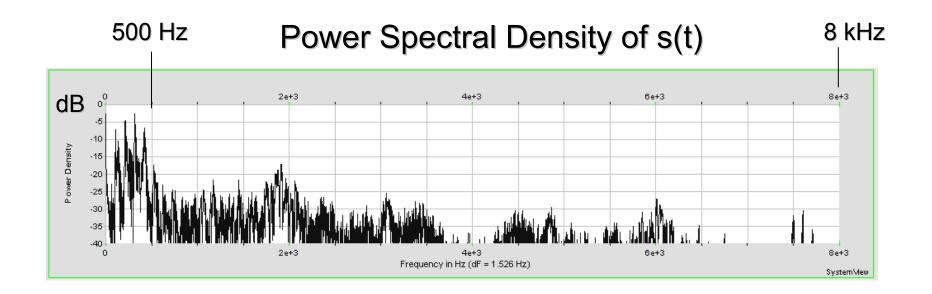


 The baseband power spectral density (PSD) spectrum of the information signal s(t) or S(f) for voice has significant components below 500 Hz and a bandwidth of < 8 kHz:

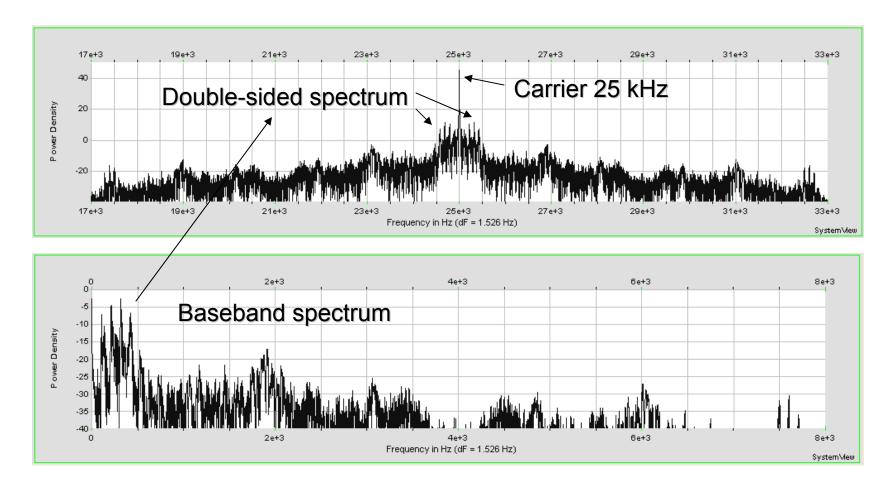
$$S(f) = F(s(t))$$

The *single-sided* spectrum of the modulated signal is:

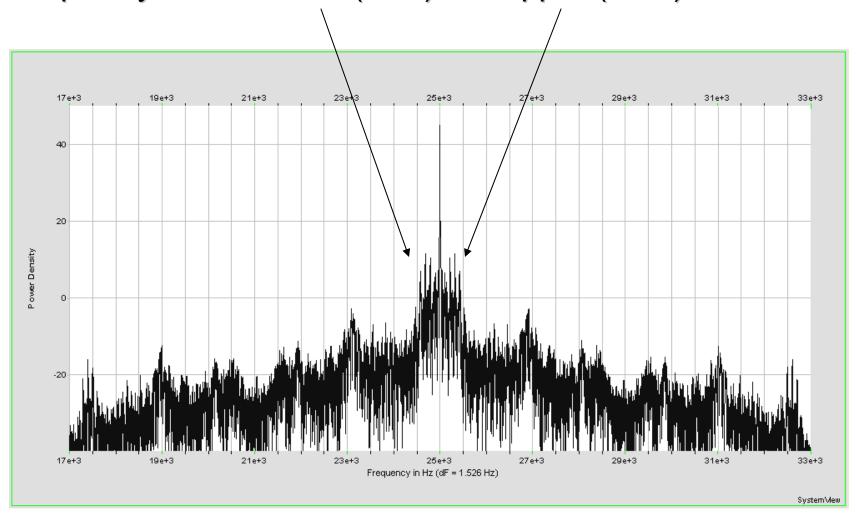
$$F(A_C (c + s(t)) cos (2\pi f_C t)) = S(f - f_C)$$



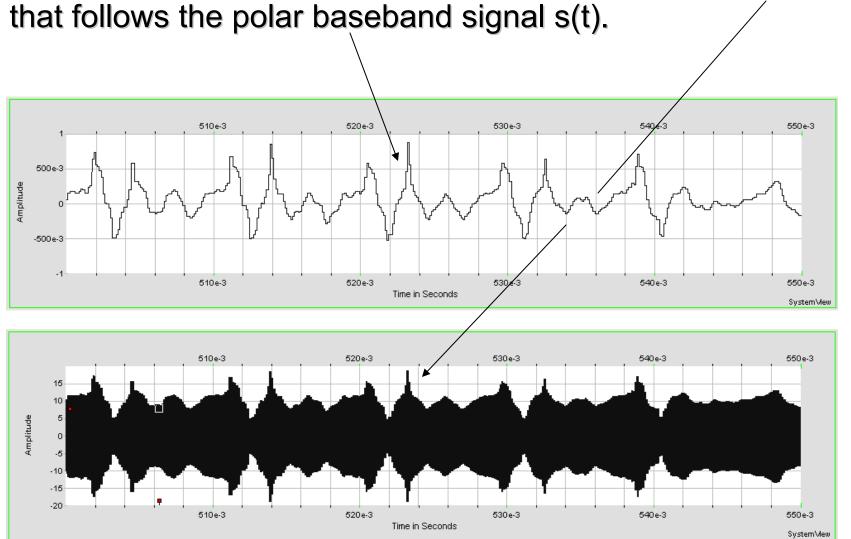
• The *single-sided* (positive frequency axis) spectrum of the *modulated signal* replicates the baseband spectrum as a *double-sided* spectrum about the carrier frequency.



• The double-sided modulated spectrum about the carrier frequency has an *lower* (*LSB*) and *upper* (*USB*) sideband.



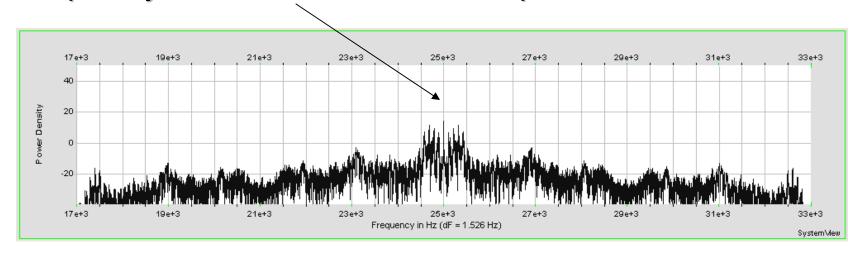
• The modulated DSB-LC AM signal shows an *outer envelope* that follows the polar baseband signal s(t).



 The analytical signal for double sideband, suppressed carrier amplitude modulation (DSB-SC AM) is:

$$s_{DSB-SC AM}(t) = A_C s(t) cos (2\pi f_C t)$$

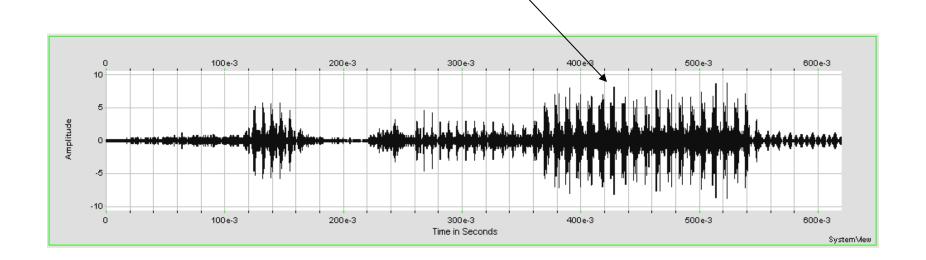
where $A_{\rm C}$ is the carrier amplitude. The single-sided spectrum of the modulated signal replicates the baseband spectrum as a double-sided spectrum about the carrier frequency but *without* a carrier component.



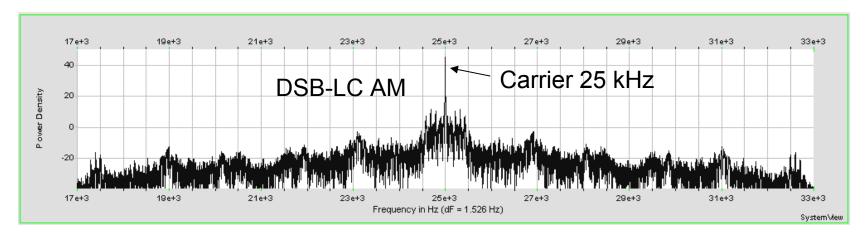
 The analytical signal for double sideband, suppressed carrier amplitude modulation (DSB-SC AM) is:

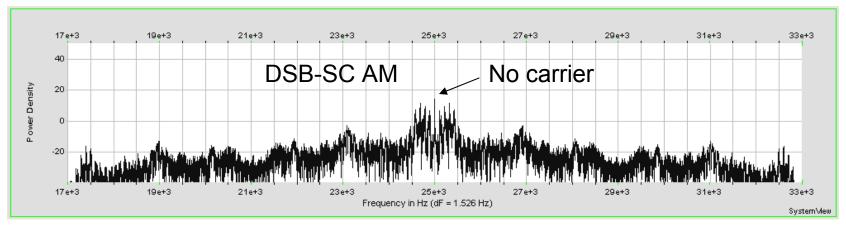
$$s_{DSB-SC AM}(t) = A_C s(t) cos (2\pi f_C t)$$

where A_C is the carrier amplitude. The modulated signal $s_{DSB-SC\ AM}(t)$ looks similar to s(t) but has a temporal but not spectral carrier component.

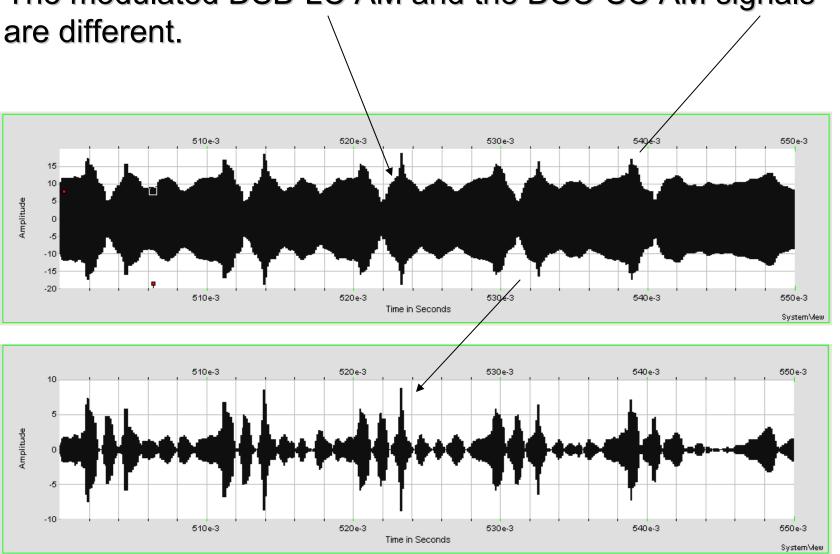


 The DSB-LC AM and the DSB-SC AM modulated signals have the same sidebands.

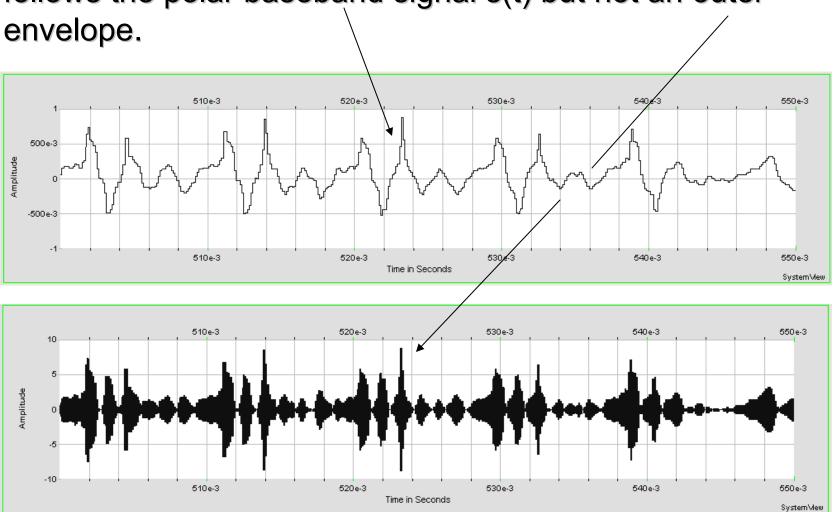




The modulated DSB-LC AM and the DSC-SC AM signals



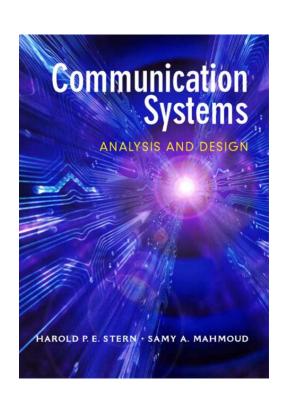
 The modulated DSB-SC AM signal has an envelope that follows the polar baseband signal s(t) but not an outer



Chapter 6

Analog Modulation and Demodulation

- Coherent Demodulation of AM Signals
- Pages 309-315



 The DSB-SC AM coherent receiver has a bandpass filter centered at f_C and with a bandwidth of twice the bandwidth of s(t) because of the LSB and USB. The output of the multiplier is lowpass filtered with a bandwidth equal to the bandwidth of s(t).

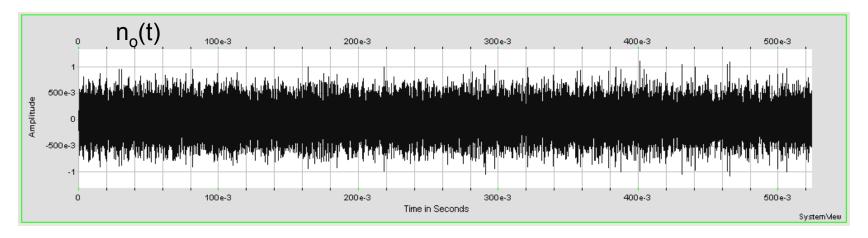
$$r(t) = \gamma s_{DSB-SC}(t) + n(t)$$

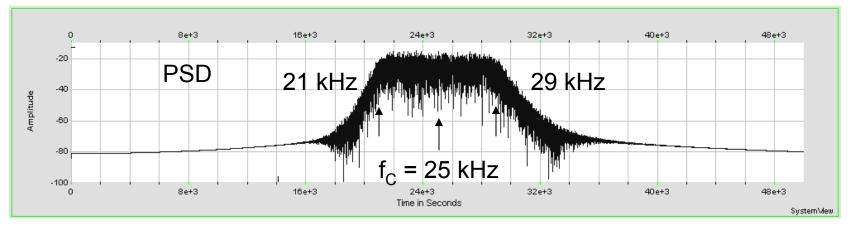
The DSB-SC AM received signal is r(t) = γ s_{DSB-SC}(t) + n(t).
 The bandpass filter passes the modulated signal but filters the noise:

$$z(t) = \gamma s_{DSB-SC}(t) + n_o(t)$$
 S&M Eq. 6.3

 $n_o(t)$ has a Gaussian distribution. The bandpass filter has a center frequency of $f_C = 25$ kHz and a -3 dB bandwidth of 8 kHz (25 ± 4 kHz).

The filter noise n_o(t) has a *flat power spectral density* within the bandwidth of the bandpass filter:





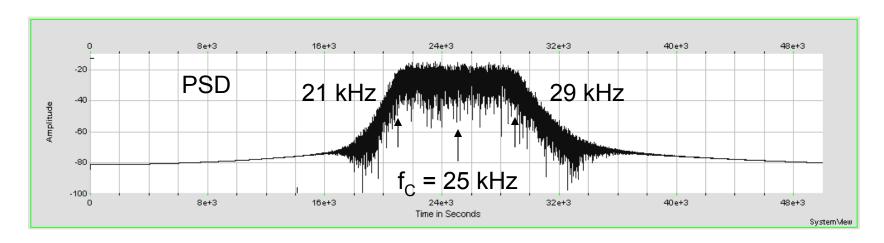
• The filter noise n_o(t) can be described as a *quadrature* representation:

$$n_o(t) = W(t) \cos (2\pi f_C t) + Z(t) \sin (2\pi f_C t)$$
 S&M Eq. 5.62R

In the coherent receiver the noise is processed:

$$n_o(t) \cos (2\pi f_C t) = \mathbf{W}(t) \cos^2 (2\pi f_C t) + S&M Eq. 6.5$$

 $\mathbf{Z}(t) \cos (2\pi f_C t) \sin (2\pi f_C t)$

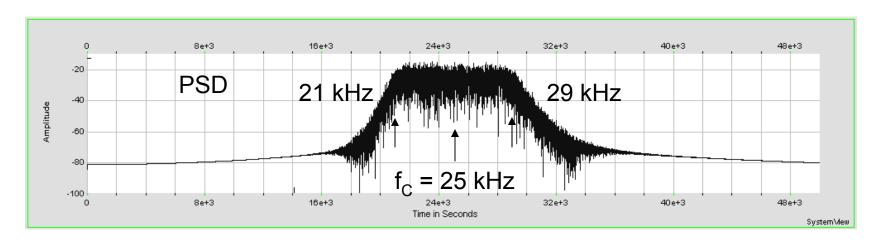


Applying the trignometric identity the filter noise n_o(t) is:

$$n_o(t)$$
 cos (2π $f_C t$) = ½ **W**(t) + ½ **W**(t) cos (4π $f_C t$) + ½ **Z**(t) sin (4π $f_C t$) S&M Eq. 6.5

After the lowpass filter in the receiver the demodulated signal is:

$$s_{demod}(t) = \frac{1}{2} \gamma A_C s(t) + \frac{1}{2} W(t)$$
 S&M Eq. 6.7

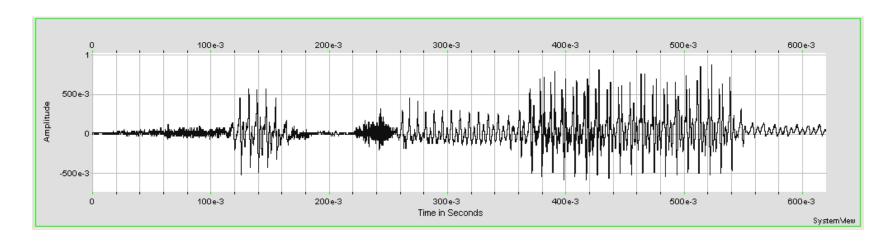


The transmitted DSB-SC AM signal is:

$$s_{DSB-SC AM}(t) = A_C s(t) cos (2\pi f_C t)$$

The average normalized bi-sided power of $s_{DSB-SC}(t)$ is found in the spectral domain with $S(f) = \mathbb{F}(s(t))$:

$$P_{trans} = A^2 \int \left| \frac{1}{2} [S(f - f_C) + S(f + f_C)] \right|^2 df$$
 S&M Eq. 6.8

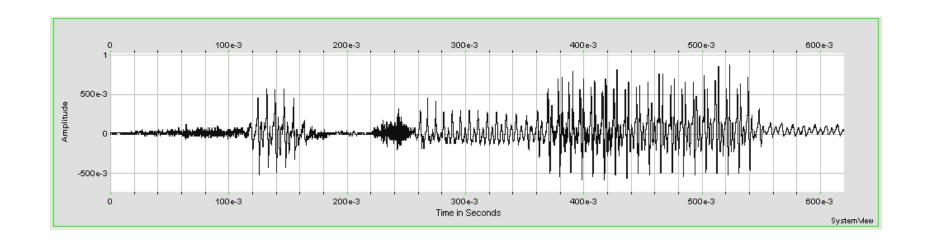


 The dual-sided spectral do not overlap (at zero frequency) and the cross terms are zero so that:

$$P_{trans} = A^{2} \int \left| \frac{1}{2} [S_{DSB-SC}(f - f_{C}) + S_{DSB-SC}(f + f_{C})] \right|^{2} df$$

$$P_{trans} = \frac{A^{2}}{2} P_{s}$$
S&M Eq. 6.9

where P_s is the average normalized power of s(t).

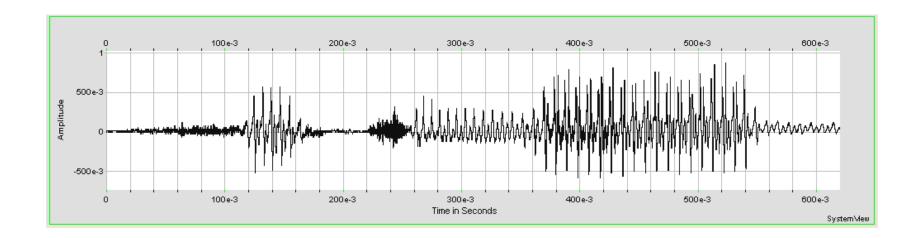


 The average normalized power of s(t) is found in the spectral domain:

$$P_s = \int |S(f)|^2 df = \int |S(f + f_C)|^2 df$$
 S&M Eq. 6.10

In a *noiseless channel* the power in the demodulated DSB-SC AM signal is:

$$P_{\text{demod, noiseless}} = \frac{1}{4} \gamma^2 A^2 P_s = \frac{\gamma^2}{2} P_{\text{trans}}$$
 S&M Eq. 6.11



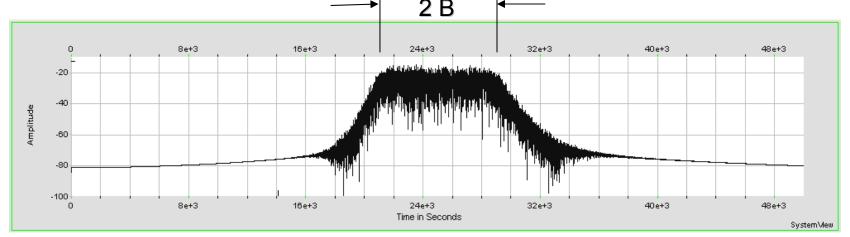
The average normalized power of the processed noise is:

$$P_{\text{processed noise}} = \frac{1}{4} N_o(2 B)$$

The signal-to-noise power ratio then is:

$$SNR_{coherent DSB-SC} = \frac{\frac{Y^2}{2}P_{trans}}{\frac{1}{4}N_o(2B)} = \frac{Y^2P_{trans}}{N_oB}$$

$$\Rightarrow 2B \qquad \Rightarrow 2B \qquad \Rightarrow 3B$$
S&M Eq. 6.12



 The DSB-SC AM coherent receiver requires a phase and frequency synchronous reference signal. If the reference signal has a phase error φ then:

$$\frac{\mathsf{SNR}_{\mathsf{coherent DSB-SC \, phase \, error}}}{\mathsf{V}^2 \, \mathsf{cos}^2 \, \varphi \, \mathsf{P}_{\mathsf{trans}}}} = \frac{\mathsf{S\&M Eq. 6.17}}{\mathsf{N_0 \, B}}$$

 The DSB-SC AM coherent receiver requires a phase and frequency synchronous reference signal. If the reference signal has a frequency error Δf then:

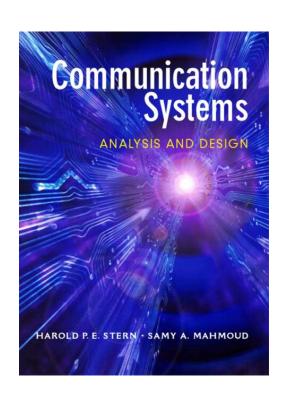
$$S_{\text{demod frequency error}}(t) = \frac{1}{2} \gamma A_{\text{C}} s(t) \cos (2\pi \Delta f t) + \frac{1}{2} \mathbf{X}(t) \cos (2\pi \Delta f t) + \frac{1}{2} \mathbf{Y}(t) \sin (2\pi \Delta f t) + \frac{1}{2} \mathbf{Y}(t) \sin (2\pi \Delta f t) + \frac{1}{2} \mathbf{X}(t) + \frac{1}{2} \mathbf{X}(t) + \frac{1}{2} \mathbf{X}(t) + \frac{1}{2} \mathbf{X}(t) + \frac{1}{2}$$

• Although the noise component remains the same, the amplitude of the demodulated signal varies with Δf:

Chapter 6

Analog Modulation and Demodulation

- Non-coherent Demodulation of AM Signals
- Pages 315-326



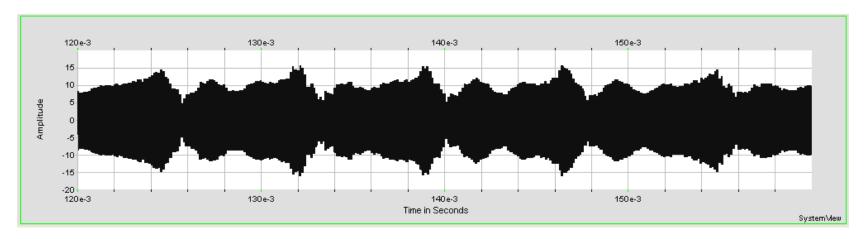
 The non-coherent AM (DSB-LC) receiver uses an envelope detector implemented as a semiconductor diode and a lowpass filter:

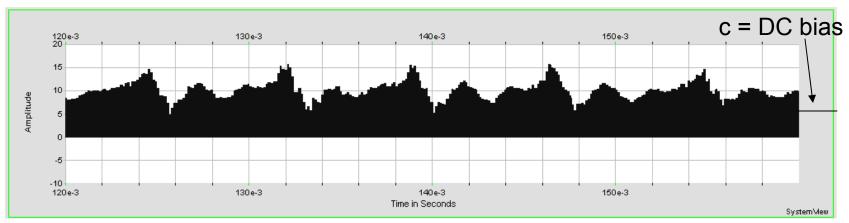
The DSB-LC AM analytical signal is:

$$s_{DSB-LC AM}(t) = A_C (c + s(t)) cos (2\pi f_C t)$$

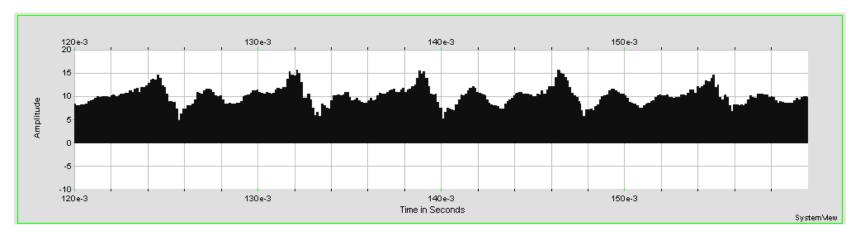
where c is the DC bias (offset).

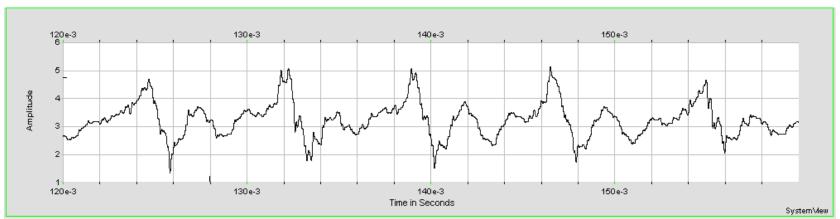
 The envelope detector is a half-wave rectifier and provides a DC bias (c) to the processed DSB-LC AM signal:





 The output of the half-wave diode rectifier is low-pass filtered to remove the carrier frequency and outputs the envelope which is the information:





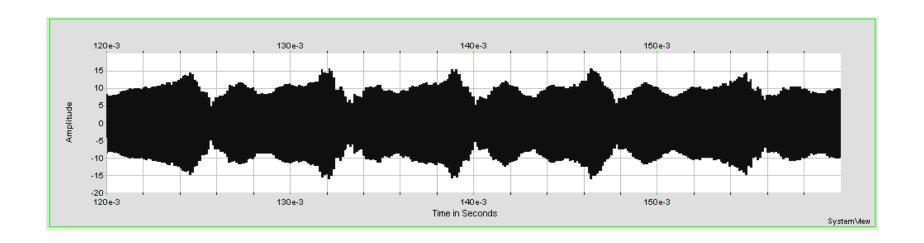
The DSB-LC AM signal can be decomposed as:

$$s_{DSB-LC AM}(t) = s(t) cos (2\pi f_C t) + A_C c cos (2\pi f_C t)$$

S&M Eq. 6.20R

The average normalized power of the information term:

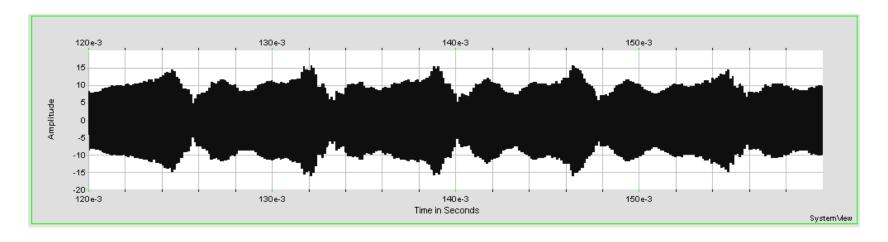
$$P_{info term} = \frac{A_C^2}{2} P_S$$
 S&M Eq. 6.23



The average normalized transmitted power is:

$$\begin{aligned} P_{\text{carrier term}} &= \frac{1}{T} \int_{0}^{T} \left[A_{\text{C}} c \cos(2\pi f_{\text{C}} t) \right]^{2} dt \\ P_{\text{carrier term}} &= \frac{A_{\text{C}}^{2} c^{2}}{2} \end{aligned}$$
 S&M Eq. 6.24

Since s(t) + c must be >= 0 to avoid distortion in the DSB-LC AM signal: $c \ge |\min[s(t)]|$ or $c^2 \ge s^2(t)$ for all t.



• Therefore $c^2 \ge P_s$ and for DSB-LC AM:

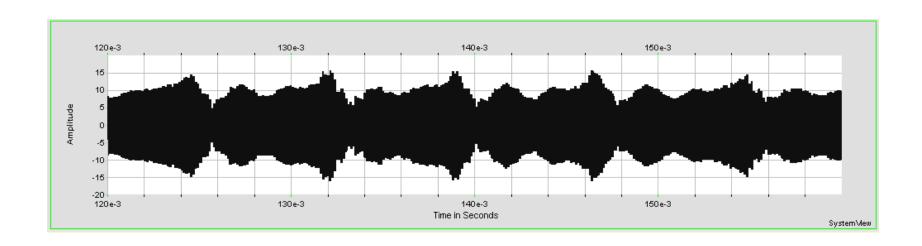
$$P_{\text{carrier term}} \ge P_{\text{info term}}$$

S&M Eq. 6.28

The power efficiency η of a DSB-LC AM signal is:

$$\eta = \frac{P_{\text{info term}}}{P_{\text{carrier term}} + P_{\text{info term}}} = \frac{P_{\text{info term}}}{P_{\text{trans DSB-LC AM term}}} \le 0.5$$

S&M Eq. 6.29



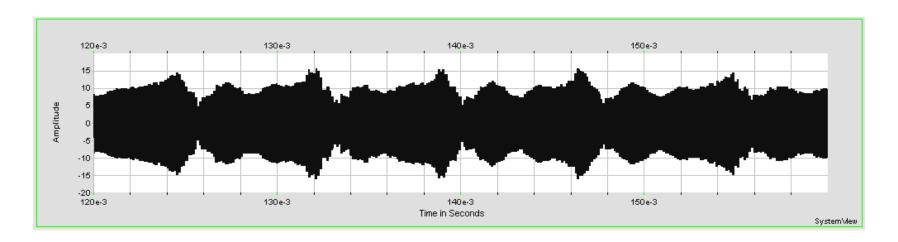
 The DSB-LC AM signal wastes at least half the transmitted power because the power in the carrier term has no information:

$$P_{carrier term} \ge P_{info term}$$
 $\eta \le 0.5$

The modulation index m is defined as:

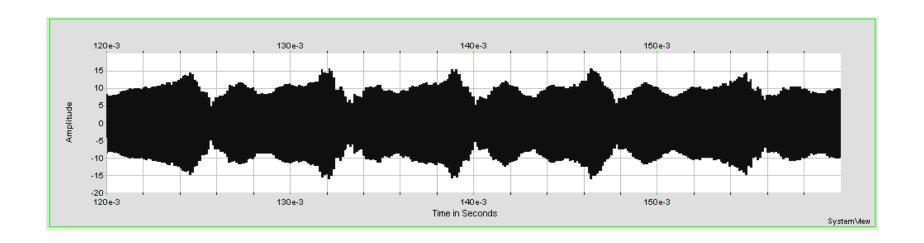
$$m = \frac{max[s(t)+c]-min[s(t)+c]}{max[s(t)+c]+min[s(t)+c]}$$

S&M Eq. 6.30



 The modulation index m defines the power efficiency but m must be less than 1. If m > 1 then min [s(t) + c] < 0 and distortion occurs.

$$m = \frac{\max[s(t) + c] - \min[s(t) + c]}{\max[s(t) + c] + \min[s(t) + c]}$$
 S&M Eq. 6.30



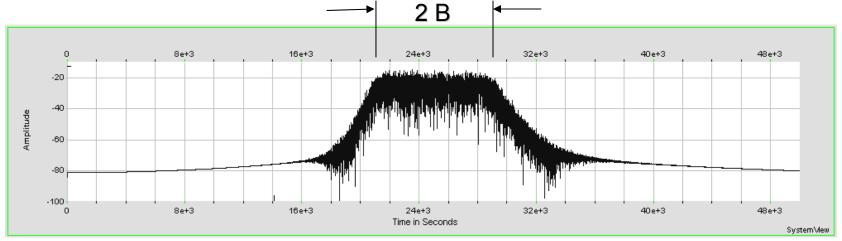
 The average normalized power of the demodulation noiseless DSB-LC AM signal is:

$$P_{\text{demod, noiseless}} = 2 \gamma^2 P_{\text{info term}}$$
 S&M Eq. 6.39

Then the signal-to-noise power ratio for the DSB-LC AM signal is:

S&M Eq. 6.40

$$SNR_{\text{noncoherent DSB-LC}} = \frac{2\,\gamma^2\,P_{\text{info term}}}{N_o\,(2\,B)} = \frac{\gamma^2\,P_{\text{trans DSB-LC}}}{N_o\,B}\eta$$



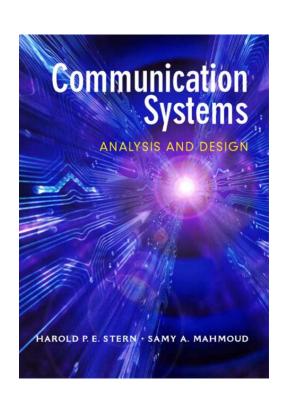
• The non-coherent AM (DSB-LC) receiver is the crystal radio which needs no batteries! Power for the high-impedance ceramic earphone is obtained directly from the transmitted signal. For simplicity, the RF BPF is omitted and the audio frequency filter is a simple RC network.



Chapter 6

Analog Modulation and Demodulation

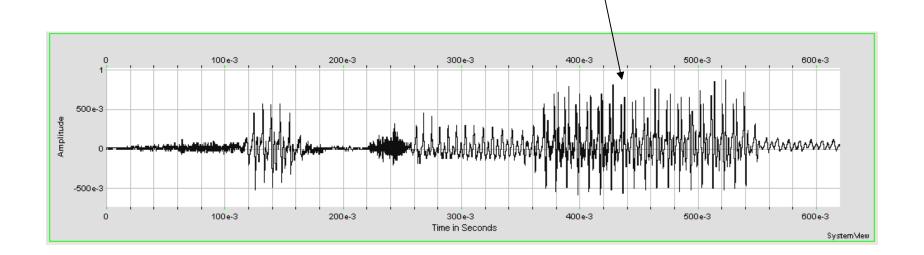
- Frequency Modulation and Phase Modulation
- Pages 334-343



 The analytical signal for an analog phase modulated (PM) signal is:

$$s_{PM}(t) = A_{c} \cos [2\pi f_{c} t + \alpha s(t)]$$
 S&M Eq. 6.53

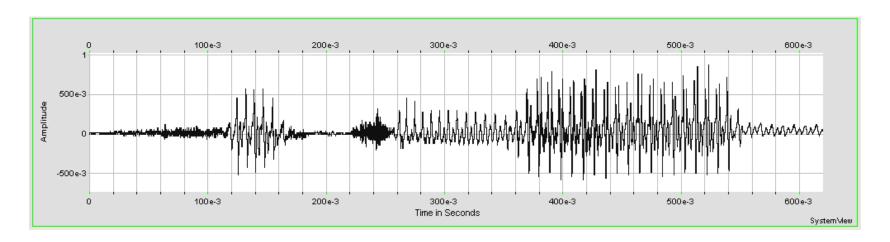
where α is the *phase modulation constant* rad/V and A_C is the carrier amplitude. The continuous analog signal s(t) is a baseband signal with the information content (voice or music) to be transmitted.



 The analytical signal for an analog frequency modulated (FM) signal is:

$$s_{FM}(t) = A_C \cos\{2\pi [f_C + k s(t)] t + \phi\}$$
 S&M Eq. 6.53

where k is the frequency modulation constant Hz / V, A_C is the carrier amplitude and φ is the initial phase angle at t = 0. The continuous analog signal s(t) is a baseband signal with the information content.



The instantaneous phase of the PM signal is:

$$\Psi_{PM}(t) = 2\pi f_C t + \alpha s(t)$$
 S&M Eq. 6.56

The *instantaneous phase* of the FM signal is:

$$\Psi_{FM}(t) = 2\pi [f_C + k s(t)] t + \phi]$$
 S&M Eq. 6.57

The instantaneous phase is also call the *angle* of the signal. The *instantaneous frequency* is the time rate of change of the angle:

$$f(t) = (1/2\pi) d\Psi(t) / dt$$
 S&M Eq. 6.58

 The instantaneous frequency of the unmodulated carrier signal is:

$$f_{carrier}(t) = d\Psi_{carrier}(t) / dt = d/dt \{2\pi f_C t + \phi\}$$
 S&M Eq. 6.59

The instantaneous phase is also:

$$\Psi(t) = \int_{-\infty}^{t} f(\lambda) d\lambda = \int_{0}^{t} f(\lambda) d\lambda + \phi$$
 S&M Eq. 6.60

There are *practical limits* on instantaneous frequency and instantaneous phase. To avoid ambiguity and distortion in FM signals due to *phase wrapping*:

$$k s(t) \le f_C$$
 for all t

S&M Eq. 6.61

 To avoid ambiguity and distortion in PM signals due to phase wrapping:

$$-\pi < \alpha s(t) \le \pi$$
 radians for all t

S&M Eq. 6.61

Since FM and PM are both change the angle of the carrier signal as a function of the analog information signal s(t), FM and PM are called *angle modulation*.

For example, is this signal FM, PM or neither:

t
x(t) = A_C cos { 2π f_Ct +
$$\int$$
 k s(λ) dλ + φ} S&M Eq. 6.60

The instantaneous phase of the signal is:

$$\Psi_{x}(t) = 2\pi f_{C}t + \int_{-\infty}^{t} k s(\lambda) d\lambda + \varphi$$
 S&M Eq. p. 336

which is *not* a linear function of s(t) so the signal is *not PM*. The instantaneous frequency of the signal is:

$$f_x(t) = (1/2\pi) d\Psi_x(t) / dt = f_C + k s(t) / 2\pi$$

and the frequency difference $f_x - f_C$ is a linear function of s(t) so the signal is FM.

The maximum phase deviation of a PM signal is max | $\alpha s(t)$ |. The maximum frequency deviation of a FM signal is $\Delta f = \max | k s(t) |$.

 The spectrum of a PM or FM signal can be developed as follows: S&M Eqs. 6.64 through 6.71

$$\begin{split} v(t) &= A_{_C} \sin(2\pi \, f_{_C} t + \beta \sin 2\pi \, f_{_m} t) \\ v(t) &= \text{Re} \, \big\{ \exp(j \, 2\pi \, f_{_C} t + j \, \beta \sin 2\pi \, f_{_m} t) \big\} \\ now &= \exp(j \, 2\pi \, f_{_C} t + j \, \beta \sin 2\pi \, f_{_m} t) = \\ &= \cos(2\pi \, f_{_C} t + \beta \sin 2\pi \, f_{_m} t) + j \sin(2\pi \, f_{_C} t + \beta \sin 2\pi \, f_{_m} t) \\ v(t) &= \text{Im} \, \big\{ A_{_C} \, \exp(2\pi \, f_{_C} t + j \, \beta \sin 2\pi \, f_{_m} t) \big\} \\ now &= \exp(j \, \beta \sin 2\pi \, f_{_m} t) = \sum_{n = -\infty}^{\infty} c_n \exp(j \, 2\pi \, n \, f_{_m} t) \\ \textit{after further development} &= \text{Bessel function of the first kind} \\ &= \exp(j \, \beta \sin 2\pi \, f_{_m} t) = \sum_{n = -\infty}^{\infty} J_n(\beta) \exp(j \, 2\pi \, n \, f_{_m} t) \end{split}$$

n

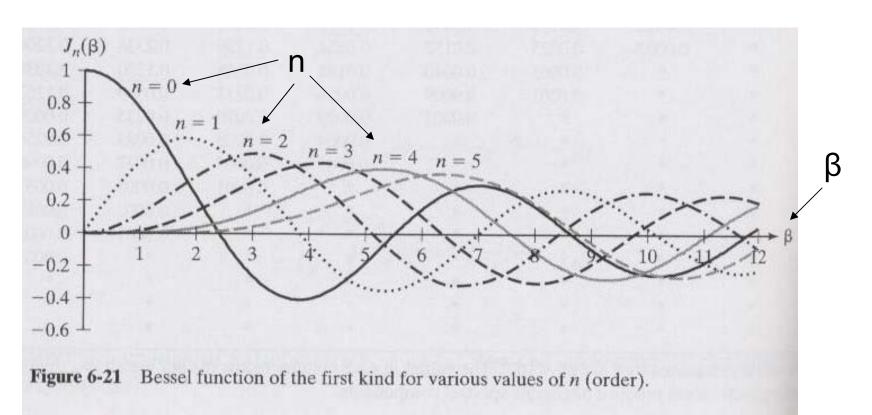
• Bessel functions of the first kind $J_n(\beta)$ are tabulated for FM with single tone f_m angle modulation (S&M Table 6.1):

n	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 4$	$\beta = 5$	$\beta = 6$	$\beta = 7$	$\beta = 8$	$\beta = 9$
0	0.7652	0.2239	-0.2601	-0.3971	-0.1776	0.1506	0.3001	0.1717	-0.0903
1	0.4401	0.5767	0.3391	-0.0660	-0.3276	-0.2767	-0.0047	0.2346	0.2453
2	0.1149	0.3528	0.4861	0.3641	0.0466	-0.2429	-0.3014	-0.1130	0.1448
3	0.0196	0.1289	0.3091	0.4302	0.3648	0.1148	-0.1676	-0.2911	-0.1809
4	0.0025	0.0340	0.1320	0.2811	0.3912	0.3576	0.1578	-0.1054	-0.2655
5	0.0002	0.0070	0.0430	0.1321	0.2611	0.3621	0.3479	0.1858	-0.0550
6	*	0.0012	0.0114	0.0491	0.1310	0.2458	0.3392	0.3376	0.2043
7	*	0.0002	0.0025	0.0152	0.0534	0.1296	0.2336	0.3206	0.3275
8	*		0.0005	0.0040	0.0184	0.0565	0.1280	0.2235	0.3051
9	*	*	0.0001	0.0009	0.0055	0.0212	0.0589	0.1263	0.2149
10	*	*	*	0.0002	0.0015	0.0070	0.0235	0.0608	0.1247
11	*	*	*	*	0.0004	0.0020	0.0083	0.0256	0.0622
12	*	*	*	*	0.0001	0.0005	0.0027	0.0096	0.0274
13	*	*	*	*	*	0.0001	0.0008	0.0033	0.0108
14	*	*	*	*	*	*	0.0002	0.0010	0.0039
15	*	*	*	*	*	*	0.0001	0.0003	0.0013
16	*	*	*	*	*	*	*	0.0001	0.0004
17	*	*	*	*	*	*	*	*	0.000
18	*	*	*	*	*	*	*	*	*
19	*	*	*	*	*	*	*	*	*

^{*}The values designated by * are all $<10^{-4}$. The values in each column below the bar are all less than 0.1. Practically speaking, such values produce negligible spectral components.

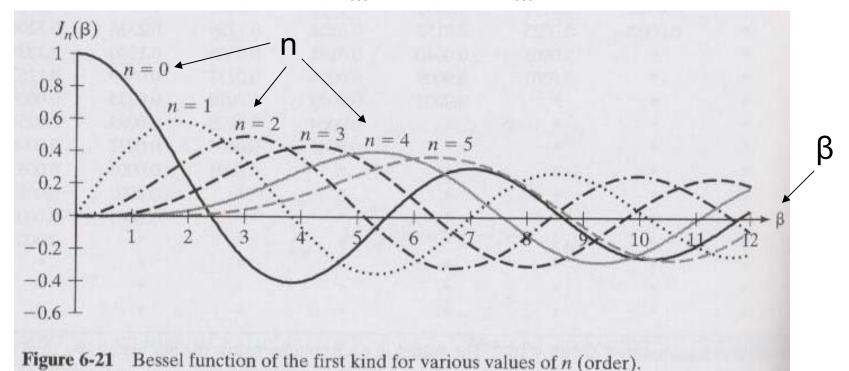
 For single tone f_m angle modulation the spectrum is periodic and infinite in extent:

$$v(t) = A_C \sum_{n=-\infty}^{\infty} J_n(\beta) \sin[2\pi (n f_m + f_C) t]$$
 S&M Eq. 6.72



• The *complexity* of the Bessel function solution for the spectrum of a single tone angle modulation can be simplified by the *Carson's Rule approximation* for the bandwidth *B*. Since $\beta = \Delta f / f_m$:

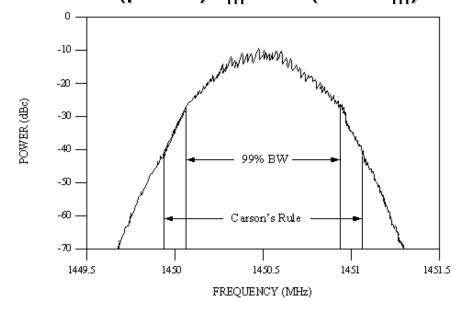
B = 2 (
$$\beta$$
 + 1) f_m = 2 (Δ f + f_m) Hz S&M Eq. 6.74

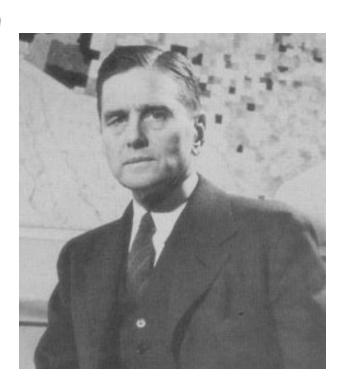


 Carson's Rule for the approximate bandwidth of an angle modulated signal was developed by John R. Carson in 1922 while he worked at AT&T. Prior to this in 1915 he

presaged the concept of bandwidth efficiency in AM by proposing the suppression of a sideband (see S&M p. 326-333):

$$B = 2 (\beta + 1) f_m = 2 (\Delta f + f_m) Hz$$



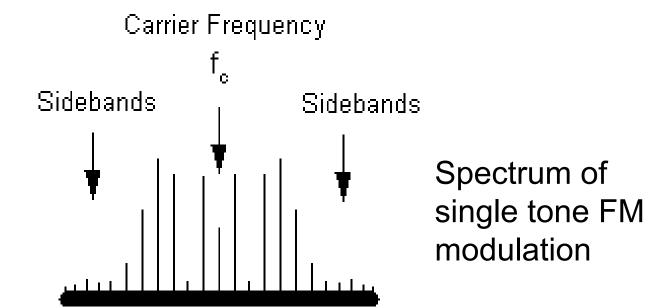


1886-1940

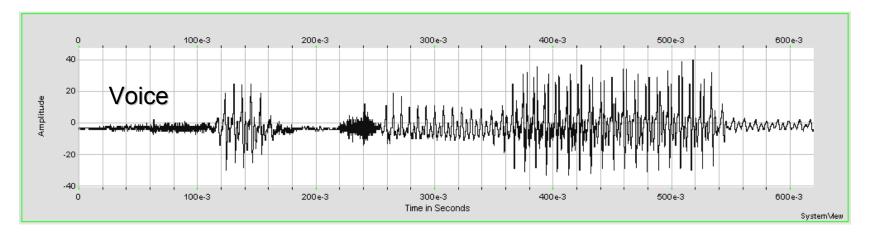
 The normalized power within the Carson's Rule bandwidth for a single tone angle modulated signals is:

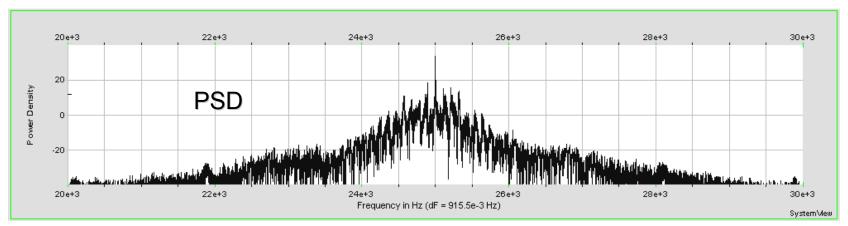
$$P_{\text{in-band, sinusoid}} = \frac{A_C^2}{2} \sum_{n=-(\beta+1)}^{\beta+1} J_n^2(\beta)$$
 S&M Eq. 6.75

Note that $J_{-n}(\beta) = \pm J_n(\beta)$ so that $J_{-n}^2(\beta) = J_n^2(\beta)$ and for the normalized power calculation the sign of $J(\beta)$ is not used.

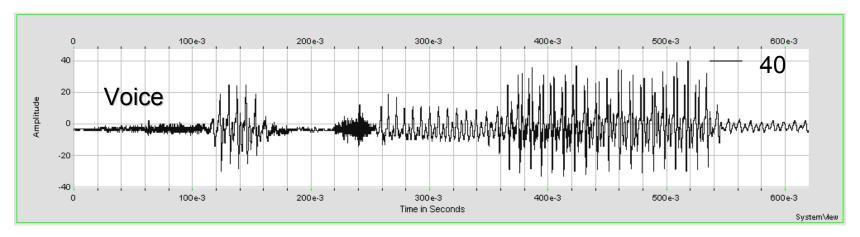


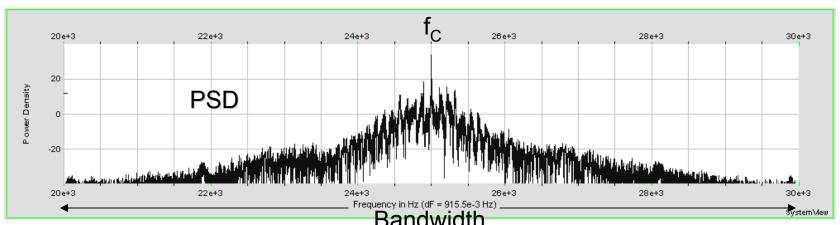
 The analog FM power spectral density PSD of the voice signal has a bandwidth predicted only by Carson's Rule since it is not a single tone.



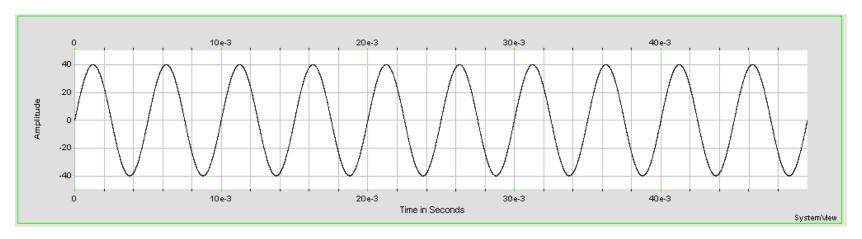


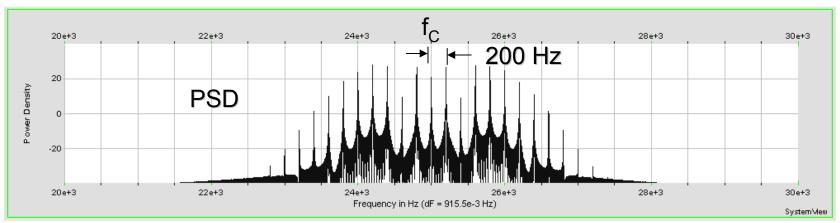
• Here f_{max} = 4 kHz, k = 25 Hz/V and Δf_{max} = 40(25) = 1 kHz. The Carson's Rule approximate maximum bandwidth B = 2 (Δf + f_m) = 10 kHz or ± 5 kHz (but seems wrong!)



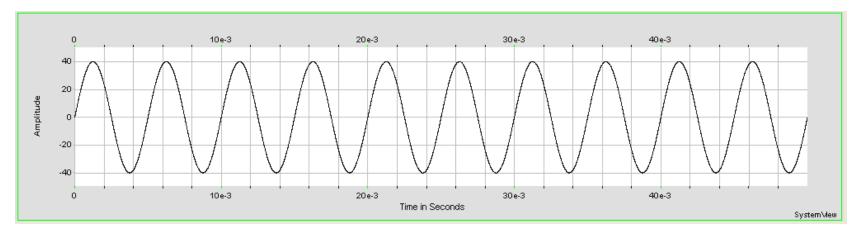


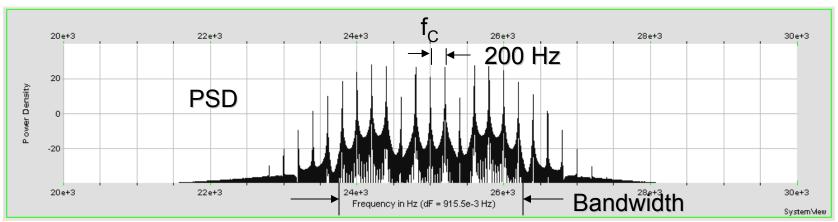
• A 200 Hz single tone FM signal has a PSD with periodic terms at $f_C \pm n f_m = 25 \pm 0.2 n \text{ kHz}$.



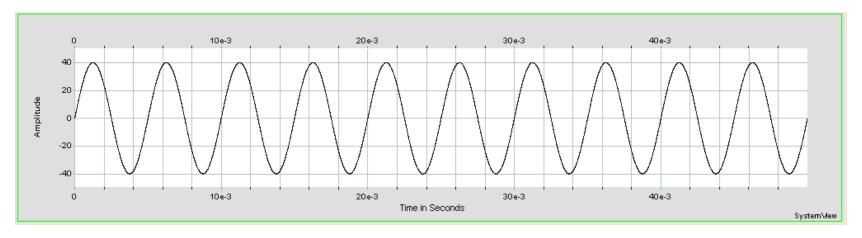


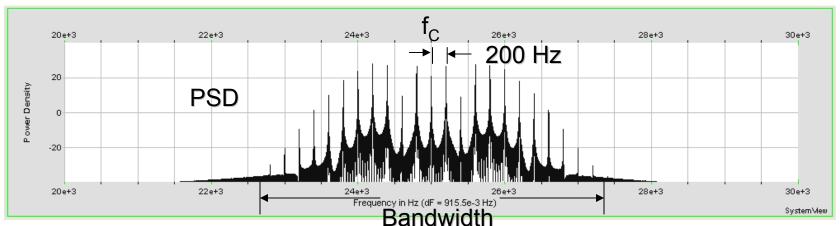
• Here f_m = 200 Hz, k = 25 Hz/V and Δf_{max} = 40(25) = 1 kHz. The Carson's Rule approximate maximum bandwidth B = 2 (Δf + f_m) = 2.4 kHz or ± 1.2 kHz:





• Since $\beta = \Delta f / f_m = 1 \text{ kHz} / 0.2 \text{ kHz} = 5 \text{ and the Bessel function predicts a bandwidth of 2 n } f_m = 2(12)(200) = 4.8 \text{ kHz} (since n = 12 \text{ for } \beta = 5 \text{ from Table 6.1}):$

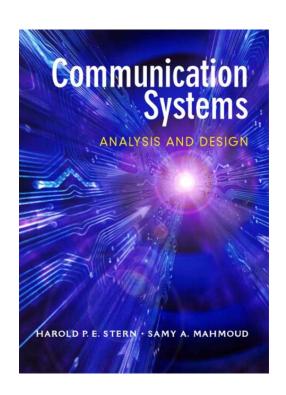




Chapter 6

Analog Modulation and Demodulation

- Noise in FM and PM Systems
- Pages 347-355

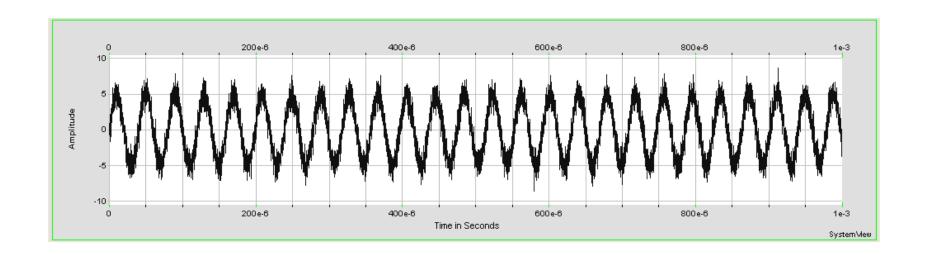


A general angle modulated transmitted signal, where Ψ(t) is the instantaneous phase, is:

$$s_{angle-modulated}(t) = A_C \cos [\Psi(t)]$$
 S&M Eq. 6.86

The received signals is:

$$r_{angle-modulated}(t) = \gamma A_C \cos [\Psi(t)] + n(t)$$
 S&M Eq. 6.87



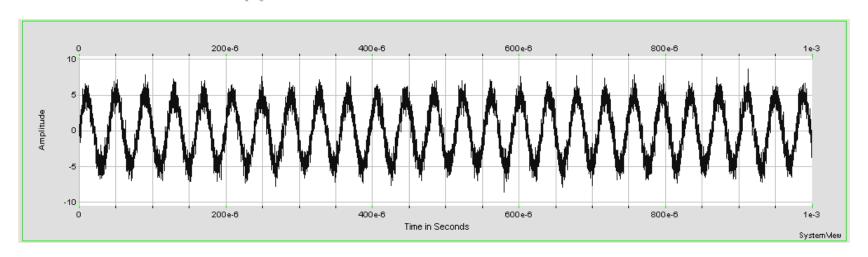
The analytical signal for PM is:

$$s_{PM}(t) = A_C \cos [\Psi(t)] = A_C \cos [2\pi f_C t + \alpha s(t)]$$

S&M Eq. 6.53
After development the SNR for demodulated PM is:

$$SNR_{PM} = (\alpha \gamma A_C)^2 P_S / (2 N_0 f_{max})$$
 S&M Eq. 6.98

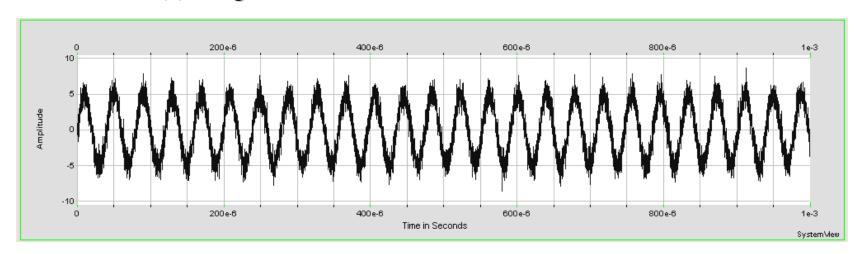
where $-\pi < \alpha s(t) \le \pi$ for all t.



The analytical signal for FM is:

$$s_{FM}(t) = A_C \cos [\Psi(t)] = A_C \cos [2\pi f_C t + \int k s(\lambda) d\lambda]$$
 S&M Eq. 6.53 After development the SNR for demodulated FM is:

SNR_{FM} = 1.5 (k γ A_C /(2 π))² P_S / (N_o f_{max}³) S&M Eq. 6.98 where k s(t) \leq f_C for all t.



End of Chapter 6 Analog Modulation and Demodulation

