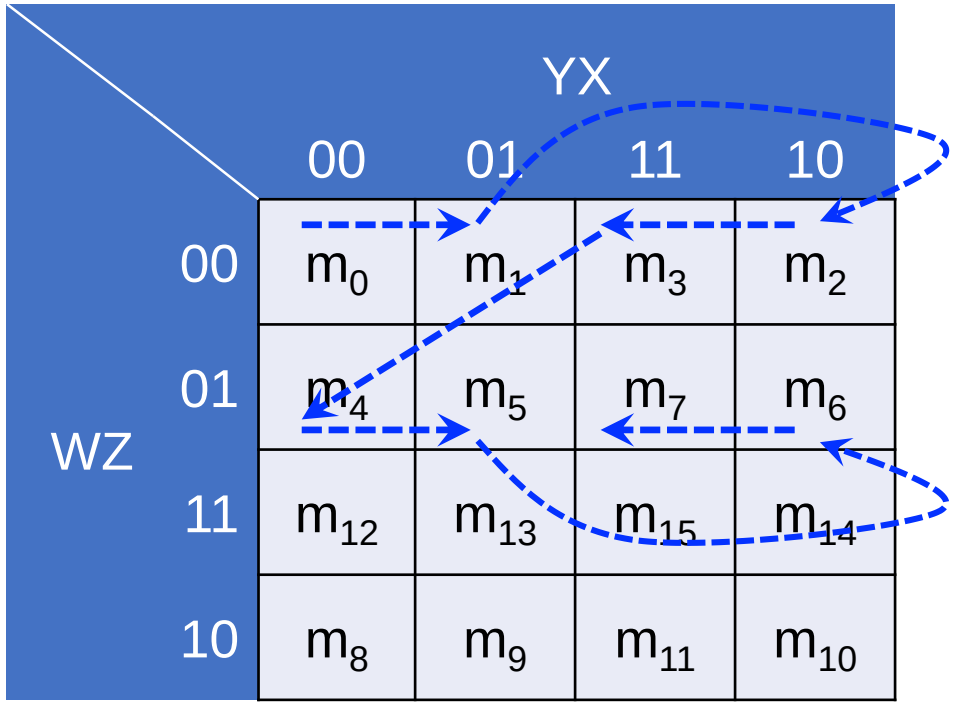
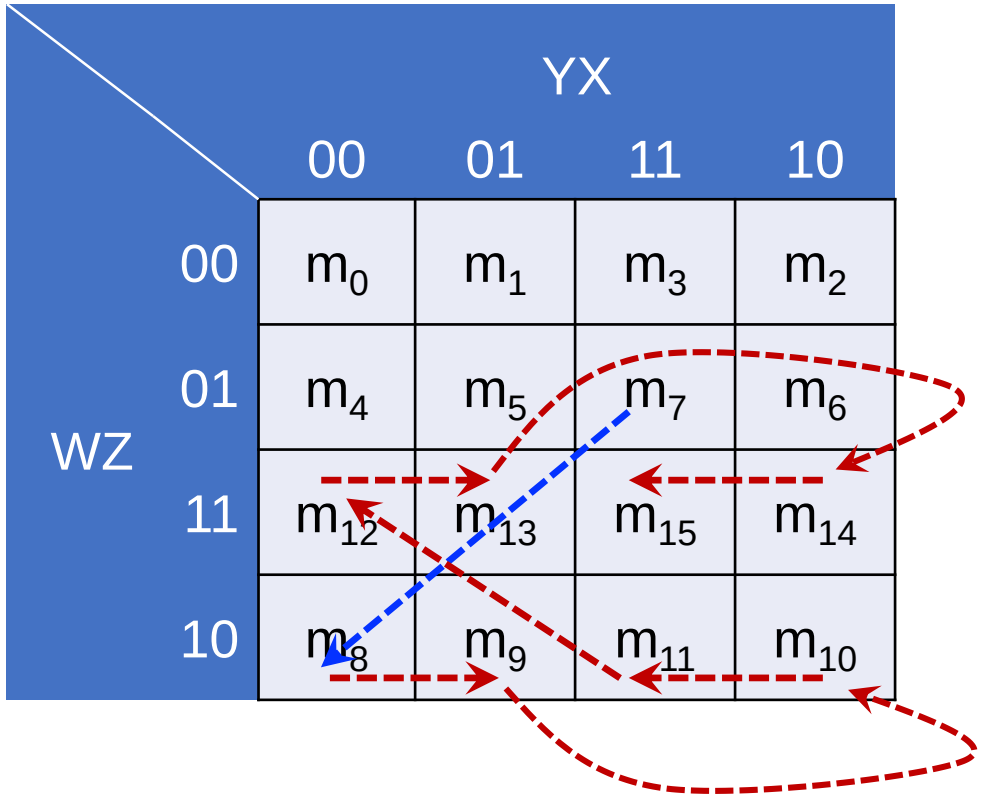
4-Variable KARNAUGH MAP

WZ \ YX	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}


YX
00
01
11
10
WZ
00
01
11
10
m_0
m_1
m_3
m_2
m_4
m_5
m_7
m_6
m_{12}
m_{13}
m_{15}
m_{14}
m_8
m_9
m_{11}
m_{10}

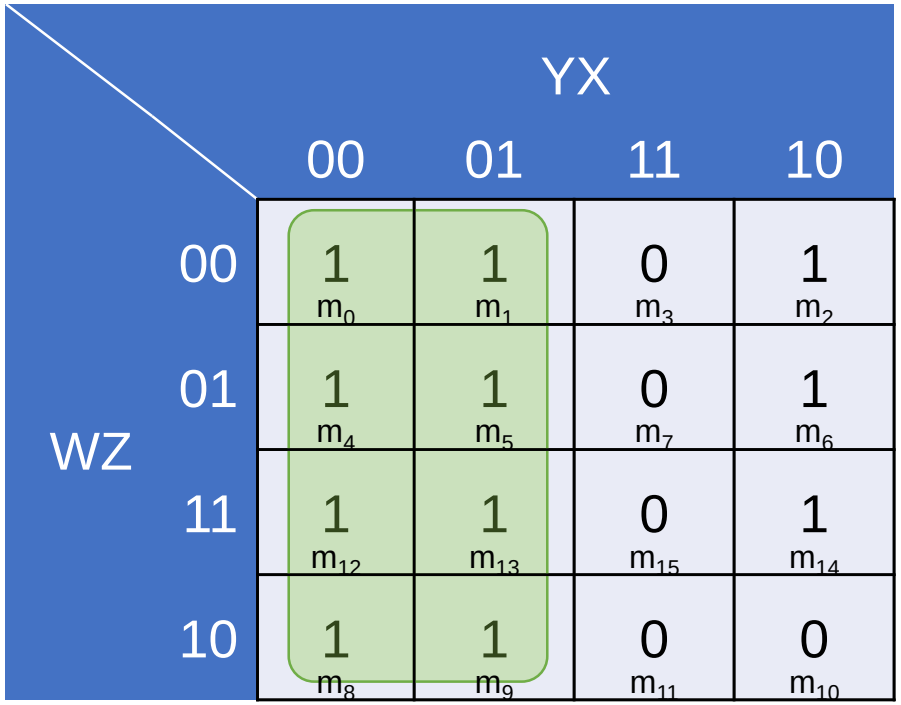

YX
00
01
11
10
WZ
00
01
11
10
m0
m1
m3
m2
m4
m5
m7
m6
m12
m13
m15
m14
m8
m9
m11
m10

WZ \ YX	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

$$F(W,Z,Y,X) = \sum m(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$

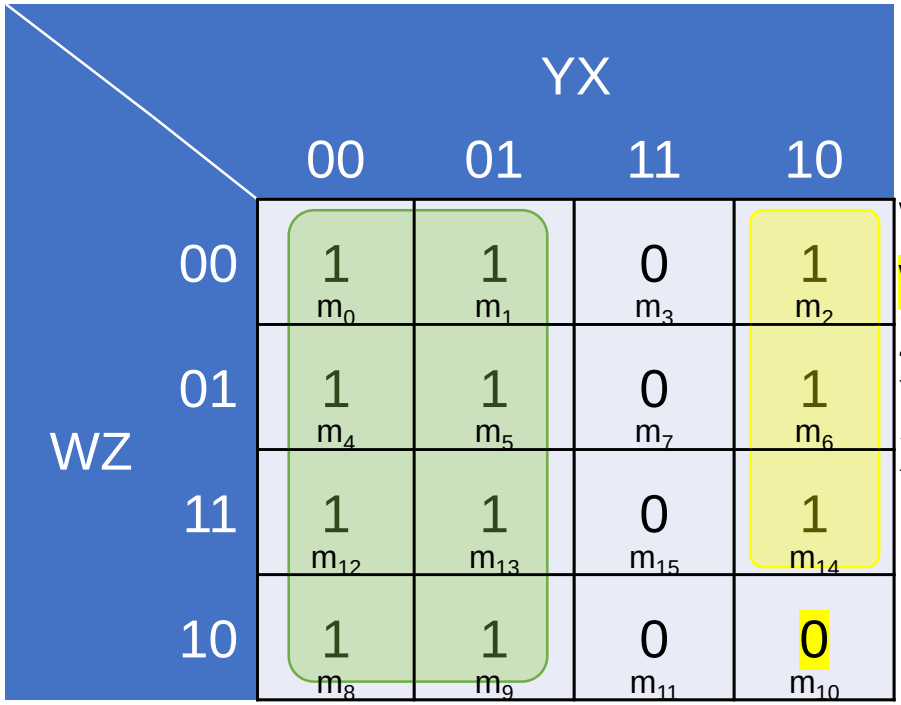
WZ \ YX	00	01	11	10
00	1 m_0	1 m_1	0 m_3	1 m_2
01	1 m_4	1 m_5	0 m_7	1 m_6
11	1 m_{12}	1 m_{13}	0 m_{15}	1 m_{14}
10	1 m_8	1 m_9	0 m_{11}	0 m_{10}

$$F(W,Z,Y,X) = \sum m(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$



$$F(W,Z,Y,X) = \sum m(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$

$$= Y' +$$



Warning!

W={0,1}, the value of W matters in m_{10}

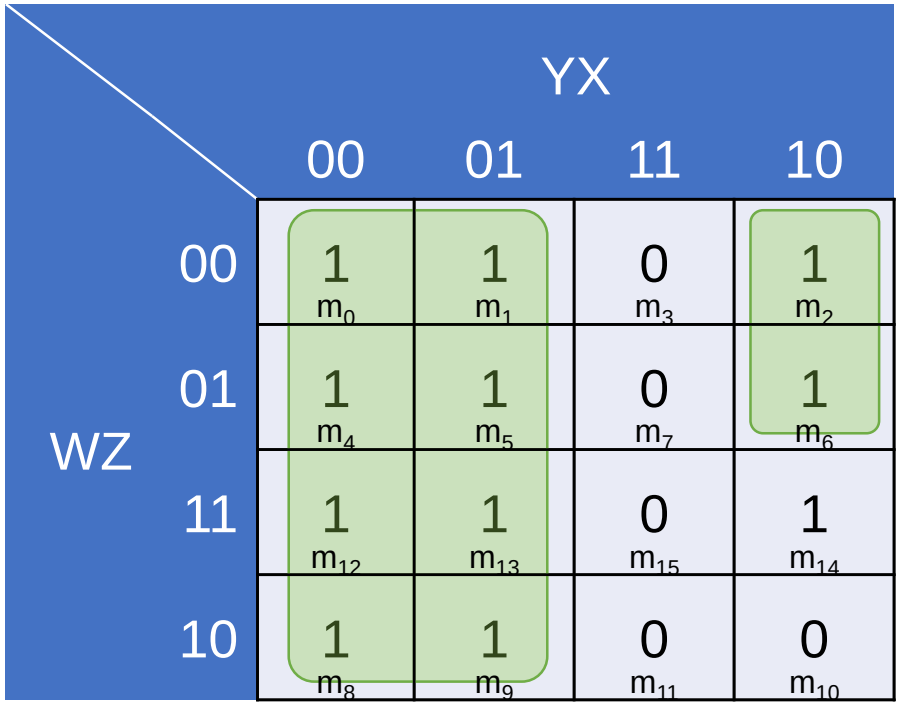
Z={0,1}

Y={1}

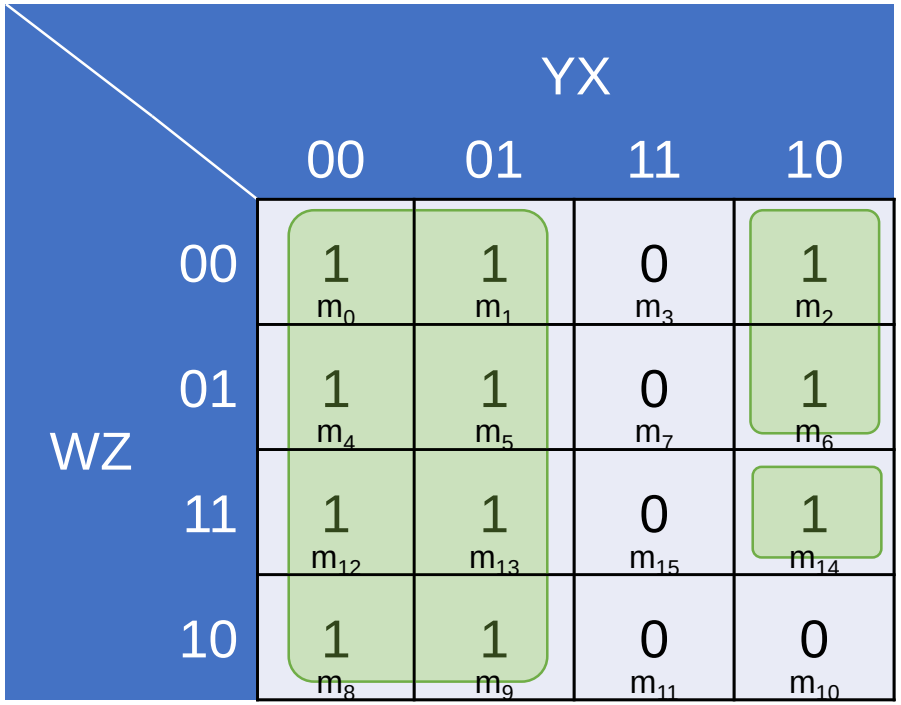
X={0}

$F(W,Z,Y,X) = \sum m(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$

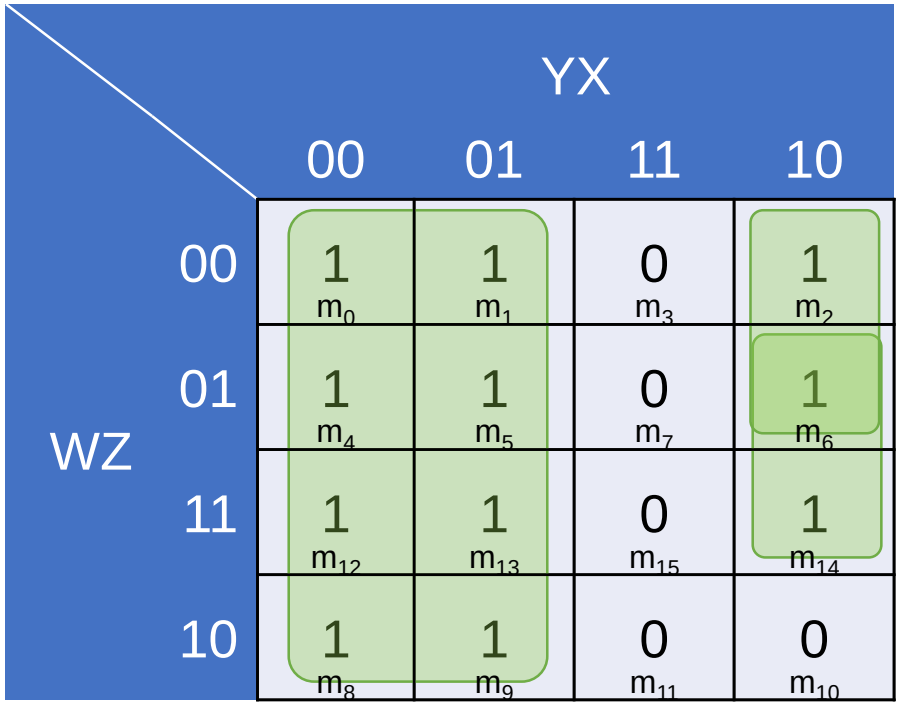
$= Y' +$



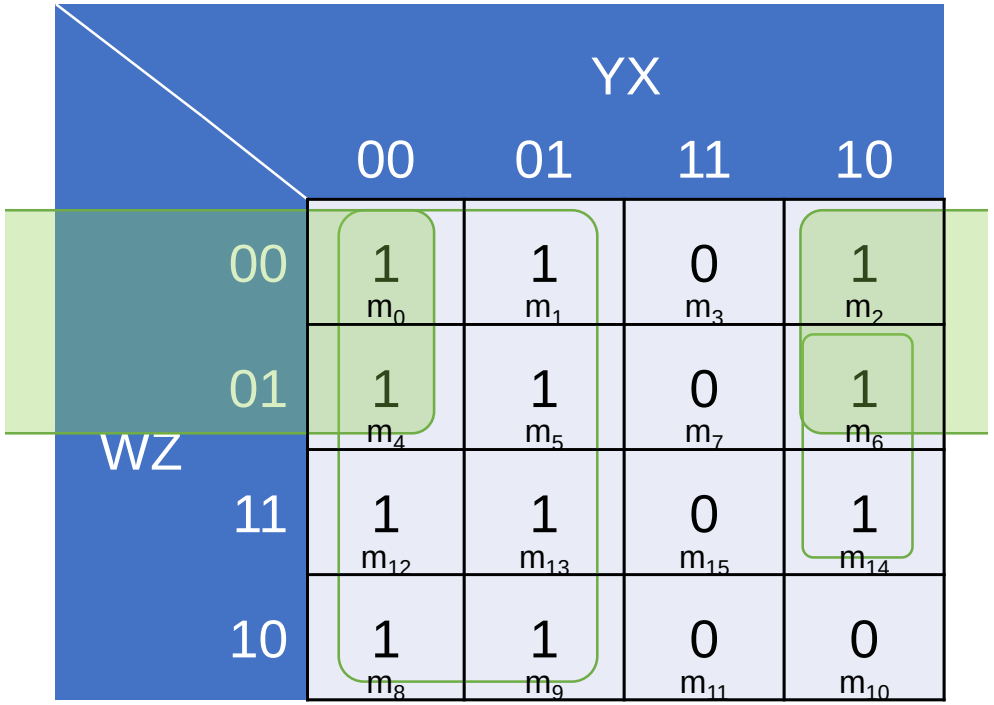
$$F(W,Z,Y,X) = \sum m(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$
$$= Y' + W'YX'$$



$$F(W,Z,Y,X) = \sum m(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$
$$= Y' + W'YX' + WZYX'$$



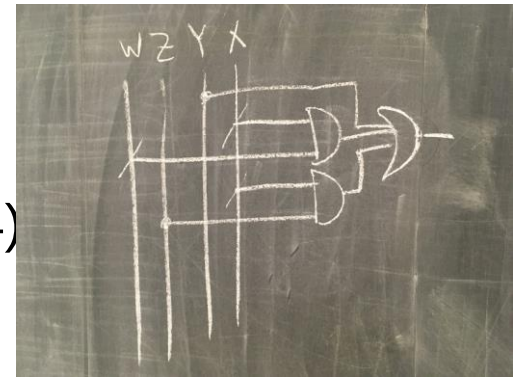
$$F(W,Z,Y,X) = \sum m(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$
$$= Y' + W'YX' + ZYX'$$



$$F(W,Z,Y,X) = \sum m(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$
$$= Y' + W'X' + WYX'$$

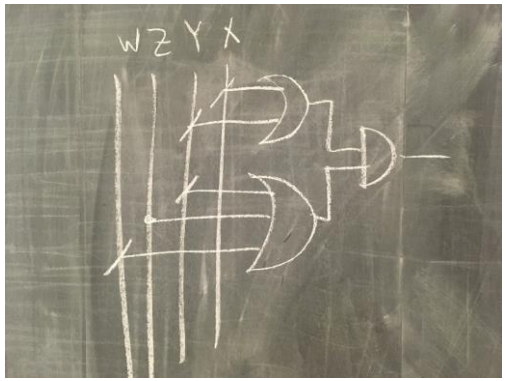
WZ \ YX	00	01	11	10
00	1 m_0	1 m_1	0 m_3	1 m_2
01	1 m_4	1 m_5	0 m_7	1 m_6
11	1 m_{12}	1 m_{13}	0 m_{15}	1 m_{14}
10	1 m_8	1 m_9	0 m_{11}	0 m_{10}

$$F(W,Z,Y,X) = \sum m(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$
$$= Y' + W'X' + ZX'$$



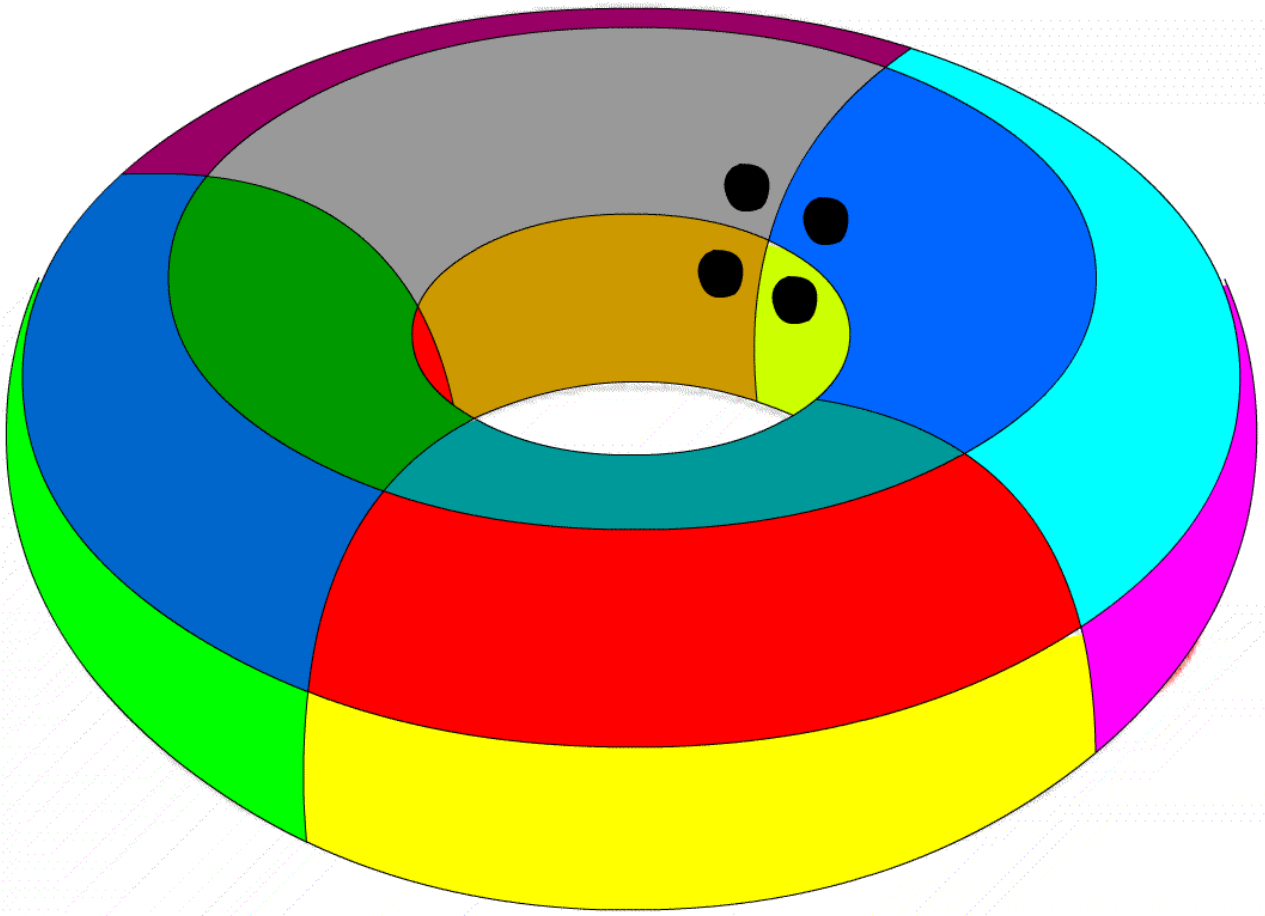
WZ \ YX	00	01	11	10
00	1 m_0	1 m_1	0 m_3	1 m_2
01	1 m_4	1 m_5	0 m_7	1 m_6
11	1 m_{12}	1 m_{13}	0 m_{15}	1 m_{14}
10	1 m_8	1 m_9	0 m_{11}	0 m_{10}

$$F(W,Z,Y,X) = \prod M(3, 7, 10, 11, 15)$$
$$= (YX)'(WZ'Y)'$$
$$= (Y'+X')(W'+Z+Y')$$



Click to Play!

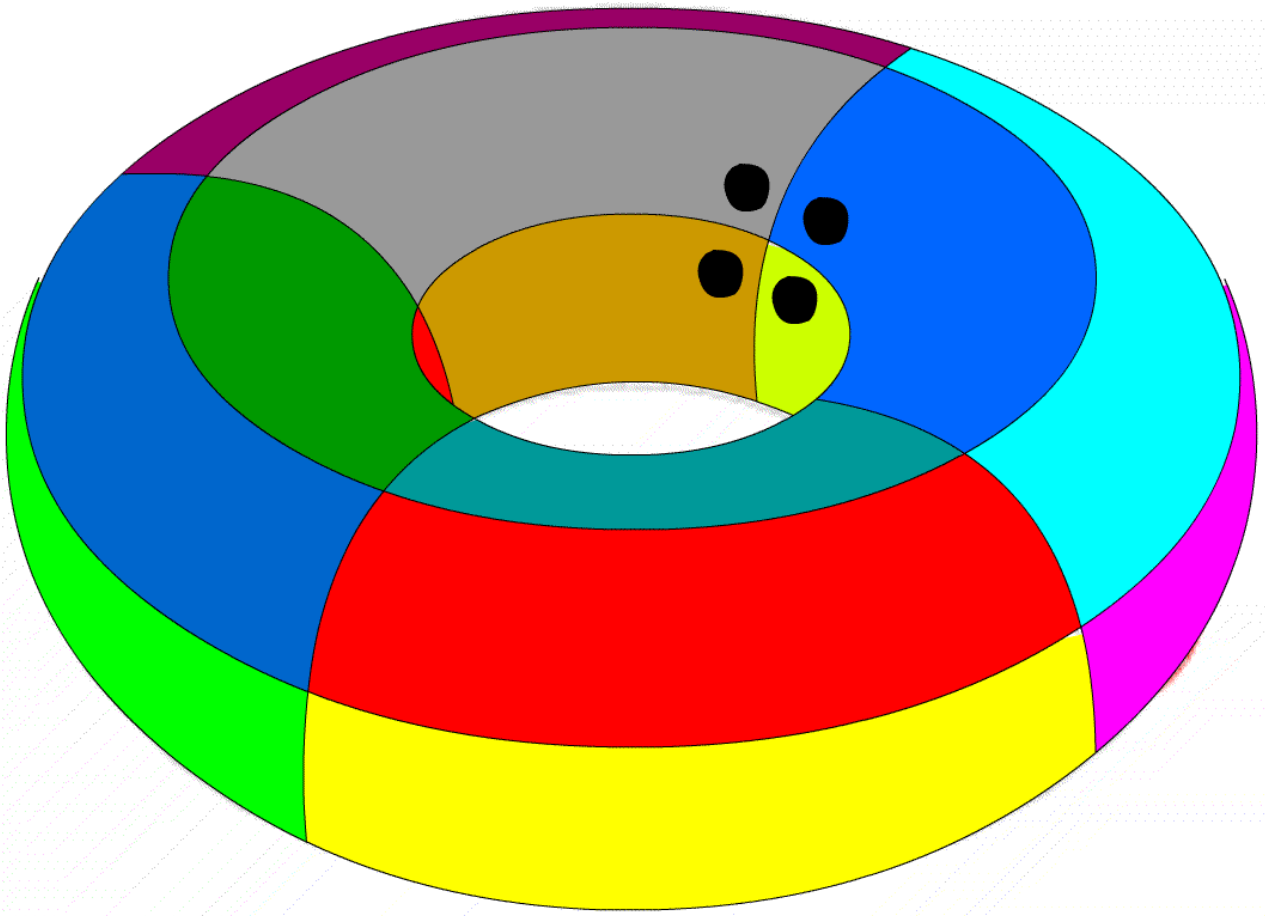
https://en.wikipedia.org/wiki/Karnaugh_map#/media/File:Torus_from_rectangle.gif



torus

WZ \ YX	00	01	11	10
00	1 m_0	0 m_1	0 m_3	1 m_2
01	0 m_4	0 m_5	0 m_7	0 m_6
11	0 m_{12}	0 m_{13}	0 m_{15}	0 m_{14}
10	1 m_8	0 m_9	0 m_{11}	1 m_{10}

$$F(W,Z,Y,X) = \sum m(0, 2, 8, 10)$$
$$= Z'X'$$



torus

WZ \ YX	00	01	11	10
00	1 m_0	0 m_1	0 m_3	1 m_2
01	0 m_4	0 m_5	0 m_7	0 m_6
11	0 m_{12}	0 m_{13}	0 m_{15}	0 m_{14}
10	1 m_8	0 m_9	0 m_{11}	1 m_{10}

$F(W,Z,Y,X) = \sum m(0, 2, 8, 10)$

$= Z'X'$

$= \prod M(1,3\text{-}7,9,11\text{-}15)$

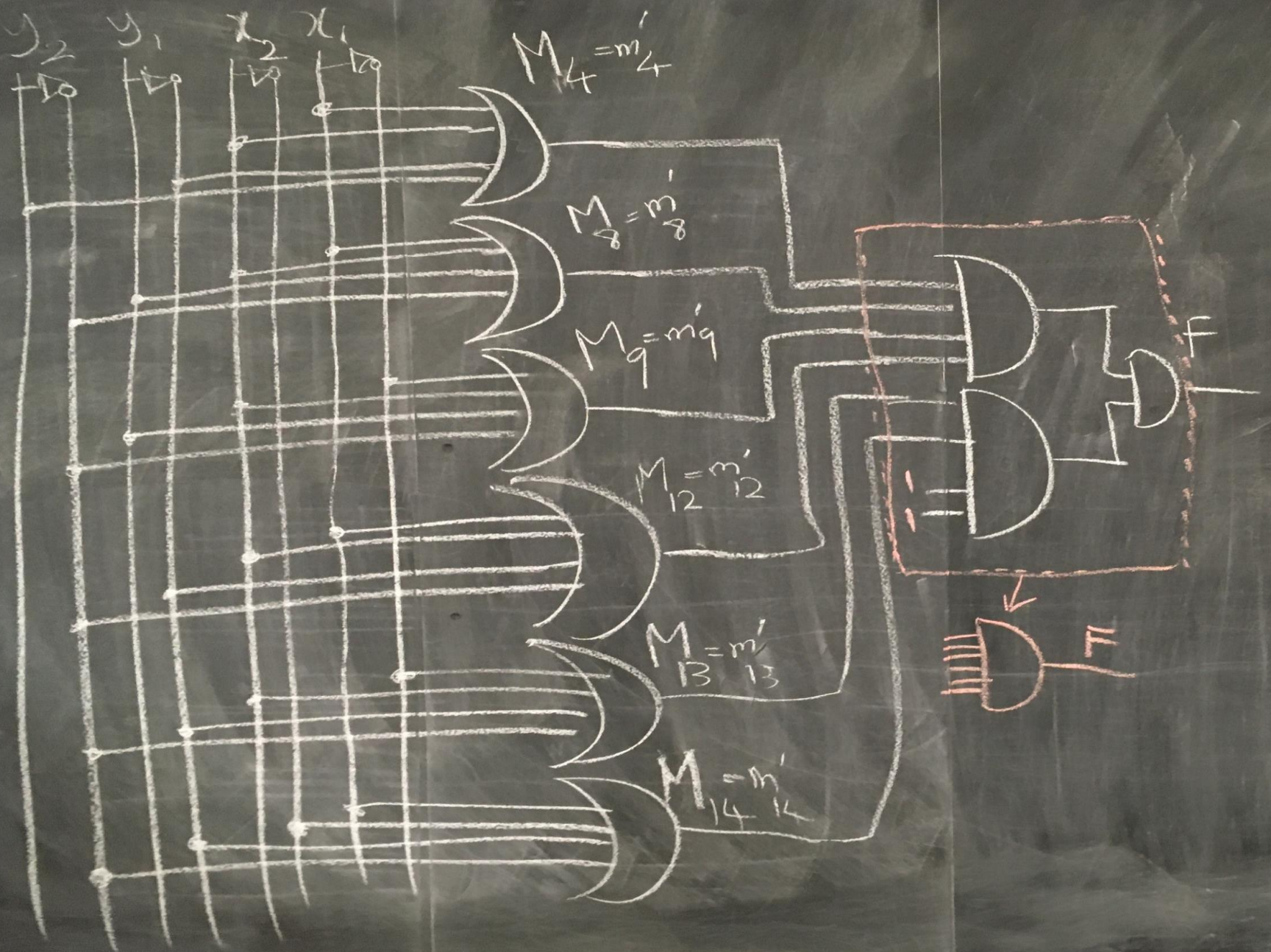
$= (X)'(Z)'$

$= X'Z'$

Given two unsigned numbers x and y, design a logic circuit to see

$$x \geq^{?} y$$

Y2	Y1	X2	X1	F(Y2,Y1,X2,X1)=Σ m(0,1,2,3,5,6,7,10,11,15)	F(Y2,Y1,X2,X1)=Π M(4,8,9,12,13,14)
0	0	0	0	1	1
0	0	0	1	1	1
0	0	1	0	1	1
0	0	1	1	1	1
0	1	0	0	0	0
0	1	0	1	1	1
0	1	1	0	1	1
0	1	1	1	1	1
1	0	0	0	0	0
1	0	0	1	0	0
1	0	1	0	1	1
1	0	1	1	1	1
1	1	0	0	0	0
1	1	0	1	0	0
1	1	1	0	0	0
1	1	1	1	1	1



$$F = \prod M(4,8,9,12,13,14)$$

$$= M_4 M_8 M_9 M_{12} M_{13} M_{14}$$

$$= m'_4 m'_8 m'_9 m'_{12} m'_{13} m'_{14}$$

$(y'_2 y'_1 x'_2 x'_1)'$ $m'_4 \rightarrow y_2 + y'_1 + x_2 + x_1$

$(y_2 y'_1 x'_2 x'_1)'$ $m'_8 \rightarrow y'_2 + y_1 + x_2 + x_1$

$(y_2 y'_1 x_2 x'_1)'$ $m'_9 \rightarrow y'_2 + y_1 + x_2 + x'_1$

$(y_2 y'_1 x_2 x_1)'$ $m'_{12} \rightarrow y'_2 + y'_1 + x_2 + x'_1$

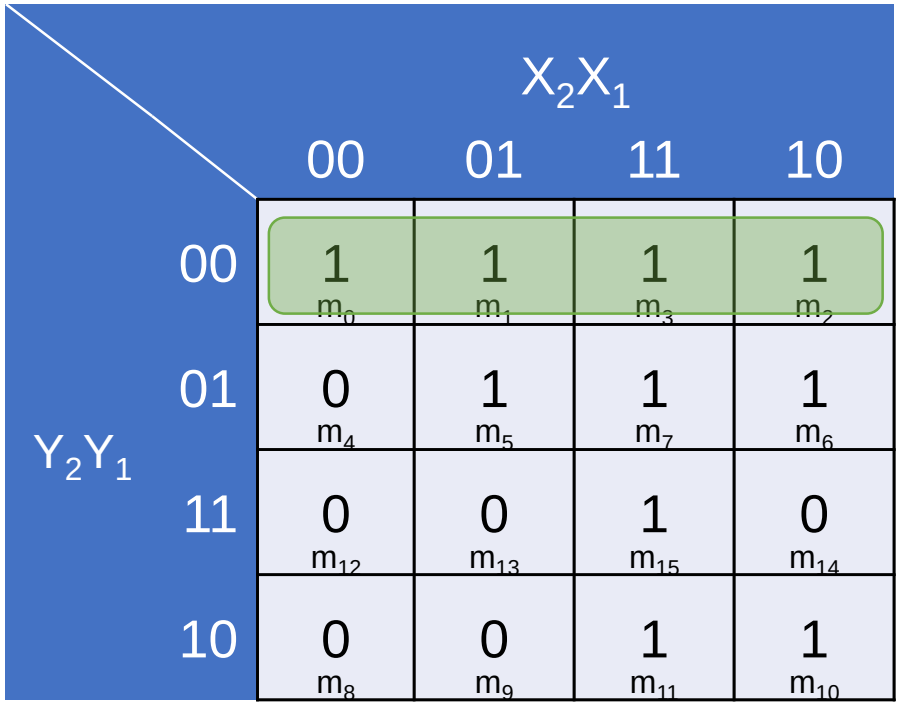
$(y_2 y_1 x'_2 x_1)'$ $m'_{13} \rightarrow y'_2 + y'_1 + x_2 + x'_1$

$(y_2 y_1 x_2 x'_1)'$ $m'_{14} \rightarrow y'_2 + y'_1 + x'_2 + x_1$

Y_2Y_1 \ X_2X_1	00	01	11	10
00	1 m_0	1 m_1	1 m_3	1 m_2
01	0 m_4	1 m_5	1 m_7	1 m_6
11	0 m_{12}	0 m_{13}	1 m_{15}	0 m_{14}
10	0 m_8	0 m_9	1 m_{11}	1 m_{10}

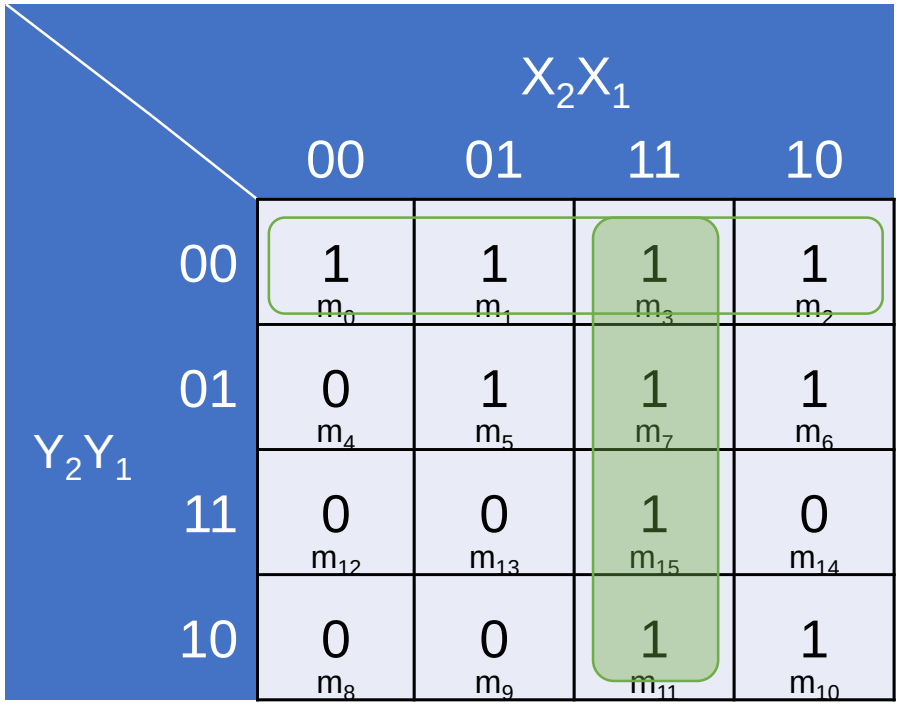
$F(Y_2,Y_1,X_2,X_1)=\mathbf{\Sigma}\ m(0,1,2,3,5,6,7,10,11,15)$

$F(Y_2,Y_1,X_2,X_1)=\mathbf{\Pi}\ M(4,8,9,12,13,14)$

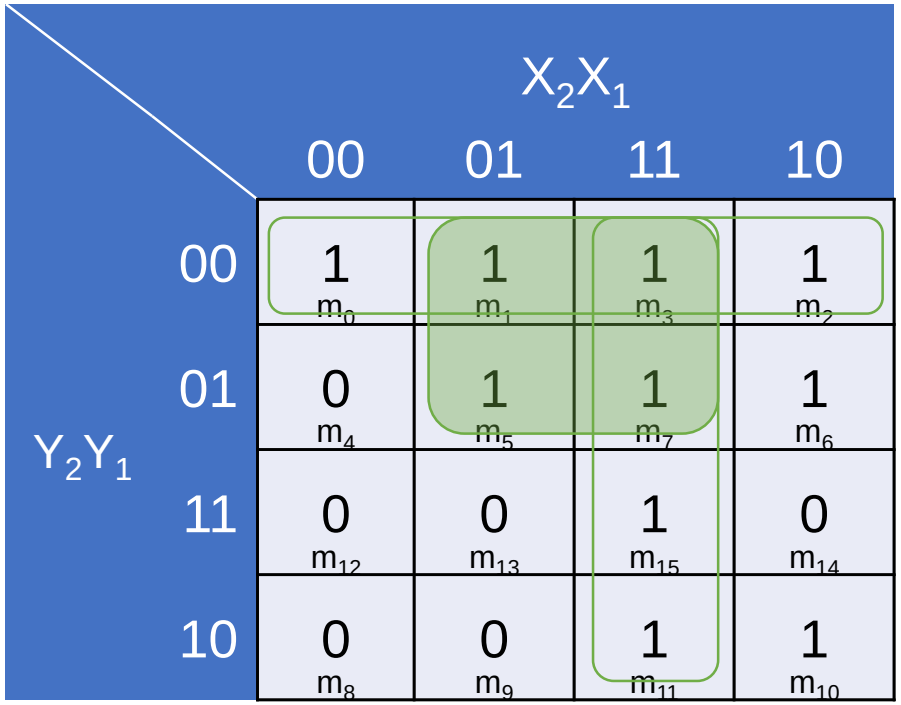


$F(Y_2,Y_1,X_2,X_1)=\Sigma\, m(0,1,2,3,5,6,7,10,11,15)$

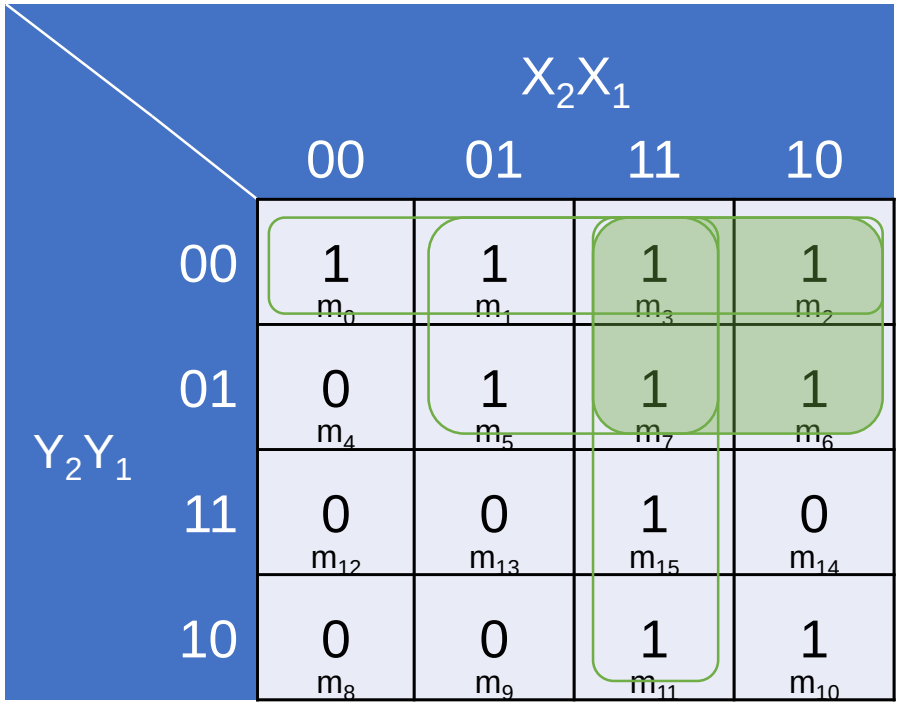
$=Y'_2Y'_1 +$



$$F(Y_2,Y_1,X_2,X_1)=\Sigma\, m(0,1,2,3,5,6,7,10,11,15)$$
$$= Y'_2Y'_1 + X_2X_1$$

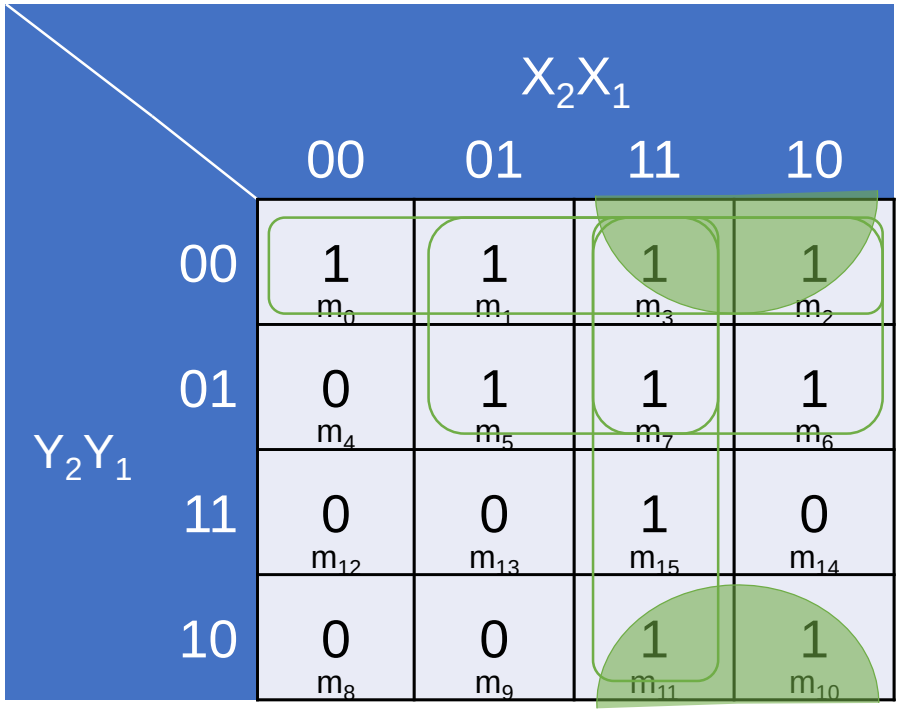


$$F(Y_2,Y_1,X_2,X_1)=\Sigma\, m(0,1,2,3,5,6,7,10,11,15)$$
$$= Y'_2Y'_1 + X_2X_1 + Y'_2X_1$$



$F(Y_2,Y_1,X_2,X_1)=\Sigma\, m(0,1,2,3,5,6,7,10,11,15)$

$= Y'_2Y'_1 + X_2X_1 + Y'_2X_1 + Y'_2X_2$



$F(Y_2,Y_1,X_2,X_1)=\Sigma\, m(0,1,2,3,5,6,7,10,11,15)$

$= Y'_2Y'_1 + X_2X_1 + Y'_2X_1 + Y'_2X_2 + Y'_1X_2$

Change of Variable:

X1 → X

X2 → Y

Y1 → Z

Y2 → W

WZ \ YX	00	01	11	10
00	1 m_0	1 m_1	1 m_3	1 m_2
01	0 m_4	1 m_5	1 m_7	1 m_6
11	0 m_{12}	0 m_{13}	1 m_{15}	0 m_{14}
10	0 m_8	0 m_9	1 m_{11}	1 m_{10}

$F(Y_2,Y_1,X_2,X_1)=\Pi\ M(4,8,9,12,13,14)$

Change of Variable:

X1 → X

X2 → Y

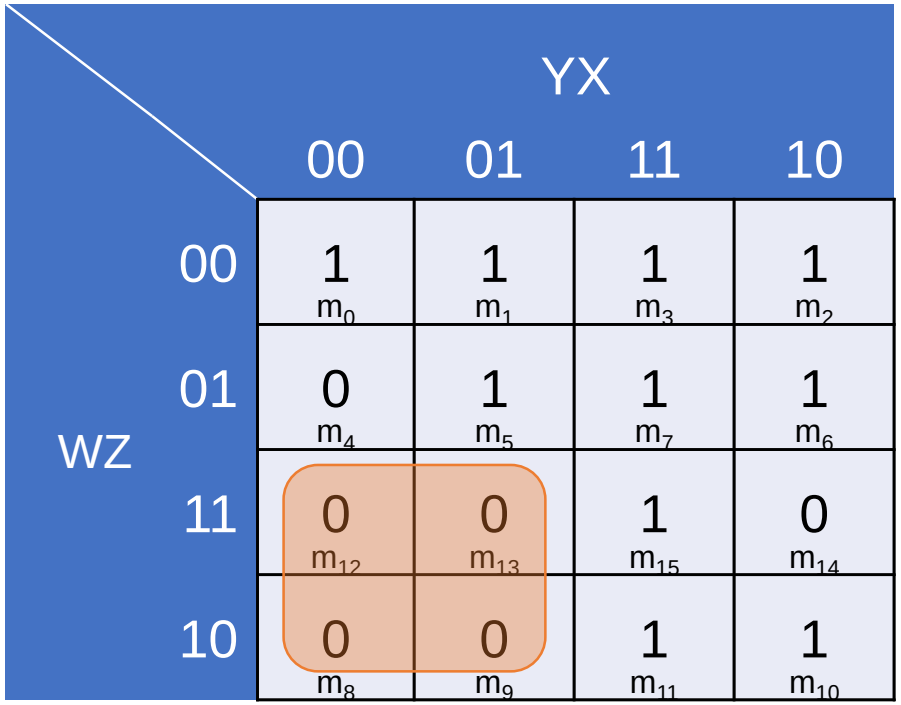
Y1 → Z

Y2 → W

WZ \ YX	00	01	11	10
00	1 m_0	1 m_1	1 m_3	1 m_2
01	0 m_4	1 m_5	1 m_7	1 m_6
11	0 m_{12}	0 m_{13}	1 m_{15}	0 m_{14}
10	0 m_8	0 m_9	1 m_{11}	1 m_{10}

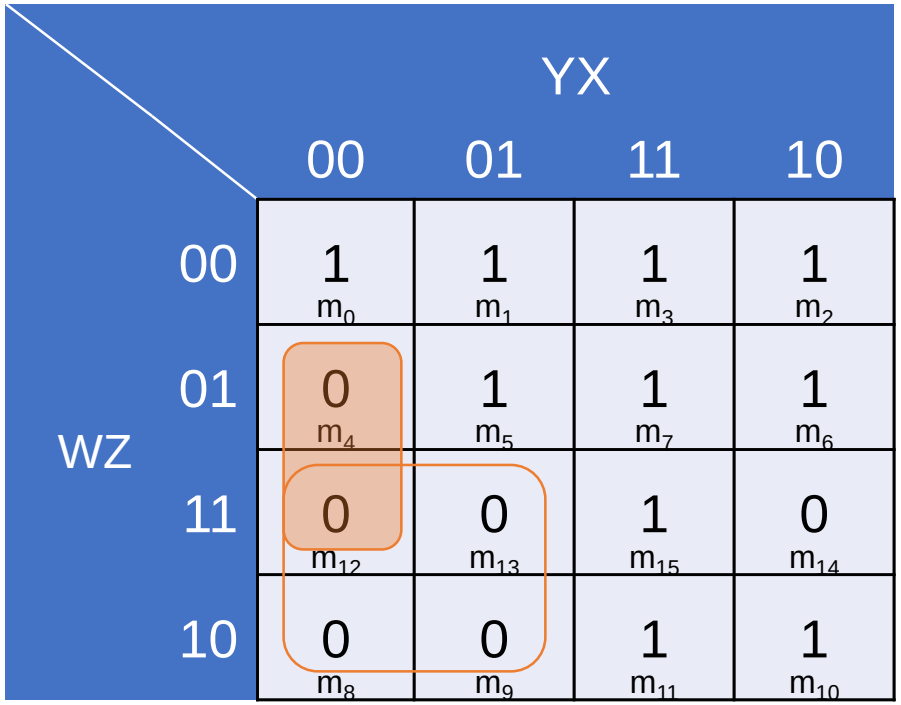
$F(Y_2,Y_1,X_2,X_1)=\Pi\ M(4,8,9,12,13,14)$

$F(W,Z,Y,X)=()'$



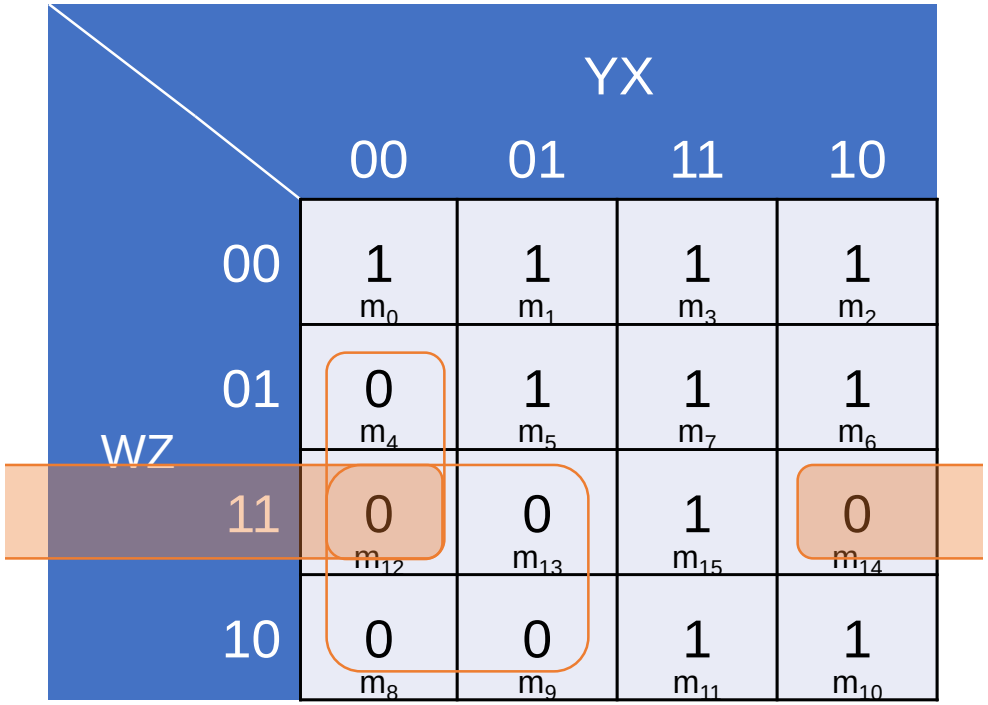
$F(Y_2,Y_1,X_2,X_1)=\Pi\ M(4,8,9,12,13,14)$

$F(W,Z,Y,X) = (WY' +)'$



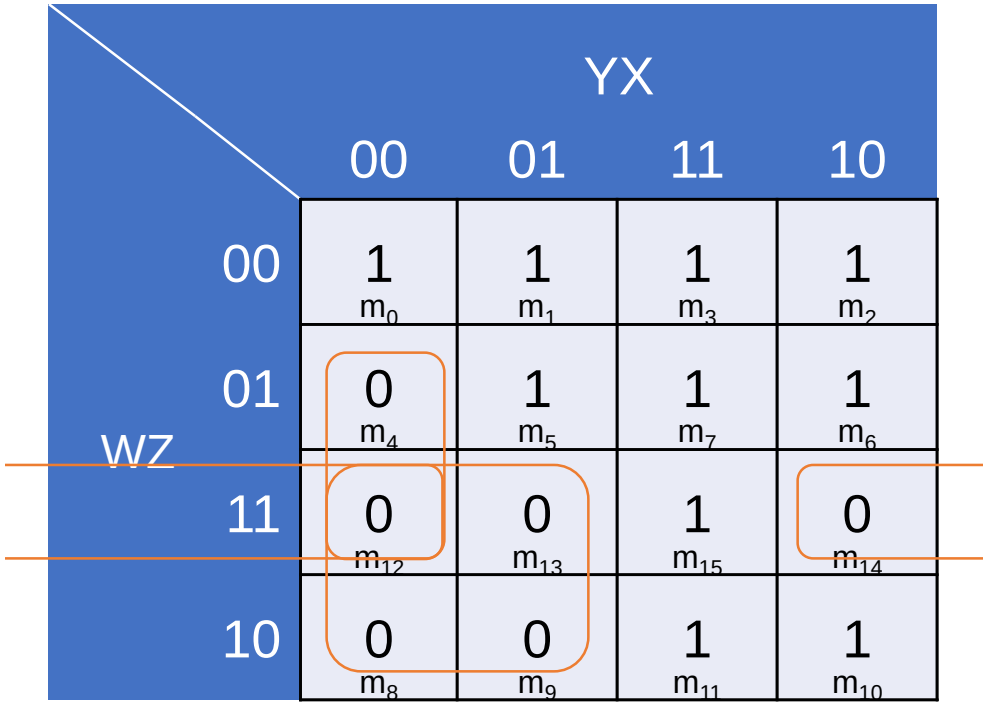
$F(Y_2,Y_1,X_2,X_1)=\Pi\ M(4,8,9,12,13,14)$

F(W,Z,Y,X) =(WY’ + ZY’X’ +)’



$F(Y_2,Y_1,X_2,X_1)=\Pi\ M(4,8,9,12,13,14)$

$F(W,Z,Y,X) = (WY' + ZY'X' + WZX')'$



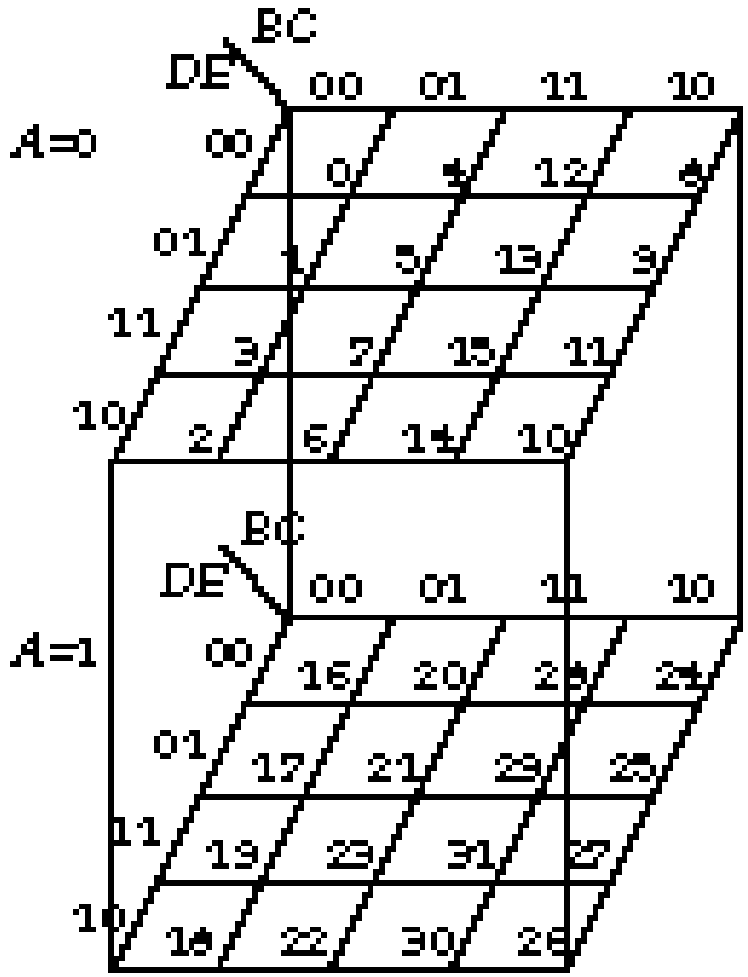
$F(Y_2,Y_1,X_2,X_1)=\Pi\, M(4,8,9,12,13,14)$

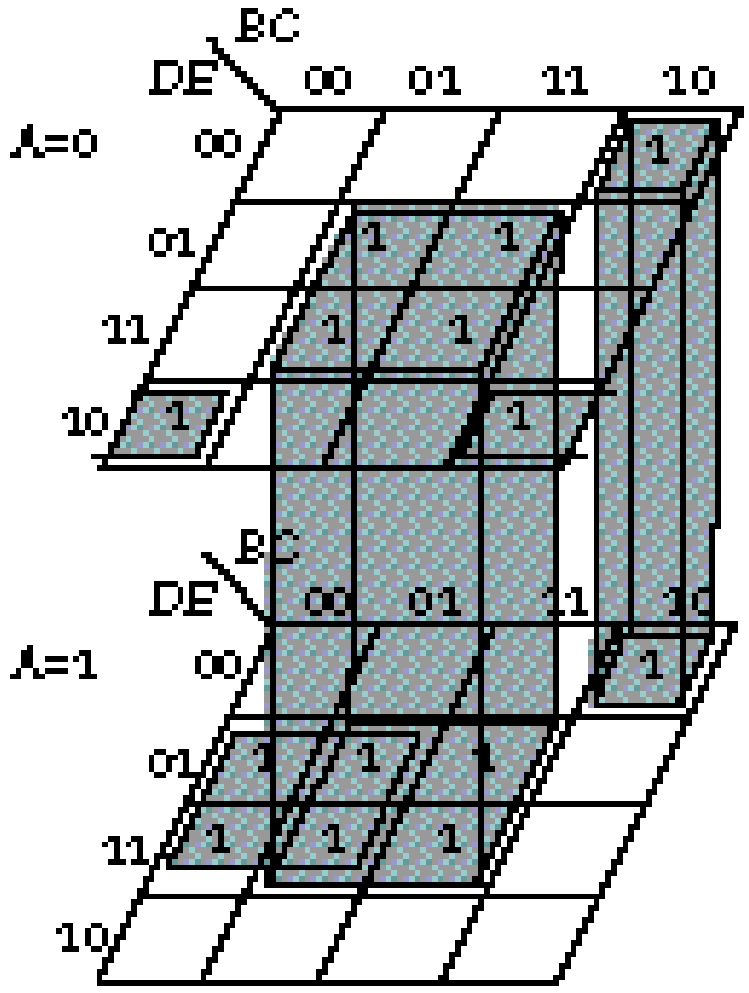
$F(W,Z,Y,X) = (WY' + ZY'X' + WZX')'$

$= (WY')'\,(ZY'X')'\,(WZX')'$

$= (W'+Y)\,(Z'+Y+X)'\,(W'+Z'+X)$

5-Variable KARNAUGH MAP


A=0
BC
DE
00
01
11
10
0
4
12
8
1
5
13
9
3
7
15
11
2
6
14
10
A=1
BC
DE
00
01
11
10
16
20
28
24
17
21
29
25
19
23
31
27
18
22
30
26


BC
DE
00
01
11
10
A=0
A=1

n-Variable ~~KARNAUGH MAP~~

n-Variable Quine–McCluskey Algorithm

https://en.wikipedia.org/wiki/Quine%E2%80%93McCluskey_algorithm

1878 ← 1937 ← 1952 ← 1956

Demo
Quine–McCluskey Algorithm

https://www.mathematik.uni-marburg.de/~thormae/lectures/ti1/code/qmc/

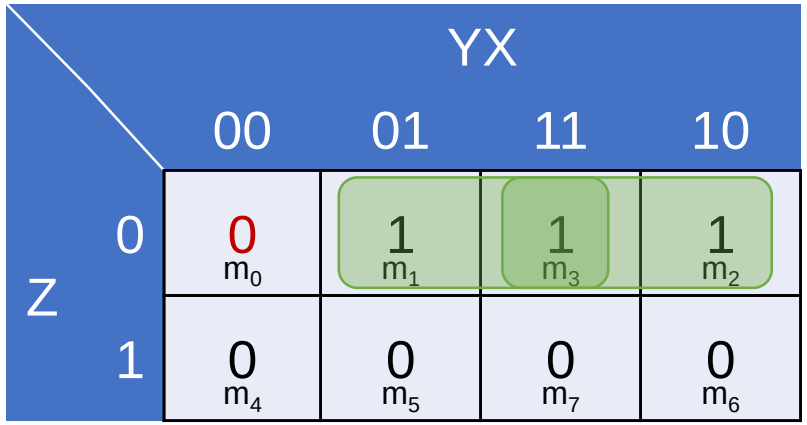
Don’t Care Conditions

In practice, in some applications the function is not specified for certain combinations of the variables.

Z	Y	X	F=if input is positive(2’s comp.) then 1 else 0
0	0	0	?
0	0	1	?
0	1	0	?
0	1	1	?
1	0	0	?
1	0	1	?
1	1	0	?
1	1	1	?

Z	Y	X	F=if input is positive(2's comp.) then 1 else 0
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

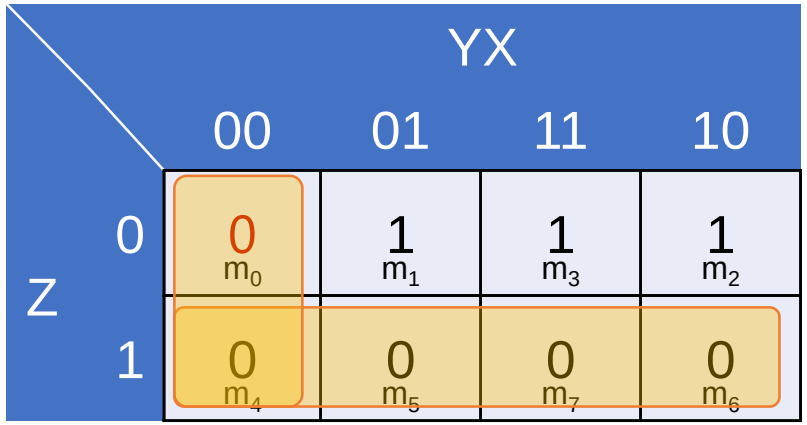
Z	Y	X	$F=\sum m(1,2,3)=\prod M(0,4,5,6,7)$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0



$F(Z,Y,X) = \sum m(1, 2, 3)$

$= Z'X + Z'Y$

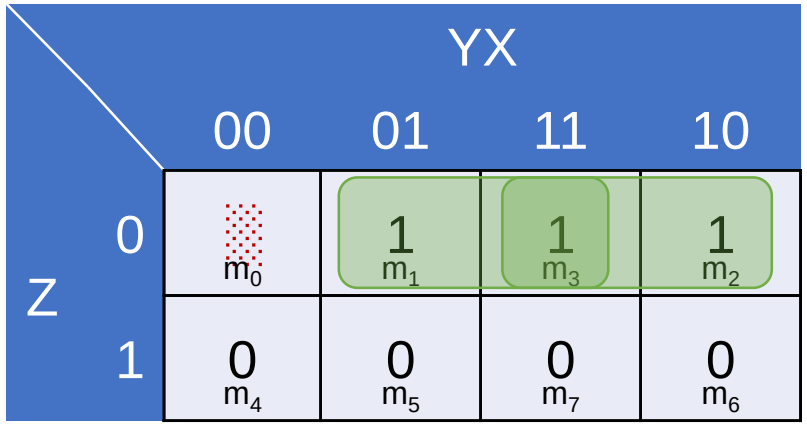
Boolean algebra → $Z'(X+Y)$



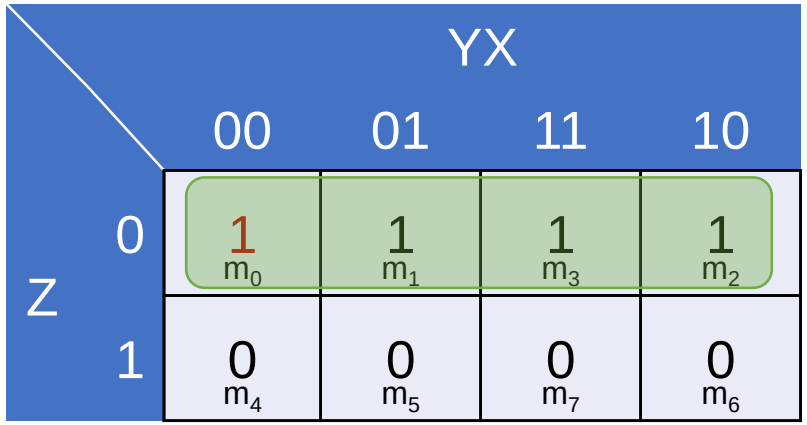
$$F(Z,Y,X) = \Pi\, M(0,4,5,6,7)$$
$$= (Z + Y'X')'$$
$$= Z' (Y+X)$$

Z	Y	X	F=if positive(2's comp.) then 1 if negative 0
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

Z	Y	X	F=if positive(2's comp.) then 1 if negative 0
0	0	0	
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

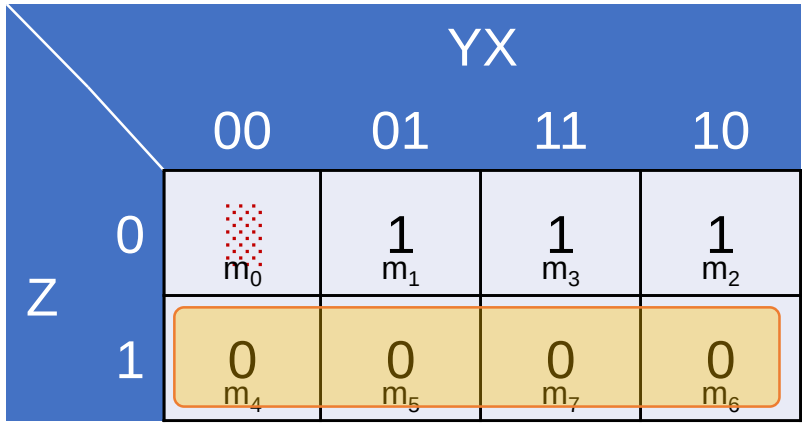


$$F(Z,Y,X) = \sum m(1, 2, 3) + \sum d(0)$$
$$= Z'X + Z'Y$$

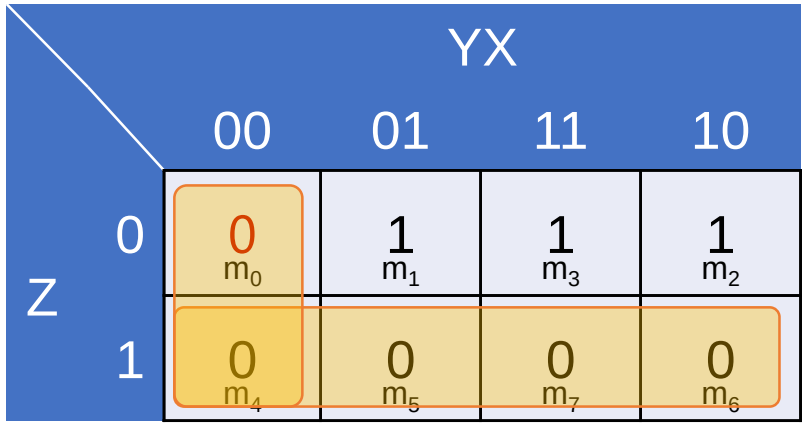


$$F(Z,Y,X) = \sum m(1, 2, 3) + \sum m(0)$$
$$= Z'$$

In this case, the don't care condition help to more simplification



$$F(Z,Y,X) = \Pi\, M(4,5,6,7) + \sum D(0)$$
$$= (Z)'$$
$$= Z'$$



$$F(Z,Y,X) = \Pi\, M(0,4,5,6,7) + \sum M(0)$$
$$= (Z + Y'X')'$$
$$= Z' (Y+X)$$

In this case, the don't care condition does NOT help to more simplification

Don’t Care Conditions

Functions that have unspecified outputs for some input combinations are called *incompletely specified functions*.

Don’t-care conditions can be used on a map to provide further simplification of the Boolean expression.

Don’t Care Conditions

To distinguish the don’t-care condition from 1’s and 0’s, an X is used.

Z \ YX	00	01	11	10
0	X m_0	1 m_1	1 m_3	1 m_2
1	0 m_4	0 m_5	0 m_7	0 m_6

$F(Z,Y,X) = \sum m(1, 2, 3) + \sum d(0)$

$F(Z,Y,X) = \prod M(4,5,6,7) + \sum D(0)$