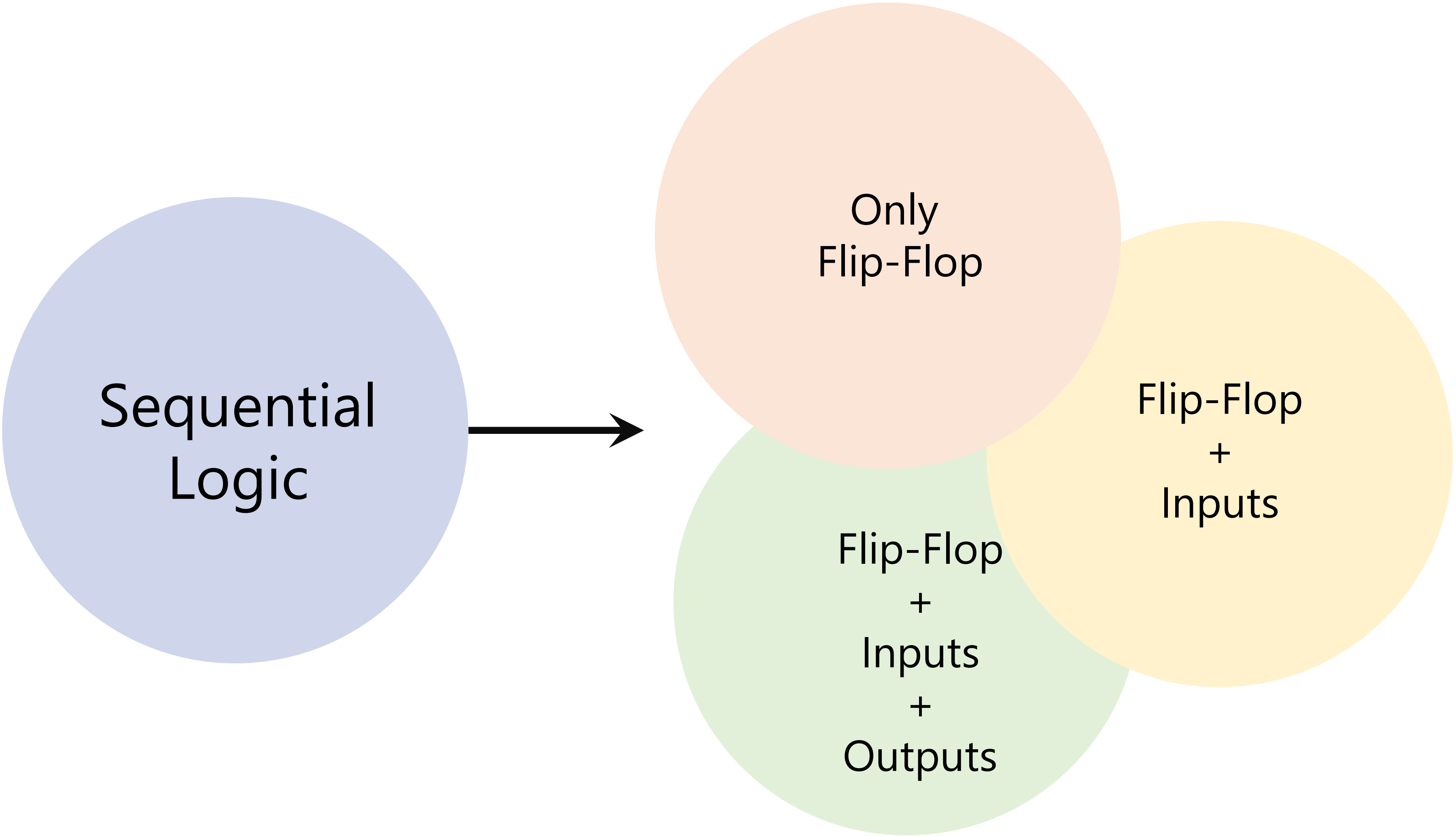
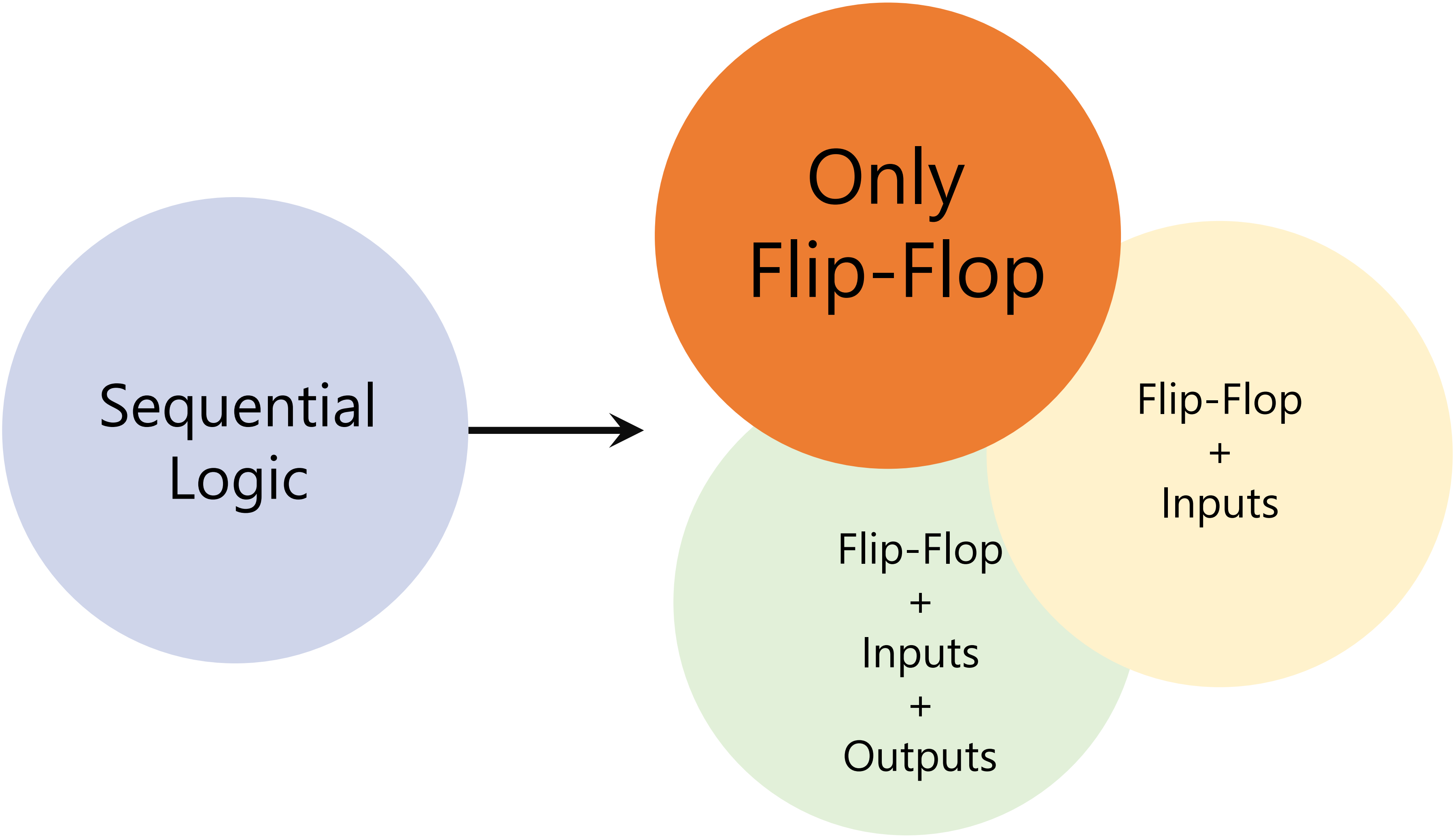
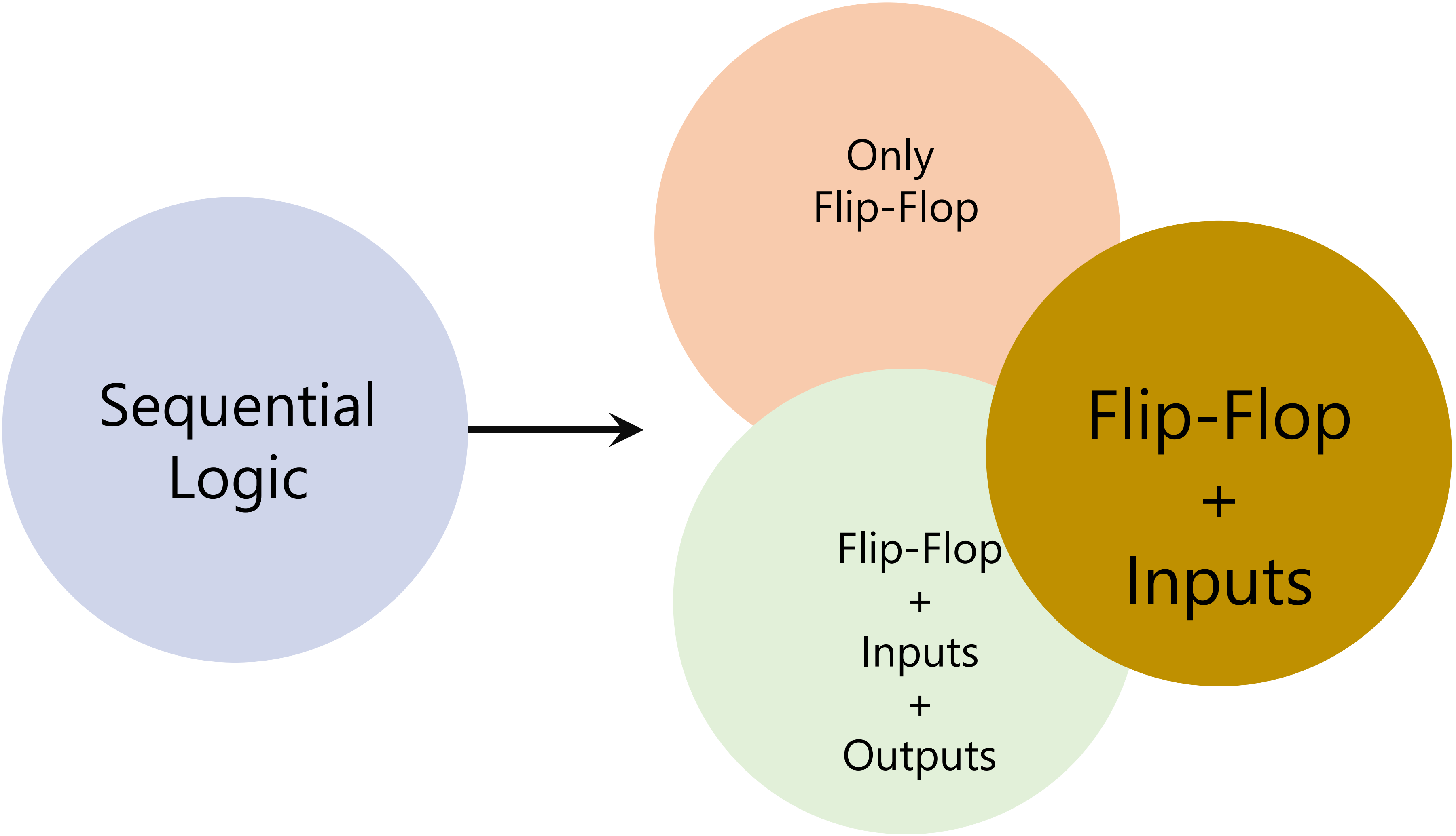
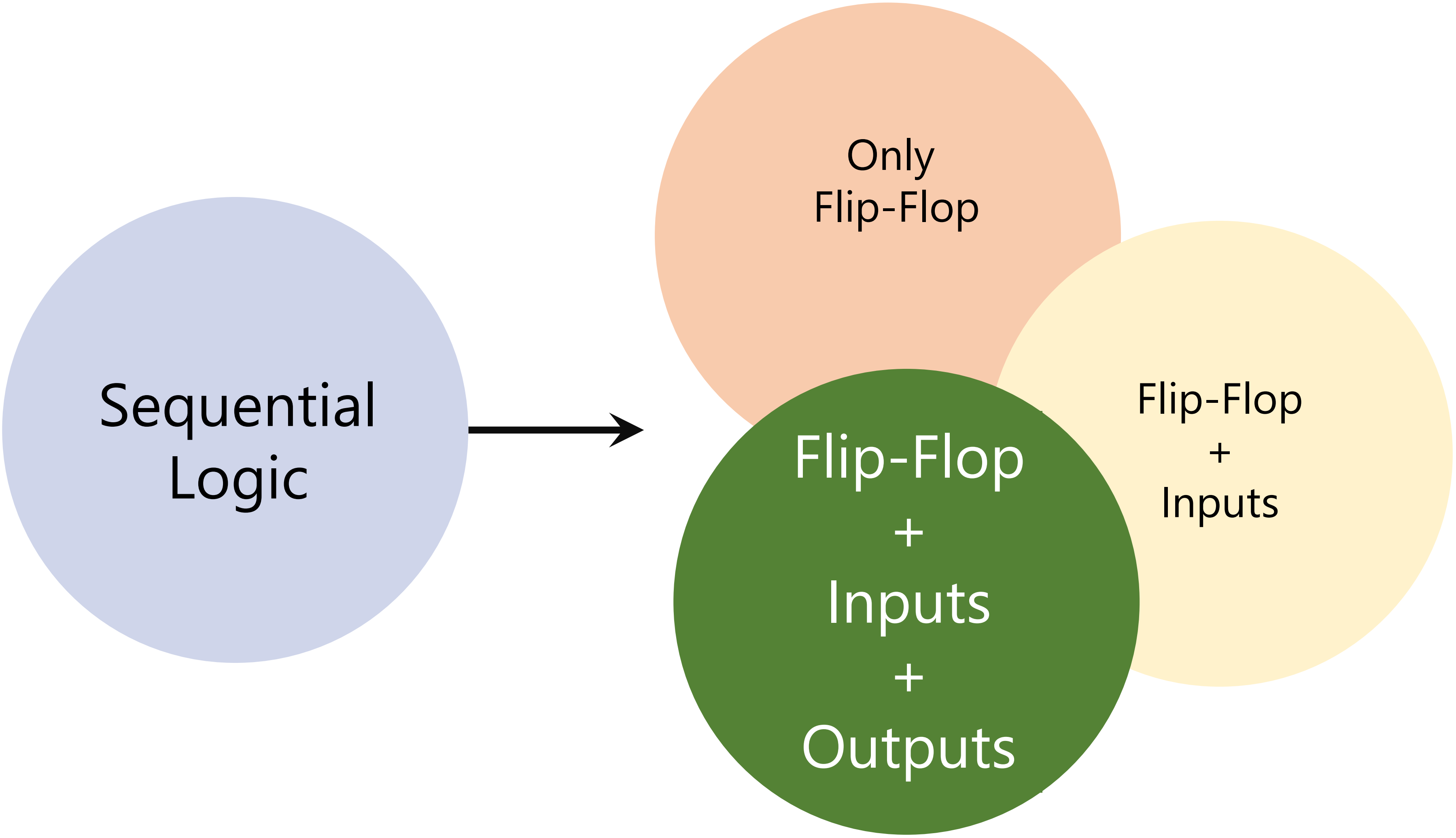
Sequential Logic


Sequential Logic
Only Flip-Flop
Flip-Flop + Inputs
Flip-Flop + Inputs + Outputs


Sequential
Logic
Only
Flip-Flop
Flip-Flop
+
Inputs
Flip-Flop
+
Inputs
+
Outputs


Sequential
Logic
Only
Flip-Flop
Flip-Flop
+
Inputs
+
Outputs
Flip-Flop
+
Inputs

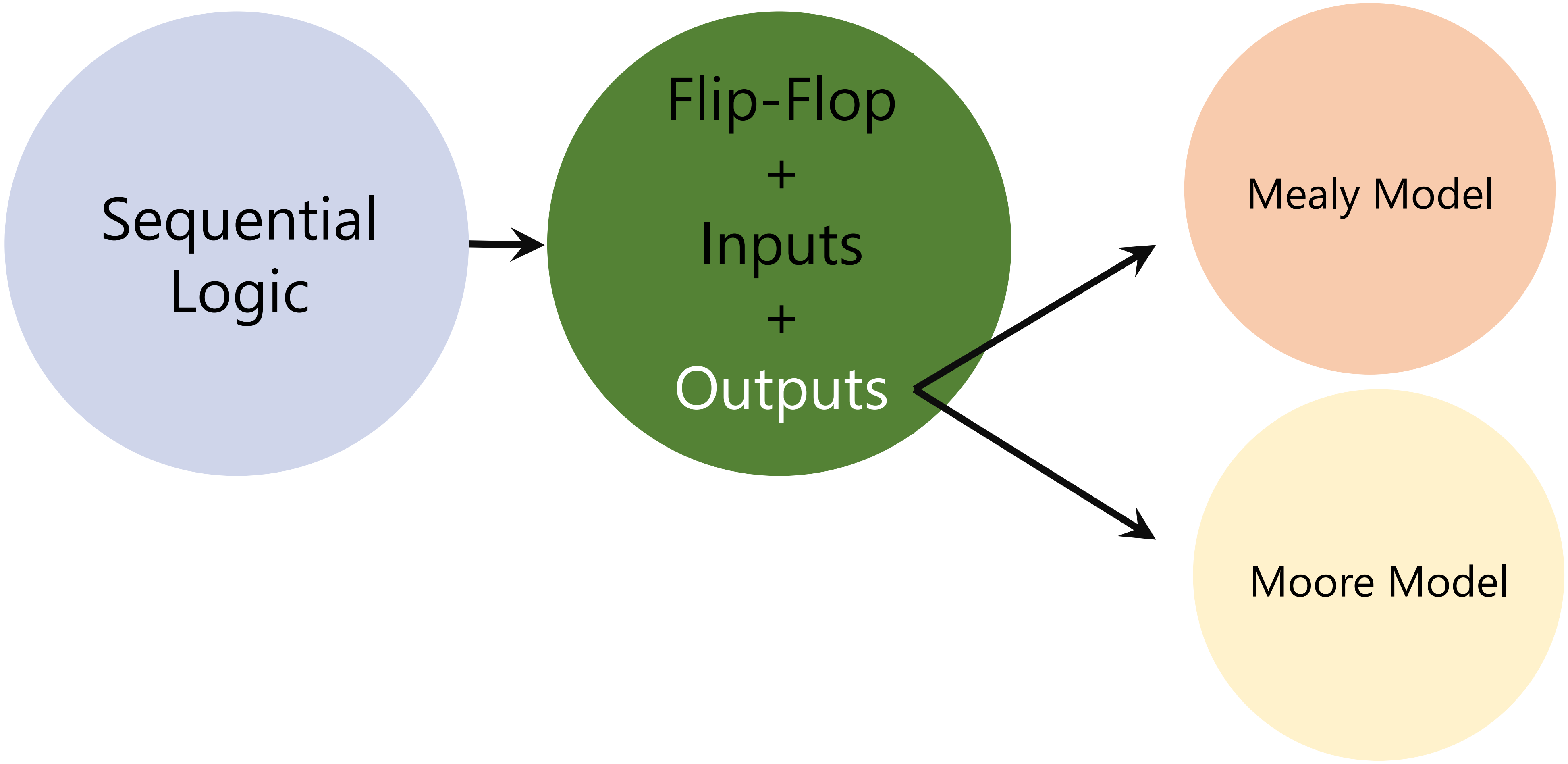

Sequential
Logic
Only
Flip-Flop
Flip-Flop
+
Inputs
Flip-Flop
+
Inputs
+
Outputs

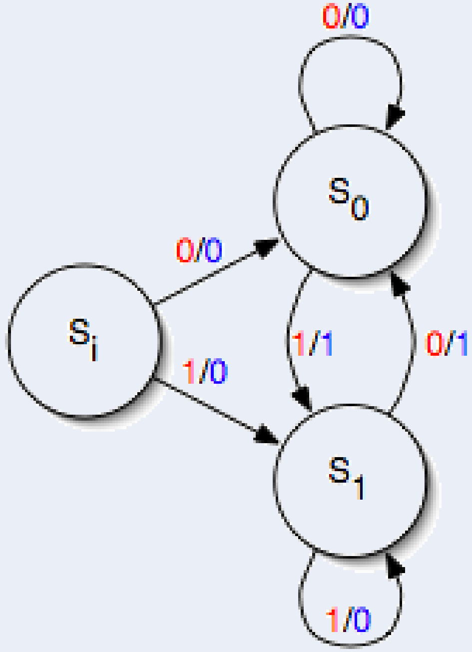
Analysis
vs.
Design

Analysis: Given a sequential circuit, show the behavior

vs.

Design: Given a behavior, build the sequential circuit


Sequential Logic
Flip-Flop
+
Inputs
+
Outputs
Mealy Model
Moore Model

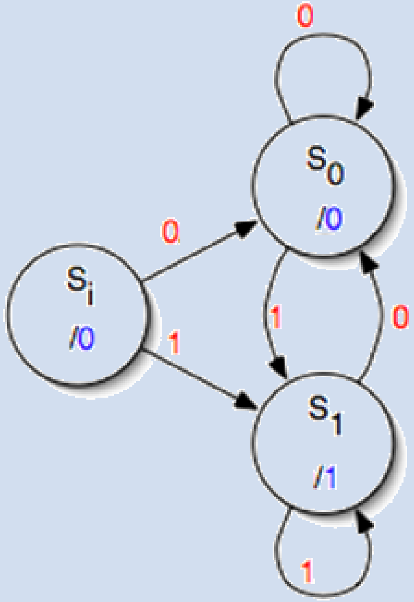


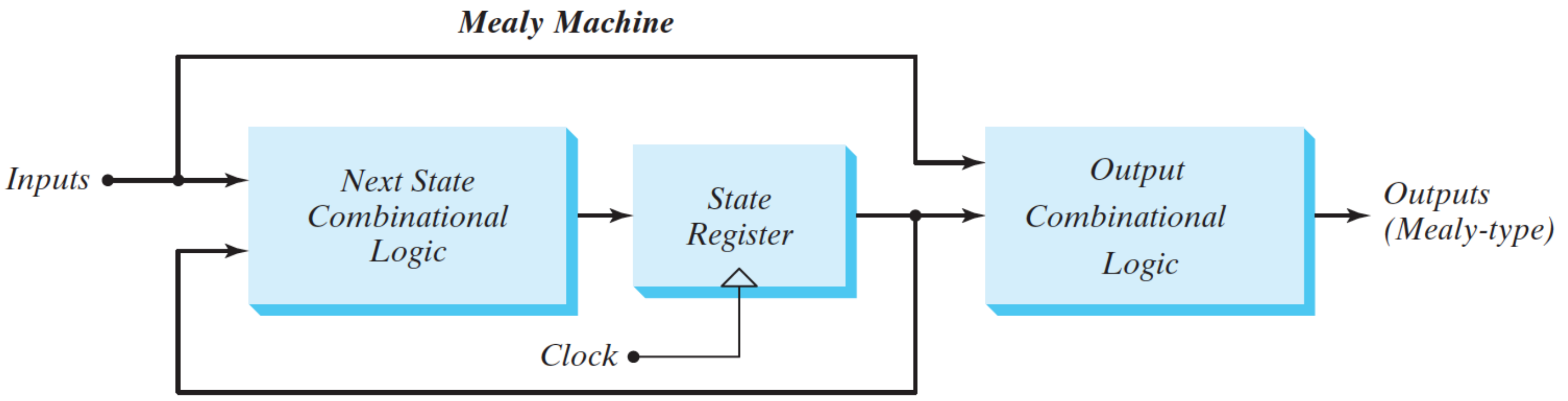
George H. Mealy
(1927 – 2010)
Mathematician and Computer Scientist
Invented Mealy Machine
Also a pioneer of modular programming

Outputs = Function(Current State, Inputs)

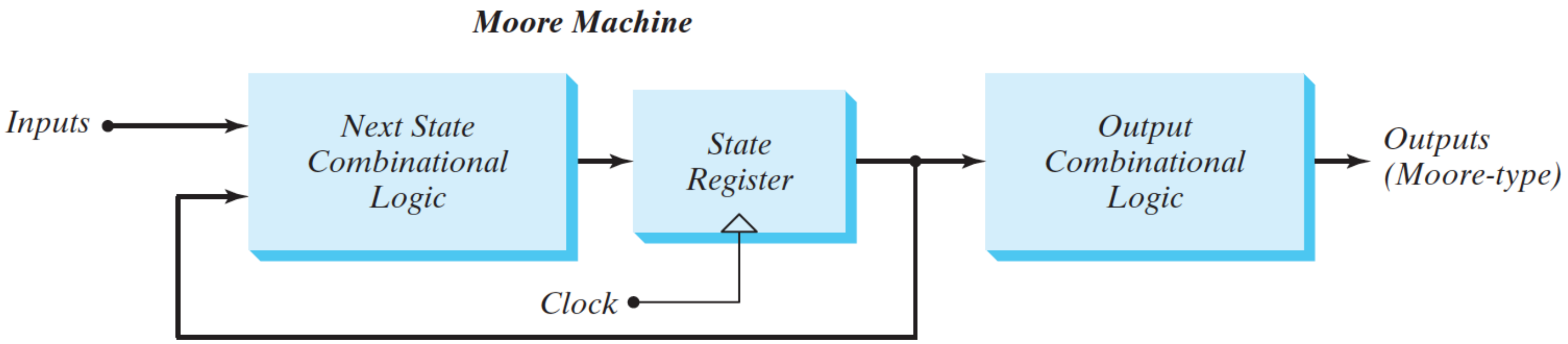
Edward Forrest Moore
(1925 – 2003)
Mathematician and Computer Scientist
Inventor of the Moore Machine
Also an early pioneer of artificial life

Outputs = Function(Current State~~, Inputs~~)





(a)



(b)

FIGURE 5.21
Block diagrams of Mealy and Moore state machines

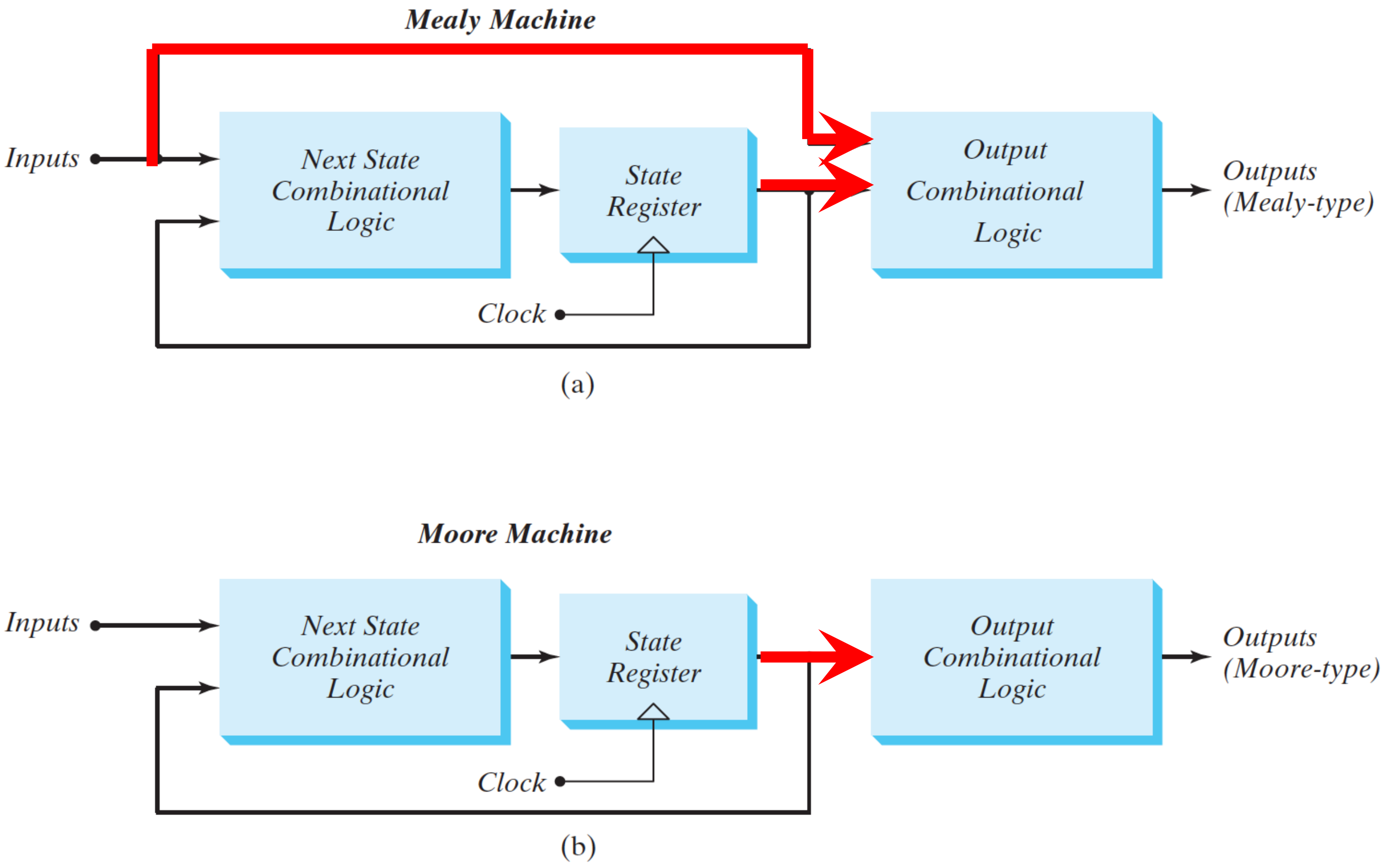
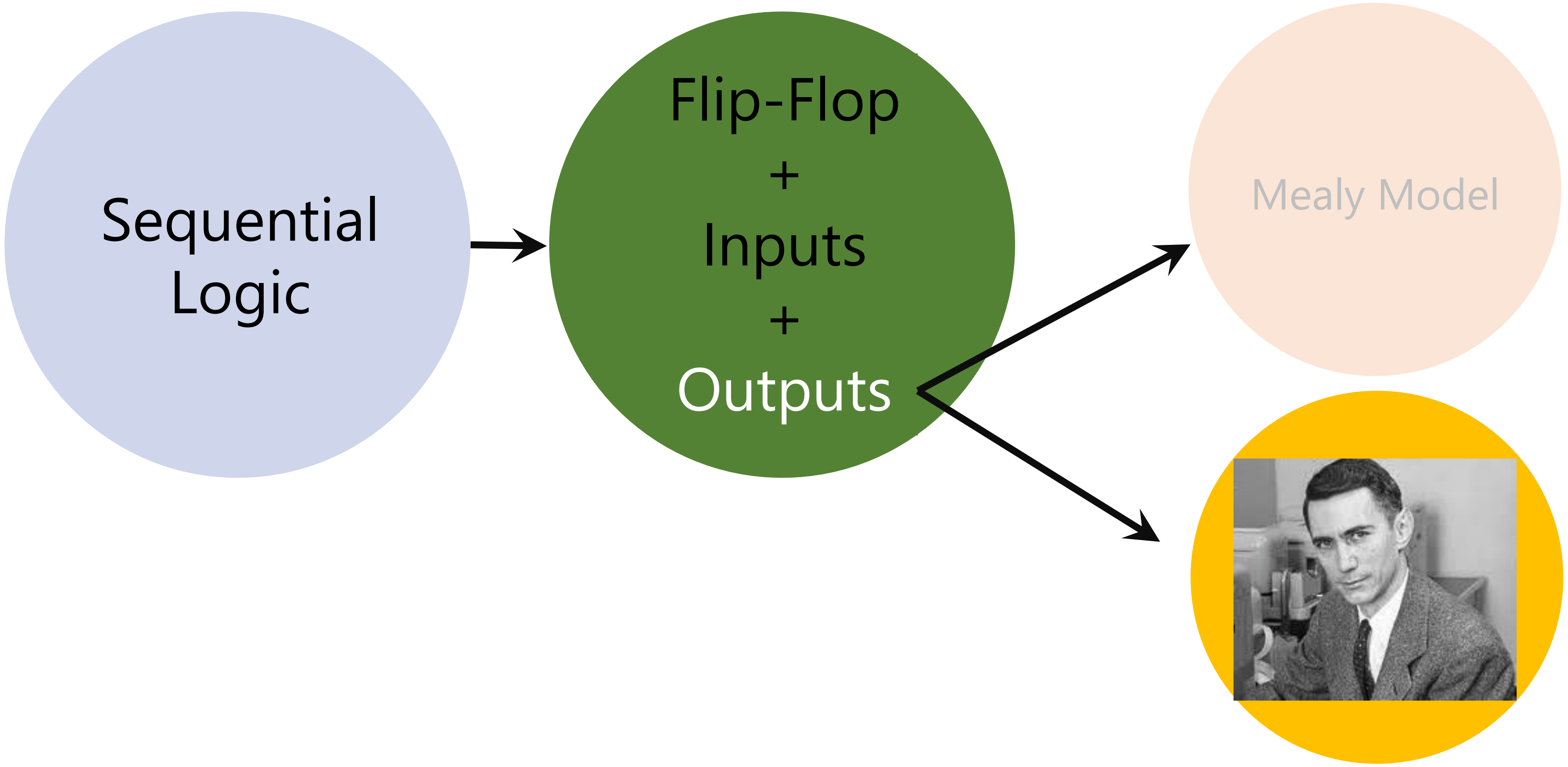


FIGURE 5.21
Block diagrams of Mealy and Moore state machines



Sequential Logic
Flip-Flop
+
Inputs
+
Outputs
Mealy Model

Analysis (Moore model in output)
by an example

Analysis (Recap)

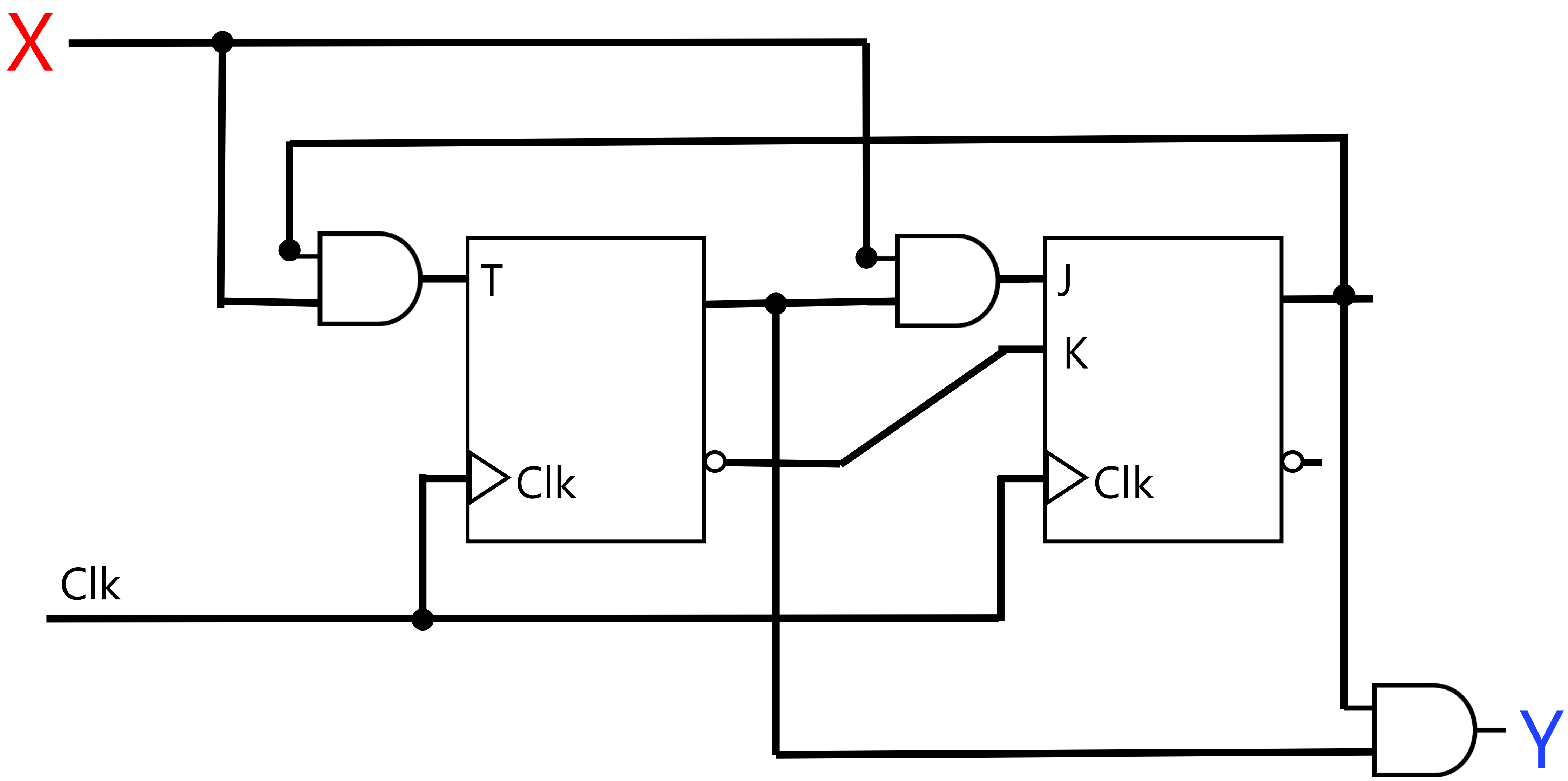
0. Is the circuit sequential or combinational? Any FF or feedback → Sequential
1. What are the flip-flops? RS, D, T, JK, or mixed (e.g., 2 JK, 1 RS, …)
2. What are the state combinations? $2^{\#FF}$
3. Form "State" table:
   a) Columns: for each FF, two columns:
      - one for current state,
      - one for next state
   b) Rows: for each state combination
      - In total: $2^{\#FF}$
4. Fill the state table for next state columns based on:
   a) the current state
   b) the inputs to the FFs
5. Form State Transition Diagram
6. (Optional) Analyze paths and states in state transition diagram

Analysis (+ Input + Moore Model Output)

0. Is the circuit sequential or combinational? Any FF or feedback → Sequential
1. What are the flip-flops? RS, D, T, JK, or mixed (e.g., 2 JK, 1 RS, …)
2. What are the state combinations? ~~$2^{\#FF}$~~
3. Form “State” table:
   a) Columns: for each FF, two columns:
      - one for current state,
      - one for next state
   b) Rows: for each state combination
      - In total: ~~$2^{\#FF}$~~
4. Fill the state table for next state columns based on:
   a) the current state
   b) the inputs to the FFs
5. Form State Transition Diagram
6. (Optional) Analyze paths and states in state transition diagram

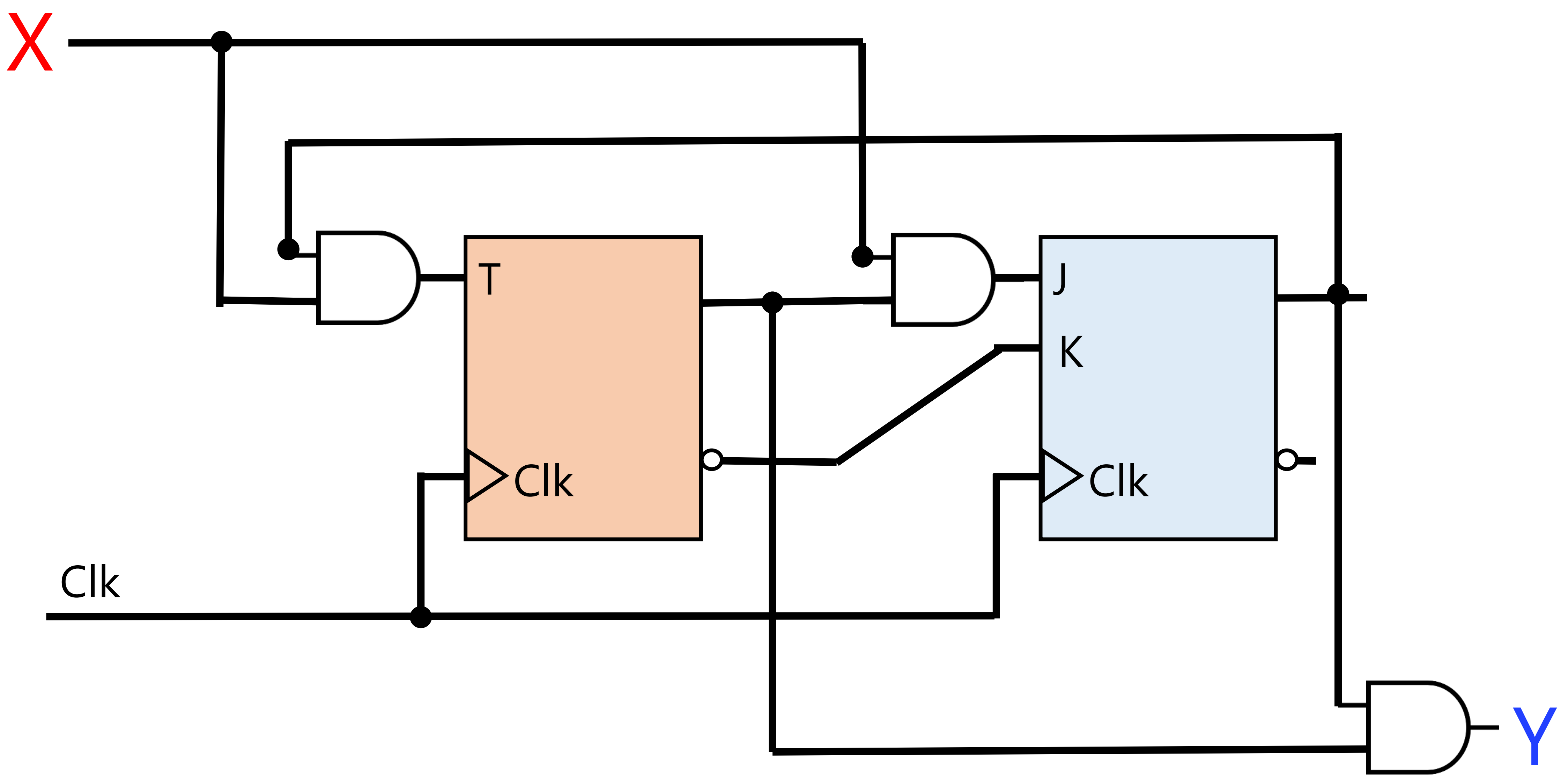
Analysis (Recap)

0. Is the circuit sequential or combinational? Any FF or feedback → Sequential


X
T
Clk
J
K
Clk
Clk
Y

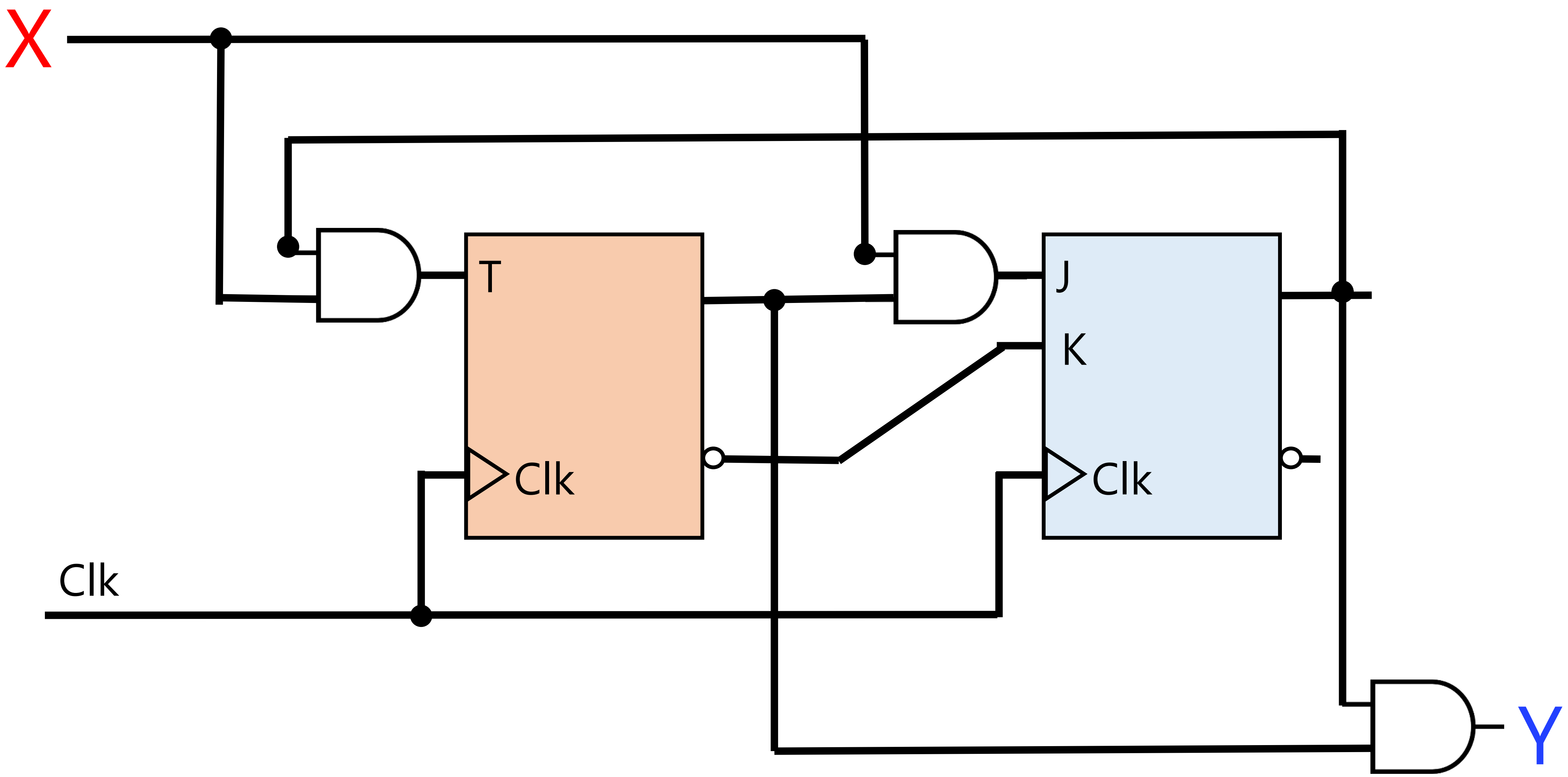
Analysis (Recap)

0. Is the circuit sequential or combinational? Sequential
1. What are the flip-flops?


X
T
Clk
J
K
Clk
Clk
Y

Analysis (Recap)

0. Is the circuit sequential or combinational? Sequential
1. What are the flip-flops? T, JK
2. What are the state combinations? $2^{\#FF} \times 2^{\#inputs} = 2^{\#FF+\#inputs}$



#FFs + #Inputs = 2+1 → 2^3 = 8 combinations

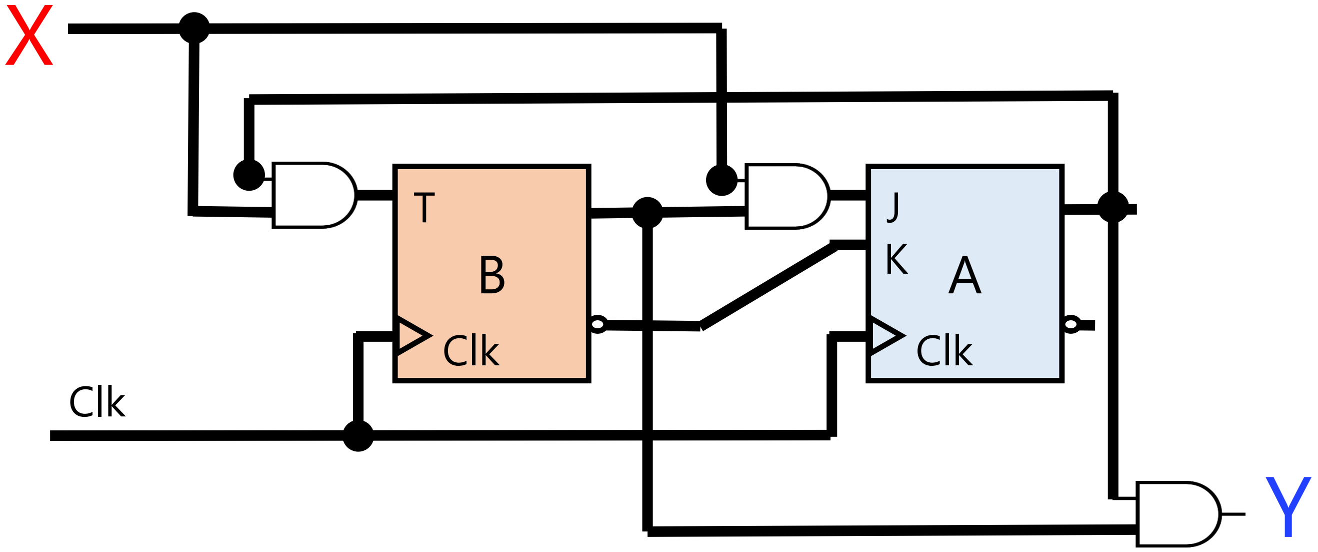
Analysis (Recap)

0. Is the circuit sequential or combinational? Sequential
1. What are the flip-flops? T, JK
2. What are the state combinations? $2^{\#FF} \times 2^{\#inputs} = 2^{\#FF+\#inputs} = 2^3 = 8$
3. Form “State” table:

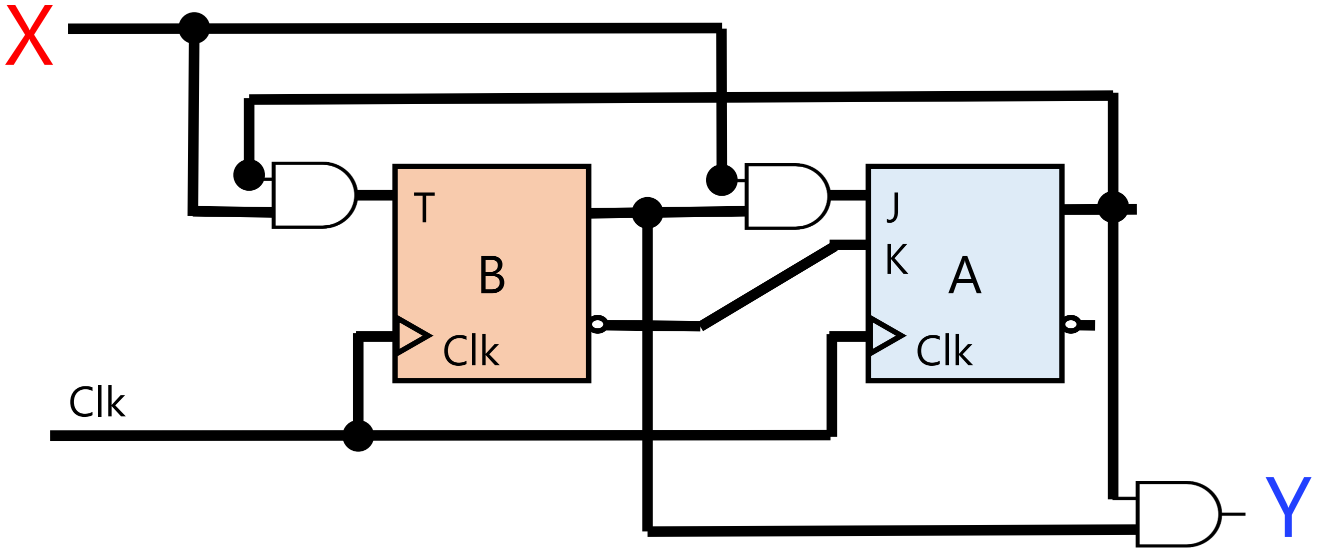
    a) Columns:

    - For each FF, two columns: one for current state, one for next state
    - For each input, one column
    - For each output, one column

    b) Rows: See item 2



Inputs	Q(T)		Q(T+1)		Outputs
X	B	A	B	A	Y



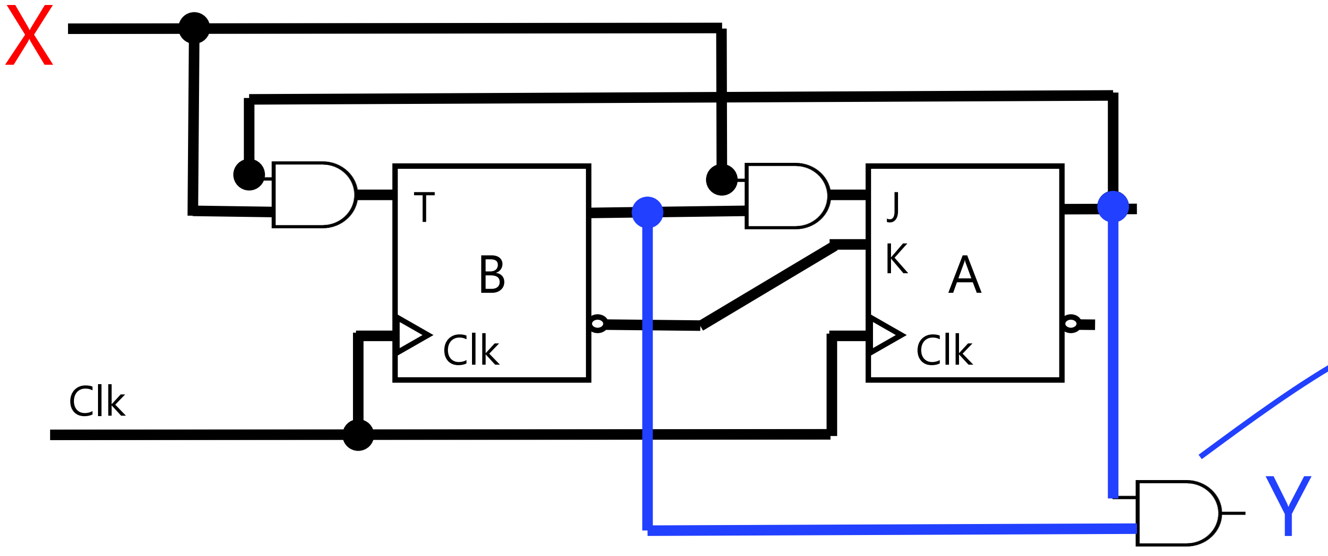
Q(T)		Q(T+1) when X=0		Q(T+1) when X=1		Outputs
B	A	B	A	B	A	Y

Alternative State Table

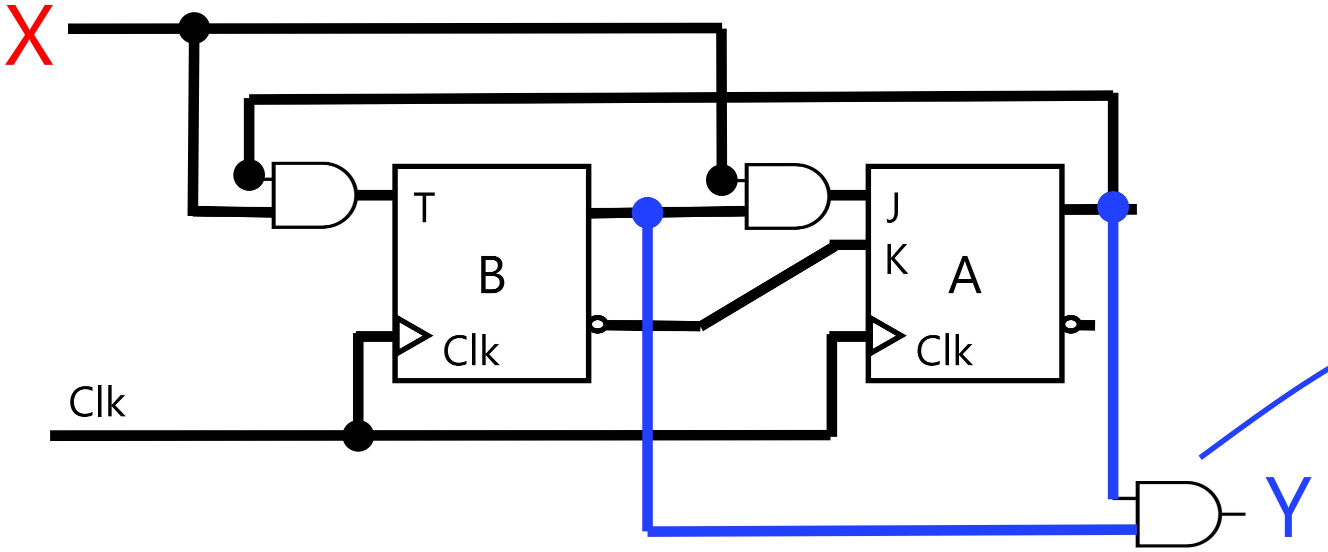
Analysis (Recap)

0. Is the circuit sequential or combinational? Sequential
1. What are the flip-flops? T, JK
2. What are the state combinations? $2^{\#FF} \times 2^{\#inputs} = 2^{\#FF+\#inputs} = 2^3 = 8$
3. Form “State” table:
   a) Columns:
      - For each FF, two columns: one for current state, one for next state
      - For each input, one column
      - For each output, one column

   b) Rows: See item 2
4. Fill the state table for
   a) next state columns
   b) the output value



Inputs	Q(T)		Q(T+1)		Outputs
X	B	A	B	A	Y=BA
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

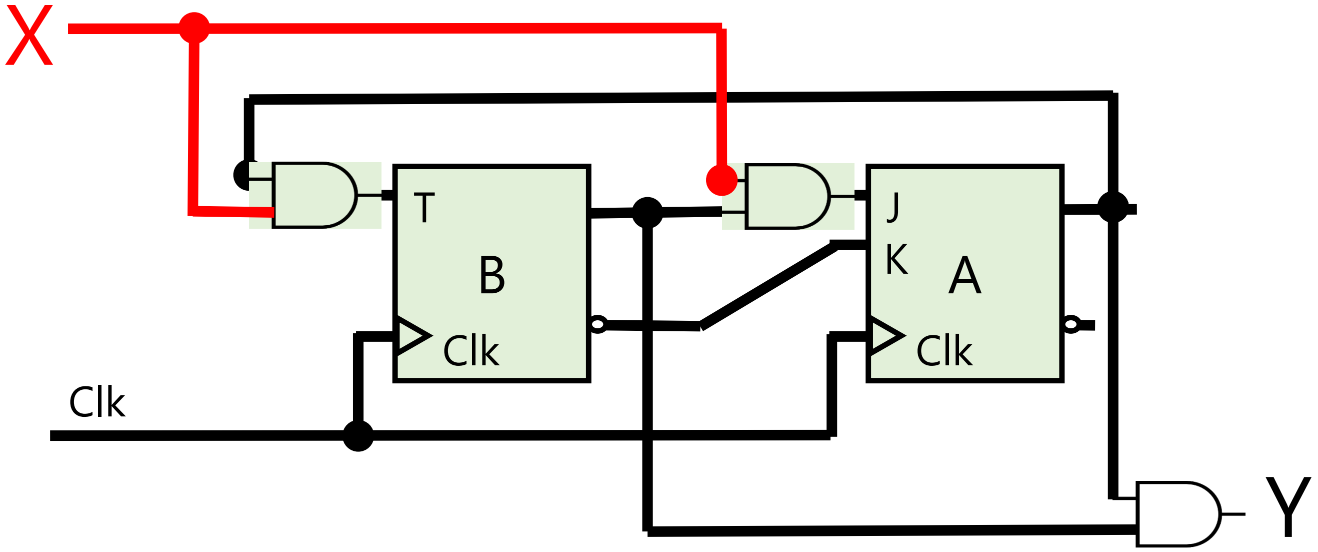


Inputs	Q(T)		Q(T+1)		Outputs
X	B	A	B	A	Y=BA
0	0	0			0
0	0	1			0
0	1				0
0	1				1
1	0				0
1	0				0
1	1	0			0
1	1	1			1

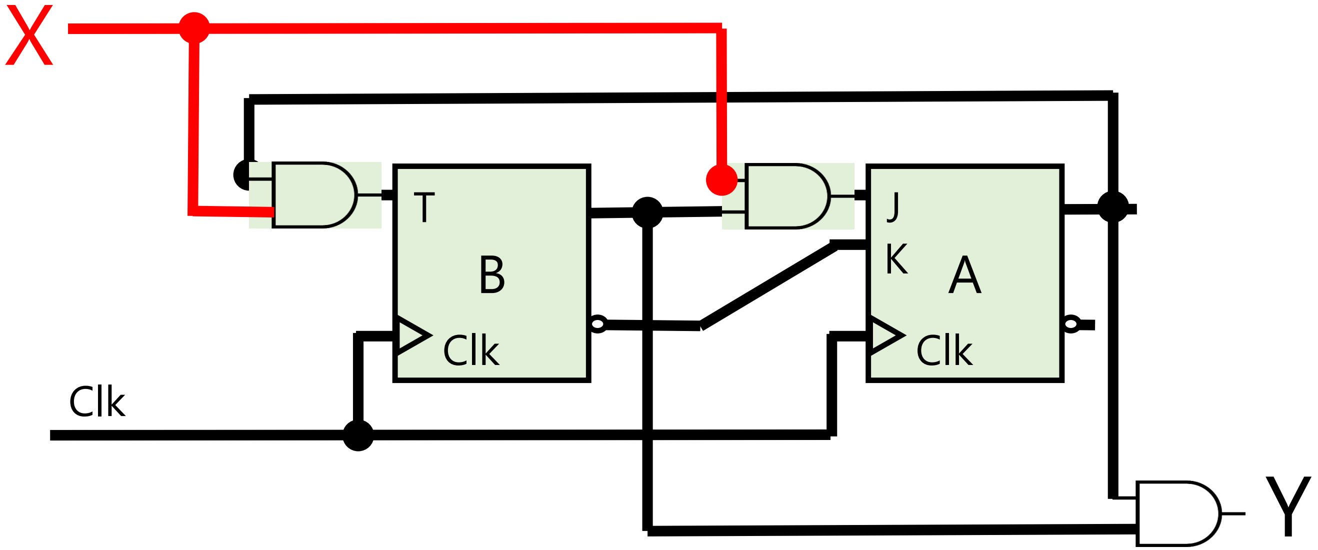
Moore Model

Only depends on current state

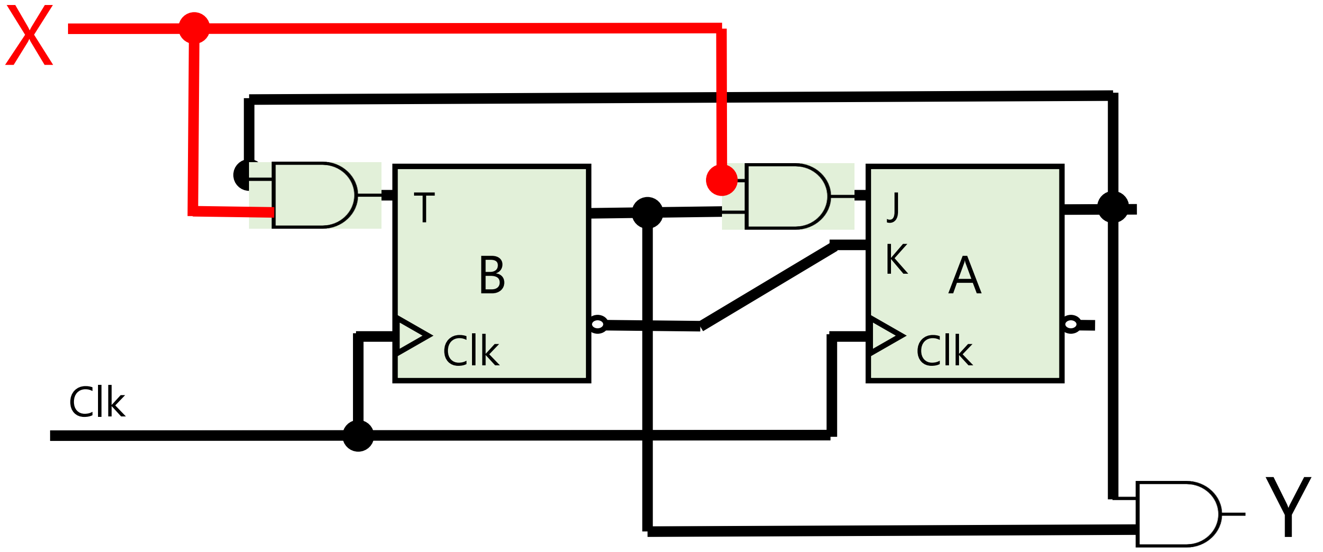
X is not involved!



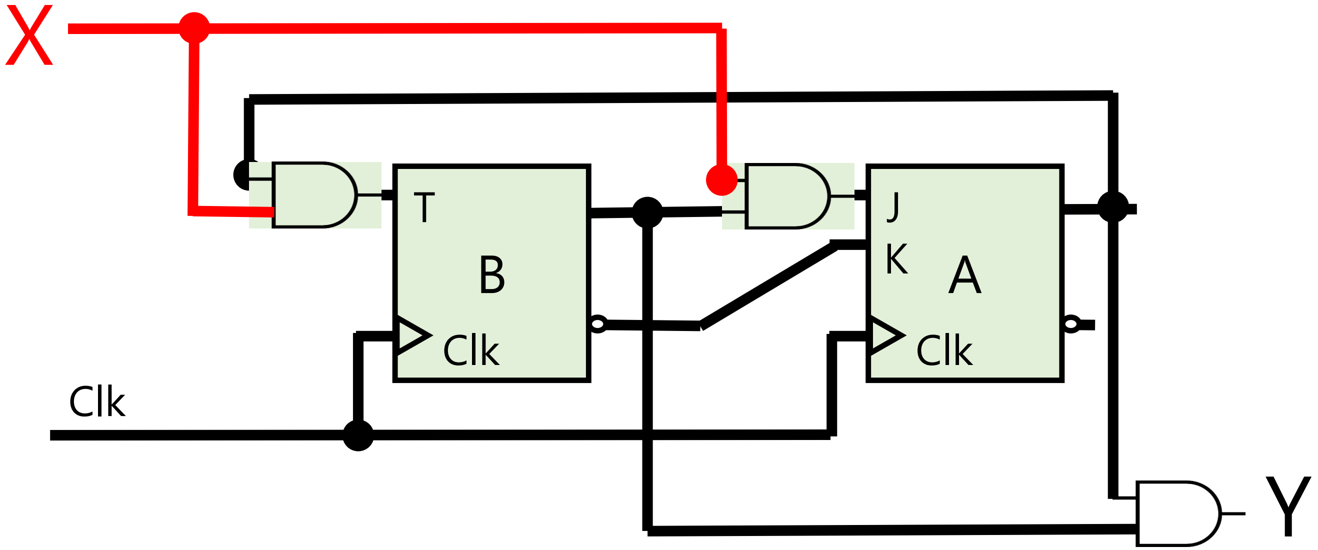
Inputs	Q(T)		Q(T+1)		Outputs
X	B	A	B	A	Y=BA
0	0	0			0
0	0	1			0
0	1	0			0
0	1	1			1
1	0	0			0
1	0	1			0
1	1	0			0
1	1	1			1



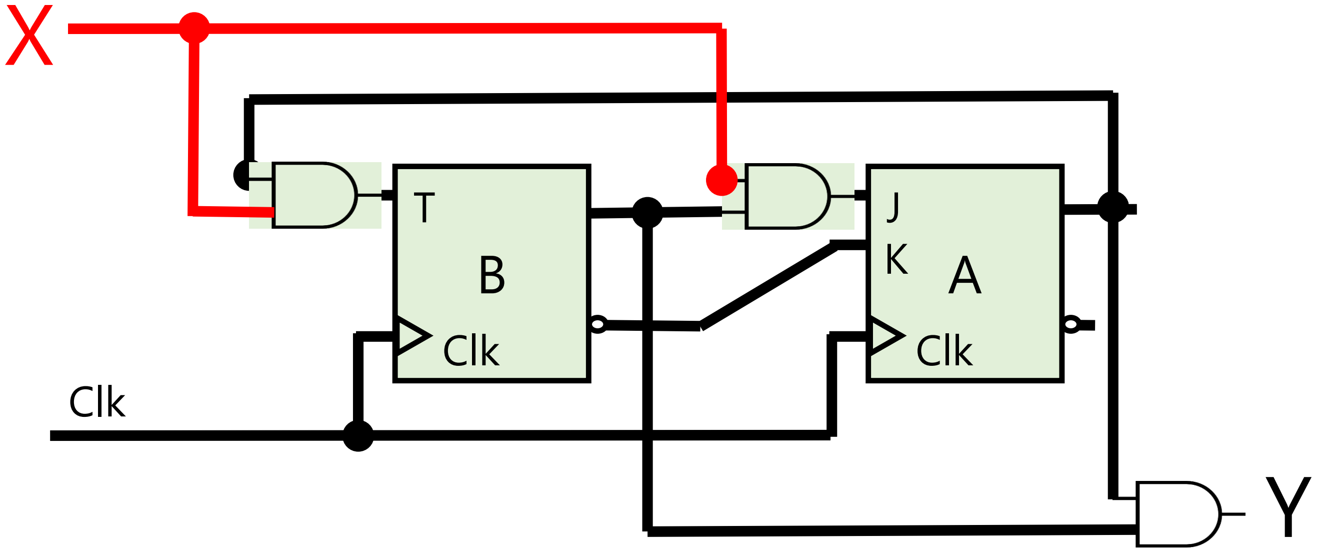
Inputs	Q(T)		Q(T+1)		Outputs
X	B	A	B	A	Y=BA
0	0	0		A=0 J_A=XB=00=0 K_A=B′=0′=1 --------------- Reset → 0	0
0	0	1			0
0	1	0			0
0	1	1			1
1	0	0			0



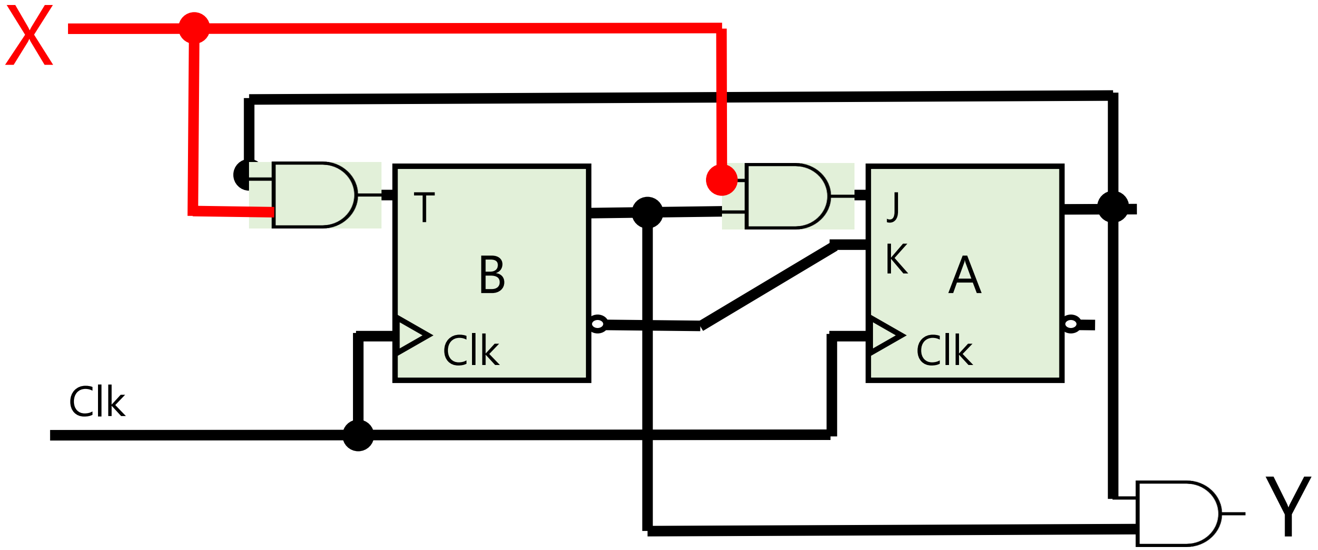
Inputs	Q(T)		Q(T+1)		Outputs
X	B	A	B	A	Y=BA
0	0	0	B=0 T_B=XA=00=0 -------------- Store → 0	0	0
0	0	1			0
0	1	0			0
0	1	1			1
1	0	0			0
1	0	1			0



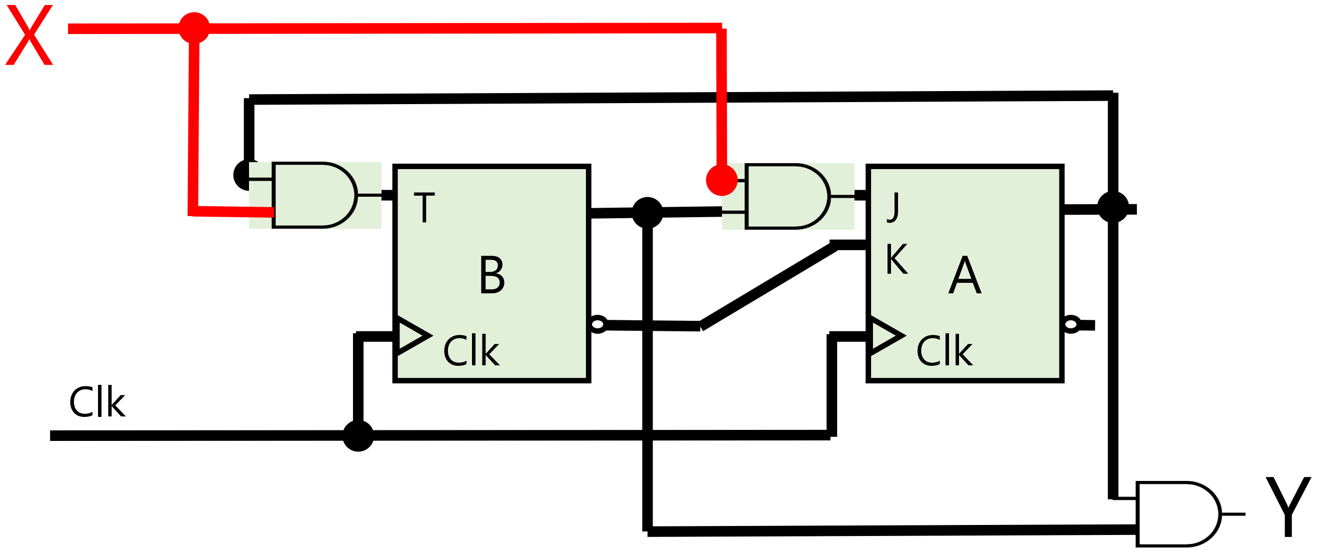
Inputs	Q(T)		Q(T+1)		Outputs
X	B	A	B	A	Y=BA
0	0	0	0	0	0
0	0	1			0
0	1	0			0
0	1	1			1
1	0	0			0
1	0	1			0
1	1	0			0
1	1	1			1



Inputs	Q(T)		Q(T+1)		Outputs
X	B	A	B	A	Y=BA
0	0	0	0	0	0
0	0	1	1	0	0
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	0	0	0
1	0	1	?	0	0
1	1	0	1	1	0
1	1	1	0	1	1



Inputs	Q(T)		Q(T+1)		Outputs
X	B	A	B	A	Y=BA
0	0	0	0	0	0
0	0	1	1	0	0
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	0	0	0
1	0	1	B=0 TB=XA=11=1 ---------- Comp. → 1	0	0

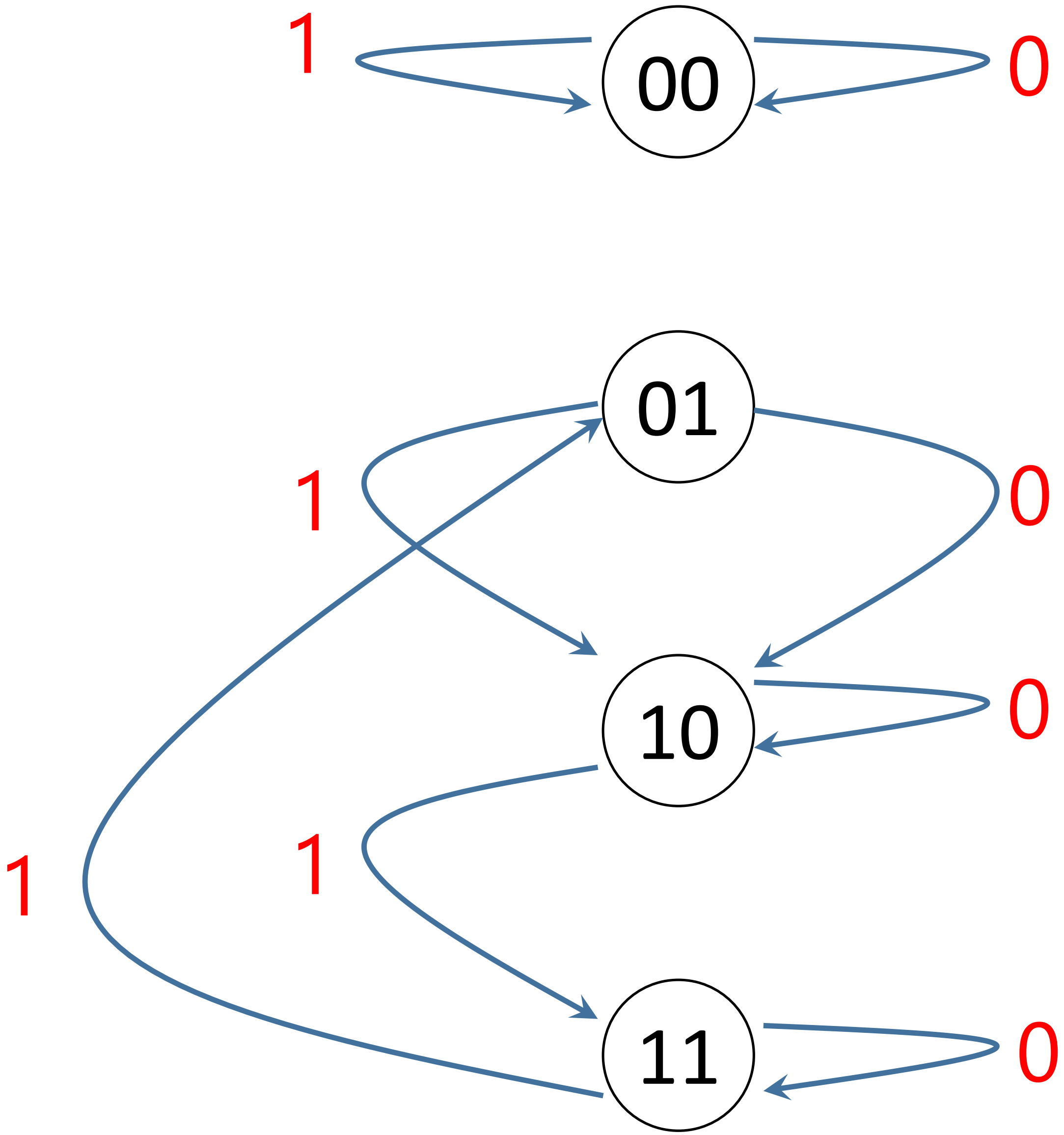


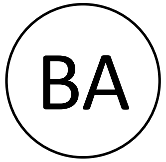
Inputs	Q(T)		Q(T+1)		Outputs
X	B	A	B	A	Y=BA
0	0	0	0	0	0
0	0	1	1	0	0
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	0	0	0
1	0	1	1	0	0
1	1	0	1	1	0
1	1	1	0	1	1

Analysis (Recap)

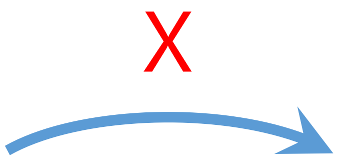
0. Is the circuit sequential or combinational? Sequential
1. What are the flip-flops? T, JK
2. What are the state combinations? $2^{\#FF} \times 2^{\#inputs} = 2^{\#FF+\#inputs} = 2^3 = 8$
3. Form "State" table:
   a) Columns:
      - For each FF, two columns: one for current state, one for next state
      - For each input, one column
      - For each output, one column

   b) Rows: See item 2
4. Fill the state table for
   a) next state columns
   b) the output value
5. Form state (transition) diagram
   a) nodes for states, directed edges for transitions between states
   b) labels for edges by the value of <u>input</u>
   c) labels for nodes by the value of <u>output</u>

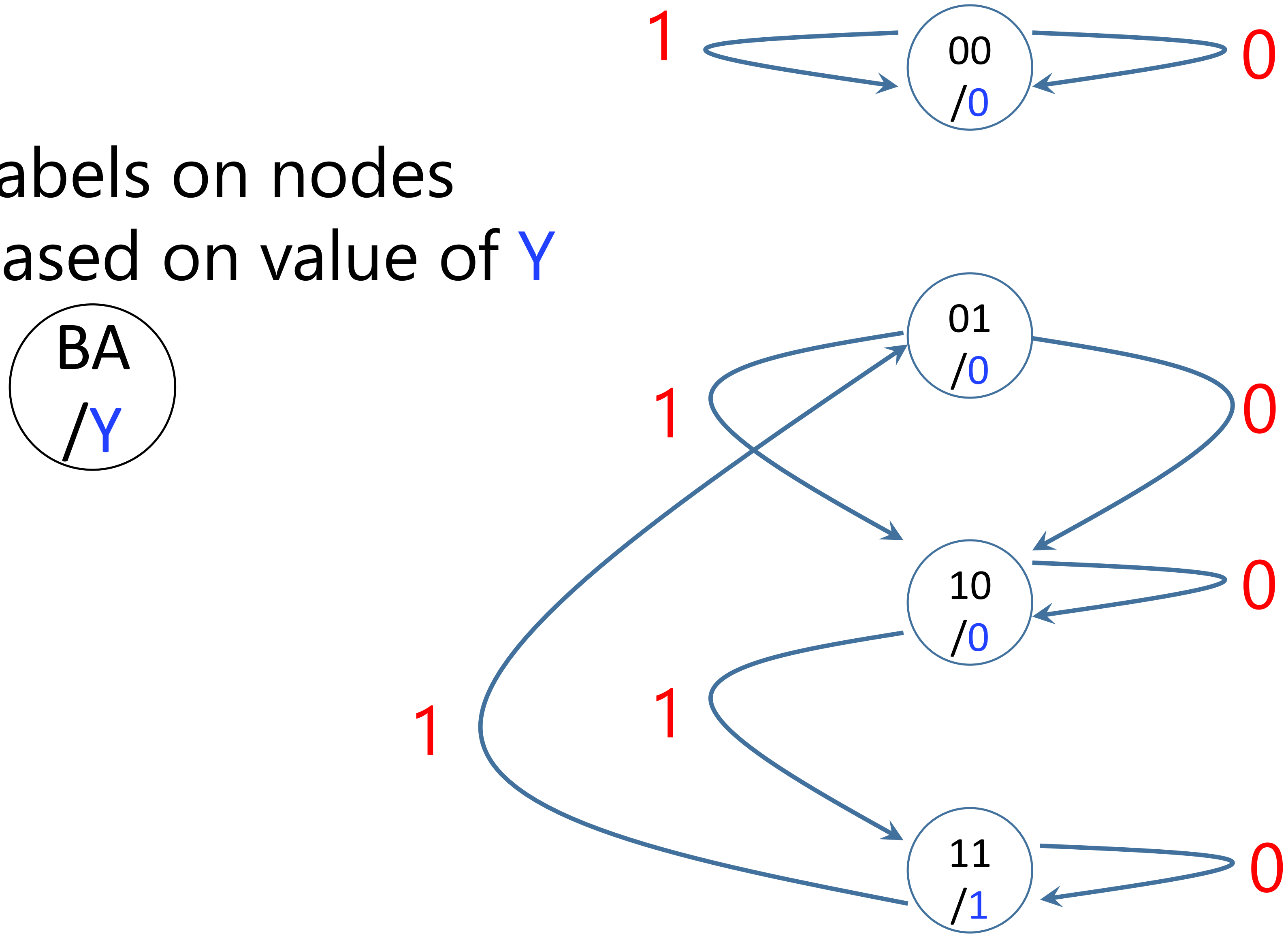




Labels on edges based on value of X



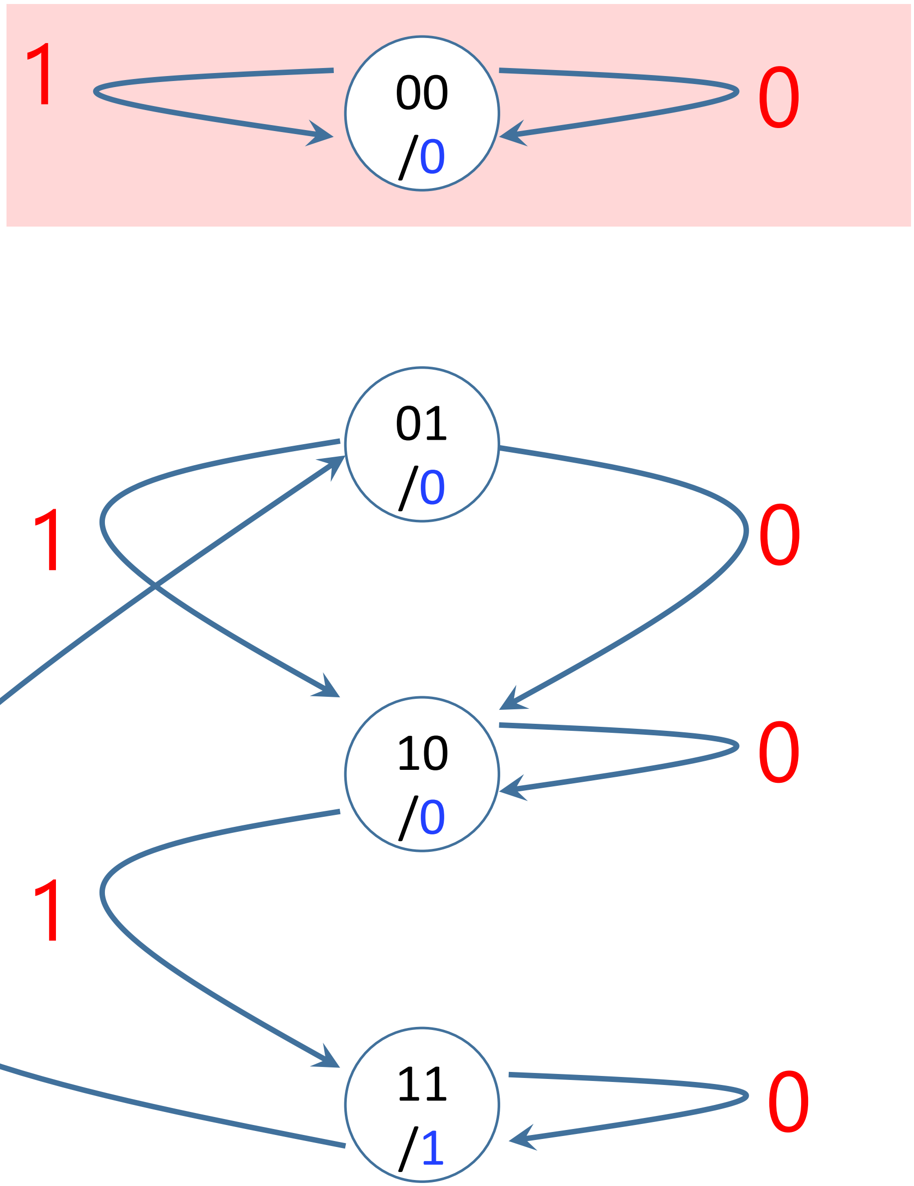
Labels on nodes based on value of Y

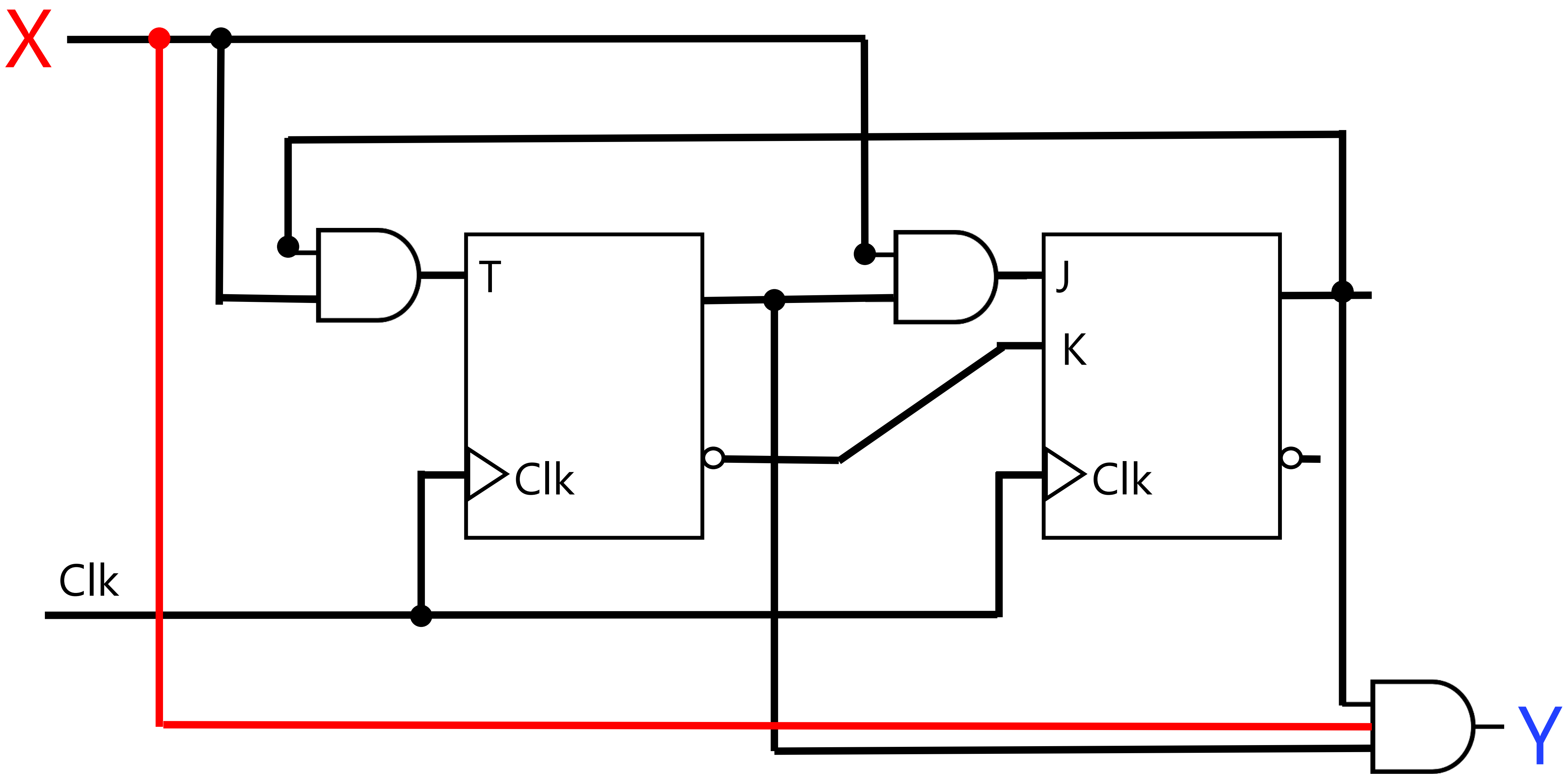


Analysis

6) (*Optional*) Path on State Transitions

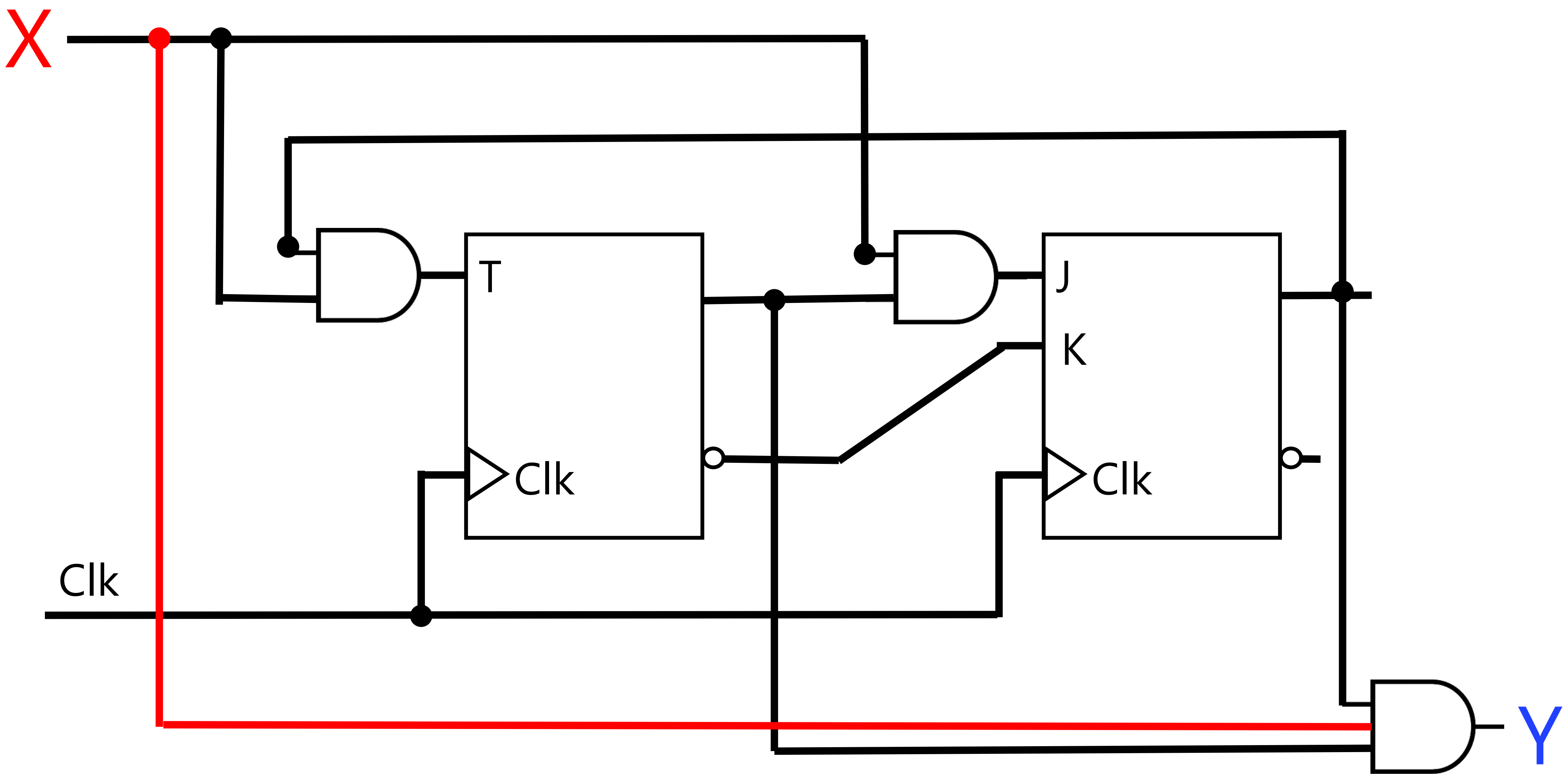
Life lock!





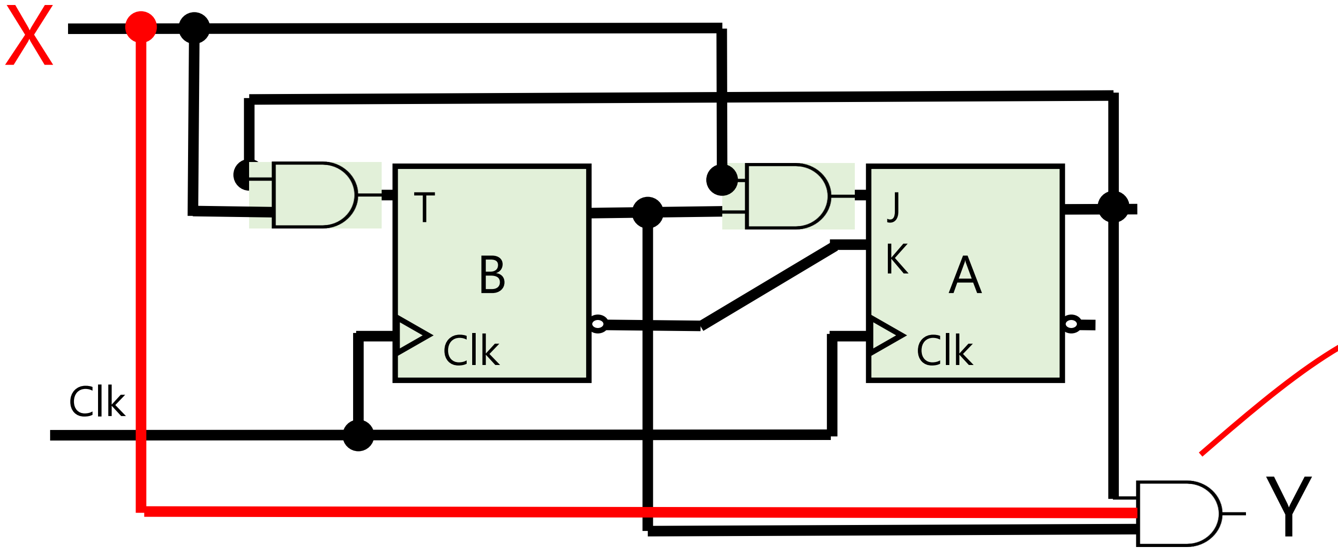
Mealy Model

Y depends on X



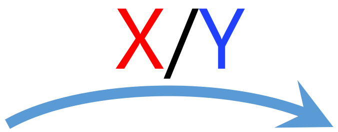
Mealy Model

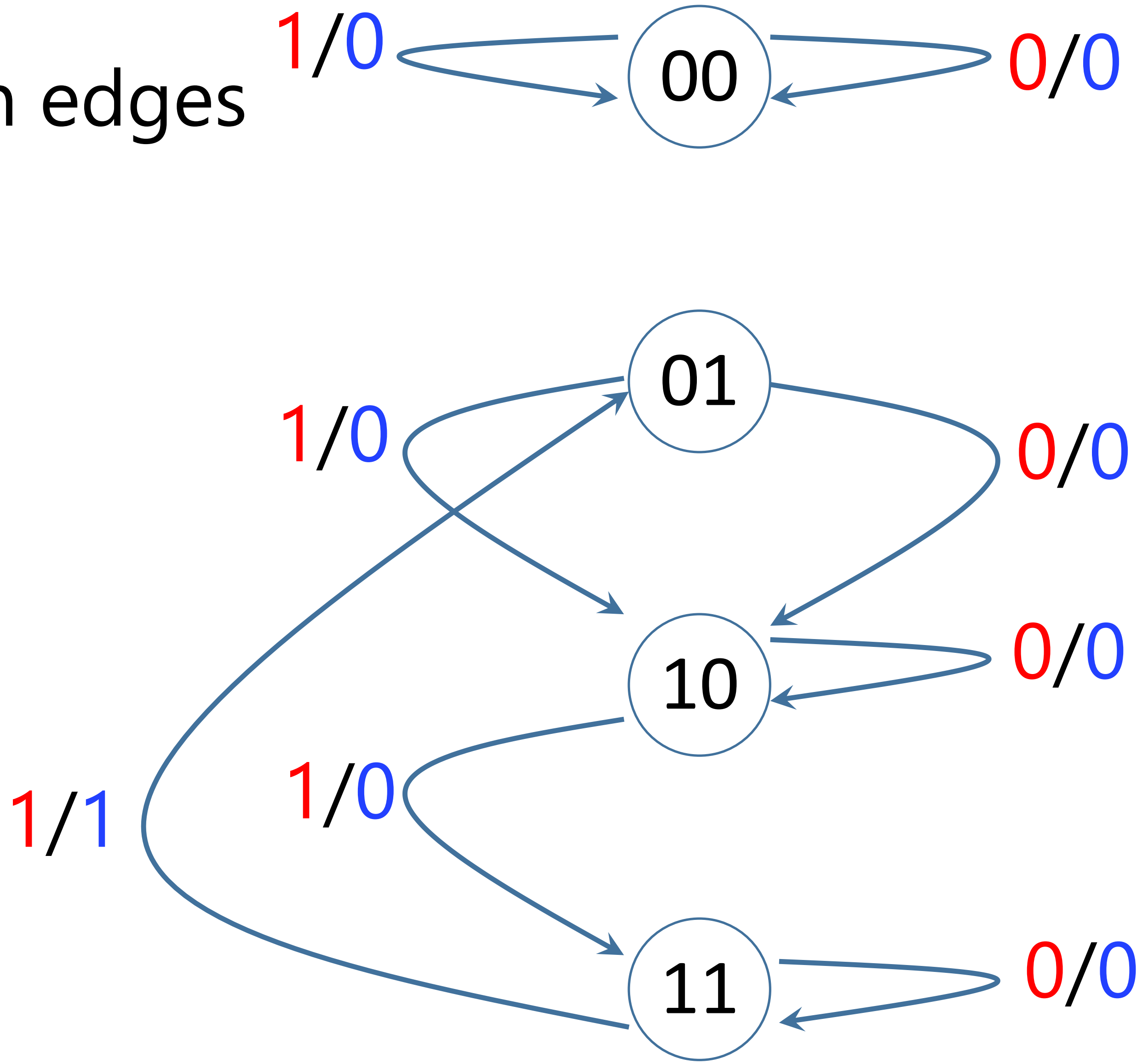
Y depends on X, so, Y can change out of Clk synchronization!



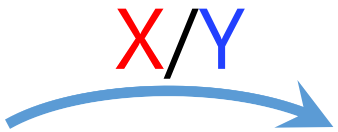
Inputs	Q(T)		Q(T+1)		Outputs
X	B	A	B	A	Y=XBA
0	0	0	0	0	0
0	0	1	1	0	0
0	1	0	1	0	0
0	1	1	1	1	0
1	0	0	0	0	0
1	0	1	1	0	0
1	1	0	1	1	0
1	1	1	0	1	1

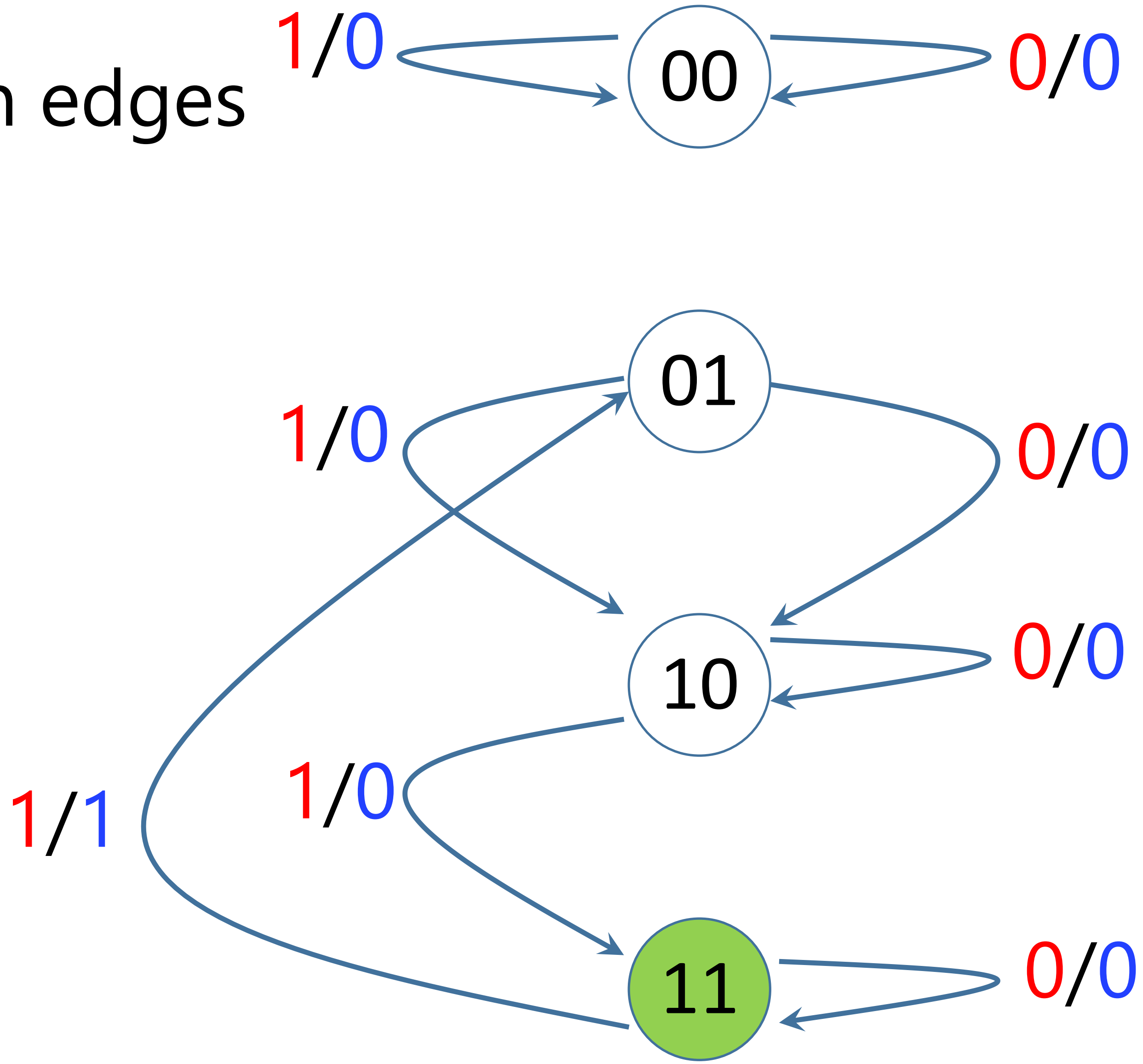
Labels go on edges for Y:

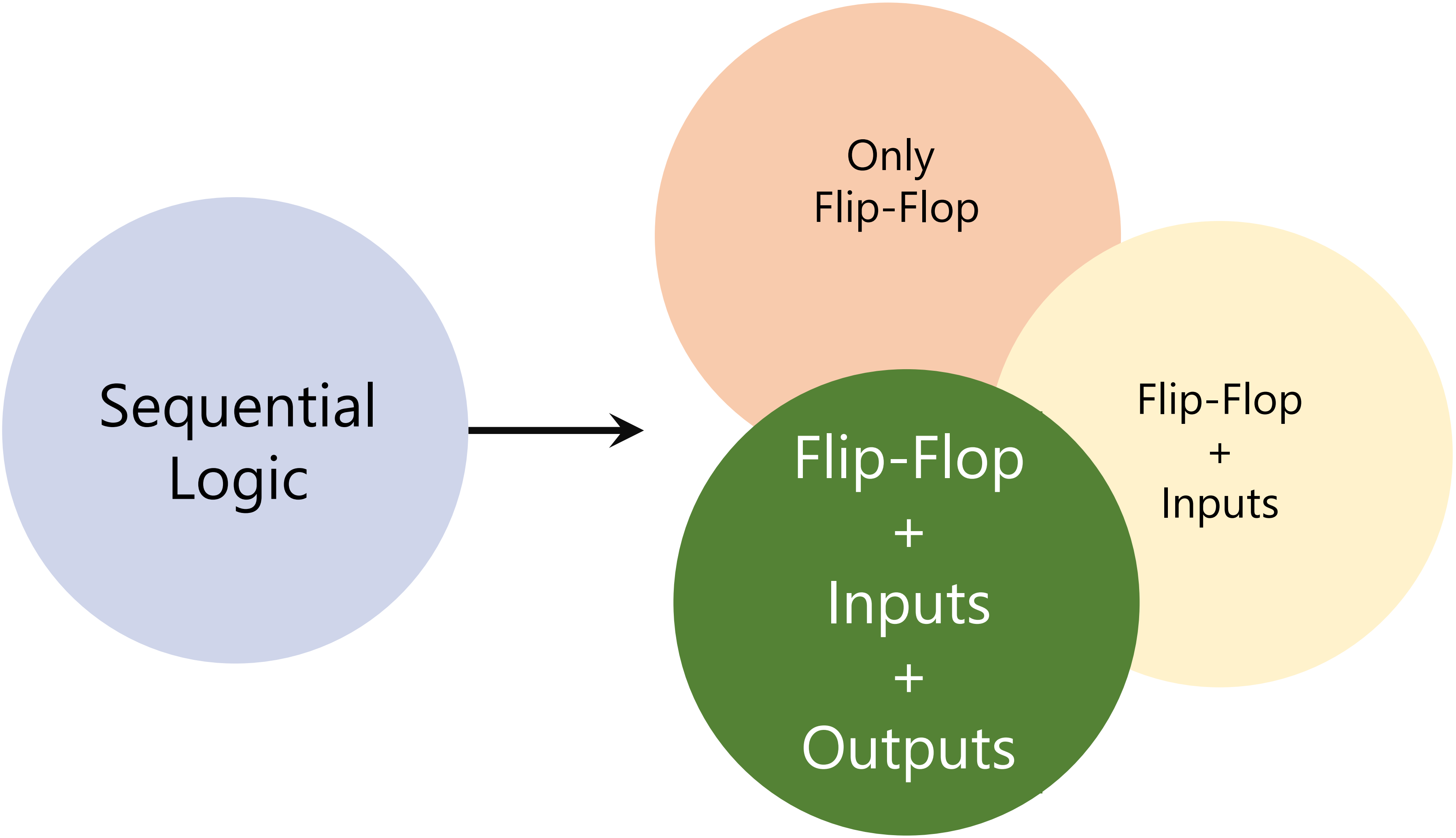




Labels go on edges for Y:






Sequential Logic
Only Flip-Flop
Flip-Flop + Inputs
Flip-Flop + Inputs + Outputs

Design
by an example

Counter Up-Down

Switch to count up from i to i+1

Switch to count down from i to i-1

Turn on the light if the current number is even

Design (Recap)

0. Do we need combinational logic or sequential logic? Do we need memory?

Counter up/down

0 → 1 → 2 → 3 → … → N-1 → N

N → N-1 → … → 3 → 2 → 1 → 0

Design (Recap)

0. Do we need combinational logic or sequential logic? Do we need memory?
1. How many storage (flip-flops)? #FF

# Counter up/down
N=7

000→001→010→011→100→101→110→111

000←001←010←011←100←101←110←111

For each intermediate state, we need 3 bits → 3 flip-flops

Design (Recap)

0. Do we need combinational logic or sequential logic? Do we need memory?
1. How many storage (flip-flops)? #FF
2. How many input and output?

# Counter up/down
N=7

000→001→010→011→100→101→110→111

000←001←010←011←100←101←110←111

We need to switch between up and down → 1 binary variable

X=0 Count Up

X=1 Count Down

Counter up/down
N=7

X=0: 000→001→010→011→100→101→110→111

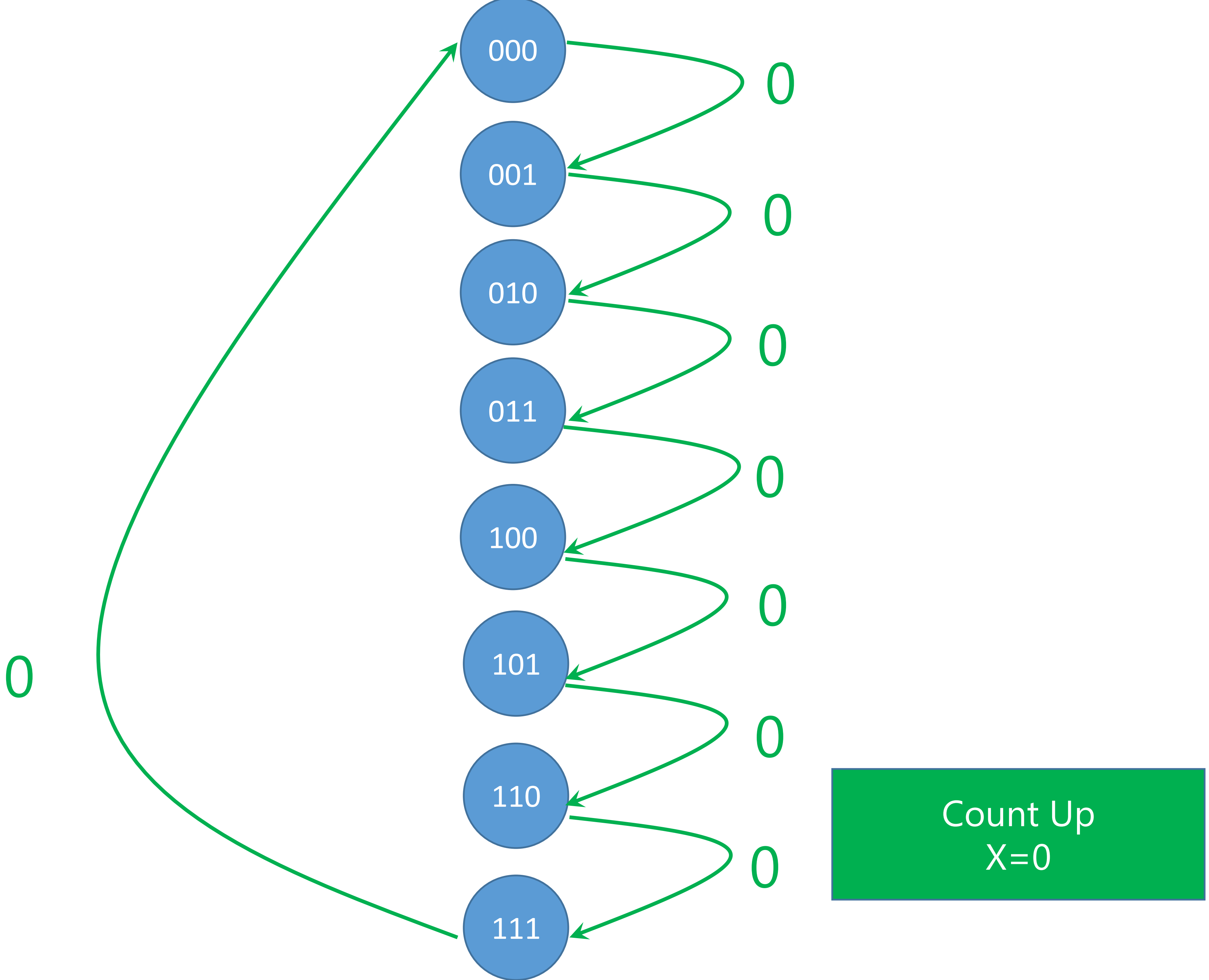
X=1: 111→110→101→100→011→010→001→000

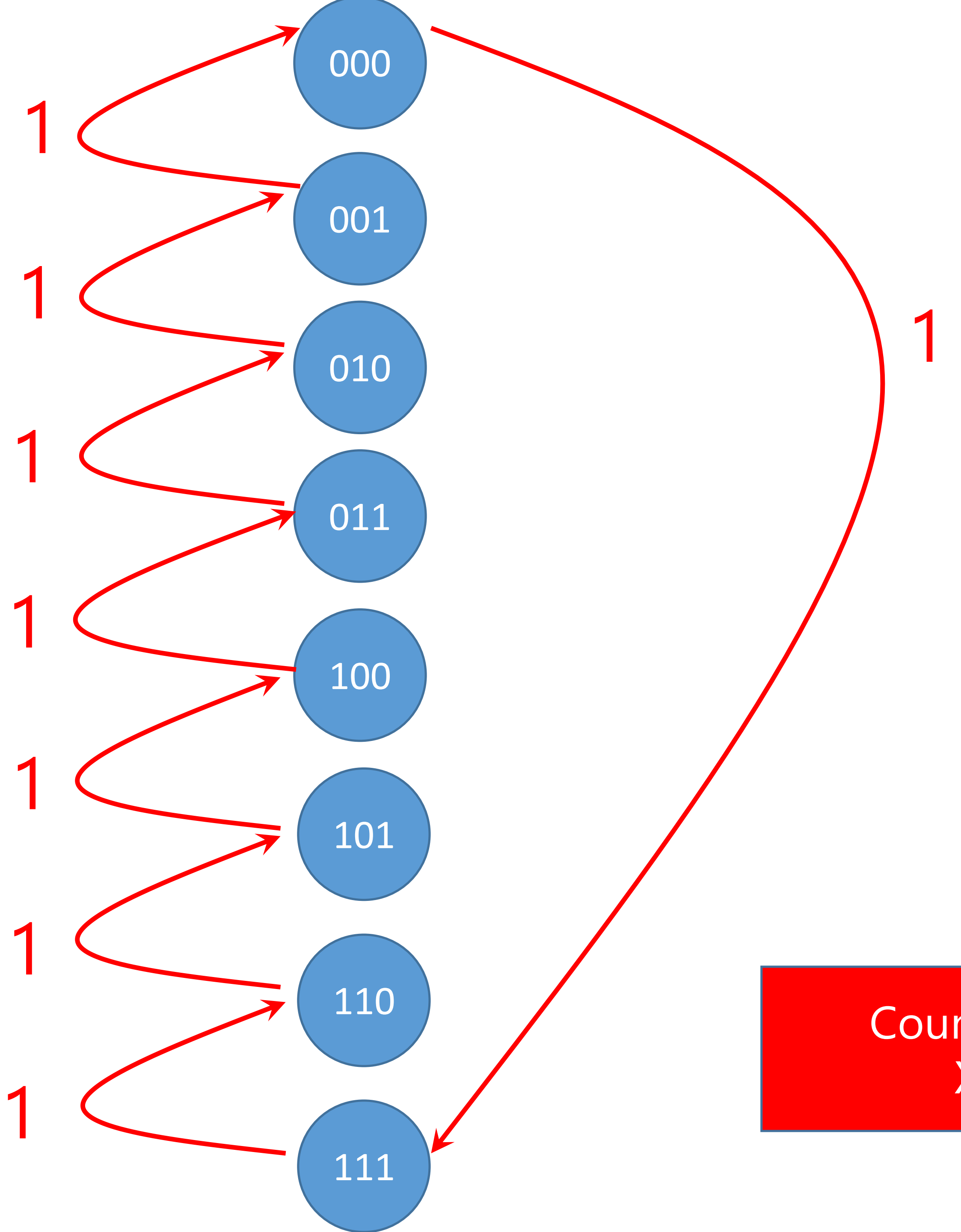
We need to turn on the light when the number is even

→ 1 binary variable → Y

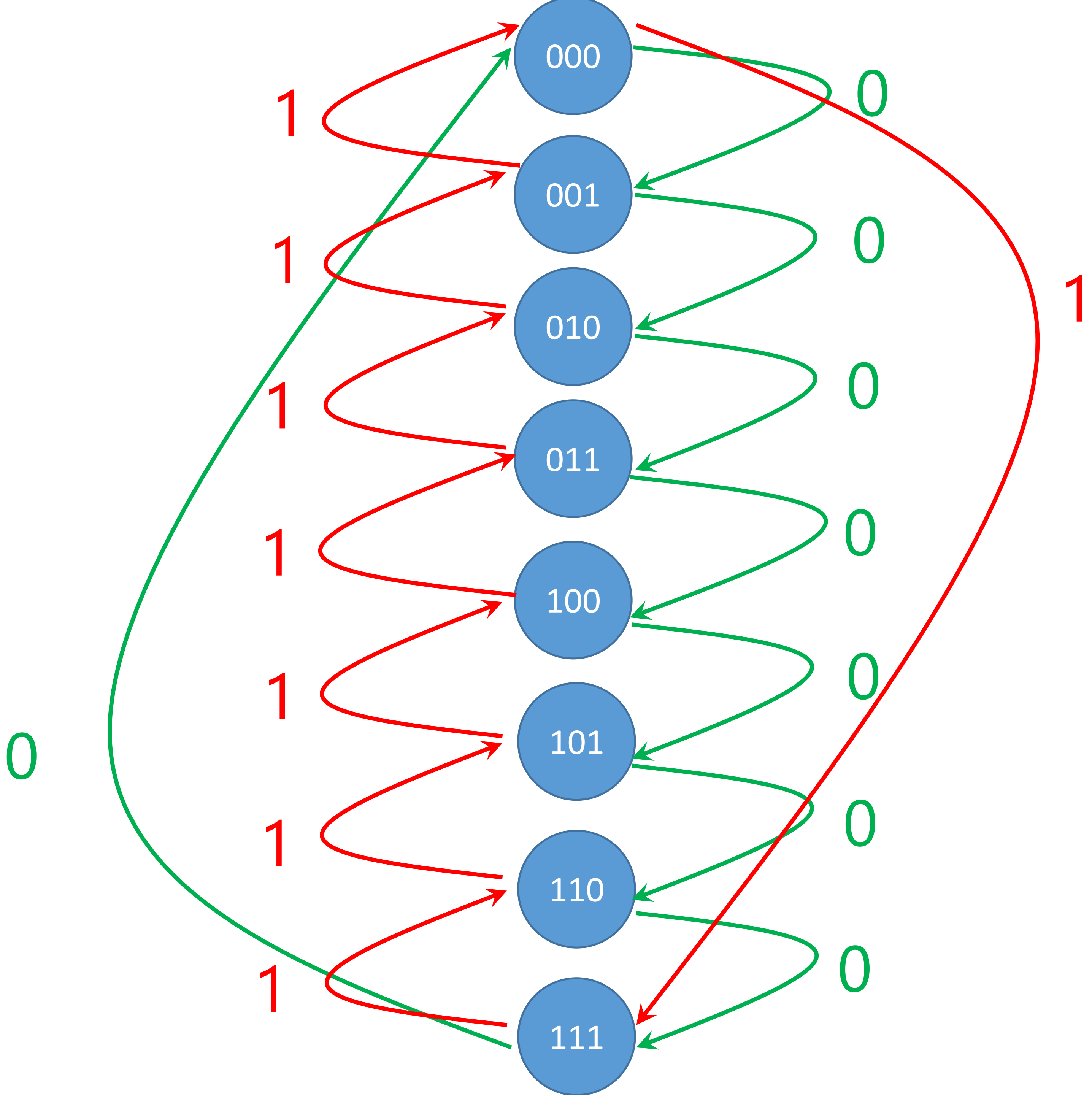
Design (Recap)

0. Do we need combinational logic or sequential logic? Do we need memory?
1. How many storage (flip-flops)? #FF
2. How many input and output?
3. Form the state (transition) diagram


000
001
010
011
100
101
110
111
0
0
0
0
0
0
0
0
Count Up
X=0



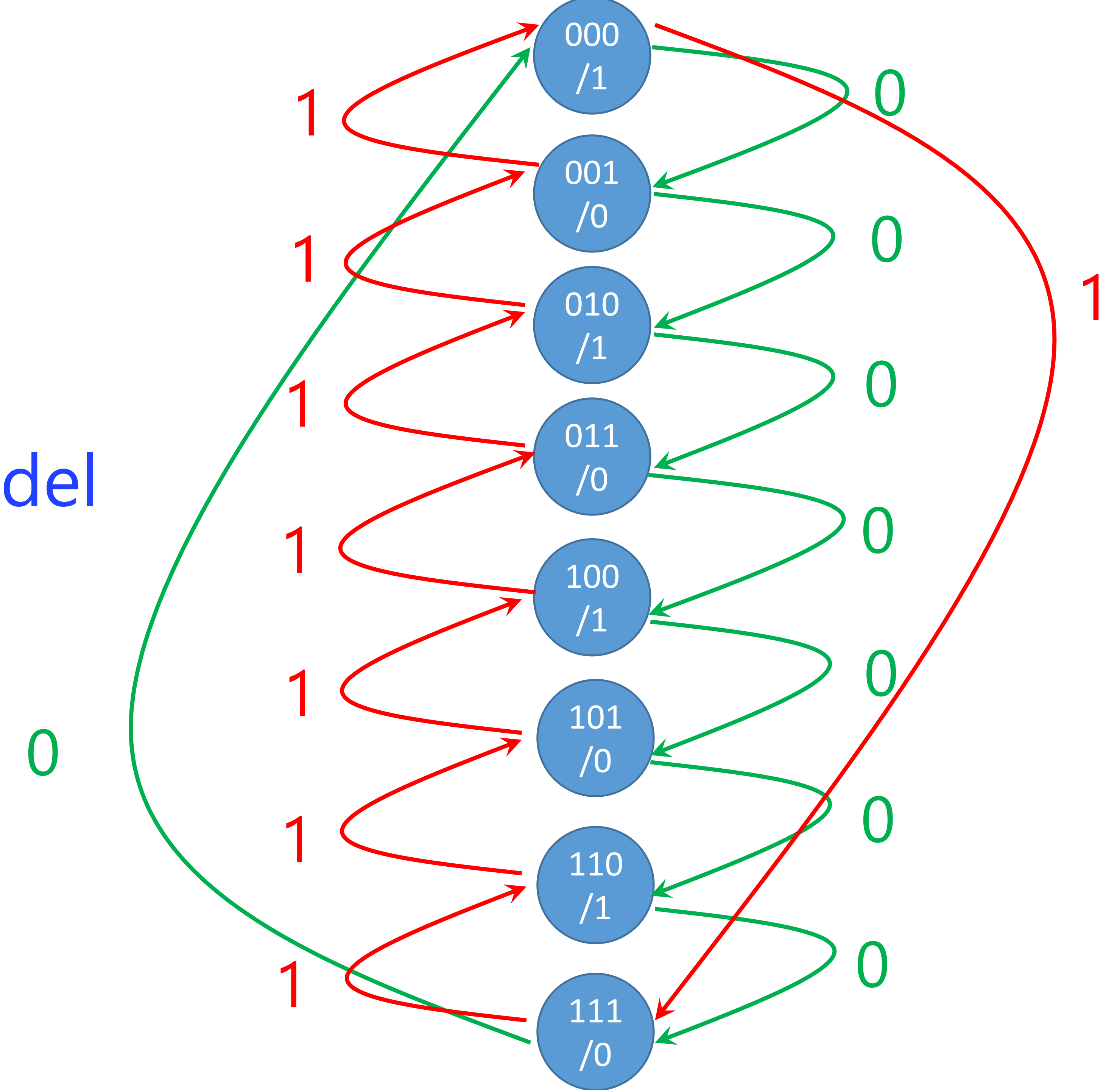
Count Down
X=1


000
001
010
011
100
101
110
111
1
1
1
1
1
1
1
1
0
0
0
0
0
0
0
0

Y=1 : even

Y=0 : else

Moore model



Design (Recap)

0. Do we need combinational logic or sequential logic? Do we need memory?
1. How many storage (flip-flops)? #FF
2. How many input and output?
3. Form the state (transition) diagram
4. Form the state table
5. Fill the state table

Inputs	Q(T)			Q(T+1)			Outputs
X	C	B	A	C	B	A	Y
0	0	0	0	0	0	1	
0	0	0	1	0	1	0	
0	0	1	0	0	1	1	
0	0	1	1	1	0	0	
0	1	0	0	1	0	1	
0	1	0	1	1	1	0	
0	1	1	0	1	1	1	
0	1	1	1	0	0	0	
1	0	0	0	1	1	1	
1	0	0	1	0	0	0	
1	0	1	0	0	0	1	
1	0	1	1	0	1	0	
1	1	0	0	0	1	1	
1	1	0	1	1	0	0	
1	1	1	0	1	0	1	
1	1	1	1	1	1	0	

Count Up
X=0

Count Down
X=1

Inputs	Q(T)			Q(T+1)			Outputs
X	C	B	A	C	B	A	Y
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	0
0	0	1	0	0	1	1	1
0	0	1	1	1	0	0	0
0	1	0	0	1	0	1	1
0	1	0	1	1	1	0	0
0	1	1	0	1	1	1	1
0	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1
1	0	0	1	0	0	0	0
1	0	1	0	0	0	1	1
1	0	1	1	0	1	0	0
1	1	0	0	0	1	1	1
1	1	0	1	1	0	0	0
1	1	1	0	1	0	1	1
1	1	1	1	1	1	0	0

Y depends only on current state of CBA

Inputs	Q(T)			Q(T+1)			Outputs
X	C	B	A	C	B	A	Y
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	0
0	0	1	0	0	1	1	1
0	0	1	1	1	0	0	0
0	1	0	0	1	0	1	1
0	1	0	1	1	1	0	0
0	1	1	0	1	1	1	1
0	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1
1	0	0	1	0	0	0	0
1	0	1	0	0	0	1	1
1	0	1	1	0	1	0	0
1	1	0	0	0	1	1	1
1	1	0	1	1	0	0	0
1	1	1	0	1	0	1	1
1	1	1	1	1	1	0	0

Design (Recap)

0. Do we need combinational logic or sequential logic? Do we need memory?
1. How many storage (flip-flops)? #FF
2. How many input and output?
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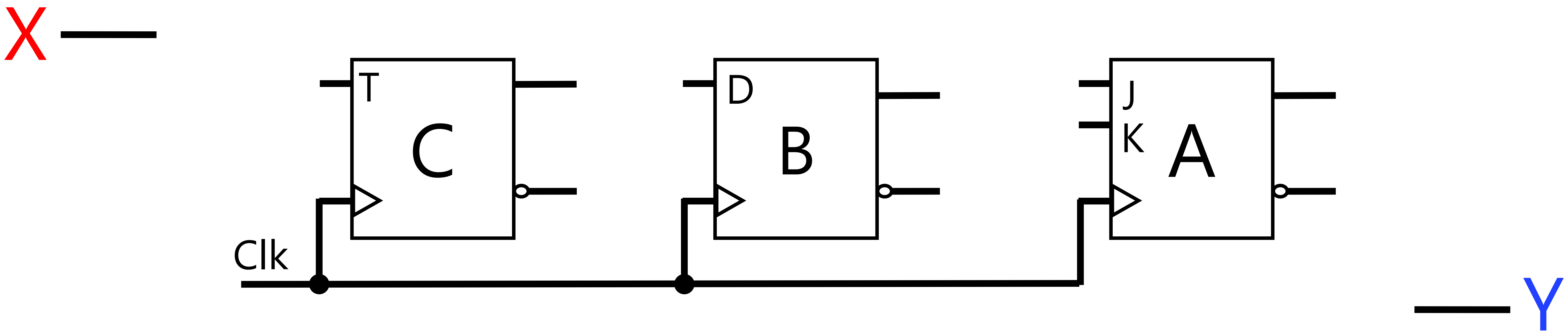
5. Fill the state table
6. What type of storage (flip-flop)? RS, D, T, JK, or Mixed

Counter Up/Down

Let’s pick mix FFs: 1×T-FF, 1×D-FF, 1×JK-FF


X
T
C
D
B
J
K
A
Clk
Y

Design (Recap)

0. Do we need combinational logic or sequential logic? Do we need memory?
1. How many storage (flip-flops)? #FF
2. How many input and output?
3. Form the state (transition) diagram
4. Form the state table
5. Fill the state table
6. What type of storage (flip-flop)? RS, D, T, JK, or Mixed
7. Input (*excitation*) equations for each FF
8. Minimization of input (*excitation*) equations

Excitation Equations		
A	$J_A=F(X, C, B, A)$	$K_A=F(X, C, B, A)$
B	$D_B=F(X, C, B, A)$	
C	$T_C=F(X, C, B, A)$	

Input

Flip-Flops

Excitation Equations		
A	J_A=F(X, C, B, A)	K_A=F(X, C, B, A)
B	D_B=	
C	T_C=	

Inputs	Q(T)			Q(T+1)			Outputs	Excitation for A		
X	C	B	A	C	B	A	Y	Action	J_A	K_A
0	0	0	0	0	0	1	1	Set/Cmp	1	x
0	0	0	1	0	1	0	0	Rst/Cmp	x	1
0	0	1	0	0	1	1	1	Set/Cmp	1	x
0	0	1	1	1	0	0	0	Rst/Cmp	x	1
0	1	0	0	1	0	1	1	Set/Cmp	1	x
0	1	0	1	1	1	0	0	Rst/Cmp	x	1
0	1	1	0	1	1	1	1	Set/Cmp	1	x
0	1	1	1	0	0	0	0	Rst/Cmp	x	1
1	0	0	0	1	1	1	1	Set/Cmp	1	x
1	0	0	1	0	0	0	0	Rst/Cmp	x	1
1	0	1	0	0	0	1	1	Set/Cmp	1	x
1	0	1	1	0	1	0	0	Rst/Cmp	x	1
1	1	0	0	0	1	1	1	Set/Cmp	1	x
1	1	0	1	1	0	0	0	Rst/Cmp	x	1
1	1	1	0	1	0	1	1	Set/Cmp	1	x
1	1	1	1	1	1	0	0	Rst/Cmp	x	1

Excitation Equations		
A	$J_A=\sum(0,2,4,6,8,10,12,14)+d(1,3,5,7,9,11,13,15)$	$K_A=\sum(1,3,5,7,9,11,13,15)+d(0,2,4,6,8,10,12,14)$
B	$D_B=$	
C	$T_C=$	

Inputs	Q(T)			Q(T+1)			Outputs	Excitation for B	
X	C	B	A	C	B	A	Y	Action	D_B
0	0	0	0	0	0	1	1	Rst	0
0	0	0	1	0	1	0	0	Set	1
0	0	1	0	0	1	1	1	Set	1
0	0	1	1	1	0	0	0	Rst	0
0	1	0	0	1	0	1	1	Rst	0
0	1	0	1	1	1	0	0	Set	1
0	1	1	0	1	1	1	1	Set	1
0	1	1	1	0	0	0	0	Rst	0
1	0	0	0	1	1	1	1	Set	1
1	0	0	1	0	0	0	0	Rst	0
1	0	1	0	0	0	1	1	Rst	0
1	0	1	1	0	1	0	0	Set	1
1	1	0	0	0	1	1	1	Set	1
1	1	0	1	1	0	0	0	Rst	0
1	1	1	0	1	0	1	1	Rst	0
1	1	1	1	1	1	0	0	Set	1

Excitation Equations		
A	$J_A=\sum(0,2,4,6,8,10,12,14)+d(1,3,5,7,9,11,13,15)$	$K_A=\sum(1,3,5,7,9,11,13,15)+d(0,2,4,6,8,10,12,14)$
B	$D_B=\sum(1,2,5,6,8,11,12,15)$ *Never "don't care condition" happens in D-FF*	
C	$T_C=$	

Inputs	Q(T)			Q(T+1)			Outputs	Excitation for C	
X	C	B	A	C	B	A	Y	Action	T_C
0	0	0	0	0	0	1	1	Store	0
0	0	0	1	0	1	0	0	Store	0
0	0	1	0	0	1	1	1	Store	0
0	0	1	1	1	0	0	0	Comp	1
0	1	0	0	1	0	1	1	Store	0
0	1	0	1	1	1	0	0	Store	0
0	1	1	0	1	1	1	1	Store	0
0	1	1	1	0	0	0	0	Comp	1
1	0	0	0	1	1	1	1	Comp	1
1	0	0	1	0	0	0	0	Store	0
1	0	1	0	0	0	1	1	Store	0
1	0	1	1	0	1	0	0	Store	0
1	1	0	0	0	1	1	1	Comp	1
1	1	0	1	1	0	0	0	Store	0
1	1	1	0	1	0	1	1	Store	0
1	1	1	1	1	1	0	0	Store	0

Excitation Equations		
A	$J_A=\sum(0,2,4,6,8,10,12,14)+d(1,3,5,7,9,11,13,15)$	$K_A=\sum(1,3,5,7,9,11,13,15)+d(0,2,4,6,8,10,12,14)$
B	$D_B=\sum(1,2,5,6,8,11,12,15)$ *Never "don't care condition" happens in D-FF*	
C	$T_C=\sum(3,7,8,12)$ *Never "don't care condition" happens in T-FF*	

Minimization for excitation equations

?-Variable K-Map

Excitation Equations		
A	$J_A=\sum(0,2,4,6,8,10,12,14)+d(1,3,5,7,9,11,13,15)$	$K_A=\sum(1,3,5,7,9,11,13,15)+d(0,2,4,6,8,10,12,14)$
B	$D_B=\sum(1,2,5,6,8,11,12,15)$ *Never "don't care condition" happens in D-FF*	
C	$T_C=\sum(3,7,8,12)$ *Never "don't care condition" happens in T-FF*	

Minimization

4-Variable K-Map

F(X, C, B, A)

XC \ BA	00	01	11	10
00	0 m_0	0 m_1	1 m_3	0 m_2
01	0 m_4	0 m_5	1 m_7	0 m_6
11	1 m_{12}	0 m_{13}	0 m_{15}	0 m_{14}
10	1 m_8	0 m_9	0 m_{11}	0 m_{10}

$$T_C = \sum(3,7,8,12)$$
$$= X'BA + XB'A'$$

XC \ BA	00	01	11	10
00	0 m_0	1 m_1	0 m_3	1 m_2
01	0 m_4	1 m_5	0 m_7	1 m_6
11	1 m_{12}	0 m_{13}	1 m_{15}	0 m_{14}
10	1 m_8	0 m_9	1 m_{11}	0 m_{10}

$$D_B = \sum(1,2,5,6,8,11,12,15)$$
$$= X'B'A + X'BA' + XB'A' + XBA$$
$$= X'(B'A + BA') + X(B'A' + BA)$$
$$= X'(B \oplus A) + X(B \odot A)$$
$$= X \oplus B \oplus A$$

XC \ BA	00	01	11	10
00	1 m_0	X m_1	X m_3	1 m_2
01	1 m_4	X m_5	X m_7	1 m_6
11	1 m_{12}	X m_{13}	X m_{15}	1 m_{14}
10	1 m_8	X m_9	X m_{11}	1 m_{10}

$$J_A = \sum(0,2,4,6,8,10,12,14) + d(1,3,5,7,9,11,13,15) = 1$$

XC \ BA	00	01	11	10
00	X m_0	1 m_1	1 m_3	X m_2
01	X m_4	1 m_5	1 m_7	X m_6
11	X m_{12}	1 m_{13}	1 m_{15}	X m_{14}
10	X m_8	1 m_9	1 m_{11}	X m_{10}

$$K_A = \sum(1,3,5,7,9,11,13,15) + d(0,2,4,6,8,10,12,14) = 1$$

Excitation Equations		
A	$J_A=1$	$K_A=1$
B	$D_B=F(X,C,B,A)=X \oplus B \oplus A$	
C	$T_C=F(X,C,B,A)=X'BA + XB'A'$	

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8. Minimization of input (*excitation*) equations
9. Boolean function for the output
10. Minimization of output variable

Inputs	Q(T)			Q(T+1)			Outputs
X	C	B	A	C	B	A	Y
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	0
0	0	1	0	0	1	1	1
0	0	1	1	1	0	0	0
0	1	0	0	1	0	1	1
0	1	0	1	1	1	0	0
0	1	1	0	1	1	1	1
0	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1
1	0	0	1	0	0	0	0
1	0	1	0	0	0	1	1
1	0	1	1	0	1	0	0
1	1	0	0	0	1	1	1
1	1	0	1	1	0	0	0
1	1	1	0	1	0	1	1
1	1	1	1	1	1	0	0

Inputs	Q(T)			Q(T+1)			Outputs
X	C	B	A	C	B	A	Y
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	0
0	0	1	0	0	1	1	1
0	0	1	1	1	0	0	0
0	1	0	0	1	0	1	1
0	1	0	1	1	1	0	0
0	1	1	0	1	1	1	1
0	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1
1	0	0	1	0	0	0	0
1	0	1	0	0	0	1	1
1	0	1	1	0	1	0	0
1	1	0	0	0	1	1	1
1	1	0	1	1	0	0	0
1	1	1	0	1	0	1	1
1	1	1	1	1	1	0	0

$$Y = F(C,B,A) = \sum(0,2,4,6)$$

Inputs	Q(T)			Q(T+1)			Outputs
X	C	B	A	C	B	A	Y
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	0
0	0	1	0	0	1	1	1
0	0	1	1	1	0	0	0
0	1	0	0	1	0	1	1
0	1	0	1	1	1	0	0
0	1	1	0	1	1	1	1
0	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1
1	0	0	1	0	0	0	0
1	0	1	0	0	0	1	1
1	0	1	1	0	1	0	0
1	1	0	0	0	1	1	1
1	1	0	1	1	0	0	0
1	1	1	0	1	0	1	1
1	1	1	1	1	1	0	0

~~Incorrect but if we make a mistake that X is involved:~~

$\cancel{Y = F(X,C,B,A) = \sum(0,2,4,6,8,10,12,14)}$

Minimization for output Y

3-Variable K-Map

~~Incorrectly 4-Variable K-Map~~

C \ BA	00	01	11	10
0	1 m_0	0 m_1	0 m_3	1 m_2
1	1 m_4	0 m_5	0 m_7	1 m_6

$Y = F(C,B,A) = \sum(0,2,4,6)$
$= A'$

Even if we consider the input, because Y does not depend on X in the state table, it will disappear in simplification!

XC \ BA	00	01	11	10
00	1 m_0	0 m_1	0 m_3	1 m_2
01	1 m_4	0 m_5	0 m_7	1 m_6
11	1 m_{12}	0 m_{13}	0 m_{15}	1 m_{14}
10	1 m_8	0 m_9	0 m_{11}	1 m_{10}

$Y = F(X,C,B,A) = \sum(0,2,4,6,8,10,12,14)$
$= A'$

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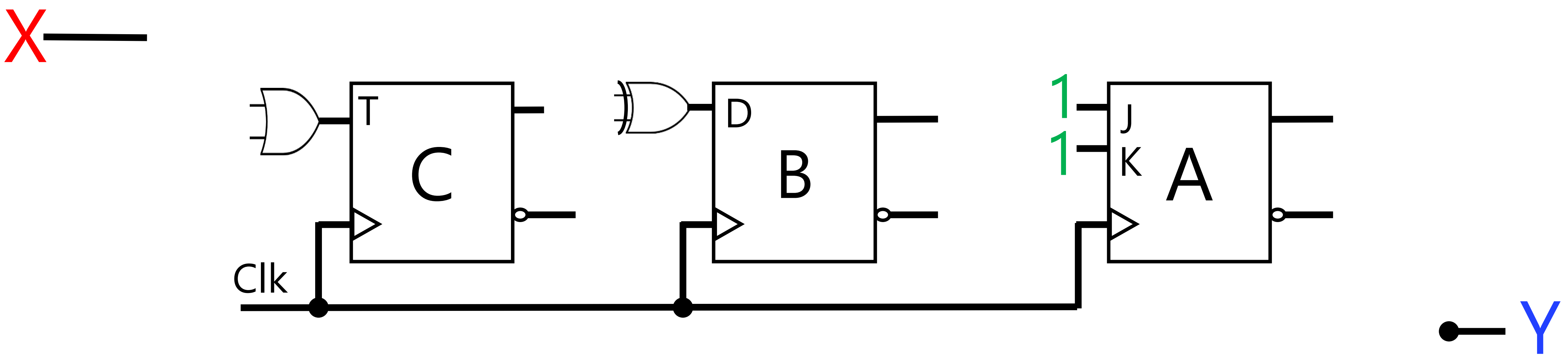
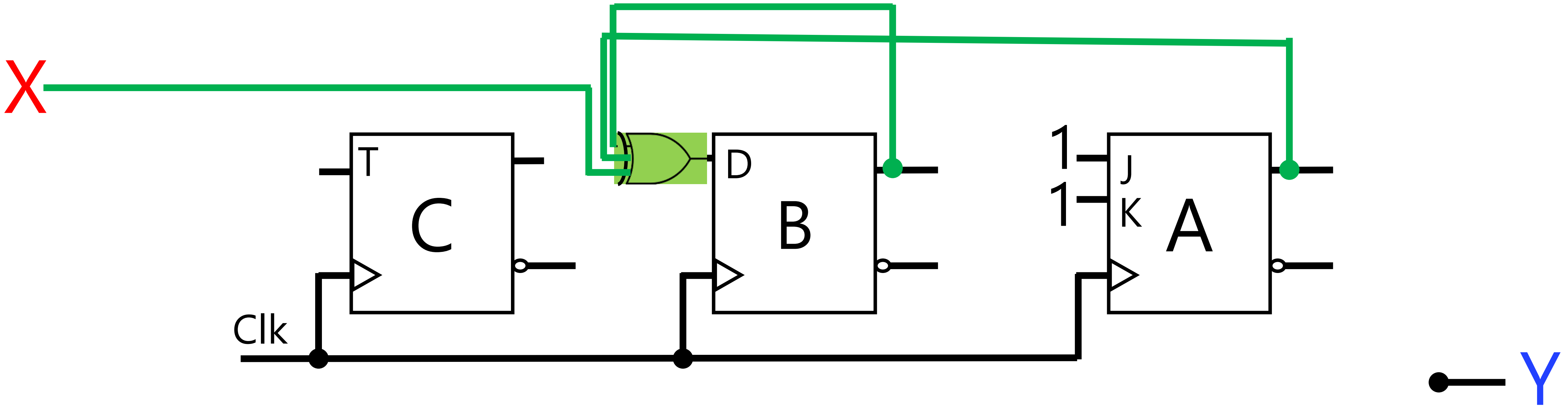
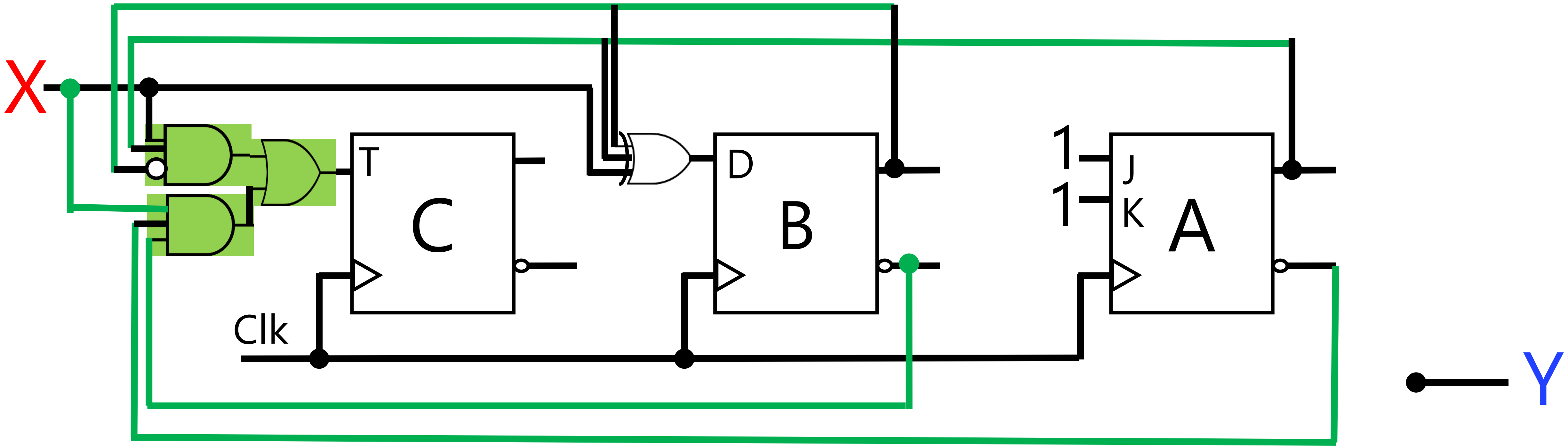
7. Input (*excitation*) equations for each FF
8. Minimization of input (*excitation*) equations
9. Boolean function for the output
10. Minimization of output variable
11. Draw/Sketch Logic Circuit
12. (Optional) Test



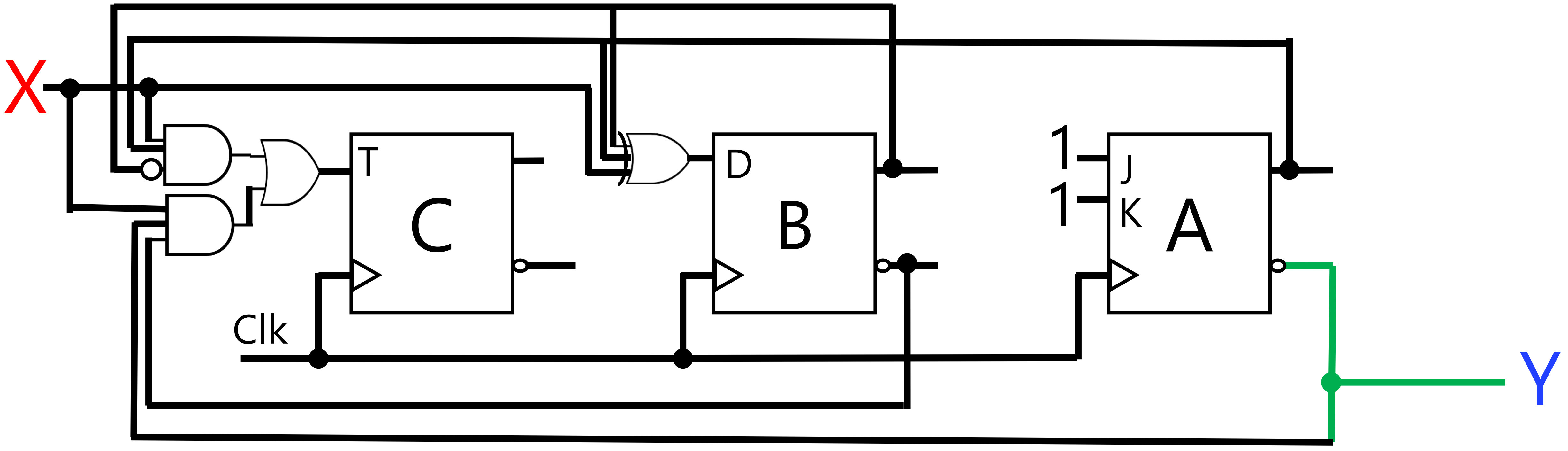
A	$J_A=1$ $K_A=1$
B	$D_B=F(X,C,B,A)=X\oplus B\oplus A$
C	$T_C=F(X,C,B,A)=X'BA + XB'A'$
Y	$F(C,B,A) = A'$



A	$J_A=1$	$K_A=1$
B	$D_B=F(X,C,B,A)=X\oplus B\oplus A$	
C	$T_C=F(X,C,B,A)=X'BA + XB'A'$	
Y	$F(C,B,A) = A'$	



A	$J_A=1$	$K_A=1$
B	$D_B=F(X,C,B,A)=X\oplus B\oplus A$	
C	$T_C=F(X,C,B,A)=X'BA + XB'A'$	
Y	$F(C,B,A) = A'$	



A	$J_A=1$	$K_A=1$
B	$D_B=F(X,C,B,A)=X\oplus B\oplus A$	
C	$T_C=F(X,C,B,A)=X'BA + XB'A'$	
Y	$F(C,B,A) = A'$	

Counter Up-Down
Example Mealy Model

Counting up and number is even
Counting up and number is odd
Counting down and number is odd
Counting down and number is even

Counter Up-Down

X=0: Count up from i to i+1

X=1: Count down from N i to i -1

Y = 1 when counting up (X=0) and number is even

Y = 0 when counting up (X=0) and number is odd

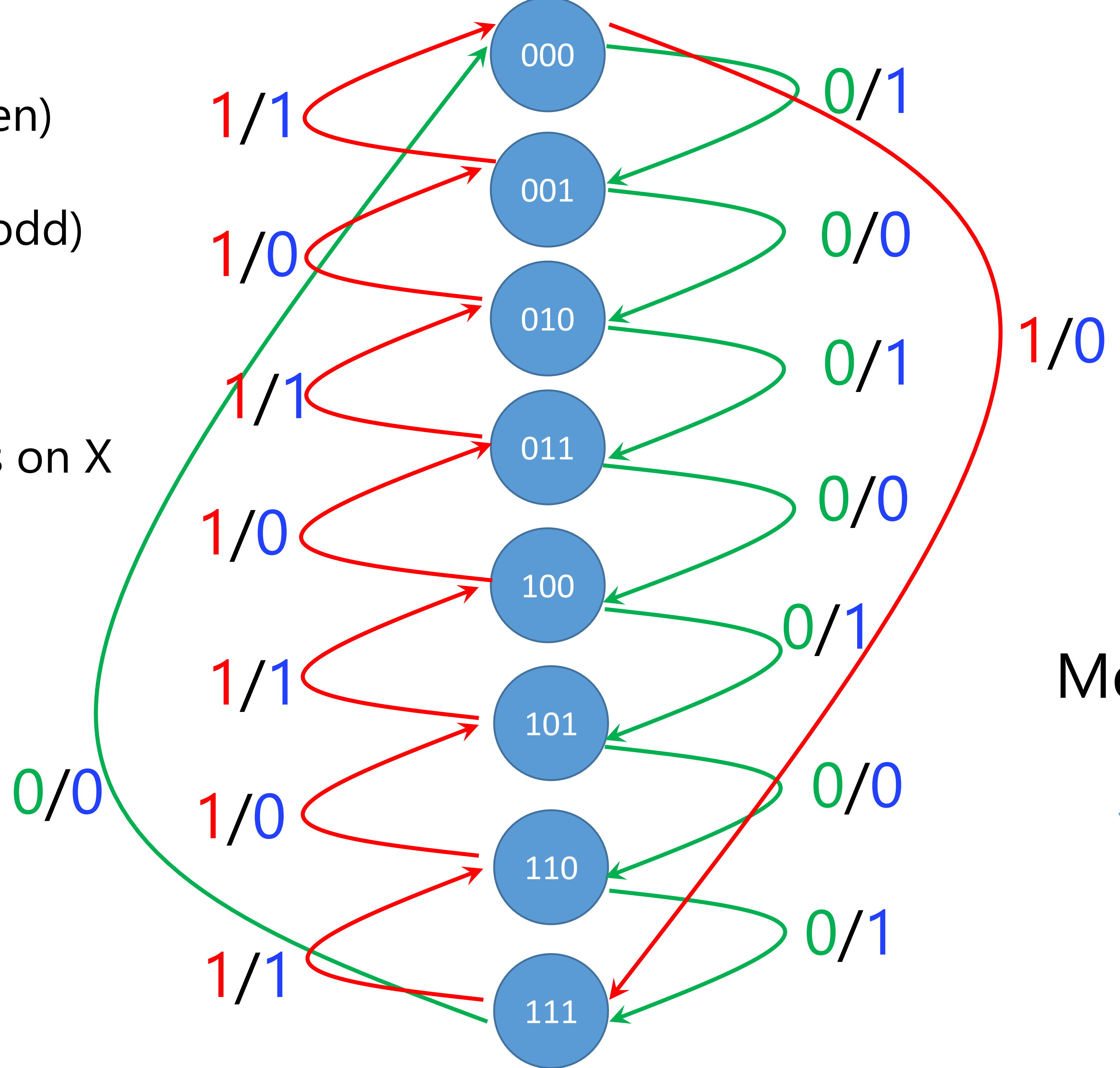
Y = 1 when counting down (X=1) and number is odd

Y = 0 when counting down (X=1) and number is even

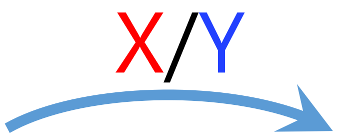
Y=1: (up AND even)
OR
(down AND odd)

Y=0 : else

Y clearly depends on X



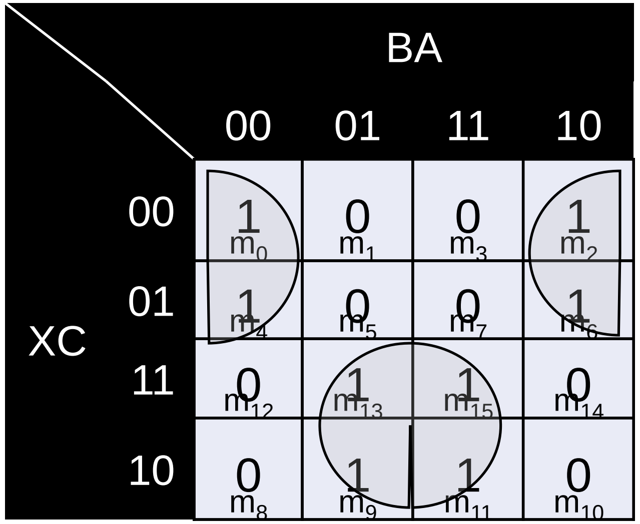
Mealy Model



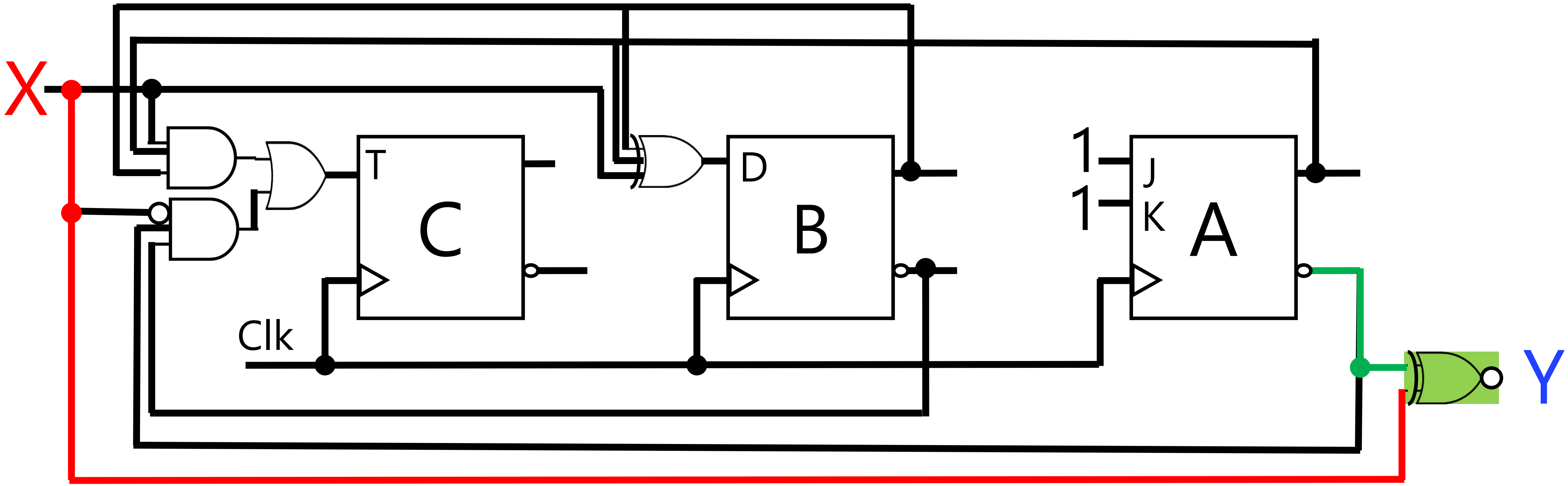
Inputs	Q(T)			Q(T+1)			Outputs
X	C	B	A	C	B	A	Y
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	0
0	0	1	0	0	1	1	1
0	0	1	1	1	0	0	0
0	1	0	0	1	0	1	1
0	1	0	1	1	1	0	0
0	1	1	0	1	1	1	1
0	1	1	1	0	0	0	0
1	0	0	0	1	1	1	0
1	0	0	1	0	0	0	1
1	0	1	0	0	0	1	0
1	0	1	1	0	1	0	1
1	1	0	0	0	1	1	0
1	1	0	1	1	0	0	1
1	1	1	0	1	0	1	0
1	1	1	1	1	1	0	1

$$Y = F(\underline{X},C,B,A) = \sum(0,2,4,6,9,11,13,15)$$

Minimization for output Y
4-Variable K-Map



$$
\begin{aligned}
Y &= F(\underline{X},C,B,A) = \sum(0,2,4,6,9,11,13,15) \\
&= X'A' + XA \\
&= X \odot A
\end{aligned}
$$



A	$J_A=1$	$K_A=1$
B	$D_B=F(X,C,B,A)=X \oplus B \oplus A$	
C	$T_C = F(X,C,B,A)=X'BA + XB'A'$	
Y	$F(C,B,A) = X \odot A$	