

### DISCRETE SYSTEMS

# POLL Click to Open the Poll

STAR	CONTINUOUS	DISCRETE
TEMPERATURE	CONTINUOUS	DISCRETE
ELECTRON	CONTINUOUS	DISCRETE
TIME	CONTINUOUS	DISCRETE
WEIGHT	CONTINUOUS	DISCRETE
SPEED	CONTINUOUS	DISCRETE
STUDENT	CONTINUOUS	DISCRETE
SOUND	CONTINUOUS	DISCRETE
IMAGE	CONTINUOUS	DISCRETE
PAIN	CONTINUOUS	DISCRETE
LIGHT	CONTINUOUS	DISCRETE
WAVE	CONTINUOUS	DISCRETE

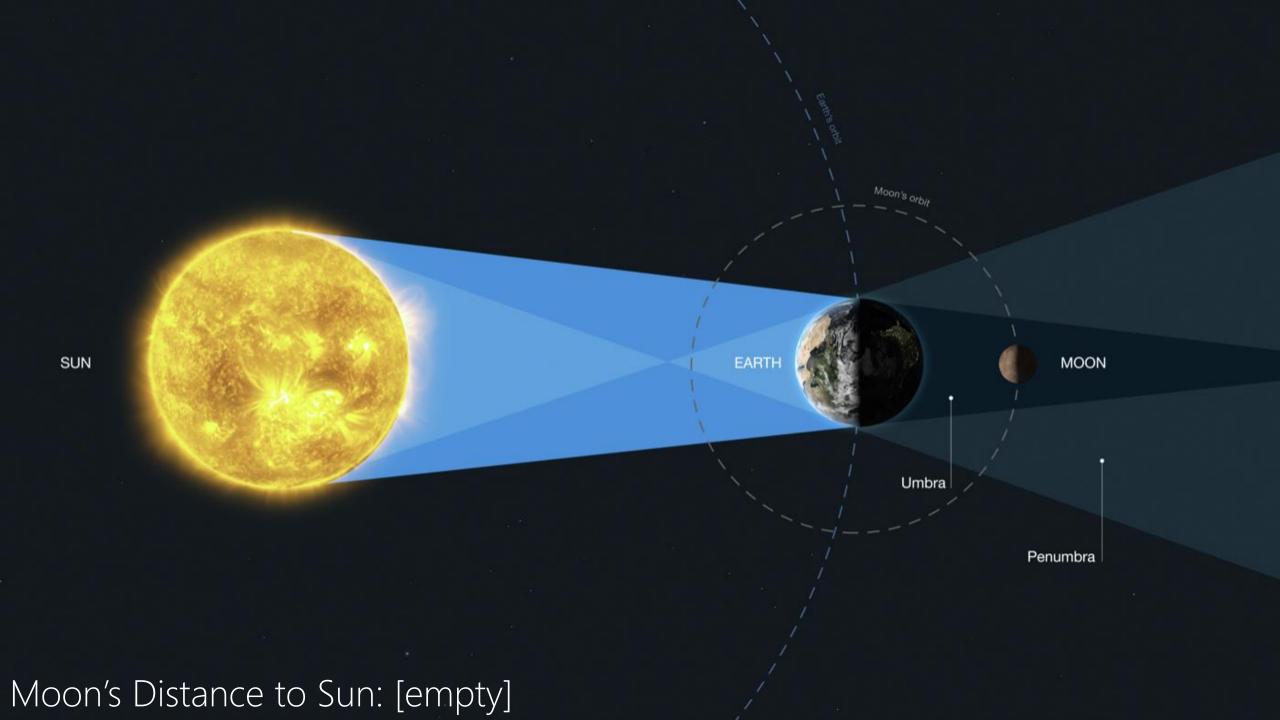
STAR		DISCRETE
TEMPERATURE	CONTINUOUS	
ELECTRON		DISCRETE
TIME	CONTINUOUS	
WEIGHT	CONTINUOUS	
SPEED	CONTINUOUS	
STUDENT		DISCRETE
SOUND	CONTINUOUS	
IMAGE	CONTINUOUS	DISCRETE
PAIN	CONTINUOUS	DISCRETE
LIGHT	CONTINUOUS	DISCRETE
WAVE	CONTINUOUS	

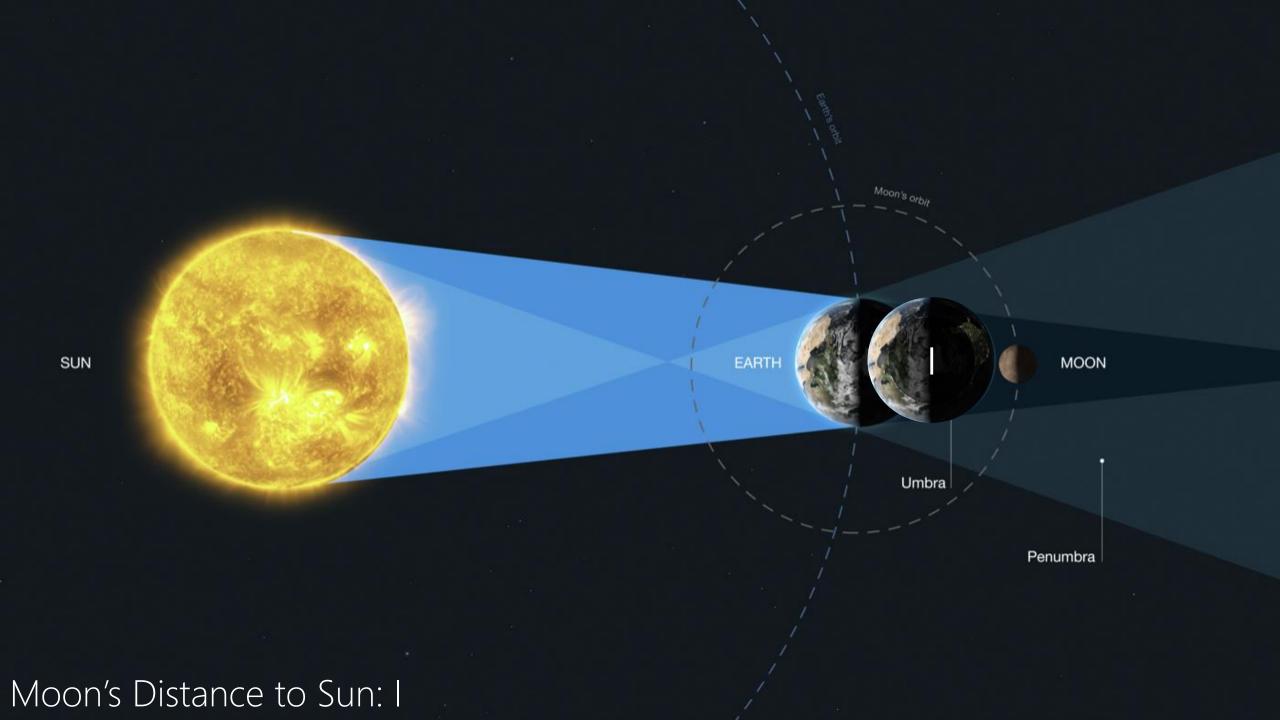


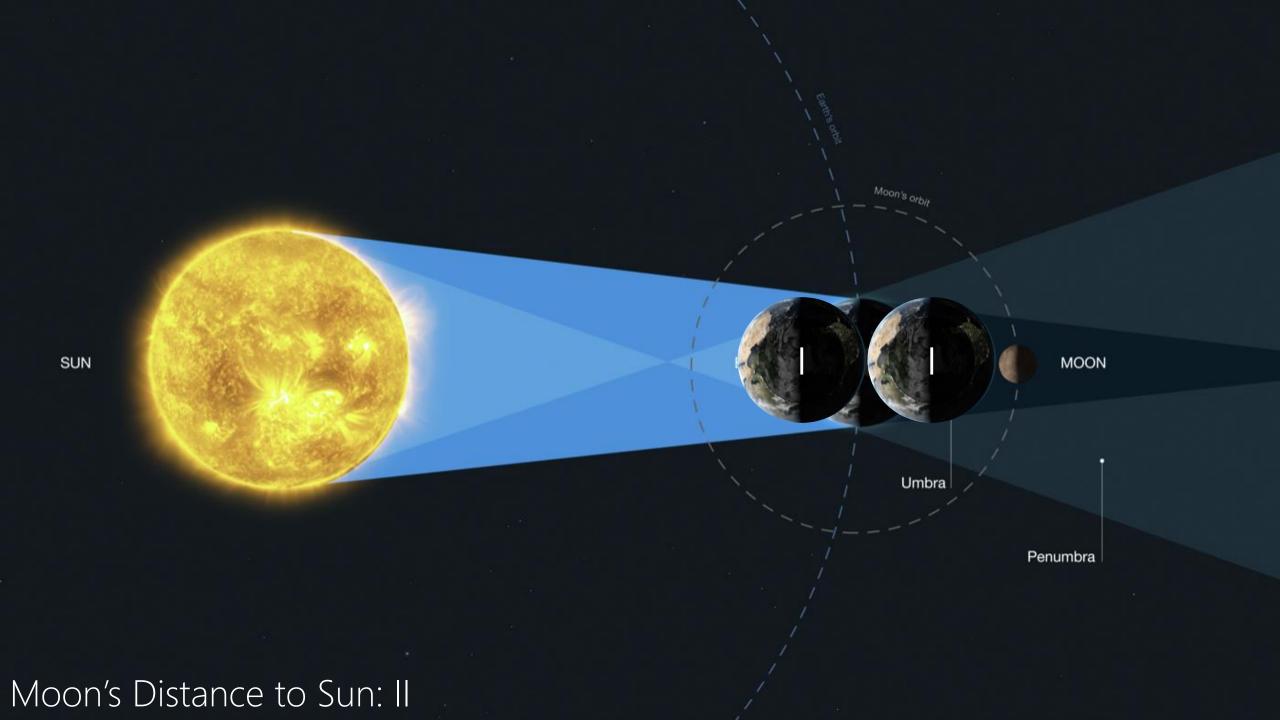
1827. Captured using a technique known as heliography, the shot was taken from an upstairs window at Niépce's estate in Burgundy. As heliography produces one-of-a-kind images, there are no duplicates of the piece, which is now part of the permanent collection at the University of Texas-Austin." 18 Famous First Photographs in History: From the Oldest Photo Ever to the World's First Instagram

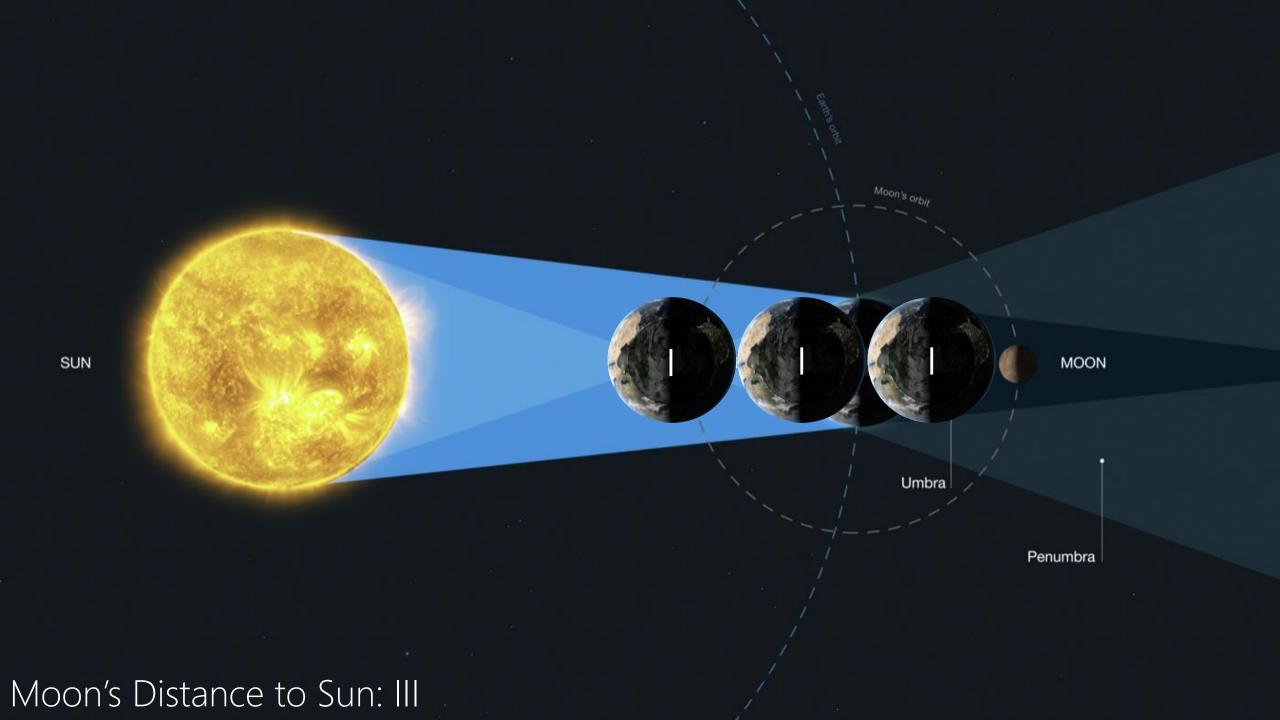


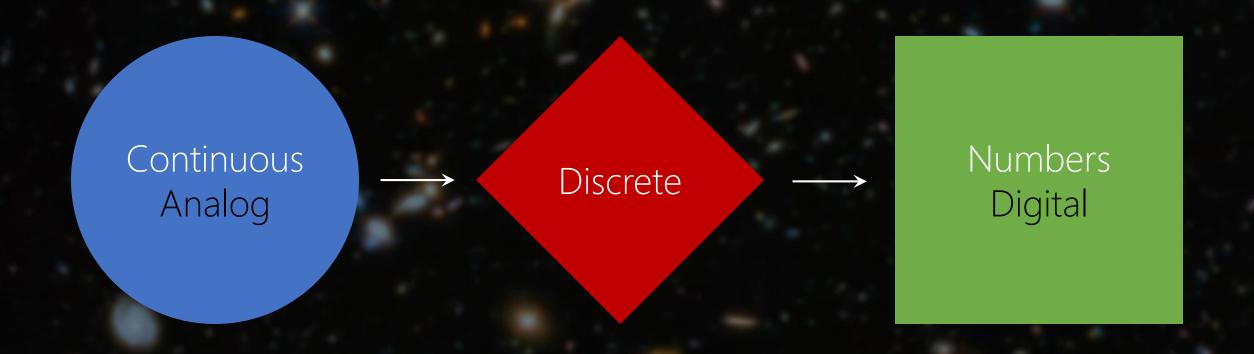












Quantization

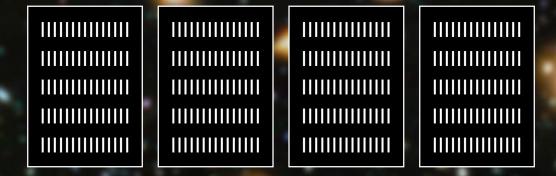
#### IIIIIIIVVVIVIIVIIIXX

Roman Numerals Originated in Ancient Rome 8th Century BC

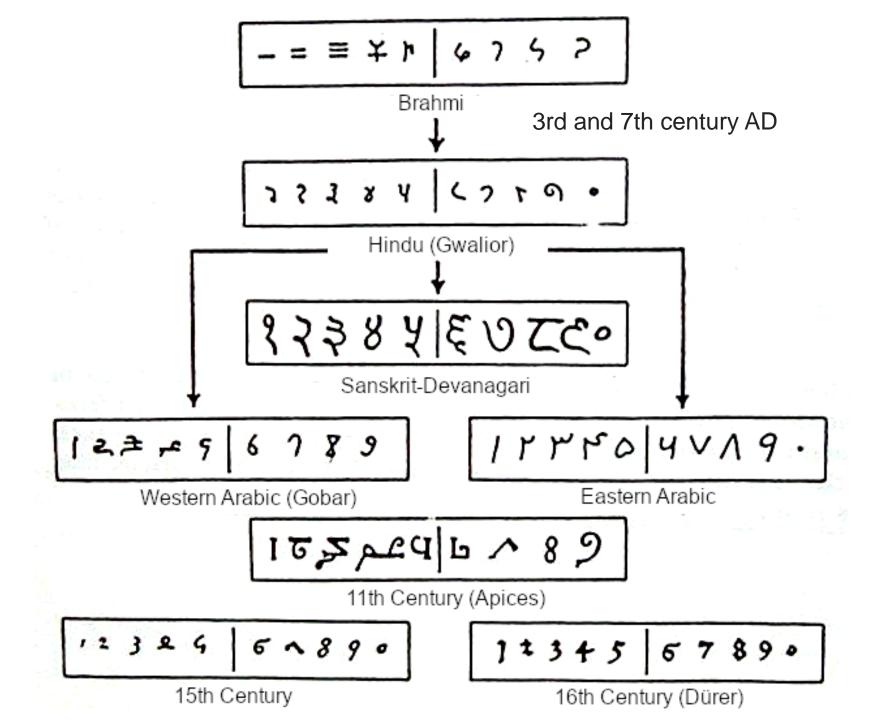
## UNARY SYSTEM aka. Base-1

how many positions to represent the moon's distance to the sun if an Oracle said it is ~150 million km and earth's diameter is ~13,000 km?

~150 million km  $\div$  ~13,000 km = ~12,000 Earth paper = ~3,000 positions 12,000  $\div$  3,000 = 4 pages!

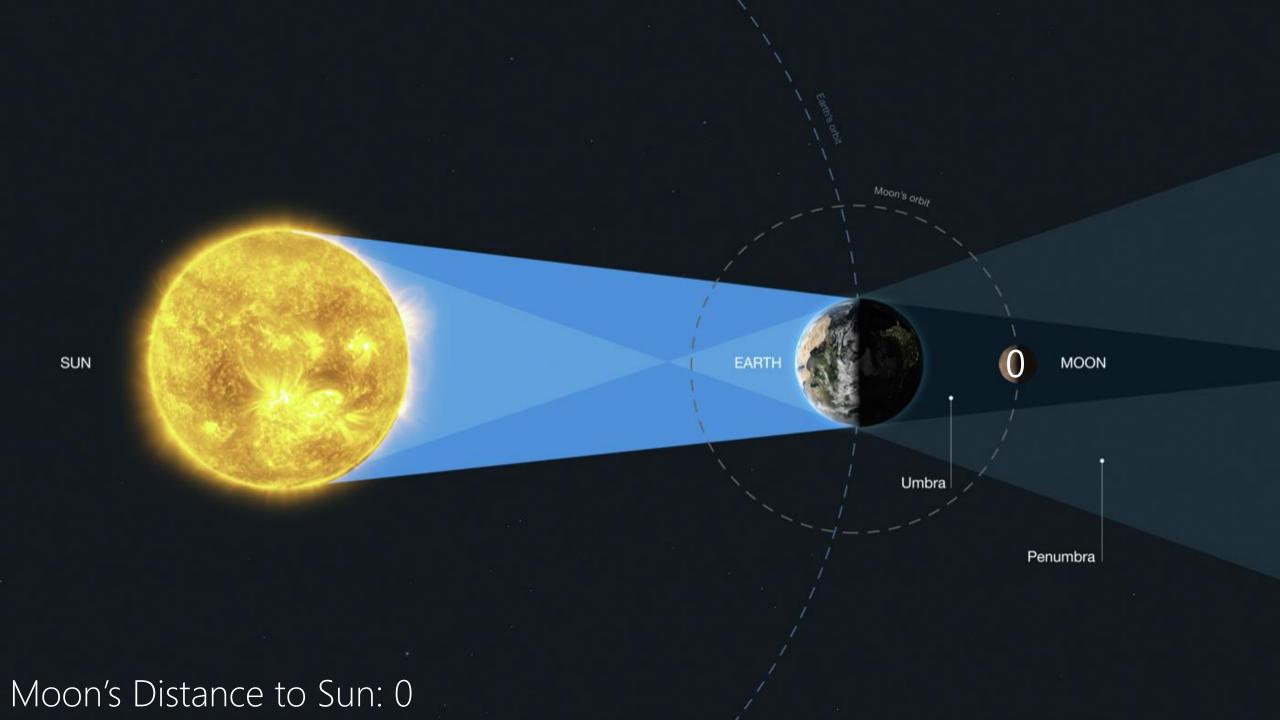


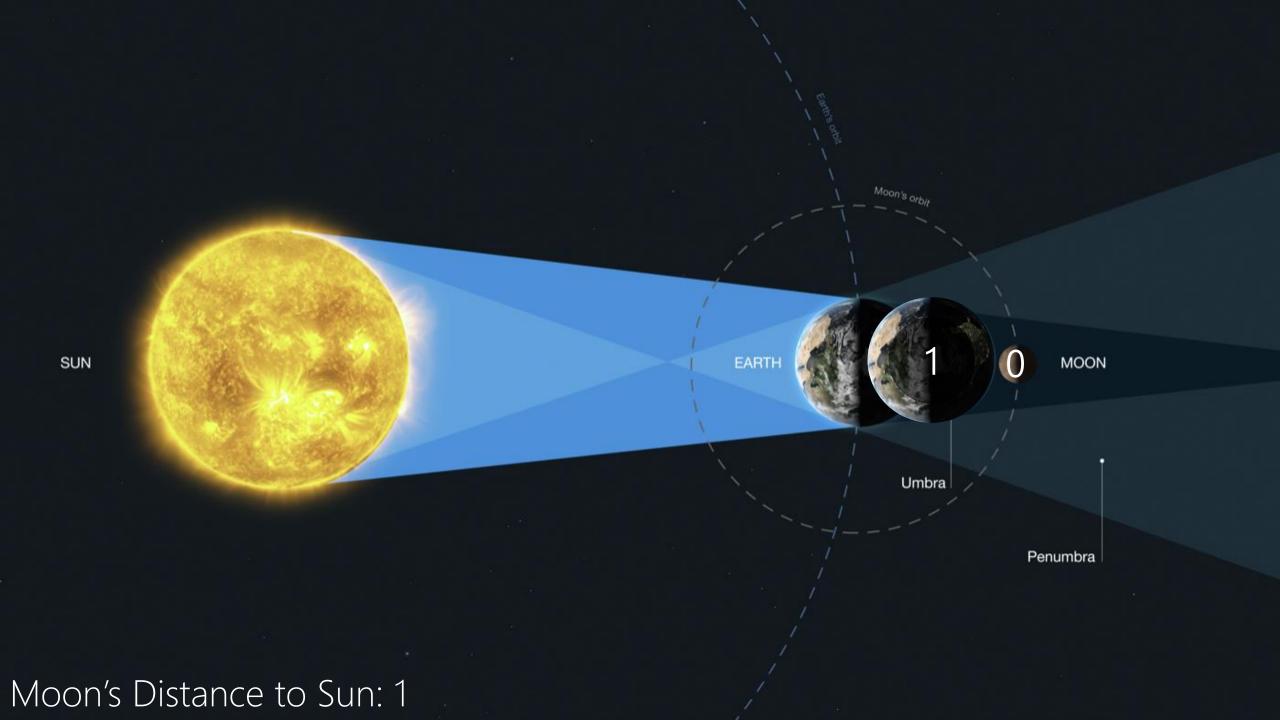
# NUMBER SYSTEMS

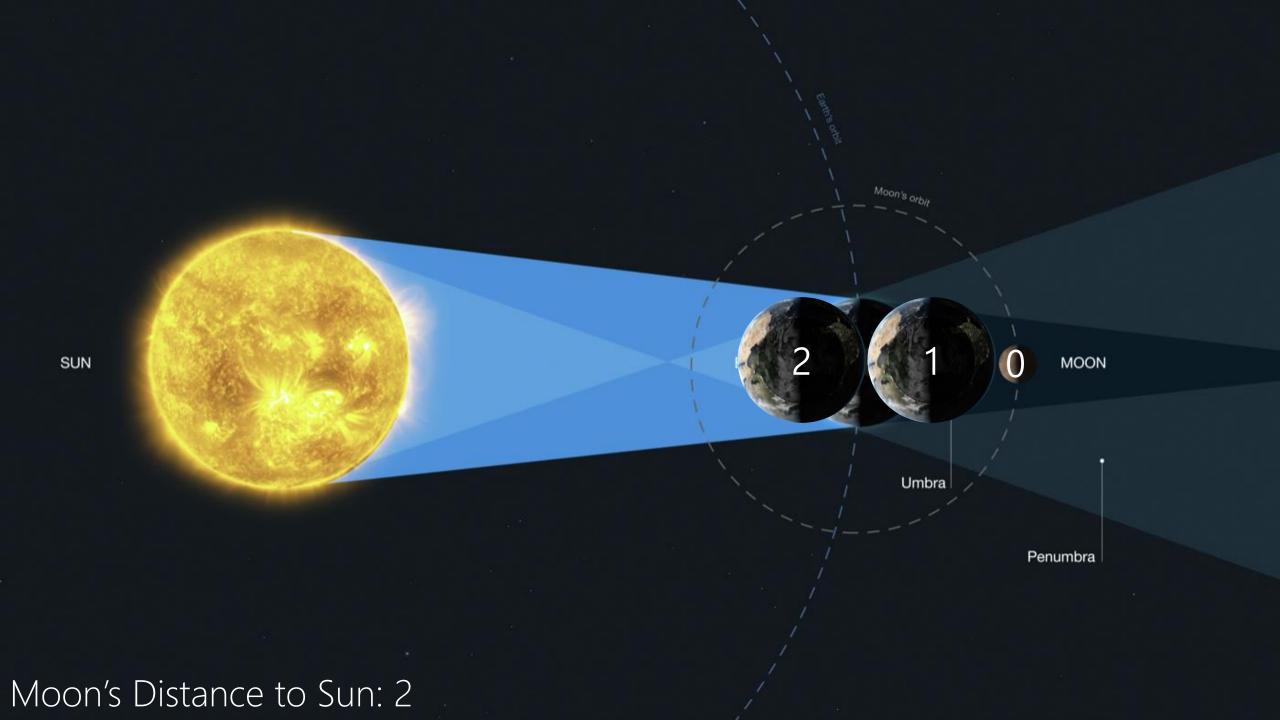


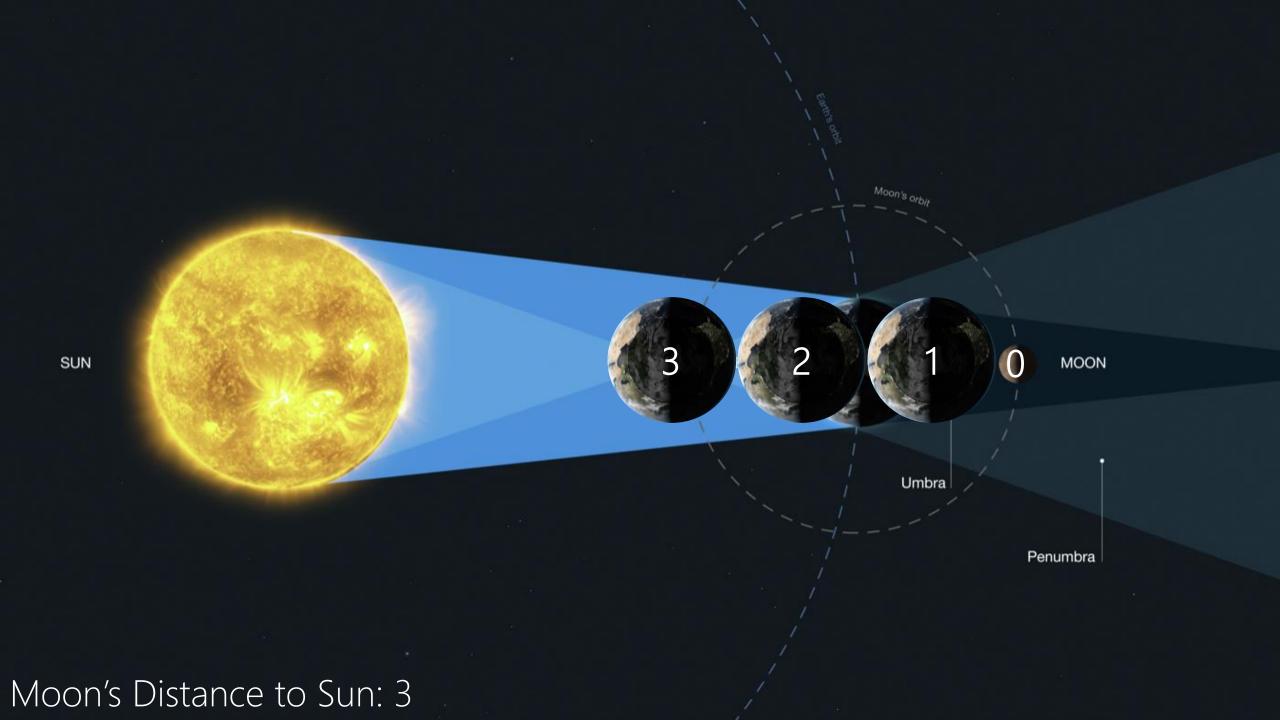
#### 0123456789

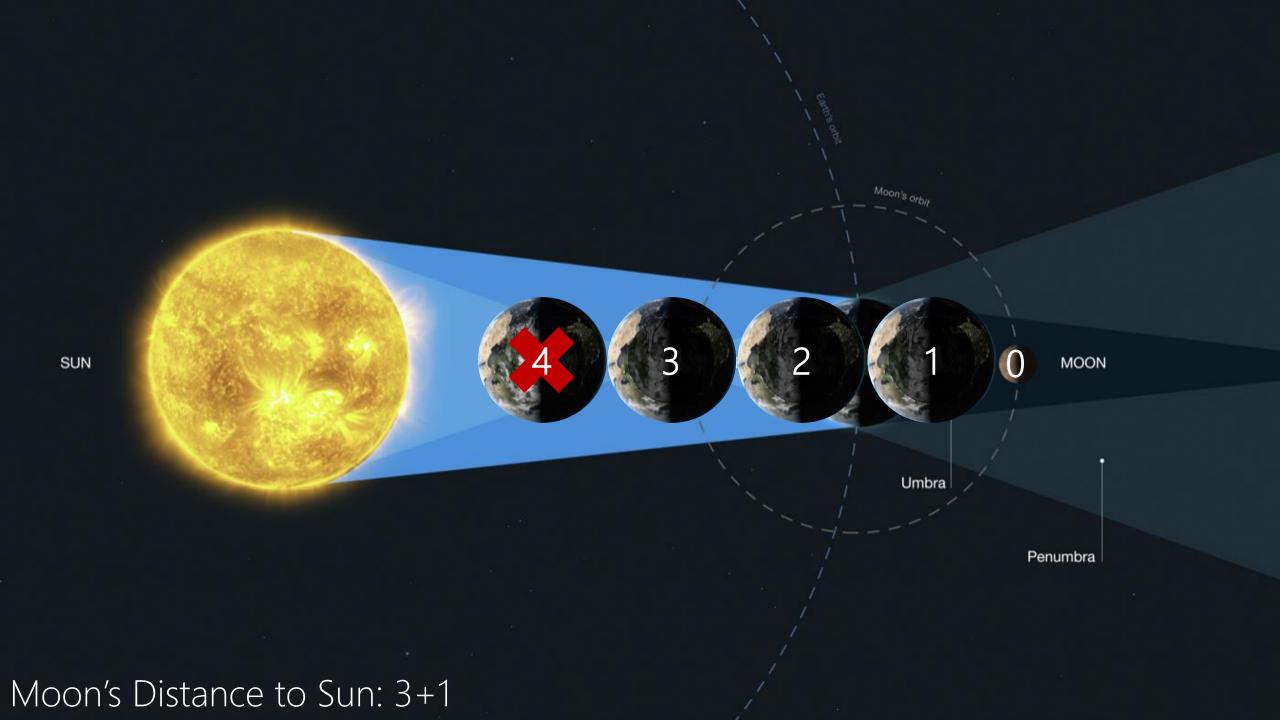
Hossein's Number System

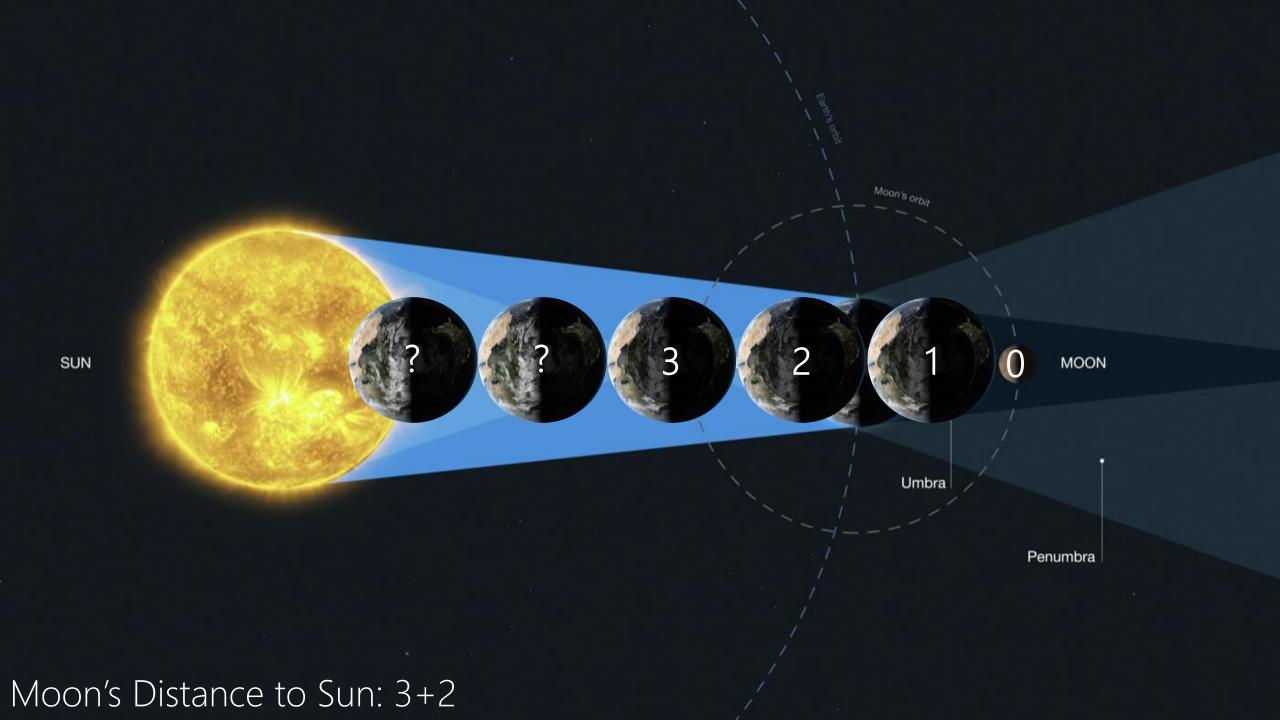


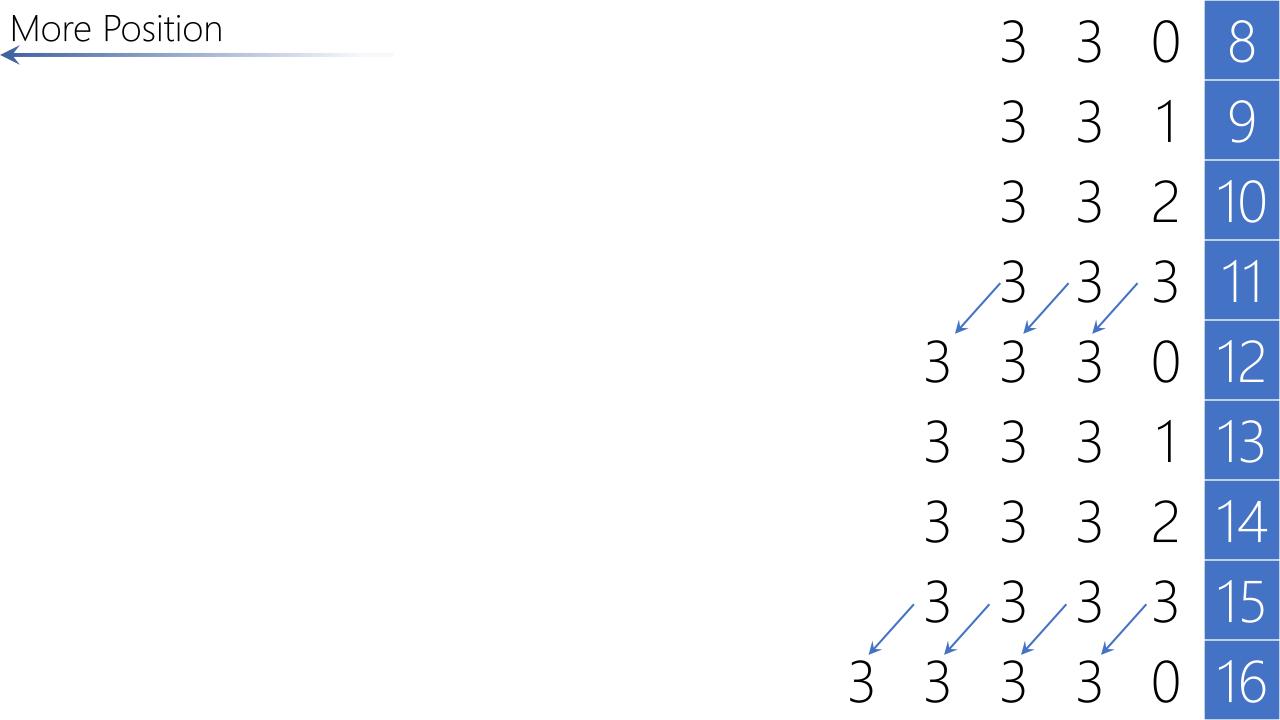




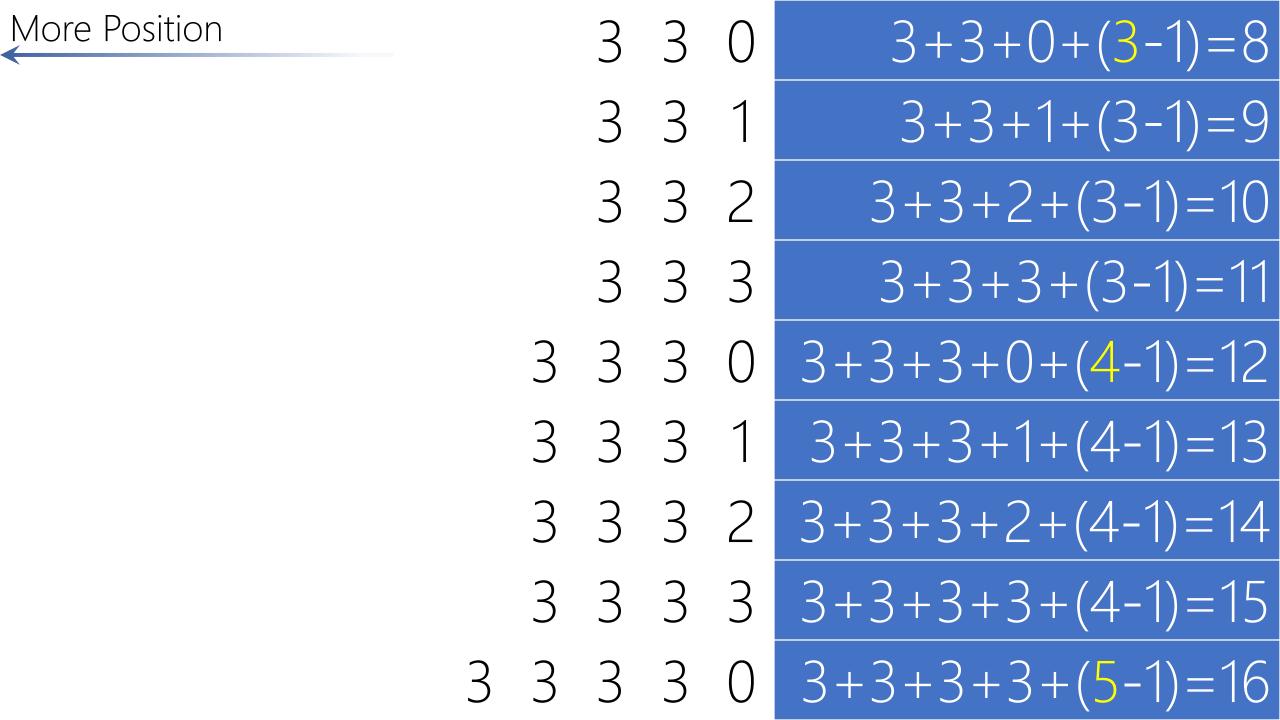








More Position	0	(1-1)=0
	1	1+(1-1)=1
	2	2+(1-1)=2
	3	3+(1-1)=3
	3 0	3+0+(2-1)=4
	3 1	3+1+(2-1)=5
	3 2	3+2+(2-1)=6
	3 3	3+3+(2-1)=7
	3 3 0	3+3+(3-1)=8

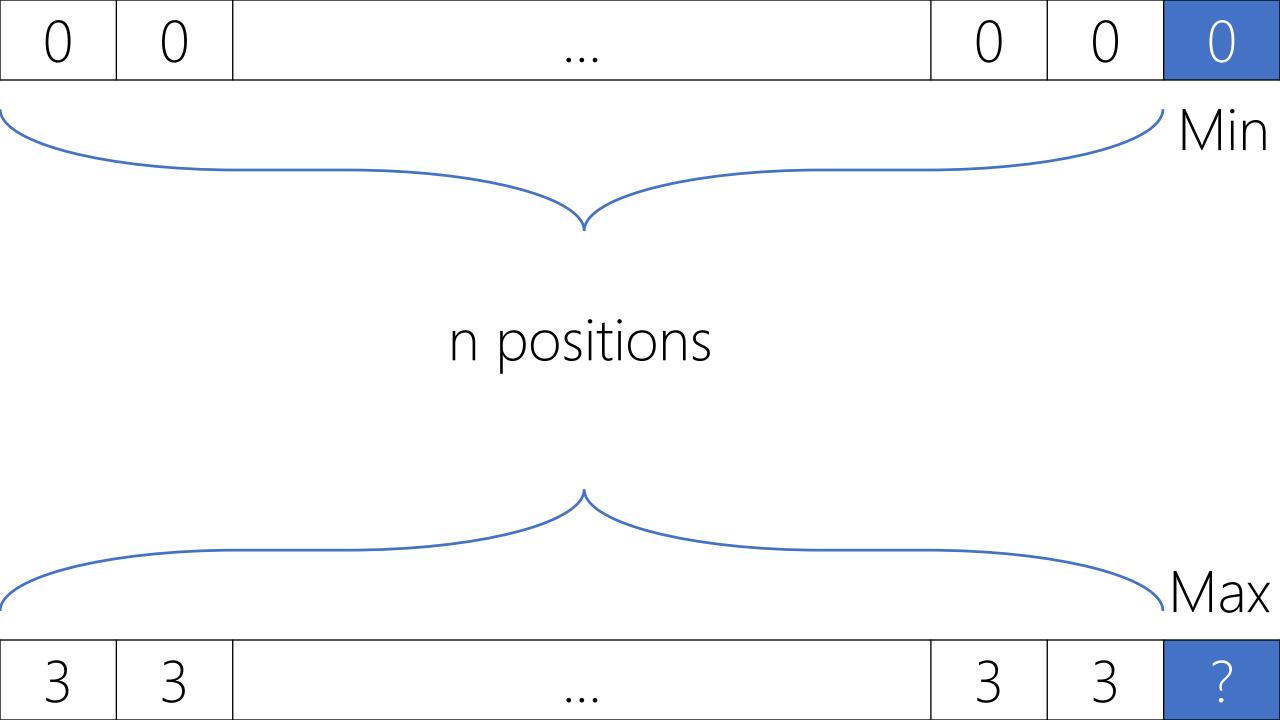


		3	0	3	0	2	1	3	1	?
					3	3	3	3	1	
				3	3	3	3	3	2	?
		3	0	0	3	3	3	3	0	?
3	3	3	3	3	3	3	3	3	3	?

		3	$\bigcirc$	$\cap$	0	2	1	3		- 1
					3	3	3	3	1	?
				3	3	3	3	3	2	?
		3	0	0	3	3	3	3	0	?
3	3	3	3	3	3	3	3	3	3	?

		3	0	3	0	2	1	3	1	_	
					3	3	3	3	1		3*4+1+(5-1)
				3	3	3	3	3	2	?	
		3	0	0	3	3	3	3	0	?	
3	3	3	3	3	3	3	3	3	3	?	

		3	0	3	0	2	1	3	1	_
					3	3	3	3	1	17
				3	3	3	3	3	2	22
		3	0	0	3	3	3	3	0	_
3	3	3	3	3	3	$\cap$	3	3	3	39



$$3+3+3+\cdots+3+(n-1) = 3\times n + (n-1)=4n-1$$

n positions

3 3 ... 3 4n+

$$Max = 4n-1$$

$$n=10 \Rightarrow (4\times10)-1 = 39$$

10 positions

					IVIAX
3	3	• • •	3	3	39

$$Max = N = 4n-1$$

$$N+1 = 4n$$

$$(N+1) \div 4 = n$$

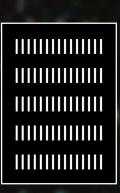
? positions

3 3 ... 3 N

how many positions to represent the moon's distance to the sun in Hossein's system if an Oracle said it is ~150 million km and earth's diameter is ~13,000 km?

~150 million km  $\div$  ~13,000 km = ~12,000 Earth N = 12,000 n = (N+1) $\div$ 4 = (12,000+1) $\div$ 4 = ~3,000 positions paper = ~3,000 positions 3,000  $\div$  3,000 = 1 pages



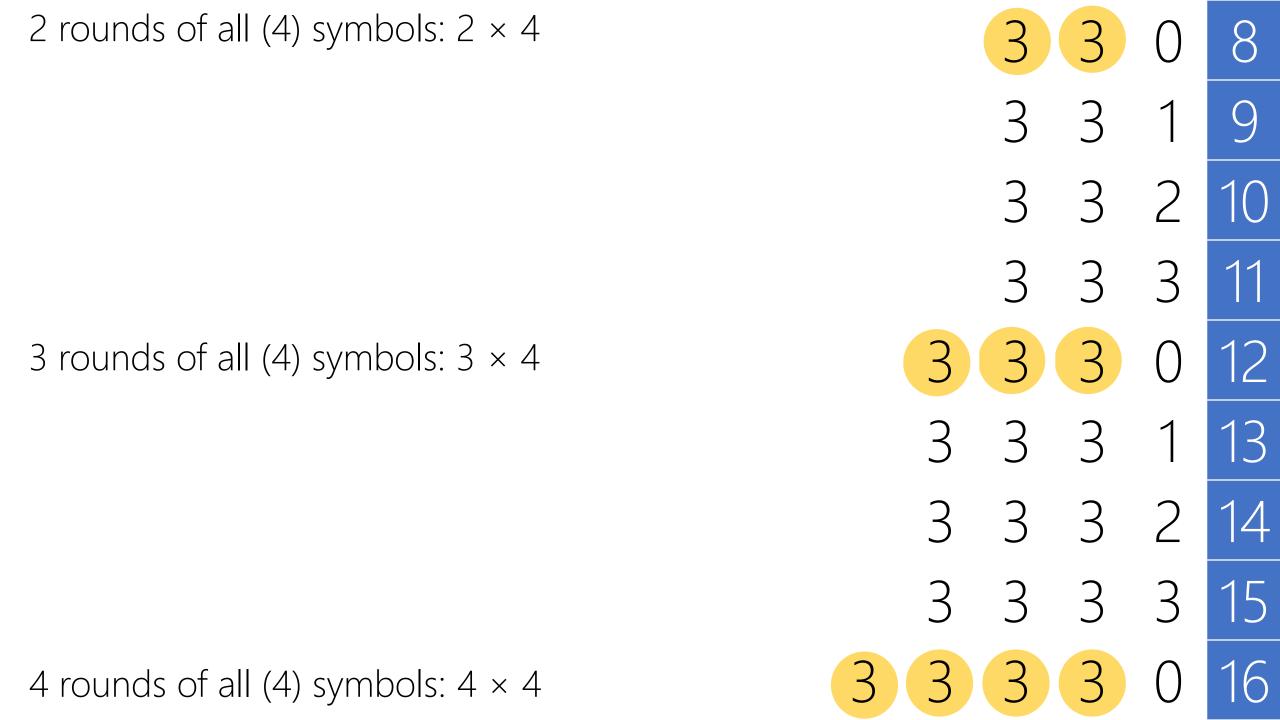


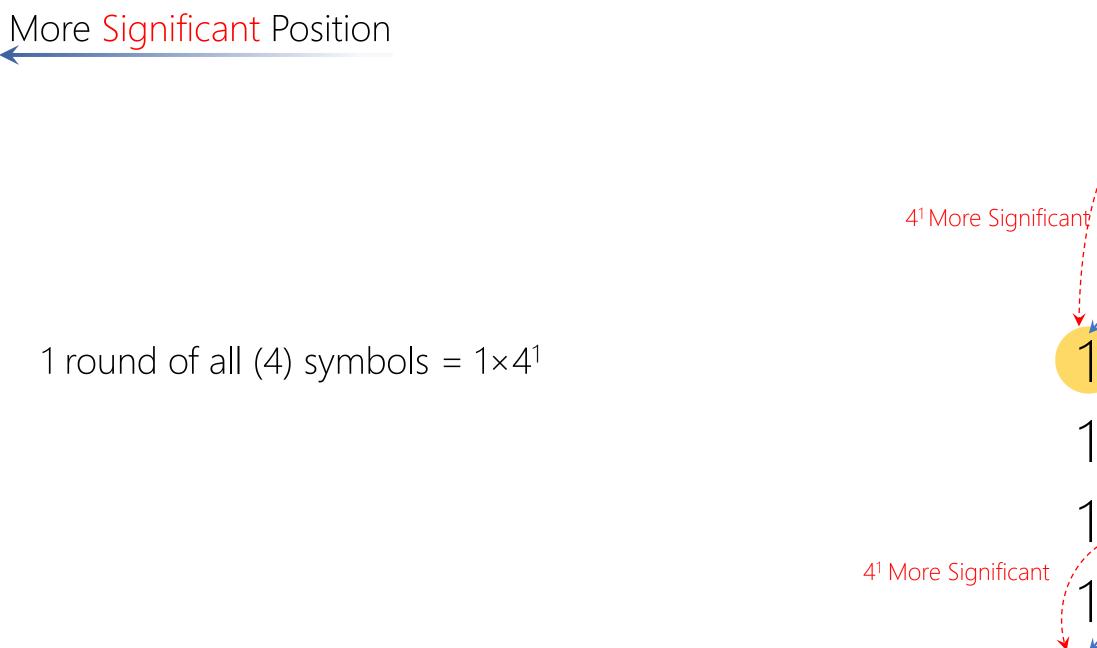




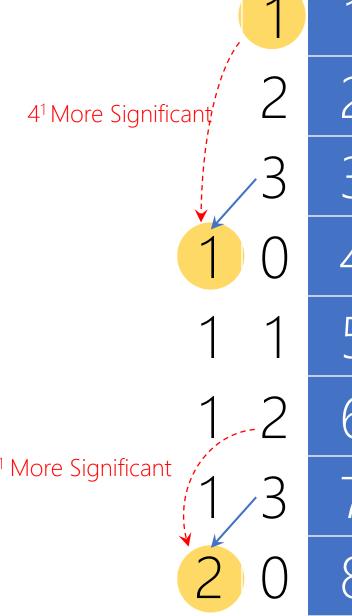


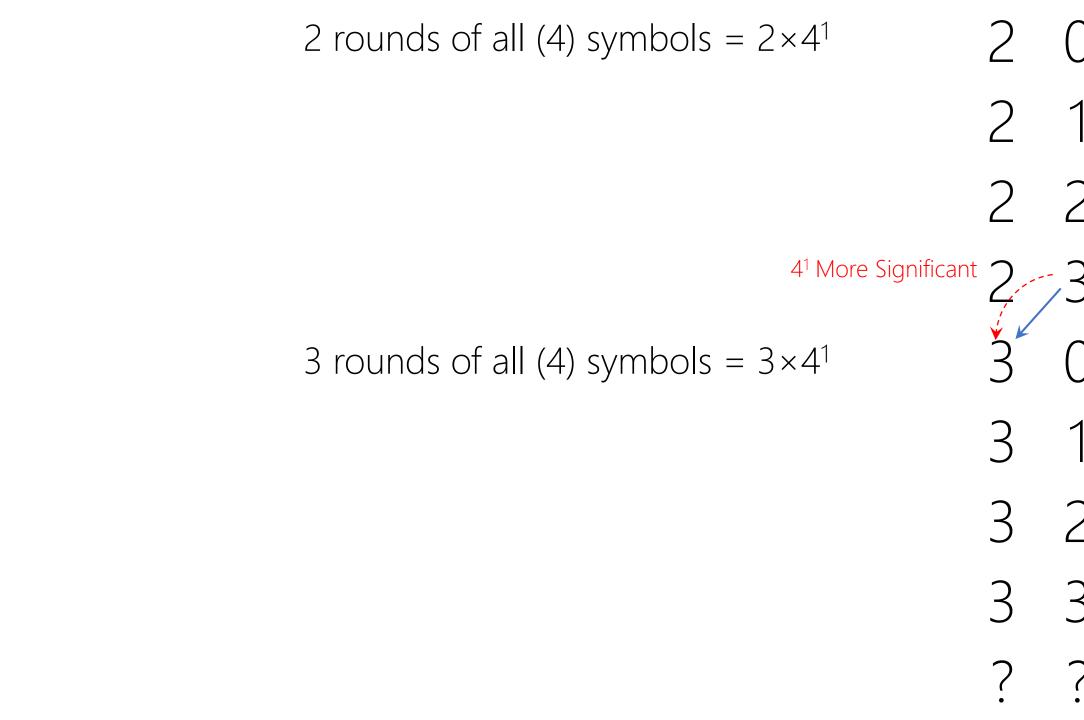
More Position		0	0
		1	1
		2	2
		3	3
1 rounds of all (4) symbols: 1 × 4	3	0	4
	3	1	5
	3	2	6
	3	3	7
2 rounds of all (4) symbols: 2 × 4	3 3	0	8

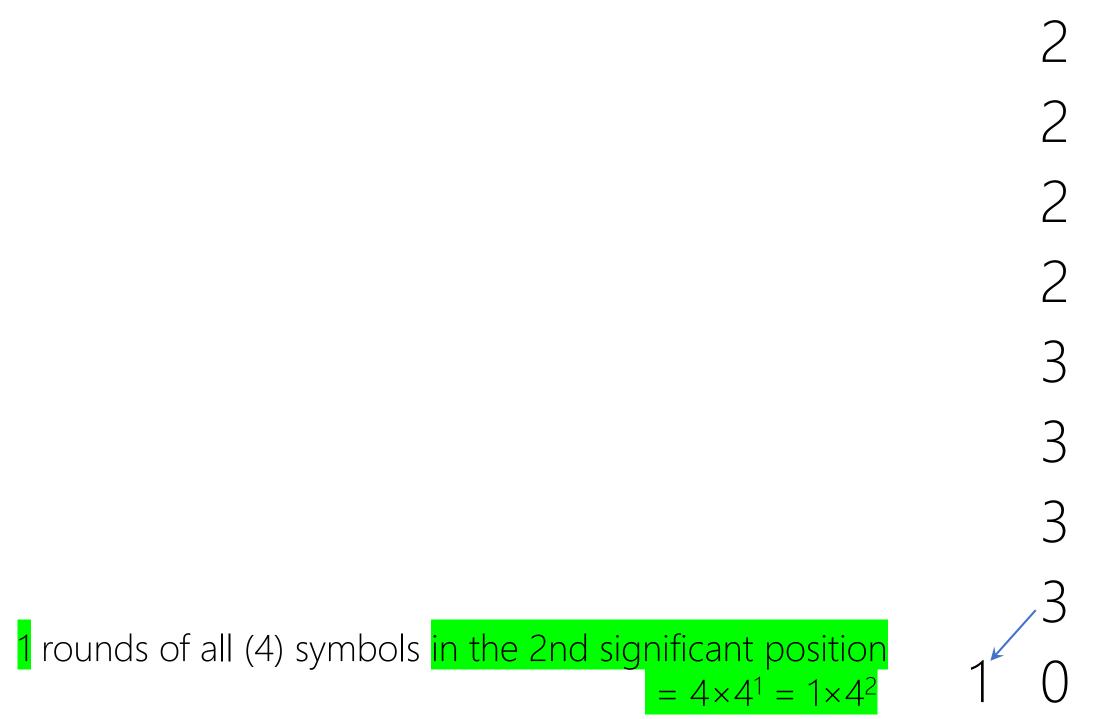


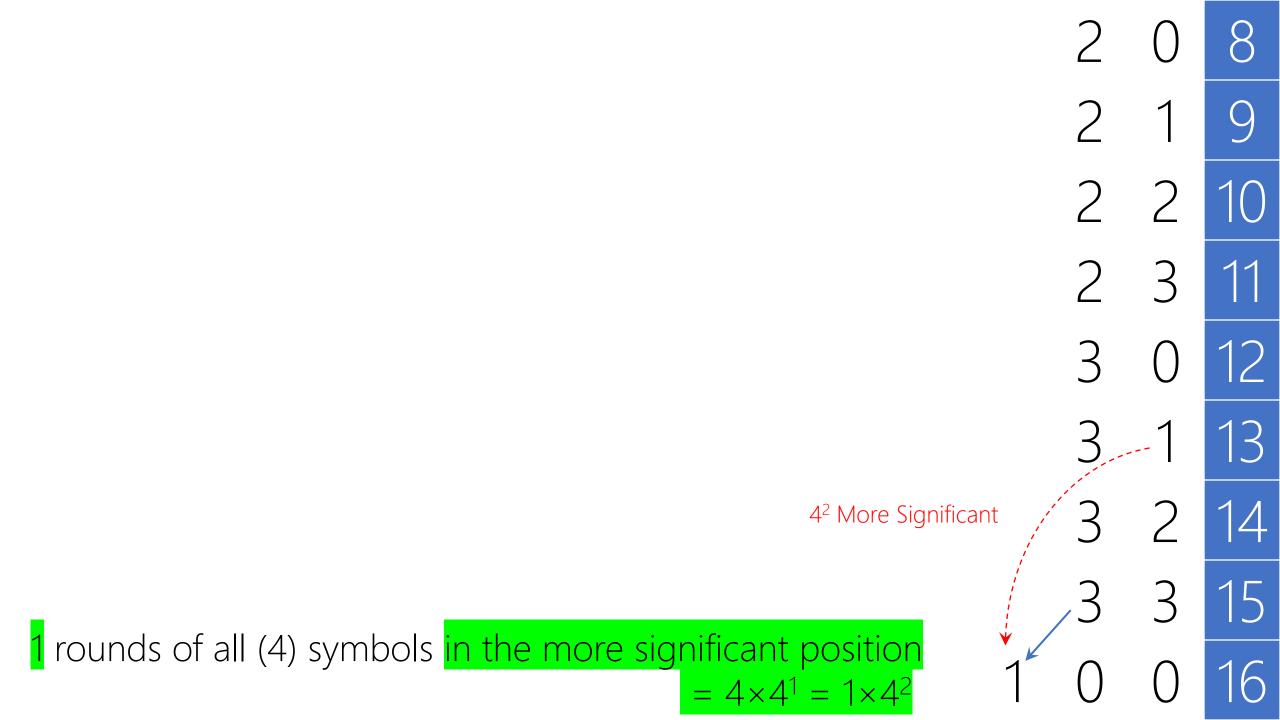


2 rounds of all (4) symbols =  $2 \times 4^{1}$ 





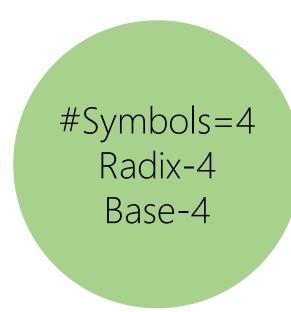




# QUATERNARY SYSTEM aka. Base-4, Radix-4

 $(0,1,2,3)_4$ 

Hindu-Arabic Numerals
Originated in India
7th Century AD



40	
0	$0 \times 4^{0} = 0$
1	$1 \times 4^{0} = 1$
2	$2 \times 4^0 = 2$
3	$3 \times 4^0 = 3$

#Symbols=4 Radix-4 Base-4

	$4^{0}$	41
$0 \times 4^{0} = 0$	0	
$1 \times 4^0 = 1$	1	
$2 \times 4^0 = 2$	2	
$3 \times 4^0 = 3$	3	
$1 \times 4^1 + 0 \times 4^0 = 4$	0	1
$1 \times 4^1 + 1 \times 4^0 = 5$	1	1
$1 \times 4^1 + 2 \times 4^0 = 6$	2	1
$1 \times 4^1 + 3 \times 4^0 = 7$	3	1
$2 \times 4^{1} + 0 \times 4^{0} = 8$	0	2

#Symbols=4 Radix-4 Base-4

42	41	40	
	2	0	$2 \times 4^{1} + 0 \times 4^{0} = 8$
	2	1	$2 \times 4^1 + 1 \times 4^0 = 9$
	2	2	$2 \times 4^{1} + 2 \times 4^{0} = 10$
	2	3	$2 \times 4^{1} + 3 \times 4^{0} = 11$
	3	0	$3 \times 4^{1} + 0 \times 4^{0} = 12$
	3	1	$3 \times 4^{1} + 1 \times 4^{0} = 13$
	3	2	$3 \times 4^{1} + 2 \times 4^{0} = 14$
	3	3	$3 \times 4^{1} + 3 \times 4^{0} = 15$
1	0	0	$1 \times 4^2 + 0 \times 4^1 + 0 \times 4^0 = 16$

	40	4'	44	
$2 \times 4^{1} + 0 \times 4^{1}$	0	2		
$2 \times 4^{1} + 1 \times 4^{1}$	1	2		
$2 \times 4^{1} + 2 \times 4^{0}$	2	2		
$2 \times 4^{1} + 3 \times 4^{0}$	3	2		
$3 \times 4^{1} + 0 \times 4^{0}$	0	3		
$3 \times 4^{1} + 1 \times 4^{0}$	1	3		
$3 \times 4^{1} + 2 \times 4^{0}$	2	3		
$3 \times 4^{1} + 3 \times 4^{0}$	3	3		
$1 \times 4^2 + 0 \times 4^1 + 0 \times 4^0$	0	0	1	

#Symbols=4 Radix-4 Base-4

		3	0	3	0	2	1	3	1	· ·
					$   \mathcal{O} $	$\cap$	$ \bigcirc $	3	1	· ·
				3	$   \mathcal{O} $	$ \bigcirc $		3	2	· ·
		3	0	0	3	3	3	3	0	?
3	3	3	3	3	3	3	3	3	3	?

		3	0	$\cap$	0	2	1	3		?
					3	3	3	3		?
				3	3	3	3	3	2	
		3	0		3	3	3	3	0	
3	3	3	3	3	3	3	3	3	3	

47	46	45	44	43	42	4 <sup>1</sup>	40
3	0	3	0	2	1	3	1

47	46	4 <sup>5</sup>	44	43	<b>4</b> <sup>2</sup>	41	40	X
3	0	3	0	2	1	3	1	
$3\times4^7$	0×4 <sup>6</sup>	$3\times4^5$	$0\times4^4$	$2\times4^3$	$1\times4^2$	$3\times4^{1}$	$1\times4^{0}$	

47	46	4 <sup>5</sup>	44	43	42	4 <sup>1</sup>	40	X
3	0	3	0	2	1	3	1	
$3\times4^7$	0×4 <sup>6</sup>	$3\times4^5$	$0\times4^4$	$2\times4^3$	$1\times4^2$	$3\times4^{1}$	1×4 <sup>0</sup>	$\sum$

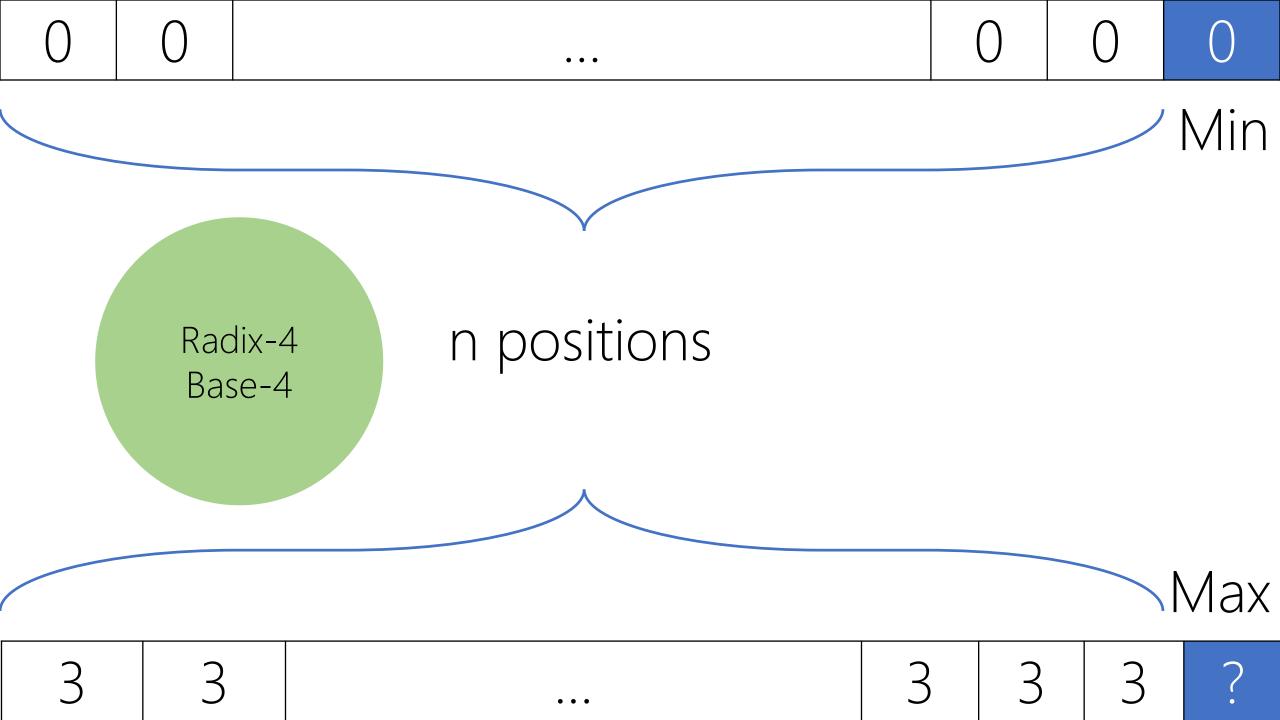
65,437

		3	0	3	0	2	1	3	1	65,437
					3	3	3	3	1	
				3	3	3	3	3	2	?
		3	0	0	3	3	3	3	0	
3	3	3	3	3	3	3	3	3	3	?

		3	0	3	0	2	1	3	1	65,437
					3	3	3	3	1	1,021
				3	3	3	3	3	2	?
		3	0	0	3	3	3	3	0	
3	3	3	3	3	3	3	3	3	3	?

		3	0	3	0	2	1	3	1	65,437
					3	3	3	3	1	1,021
				3	$   \mathcal{O} $	3	$   \mathcal{O} $	3	2	4,094
		3	0	0	3	3	3	3	0	50,172
3	3	3	3	3	3	3	3	3	3	1,048,575

										Base-4	Hossein's Number System
		3	0	$\cap$	0	2	1	3	\	65,437	_
					$\cap$	3	3	3	<b>\</b>	1,021	17
				$\cap$	$\cap$	3	3	3	2	4,094	22
		3	0	0	3	3	3	3	0	50,172	_
3	3	3	3	3	3	3	3	3	3	1,048,575	39



$$N = 3 \times 4^{n-1} + 3 \times 4^{n-2} + \dots + 3 \times 4^2 + 3 \times 4^1 + 3 \times 4^0$$

$$N = 3 \times (4^{n-1} + 4^{n-2} + \dots + 4^2 + 4^1 + 4^0)$$

$$N = 3 \times \left(\frac{4^n - 1}{4 - 1}\right)$$

$$N = 4^n - 1$$

4n − 1 Hossein's System

#### n positions

Max

4n-1	4n-2		42	41	40	
3	3	• • •	$\omega$	$\Im$	3	N

$$n = 10 \Rightarrow 410 - 1 = 1,048,575$$

10 positions

<b>/</b>				•	1 7 1 017 1
3	3	• • •	3	3	?

$$4^{n} - 1 = N$$

$$4^{n} = N + 1$$

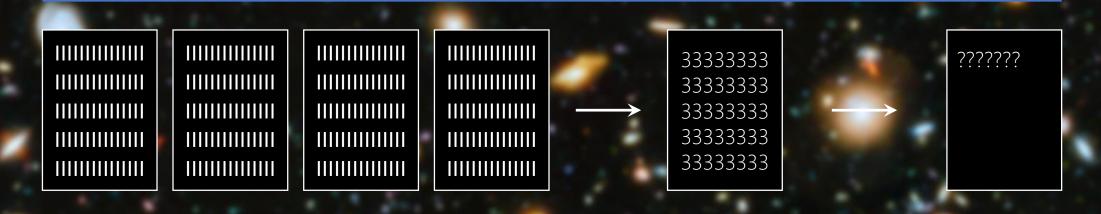
$$n = \log_{4}(N + 1)$$

? positions

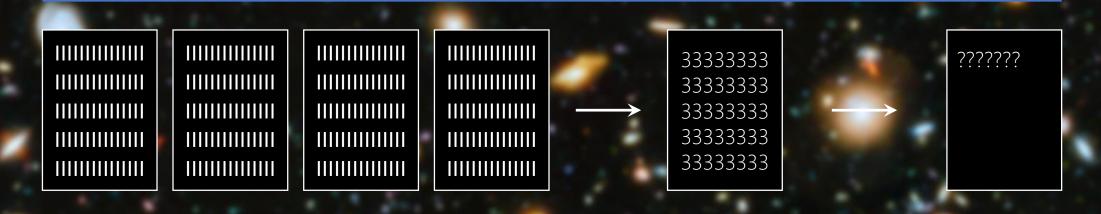
3 3 ... 3 N

how many positions to represent the moon's distance to the sun in Base-4 system if an Oracle said it is ~150 million km and earth's diameter is ~13,000 km?

~150 million km  $\div$  ~13,000 km = ~12,000 Earth N = 12,000 n = Log<sub>4</sub> (12,000+1) = Log<sub>10</sub> 12,001  $\div$  Log<sub>10</sub> 4 = 4  $\div$  0.6 = 6.79 ~ 7 positions



~150 million km  $\div$  ~13,000 km = ~12,000 Earth N = 12,000 n = Log<sub>4</sub> (12,000+1) = Log<sub>10</sub> 12,001  $\div$  Log<sub>10</sub> 4 = 4  $\div$  0.6 = 6.79 ~ 7 positions



$$N = 12,000 \rightarrow (2323200)_4$$

We'll see how to convert from decimal to base-4 or any other number systems later. Stay tuned!

