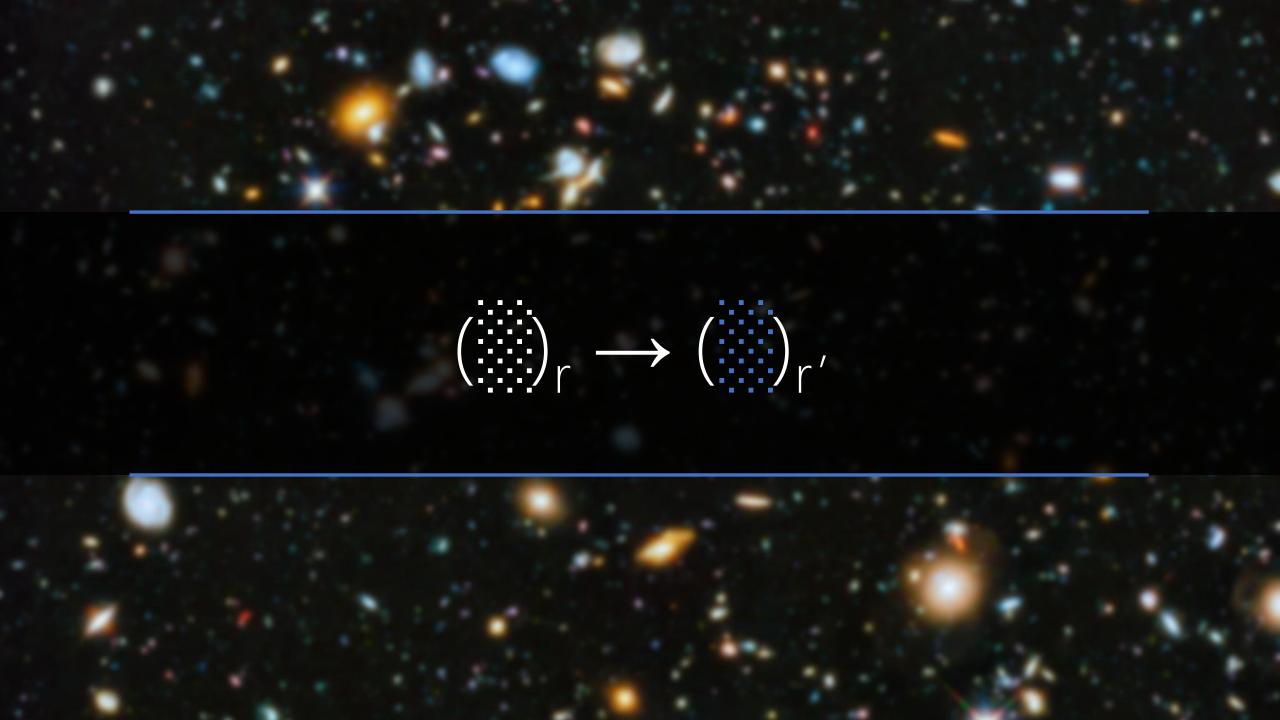
CONVERSION From Base-r to Base-r'





we already knew that: sum of the powers of r

Let $(N)_r$ be a radix-r (base-r) number in a positional weighting number system, then

$$(N)_r = (d_{n-1}r^{n-1} + \cdots + d_0r^0 \cdot d_{-1}r^{-1} + d_{-2}r^{-2} + \cdots + d_{-m}r^{-m})_{10}$$

where:

Fraction Point

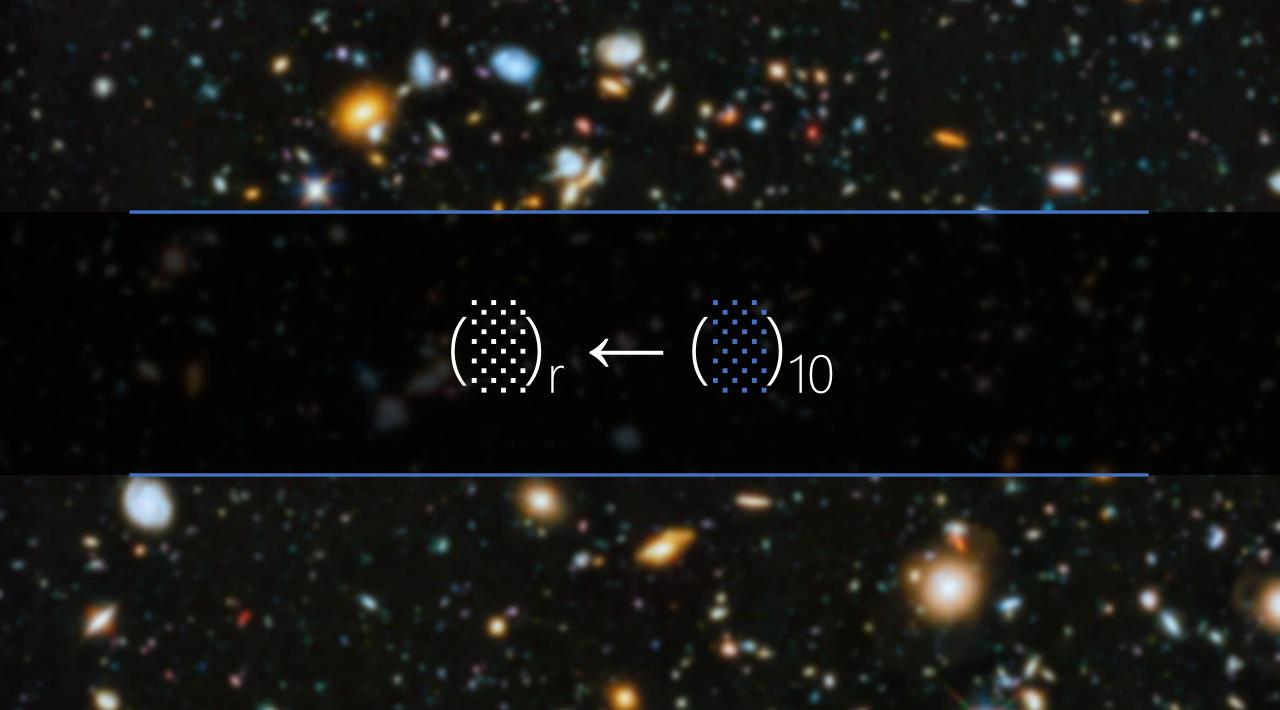
```
r = radix (base)
```

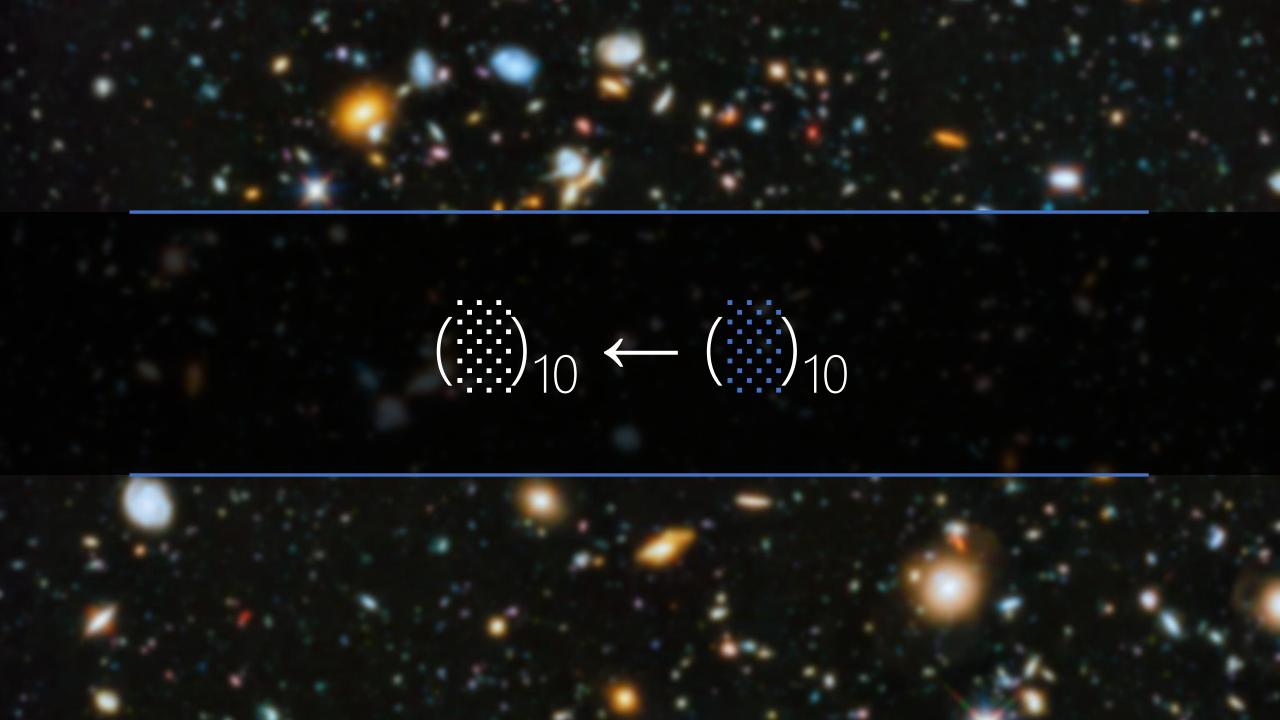
 $d_i = digit at position i, 0 \le d_i \le r - 1$

rⁱ = weight of position i

n = number of digits in integer part of N

m = number of digits in fraction part of N





What is the digit in the position with significance of 10°?

$$(30302131)_{10} \rightarrow 10^{0} \rightarrow 1$$

 $(30302131)_{10} \rightarrow 10^{1} \rightarrow 3$
 $(30302131)_{10} \rightarrow 10^{2} \rightarrow 1$
 $(30302131)_{10} \rightarrow 10^{3} \rightarrow 2$
 $(30302131)_{10} \rightarrow 10^{4} \rightarrow 0$
 $(30302131)_{10} \rightarrow 10^{5} \rightarrow 3$
 $(30302131)_{10} \rightarrow 10^{6} \rightarrow 0$
 $(30302131)_{10} \rightarrow 10^{7} \rightarrow 3$

$$(30302131)_{10} \rightarrow 10^{0} \rightarrow 1$$

$$(3030213)_{10} \rightarrow 10^{0} \rightarrow 3$$

$$(303021)_{10} \rightarrow 10^{0} \rightarrow 1$$

$$(30302)_{10} \rightarrow 10^{0} \rightarrow 2$$

$$(3030)_{10} \rightarrow 10^{0} \rightarrow 0$$

$$(303)_{10} \rightarrow 10^{0} \rightarrow 3$$

$$(30)_{10} \rightarrow 10^{0} \rightarrow 0$$

$$(30)_{10} \rightarrow 10^{0} \rightarrow 3$$

Divide by 10?

$$30,302,131 \div 10 = 3,030,213$$

 $30,302,131 \% 10 = 1$

$$\frac{30,302,131}{10} = 3,030,213 \ r \ 1$$

Divide by 10?

$$3,030,213 \div 10 = 303,021$$

 $3,030,213 \% 10 = 3$

$$\frac{3,030,213}{10} = 303,021 \ r \ 3$$

Divide by 10?

$$303,021 \div 10 = 30,302$$

 $303,021 \% 10 = 1$

$$\frac{303,021}{10} = 30,302 \ r \ 1$$

What is the digit in the position with significance of 10°?

$$\frac{30,302,131}{10} = 3,030,213 \ r \ 1$$

Remainder of the 0+1 division by 10!

$$\frac{30,302,131}{10} = \frac{3,030,213 \ r \ 1}{10} = 303,021 \ r \ 3$$

Remainder of the 1+1 division by 10!

$$\frac{30,302,131}{10} = \frac{3,030,213 \ r \ 1}{10} = \frac{3030,213 \ r \ 1}{10} = 303,021 \ r \ 3$$

Remainder of the 2+1 division by 10!

$$\frac{30,302,131}{10} = \frac{3,030,213}{10} \frac{r}{10} = \frac{303,021}{10} \frac{r}{10} = \frac{30,302}{10} \frac{r}{10} = 3,030 \frac{r}{2}$$

Remainder of the 3+1 division by 10!

$$\frac{30,302,131}{10} = \frac{3,030,213 \ r \ 1}{10} = \frac{303,021 \ r \ 3}{10} = \frac{30,302 \ r \ 1}{10} = \frac{3,030 \ r \ 2}{10} = 303 \ r \ 0$$

Remainder of the 4+1 division by 10!

$$\frac{30,302,131}{10} = \frac{3,030,213 \ r \ 1}{10} = \frac{303,021 \ r \ 3}{10} = \frac{30,302 \ r \ 1}{10} = \frac{3,030 \ r \ 2}{10} = \frac{303 \ r \ 0}{10} = 30 \ r \ 3$$

Remainder of the 5+1 division by 10!

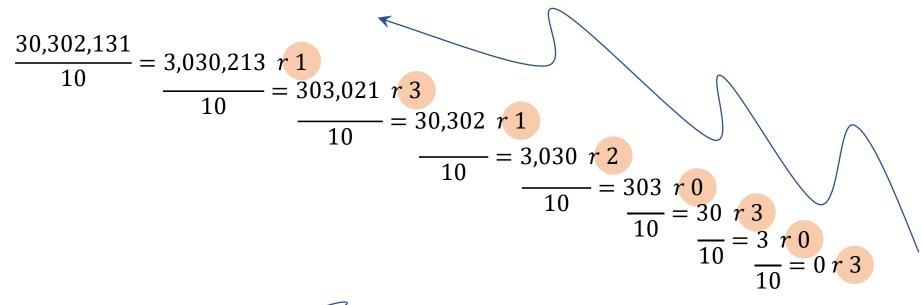
$$\frac{30,302,131}{10} = \frac{3,030,213 \ r \ 1}{10} = \frac{303,021 \ r \ 3}{10} = \frac{30,302 \ r \ 1}{10} = \frac{303 \ r \ 0}{10} = \frac{303 \ r \ 0}{10} = \frac{30 \ r \ 3}{10} = 3 \ r \ 0$$

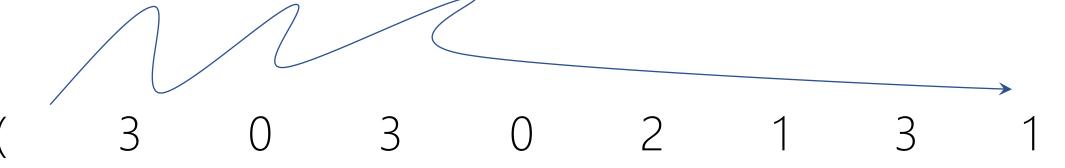
Remainder of the 6+1 division by 10!

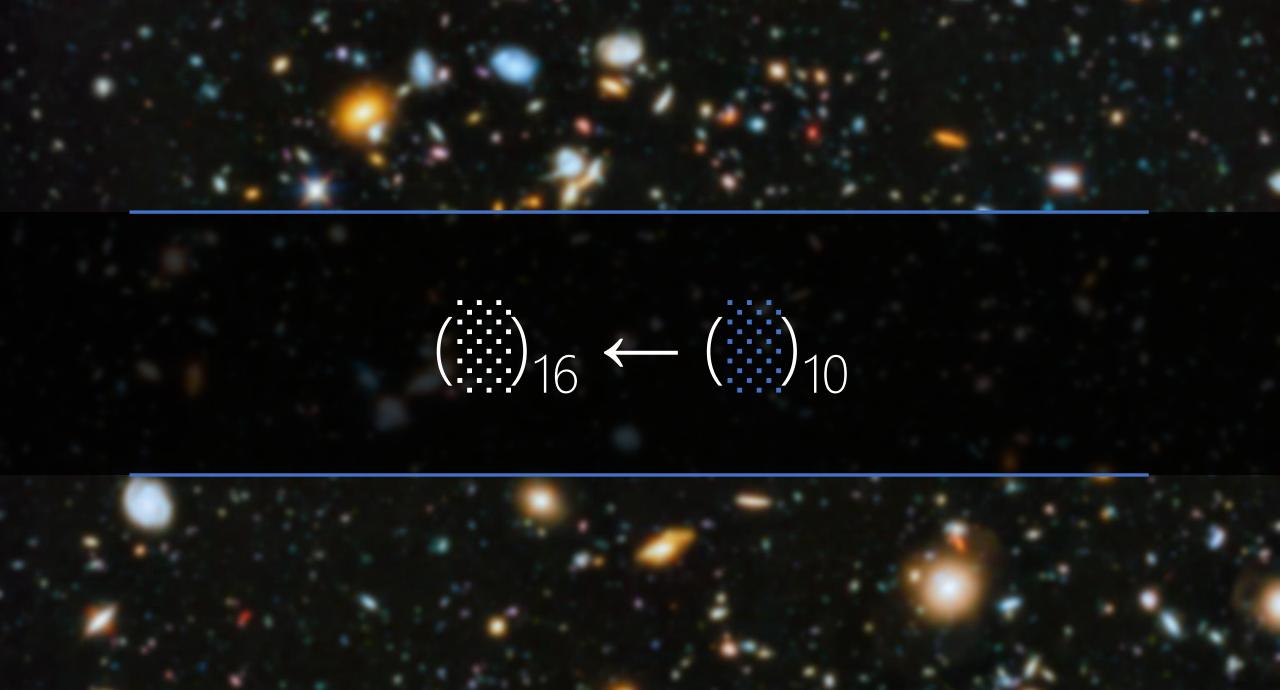
What is the digit in the position with significance of 10^7 ?

$$\frac{30,302,131}{10} = \frac{3,030,213}{10} = \frac{303,021}{10} = \frac{303,021}{10} = \frac{30,302}{10} = \frac{3030}{10} = \frac{303}{10} = \frac{30}{10} = \frac{300}{10} = \frac{300$$

Remainder of the 7+1 division by 10!







$$\frac{30,302,131}{16} = 1,893,883 \, r \, 3$$

Remainder of the 0+1 division by 16!

$$\frac{30,302,131}{16} = \frac{1,893,883 \, r \, 3}{16} = 118,367 \, r \, 11$$

Remainder of the 1+1 division by 16!

$$\frac{30,302,131}{16} = \frac{1,893,883 \, r \, 3}{16} = \frac{118,367 \, r \, 11}{16} = 7,397 \, r \, 15$$

Remainder of the 2+1 division by 16!

$$\frac{30,302,131}{16} = \frac{1,893,883 \, r \, 3}{16} = \frac{118,367 \, r \, 11}{16} = \frac{7,397 \, r \, 15}{16} = 462 \, r \, 5$$

Remainder of the 3+1 division by 16!

$$\frac{30,302,131}{16} = \frac{1,893,883 \, r \, 3}{16} = \frac{118,367 \, r \, 11}{16} = \frac{7,397 \, r \, 15}{16} = \frac{462 \, r \, 5}{16} = 28 \, r \, 14$$

Remainder of the 4+1 division by 16!

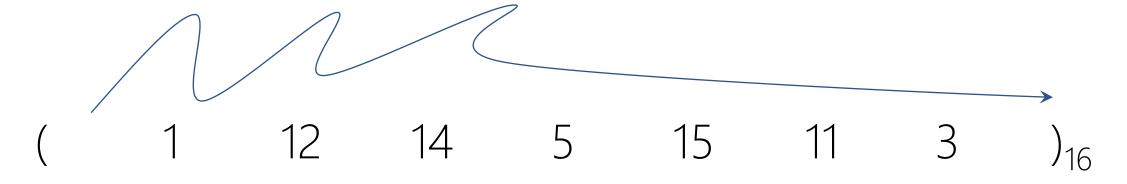
$$\frac{30,302,131}{16} = \frac{1,893,883 \, r \, 3}{16} = \frac{118,367 \, r \, 11}{16} = \frac{7,397 \, r \, 15}{16} = \frac{462 \, r \, 5}{16} = \frac{28 \, r \, 14}{16} = 1 \, r \, 12$$

Remainder of the 5+1 division by 16!

$$\frac{30,302,131}{16} = \frac{1,893,883 \, r \, 3}{16} = \frac{118,367 \, r \, 11}{16} = \frac{7,397 \, r \, 15}{16} = \frac{462 \, r \, 5}{16} = \frac{28 \, r \, 14}{16} = \frac{1}{16} \, r \, 12}$$

Remainder of the 6+1 division by 16!

$$\frac{30,302,131}{16} = \frac{1,893,883 \, r \, 3}{16} = \frac{118,367 \, r \, 11}{16} = \frac{7,397 \, r \, 15}{16} = \frac{462 \, r \, 5}{16} = \frac{28 \, r \, 14}{16} = \frac{1}{16} \, r \, 12$$



$$\frac{30,302,131}{16} = \frac{1,893,883 \, r \, 1}{16} = \frac{118,367 \, r \, 11}{16} = \frac{7,397 \, r \, 0}{16} = \frac{462 \, r \, 5}{16} = \frac{28 \, r \, 14}{16} = 0 \, r \, 1$$

 $(1 12 14 5 15 11 3)_{16}$

$$1,2,3,4,5,6,7,8,9,A = 9 + 1 = (10)_{10}$$

$$B = A + 1 = (11)_{10}$$

$$C = B + 1 = (12)_{10}$$

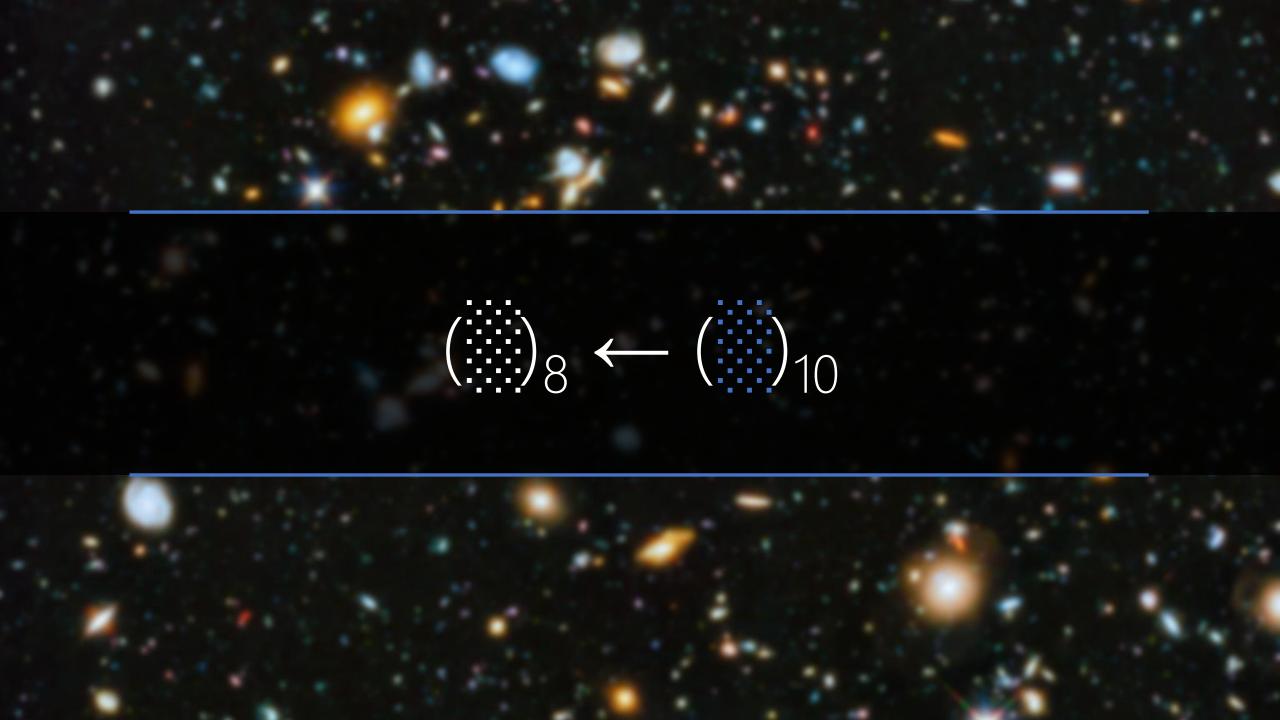
$$D = C + 1 = (13)_{10}$$

$$E = D + 1 = (14)_{10}$$

$$F = E + 1 = (15)_{10}$$

$$\frac{30,302,131}{16} = \frac{1,893,883 \, r \, 1}{16} = \frac{118,367 \, r \, 11}{16} = \frac{7,397 \, r \, 0}{16} = \frac{462 \, r \, 5}{16} = \frac{28 \, r \, 14}{16} = \frac{1}{16} \, r \, 12$$

 $(1 C E 5 F B 3)_{10}$

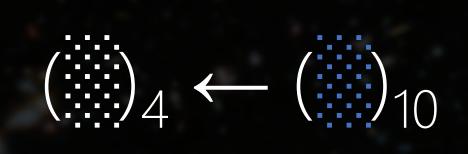


What is the digit in the position with significance of 8ⁱ?

Quotient	Remainder
30,302,131÷8	3
3787766÷8	6
473470÷8	6
59183÷8	7
7397÷8	5
924÷8	4
115÷8	3
14÷8	6
1÷8	1
0	

(1CE5FB3)₁₆ (30,302,131)₁₀

 $(163457663)_8$

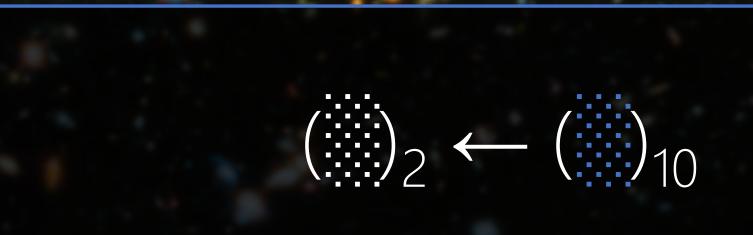


What is the digit in the position with significance of 4ⁱ?

Quotient	Remainder
30,302,131÷4	3
7575532÷4	0
1893883÷4	3
473470÷4	2
118367÷4	3
29591÷4	3
7397÷4	1
1849÷4	1
462÷4	2
115÷4	3
28÷4	0
7÷4	3
1÷4	1
0	

(1CE5FB3)₁₆ (30,302,131)₁₀ (163457663)₈

 $(1303211332303)_4$

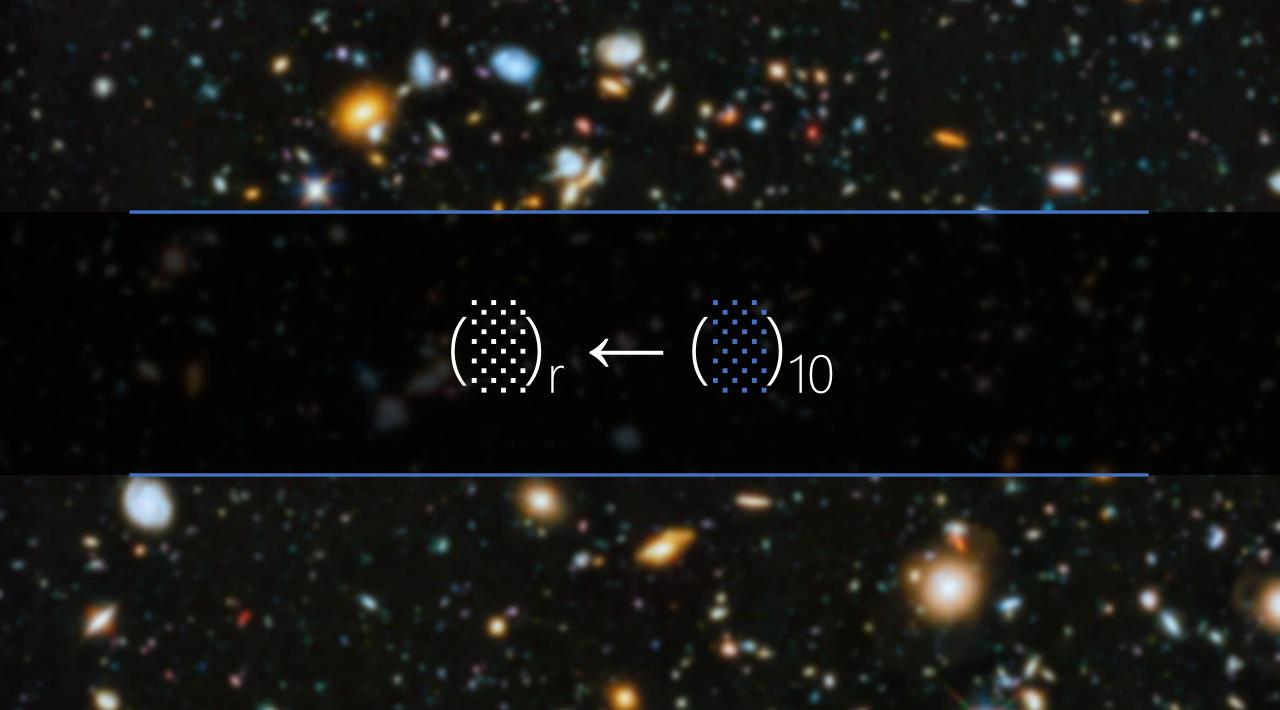


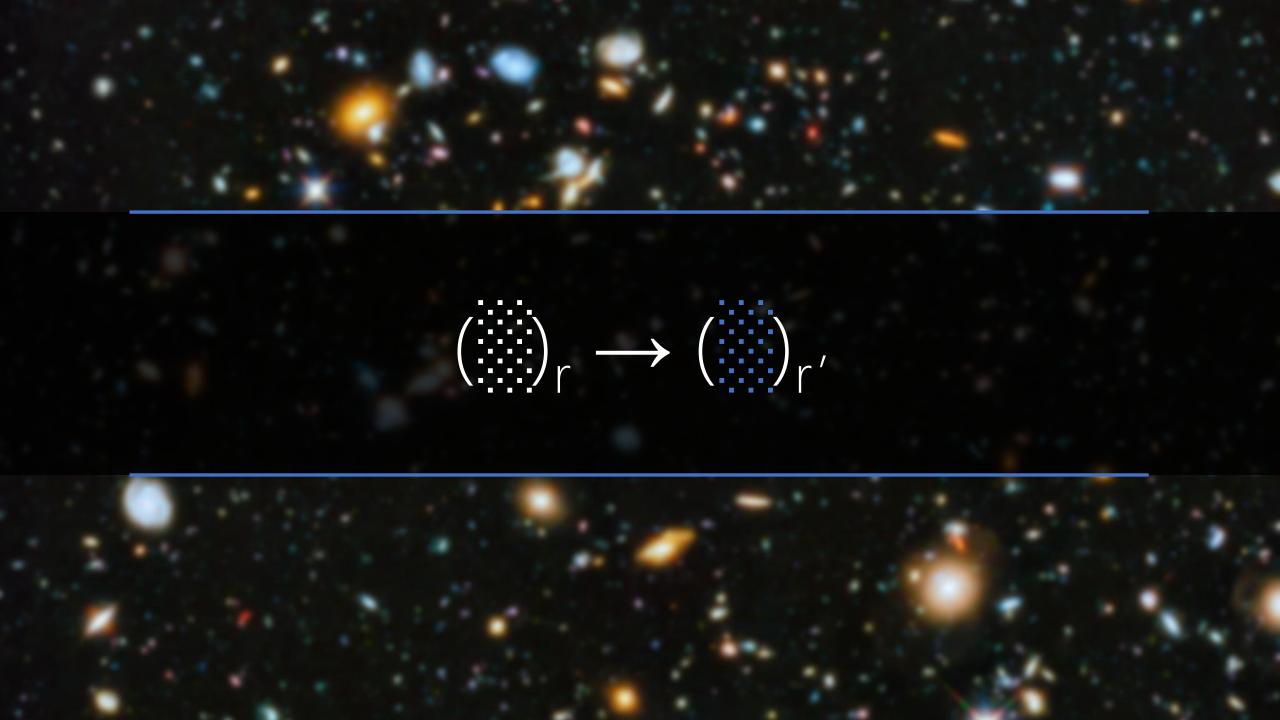
What is the digit in the position with significance of 2ⁱ?

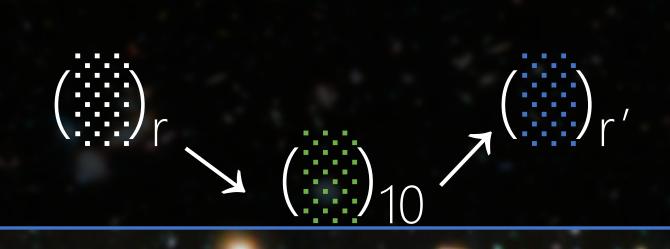
Quotient	Remainder
30,302,131÷2	1
15151065÷2	1
7575532÷2	0
3787766÷2	0
1893883÷2	1
946941÷2	1
473470÷2	0
236735÷2	1
118367÷2	1
59183÷2	1
29591÷2	1
14795÷2	1
7397÷2	1
3698÷2	0
1849÷2	1
924÷2	0
462÷2	0
231÷2	1
115÷2	1
57÷2	1
28÷2	0
14÷2	0
7÷2	1
3÷2	1
1÷2	1
0	

```
(1CE5FB3)<sub>16</sub>
(30,302,131)<sub>10</sub>
(163457663)<sub>8</sub>
(1303211332303)<sub>4</sub>
```

 $(11100111001011111110110011)_2$







$$(1CE5FB3)_{16} \rightarrow 1 \times 16^{6} + \frac{C}{C} \times 16^{5} + \frac{E}{C} \times 16^{4} + 5 \times 16^{3} + \frac{F}{C} \times 16^{2} + \frac{B}{C} \times 16^{1} + 3 \times 16^{0}$$

$$\rightarrow 1 \times 16^{6} + \frac{12}{C} \times 16^{5} + \frac{14}{C} \times 16^{4} + 5 \times 16^{3} + \frac{15}{C} \times 16^{2} + \frac{11}{C} \times 16^{1} + 3 \times 16^{0}$$

$$\rightarrow (30,302,131)_{10}$$

Quotient	Remainder
30,302,131÷8	3
3787766÷8	6
473470÷8	6
59183÷8	7
7397÷8	5
924÷8	4
115÷8	3
14÷8	6
1÷8	1
0	

 $\rightarrow (163457663)_{\rm g}$

FRACTION CONVERSION

$$(0...)_{10} \leftarrow (0...)_{10}$$

What is the digit in the position with significance of 10⁻ⁱ?

$$(0.26501)_{10} \rightarrow 10^{-1} \rightarrow 2$$

 $(0.26501)_{10} \rightarrow 10^{-2} \rightarrow 6$
 $(0.26501)_{10} \rightarrow 10^{-3} \rightarrow 5$
 $(0.26501)_{10} \rightarrow 10^{-4} \rightarrow 0$
 $(0.26501)_{10} \rightarrow 10^{-5} \rightarrow 1$

What is the digit in the position with significance of 10⁻ⁱ?

$$(0.26501)_{10} \rightarrow 10^{-1} \rightarrow 2$$

$$(0.6501)_{10} \rightarrow 10^{-1} \rightarrow 6$$

$$(0.501)_{10} \rightarrow 10^{-1} \rightarrow 5$$

$$(0.01)_{10} \rightarrow 10^{-1} \rightarrow 0$$

$$(0.1)_{10} \rightarrow 10^{-1} \rightarrow 1$$



 $(0 . 2 6 5 0 1)_{10}$

 $0.26501 \times 10 = 2.6501$

Multiply by 10?

 $(0 . 2 6 5 0 1)_{10}$

 $0.6501 \times 10 = 6.501$

Fraction	Result	Integer Part
0.26501×10	2.6501	2
0.6501×10	6.501	6
0.501×10	5.01	5
0.01×10	0.1	0
0.1×10	1.0	1
0		

$$(0....)_{16} \leftarrow (0...)_{10}$$

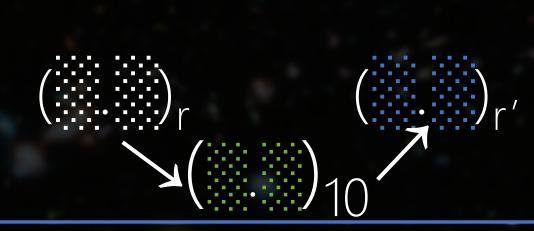
Fraction	Result	Integer Part
0.26501×16	4.24016	4
0.24016×16	3.84256	3
0.84256×16	13.48096	13 = D
0.48096×16	7.69536	7
0.69536×16	11.12576	11 = B
0.12576×16	2.01216	2
0.01216×16	0.19456	0
0.194560006×16	3.11296	3
0.1129601×16	1.807362	1
0.807361603×16	12.91779	12 = C
0.917785645×16	14.68457	14 = E
0.684570313×16	10.95313	10 = A
0.953125×16	15.25	15 = F
0.25×16	4	4
0	0	0

 $(0.26501)_{10} \rightarrow (0.43D7B2031CEAF40)_{16}$

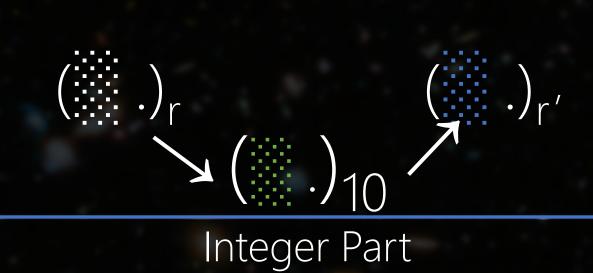
$$(0...)_{8} \leftarrow (0...)_{10}$$

 $(0...)_{4} \leftarrow (0...)_{10}$
 $(0...)_{2} \leftarrow (0...)_{10}$

MICROSOFT EXCEL SHEET



Integer Part independent of Fraction Part!





Fraction Part

$(30,302,131.26501)_{10}$ $(163457663.20753....)_{8}$

Quotient	Remainder
30,302,131÷8	3
3787766÷8	6
473470÷8	6
59183÷8	7
7397÷8	5
924÷8	4
115÷8	3
14÷8	6
1÷8	1
0	

Fraction	Result	Integer Part
0.26501×8	2.12008	2
0.12008×8	0.96064	0
0.96064×8	7.68512	7
0.68512×8	5.48096	5
0.48096×8	3.84768	3
0.84768×8	6.78144	6
0.78144×8	6.25152	6
0.25152×8	2.01216	2
0.01216×8	0.09728	0
•••	• • •	

$BASE-2 \longrightarrow BASE-[2^2, 2^3, 2^4, 2^n]$

LECTURE ASSIGNMENT

$BASE-2 \leftarrow BASE-[2^2, 2^3, 2^4, 2^n]$

LECTURE ASSIGNMENT

https://planetcalc.com/862/

ARITHMETIC

ADDITION

+		2	A	•	Е	5	4
Base-16	4	В	F	•	2	В	

PADDING

+	0	2	A	•	Е	5	4
Base-16	4	В	F	•	2	В	0

+	0	2	A	•	Е	5	4
Base-16	4	В	F	•	2	В	0
				•			4

+	0	2	A	•	Е	5	4
Base-16	4	В	F		2	B=11	0
				•		16	4

$$\frac{16}{16} = 1 r 0$$

					1		
+	0	2	A	•	Е	5	4
Base-16	4	В	F		2	В	0
				•		0	4

$$\frac{16}{16} = 1 r 0$$

					1		
+	0	2	A	•	E=14	5	4
Base-16	4	В	F	•	2	В	0
				•	17	0	4

$$\frac{17}{16} = 1 r 1$$

			1		1		
+	0	2	A	•	Е	5	4
Base-16	4	В	F	•	2	В	0
				•	1	0	4

$$\frac{17}{16} = 1 r 1$$

			1		1		
+	0	2	A=10	•	Е	5	4
Base-16	4	В	F=15	•	2	В	0
			26	•	1	0	4

$$\frac{26}{16} = 1 \, r \, 10$$

		1	1		1		
+	0	2	A	•	Е	5	4
Base-16	4	В	F	•	2	В	0
			10	•	1	0	4

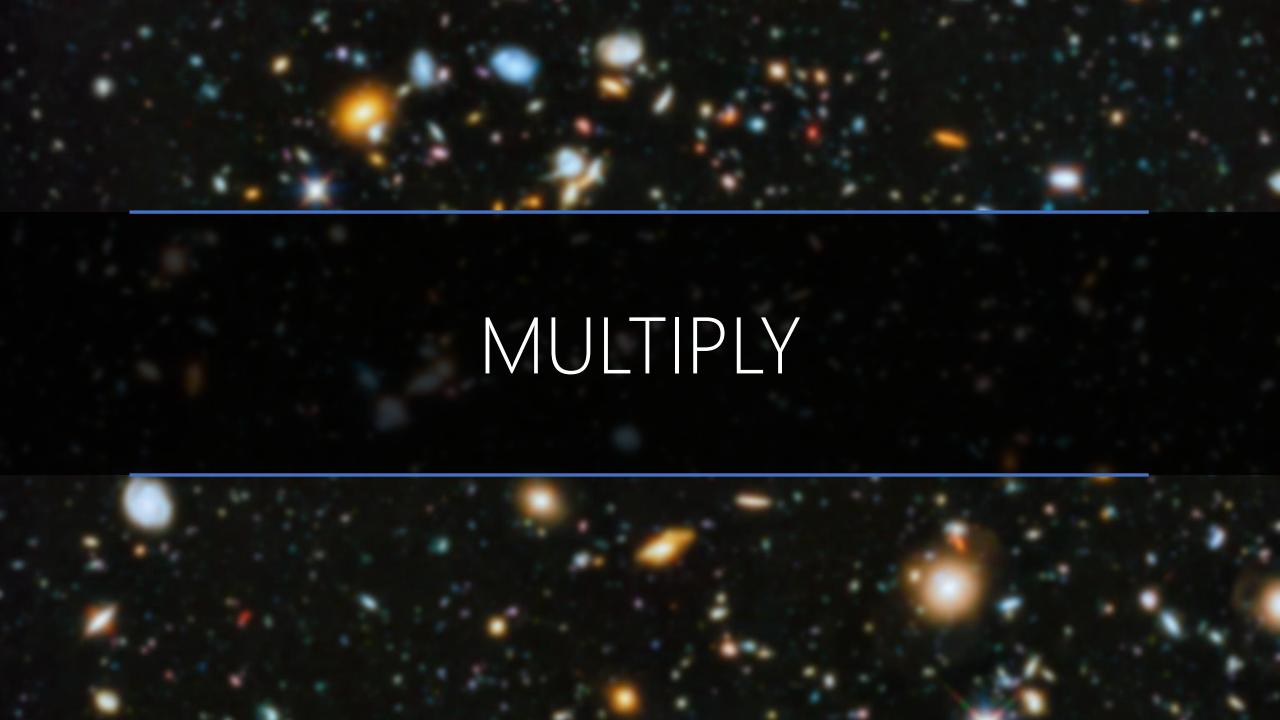
$$\frac{26}{16} = 1 \, r \, 10$$

		1	1		1		
+	0	2	A	•	Е	5	4
Base-16	4	В	F	•	2	В	0
			Α		1	0	4

		1	1		1		
+	0	2	A	•	Е	5	4
Base-16	4	B=11	F		2	В	0
		14	Α	•	1	0	4

		1	1		1		
+	0	2	A	•	Ε	5	4
Base-16	4	В	F	•	2	В	0
		Е	А	•	1	0	4

		1	1		1		
+	0	2	A	•	Е	5	4
Base-16	4	В	F	•	2	В	0
	4	Е	A	•	1	0	4



SUBTRACTION

_		2	A	•	Е	5	4
Base-16	4	В	F	•	2	В	
				•			

PADDING

_	0	2	A	•	Е	5	4
Base-16	4	В	F	•	2	В	0
				•			

_	0	2	Α	•	Е	5	4
Base-16	4	В	F	•	2	В	0
				•			4

_	0	2	A	•	E	5	4
Base-16	4	В	F	•	2	B=11	0
				•			4

			BORI	ROW	-1 —	→+16	
_	0	2	A	•	Е	5	4
Base-16	4	В	F	•	2	B=11	0
				•		= 5+16-11 = 10 = A	4

					-1	+16	
_	0	2	A	•	Е	5	4
Base-16	4	В	F	•	2	В	0
				•		А	4

					-1	+16	
_	0	2	A	•	E=14	5	4
Base-16	4	В	F	•	2	В	0
				•	B=11	Α	4

					-1	+16	
	0	2	A=10	•	Е	5	4
Base-16	4	В	F=15	•	2	В	0
				•	В	A	4

		-1	→ +16		-1	+16	
_	0	2	A=10	•	Е	5	4
Base-16	4	В	F=15	•	2	В	0
			B=11		В	А	4

		-1	+16		-1	+16	
	0	2	Α	•	Е	5	4
Base-16	4	B=11	F	•	2	В	0
			В	•	В	Α	4

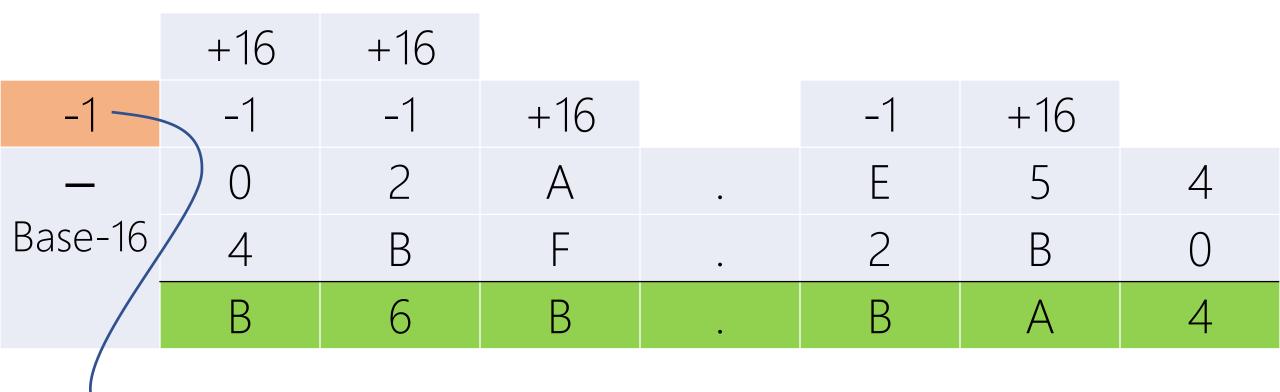
		+16					
	-1	-1	+16		-1	+16	
	0	2	A	•	Е	5	4
Base-16	4	B=11	F	•	2	В	0
		6	В	•	В	А	4

		+16					
	-1	-1	+16		-1	+16	
_	0	2	A	•	Е	5	4
Base-16	4	В	F	•	2	В	0
		6	В	•	В	A	4

	+16	+16					
-1	-1	-1	+16		-1	+16	
_	0	2	A	•	Е	5	4
Base-16	4	В	F	•	2	В	0
	B=11	6	В	•	В	А	4

	+16	+16				
-1	-1	-1	+16	-1	+16	
_	0	2	A	Е	5	4
Base-16	4	В	F	2	В	0
	В	6	В	В	А	4

02A.E54 < 4BF.2B0



Last Borrow → Negative Result

02A.E54 < 4BF.2B0

$$= (2A.E54)_{16} - (4BF.2B0)_{16} = (B6B.BA4)_{16}$$
 and last borrow!

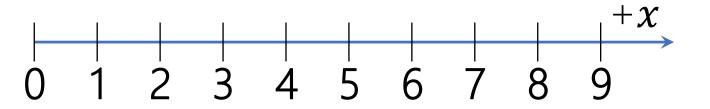
$$= (1000.000)_{16} - (B6B.BA4)_{16}$$

$$= (494.45C)_{16}$$

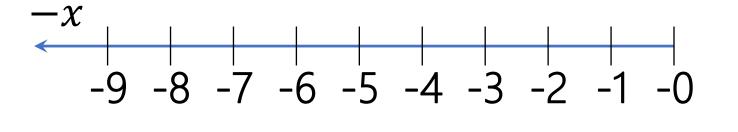
$$=$$
 $(494.45C)_{16}$

DIVISION

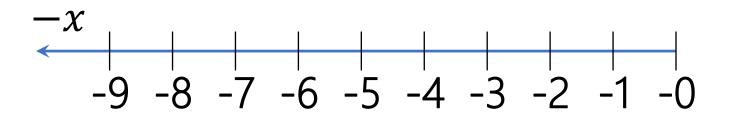
NEGATIVE NUMBERS



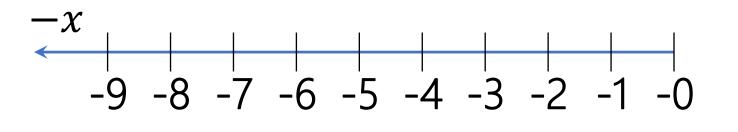
10 ¹	100
_	0
<u>—</u>	1
_	2
_	3
_	4
_	5
_	6
_	7
_	8
<u>—</u>	9

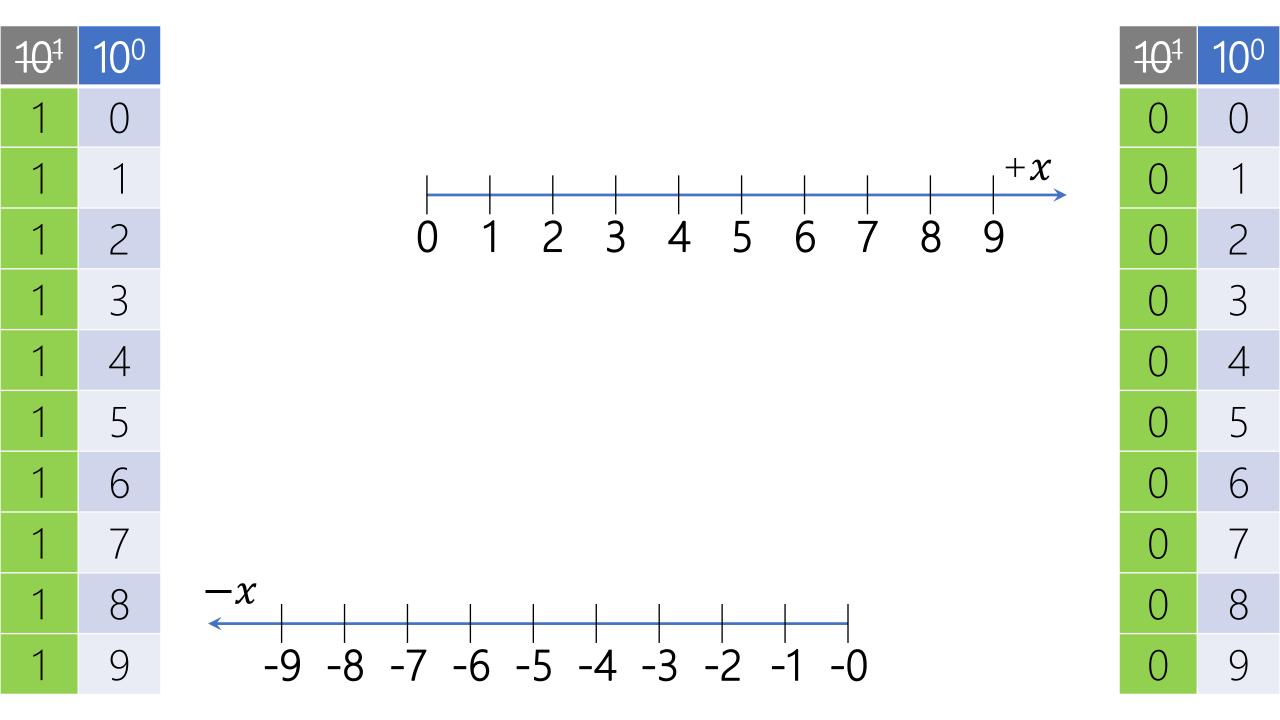


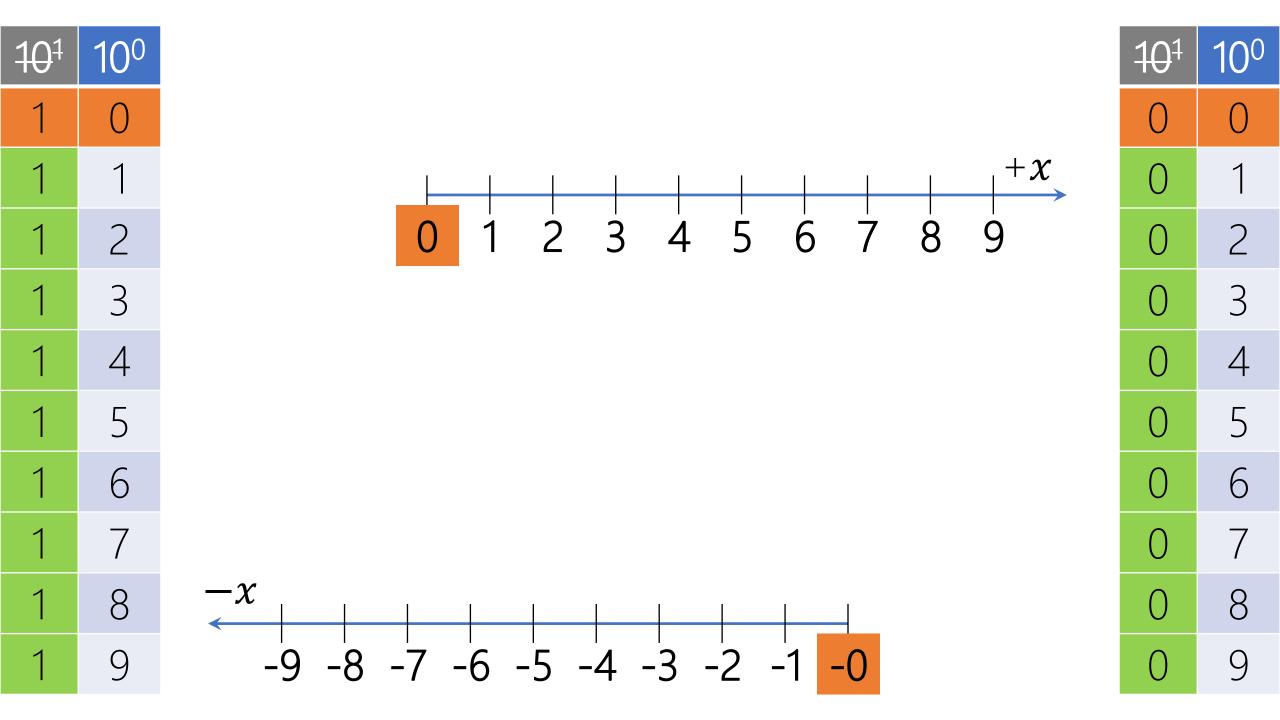
10 ¹	100
1	0
1	1
1	2
1	3
1	4
1	5
1	6
1	7
1	8 9
1	9



10 ¹	100
Nonzero	0
Nonzero	1
Nonzero	2
Nonzero	3
Nonzero	4
Nonzero	5
Nonzero	6
Nonzero	7
Nonzero	8
Nonzero	9







SIGNED MAGNITUDE

r ⁿ⁻¹	r ⁿ⁻²	r ⁿ⁻³	•••	r ²	r ¹	r ⁰
0			Positive N	Numbers		
Nonzero			Negative	Numbers		

Signed

Magnitude

Give up left most position for sign!

r ⁿ⁻¹	rn-2	r ⁿ⁻³	•••	r ²	r ¹	r ⁰
0			Positive N	Numbers		
Nonzero			Negative	Numbers		

$$0 \rightarrow Max = r^{n-1}-1 = r^{n-1}$$
Min= - $(r^{n-1}-1) \leftarrow 0$