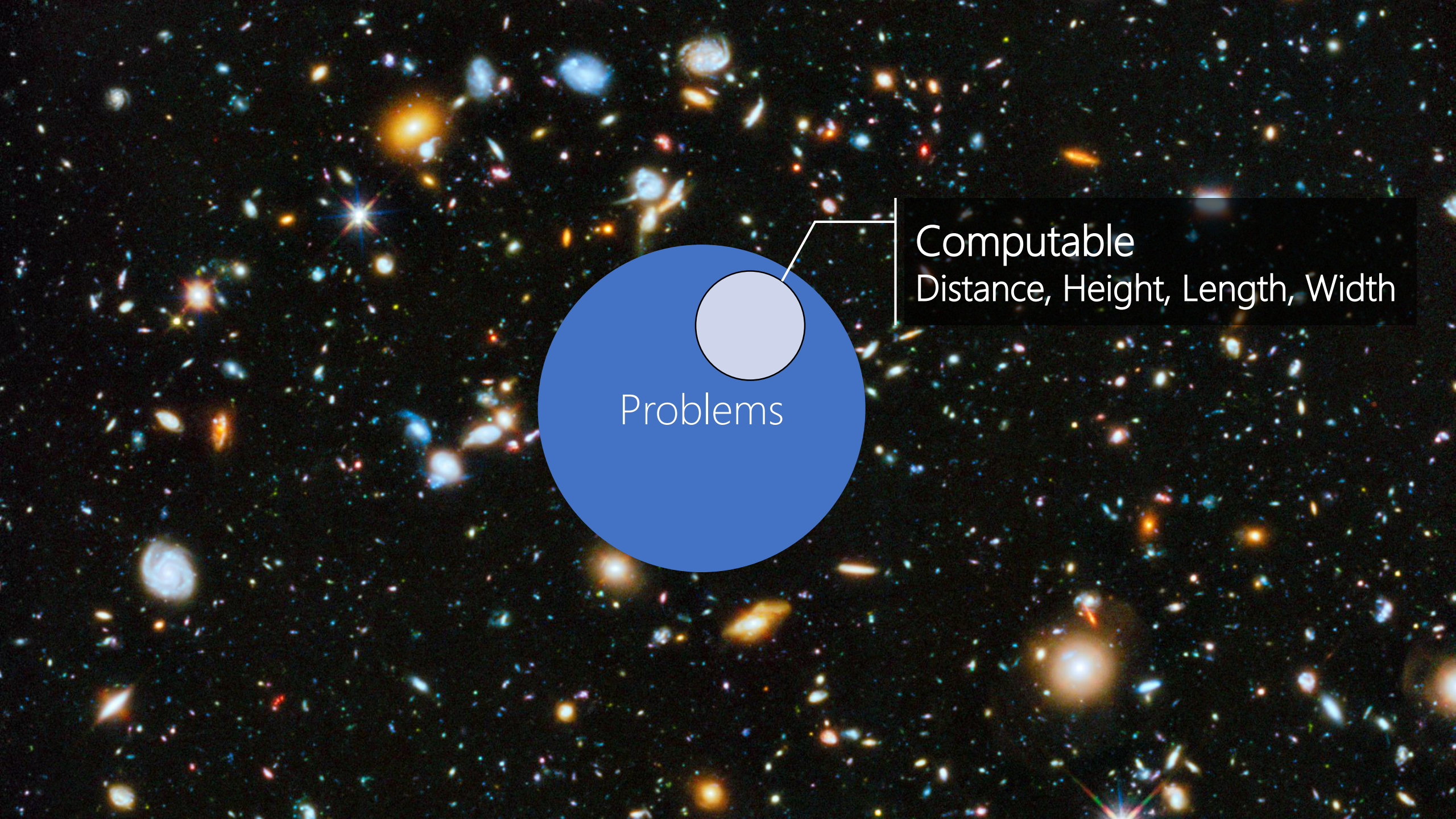
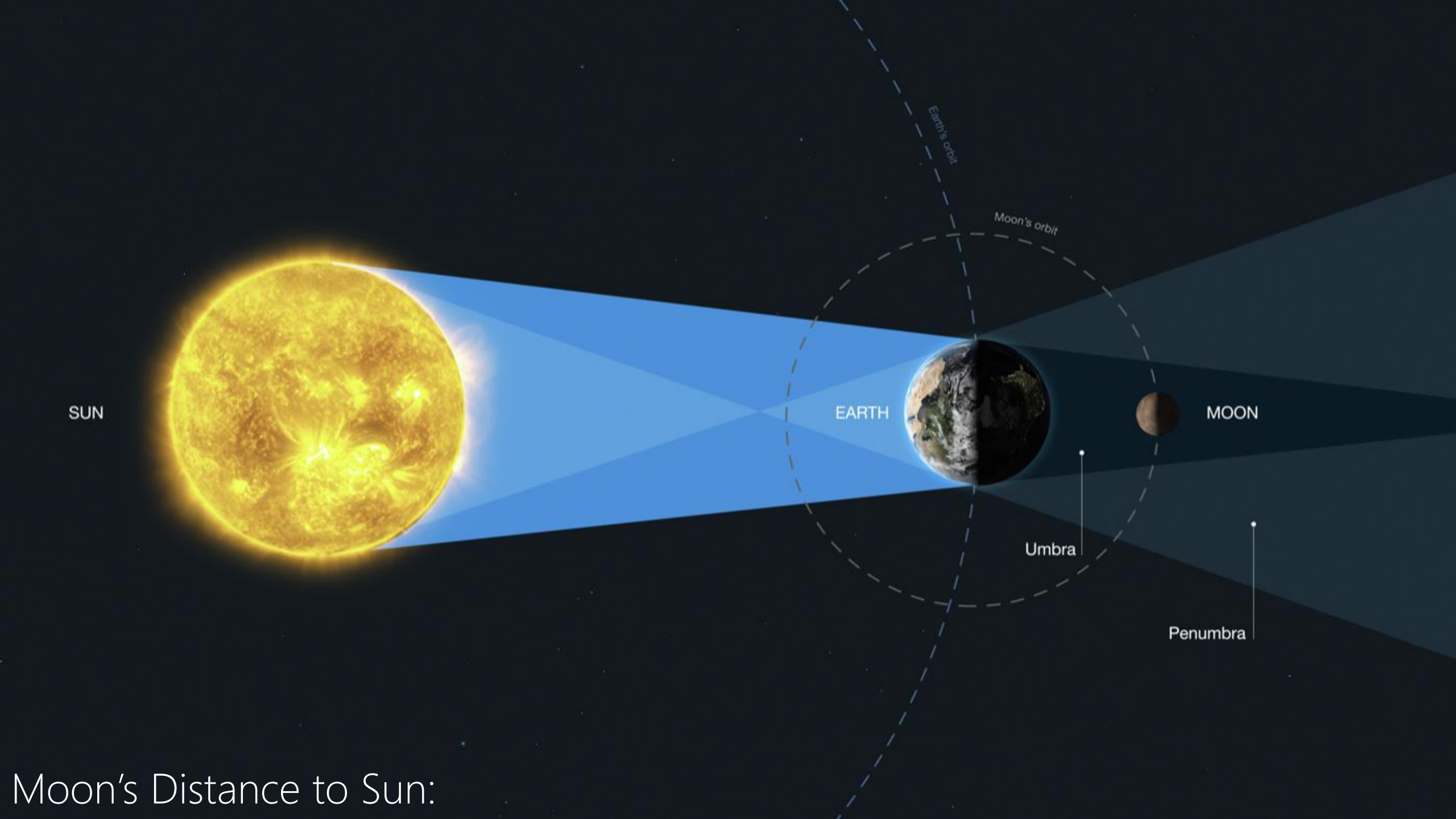
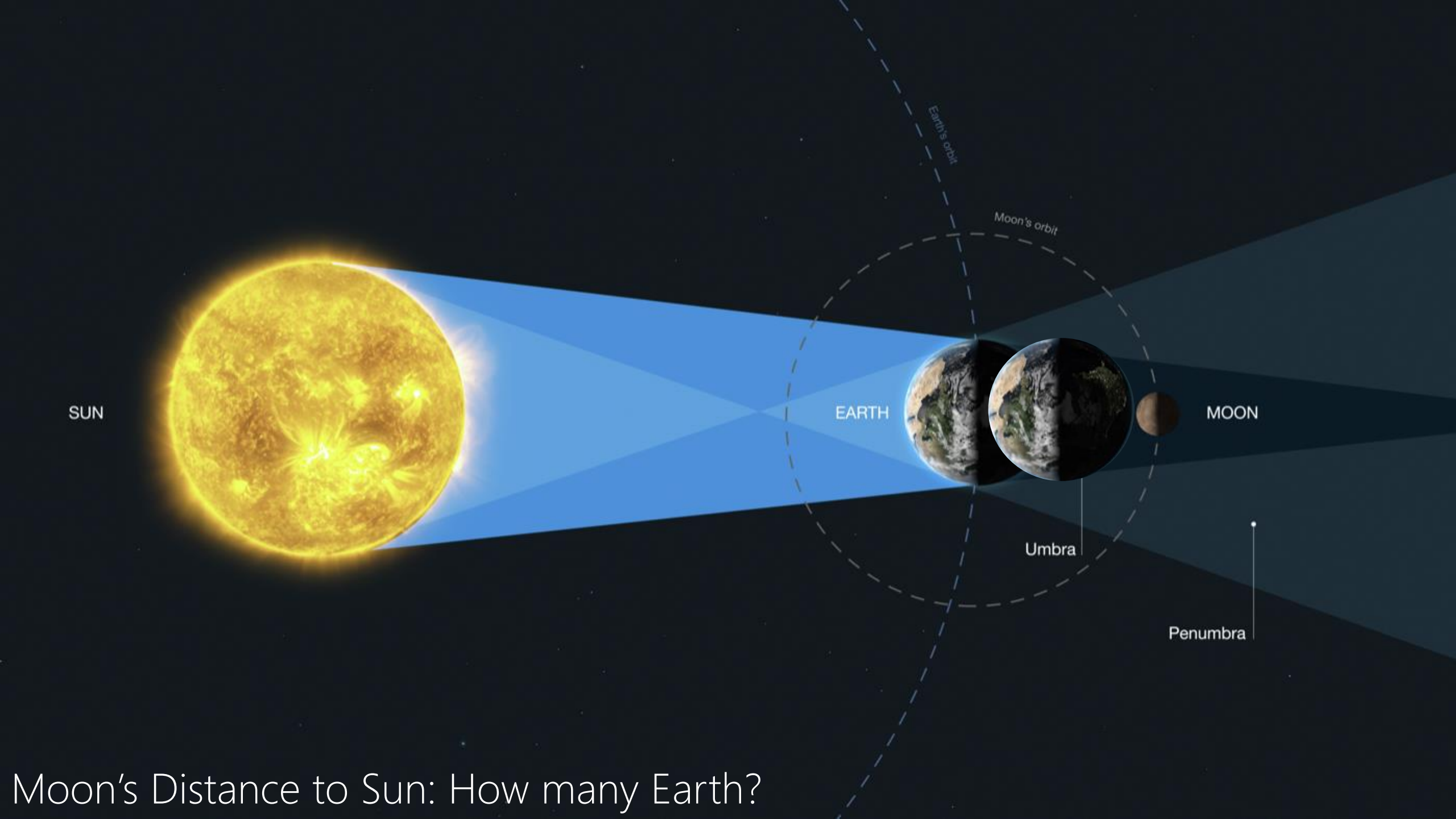
Problems

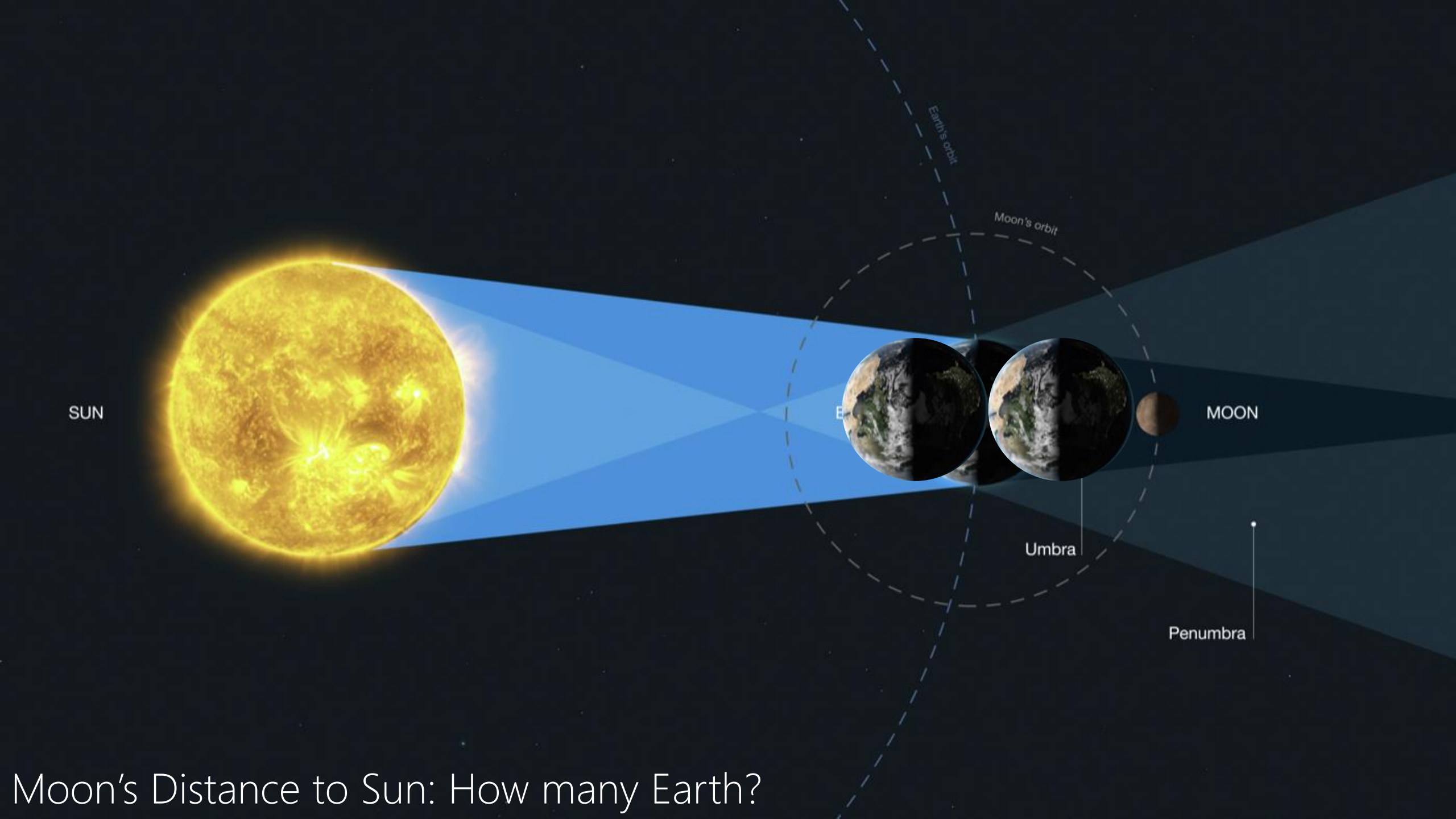

Problems
Computable
Distance, Height, Length, Width



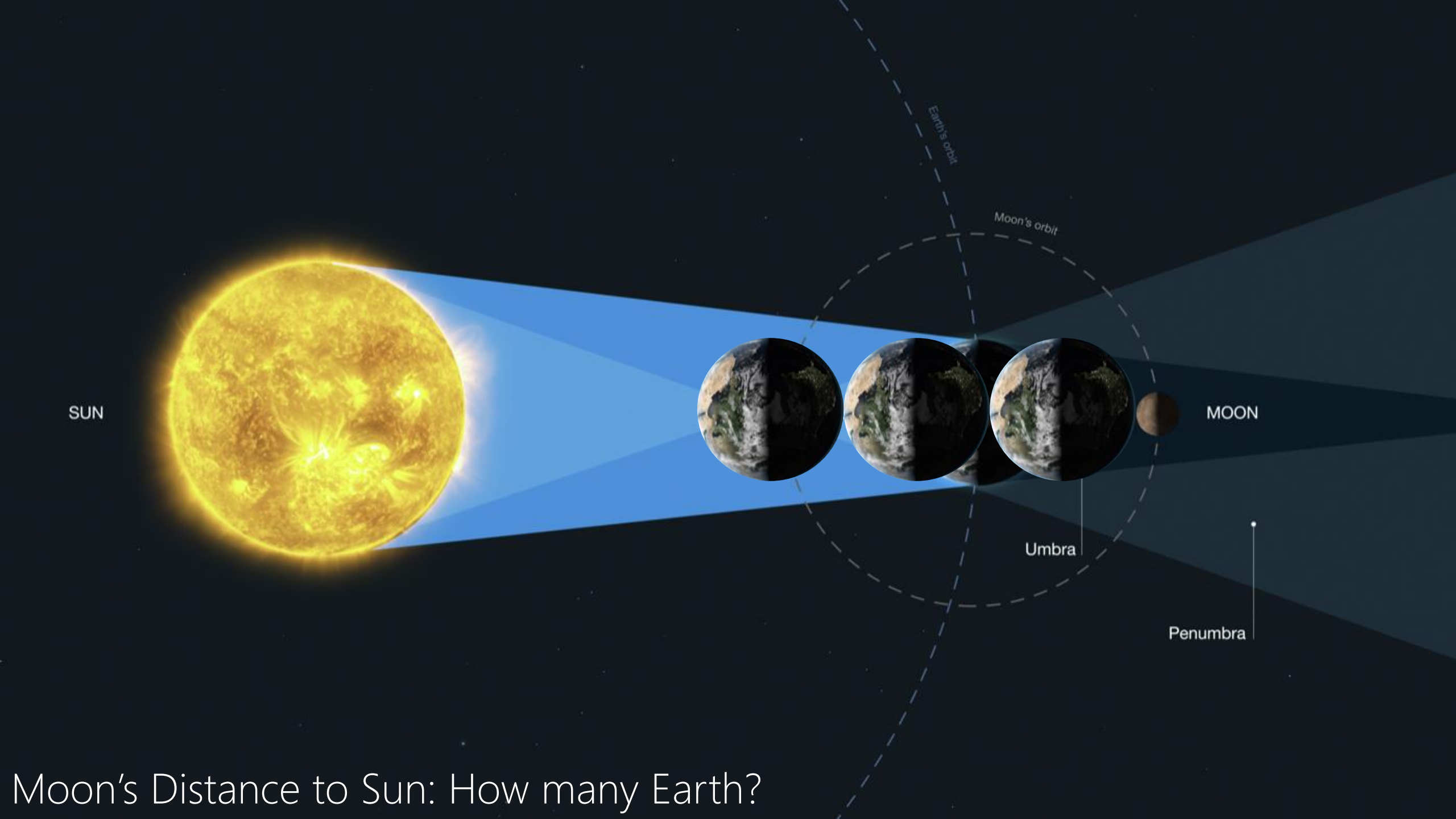
Moon's Distance to Sun:



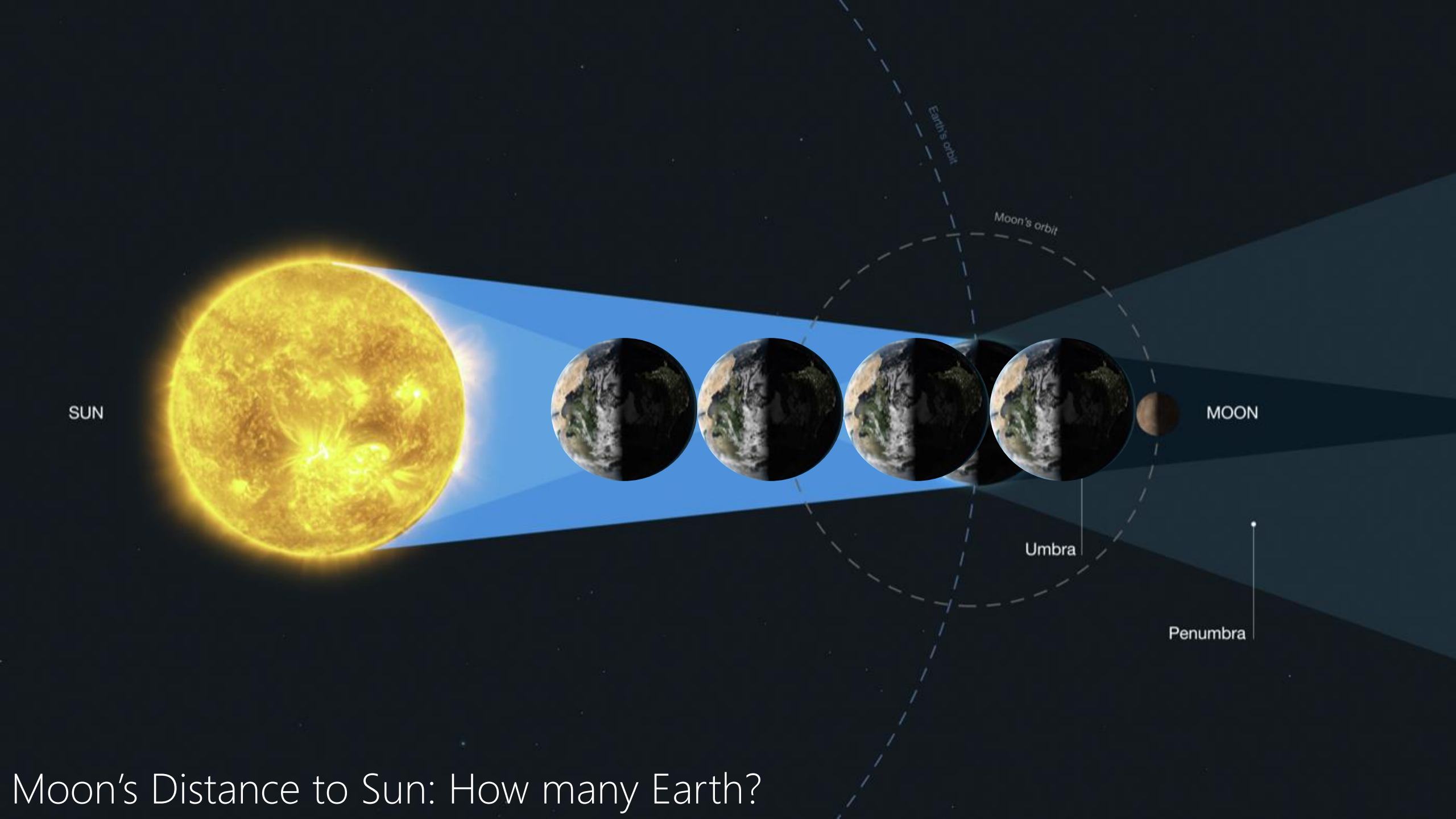
Moon's Distance to Sun: How many Earth?



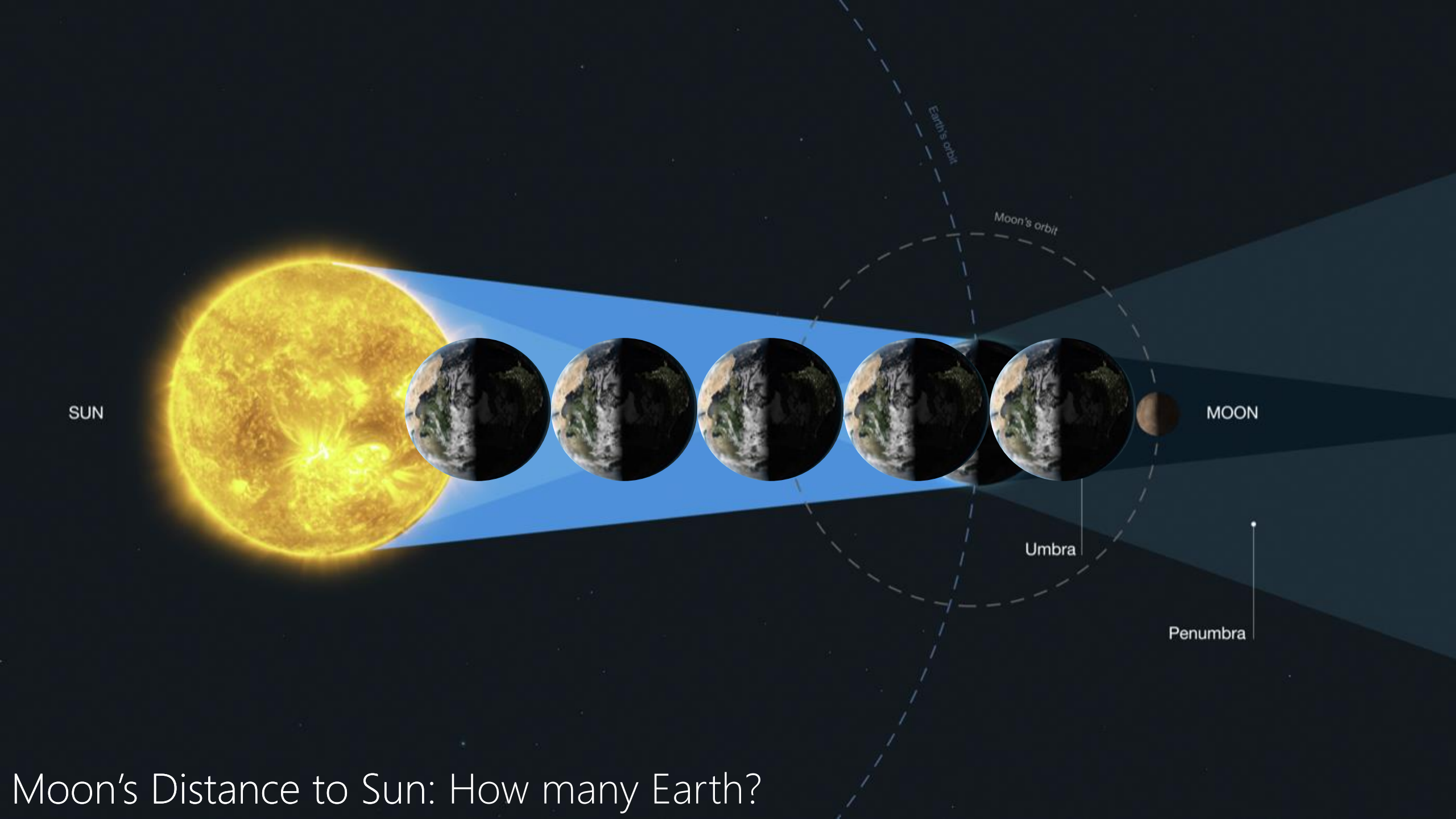
Moon's Distance to Sun: How many Earth?



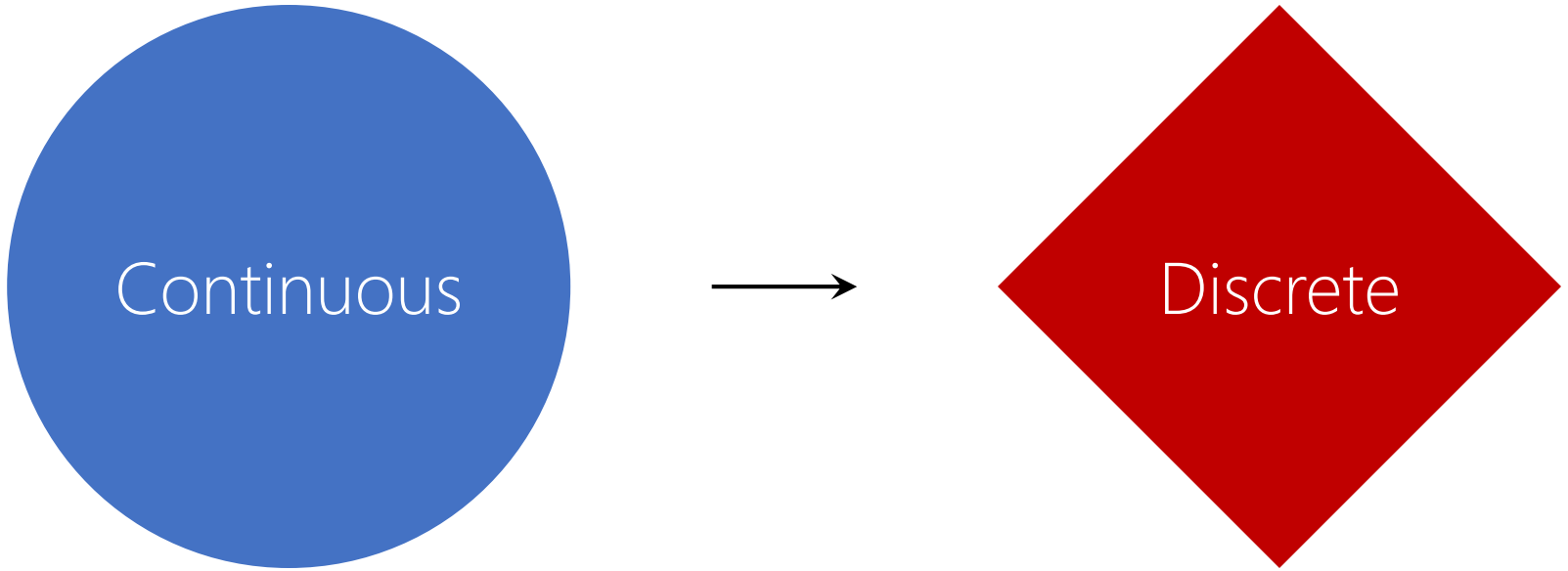
Moon's Distance to Sun: How many Earth?



Moon's Distance to Sun: How many Earth?



Moon's Distance to Sun: How many Earth?


Continuous
Discrete

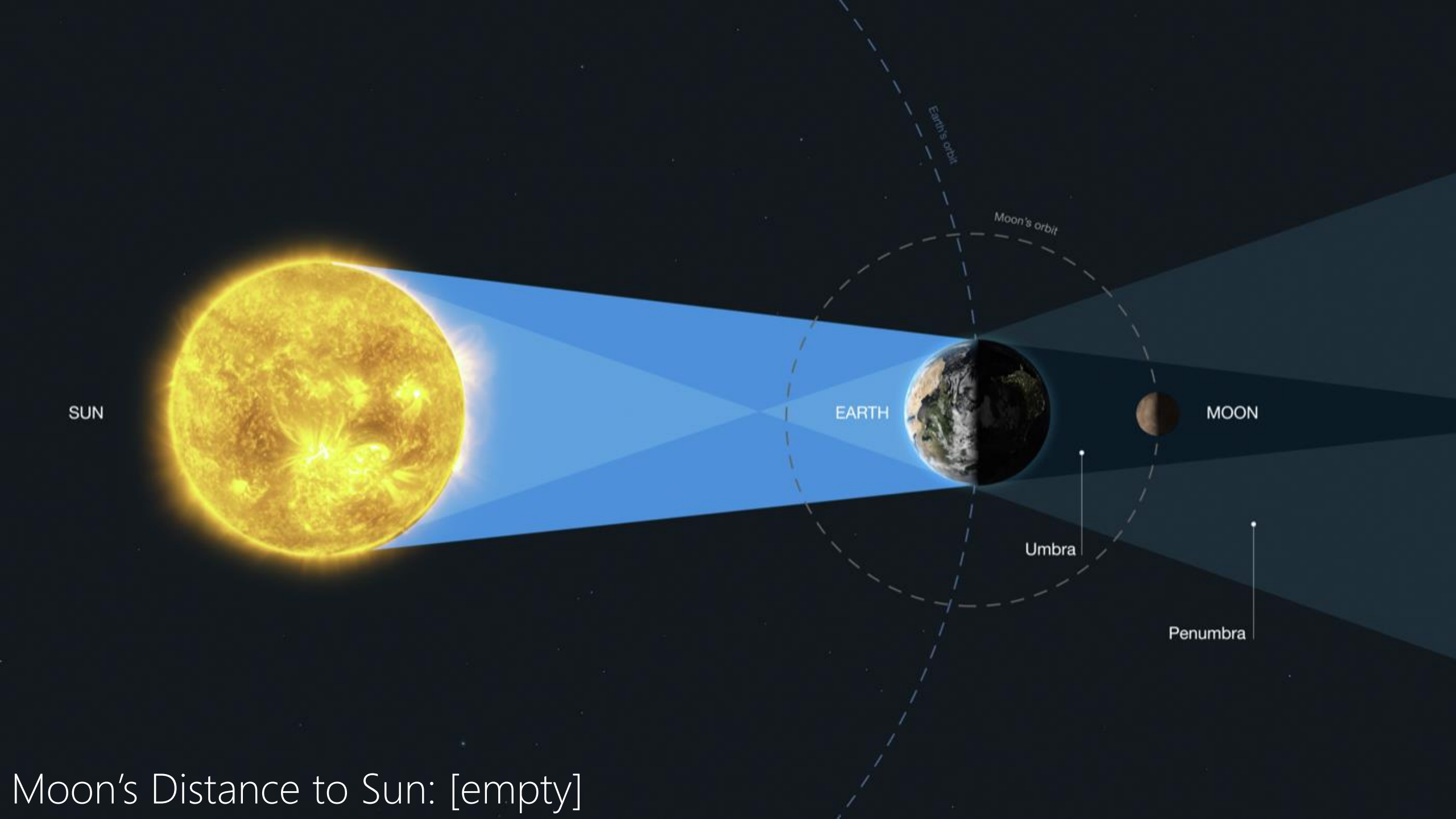
DISCRETE SYSTEMS

STAR	CONTINUOUS	DISCRETE
TEMPERATURE	CONTINUOUS	DISCRETE
ELECTRON	CONTINUOUS	DISCRETE
TIME	CONTINUOUS	DISCRETE
ATOM	CONTINUOUS	DISCRETE
WEIGHT	CONTINUOUS	DISCRETE
SPEED	CONTINUOUS	DISCRETE
STUDENT	CONTINUOUS	DISCRETE
SOUND	CONTINUOUS	DISCRETE
IMAGE	CONTINUOUS	DISCRETE
PAIN	CONTINUOUS	DISCRETE

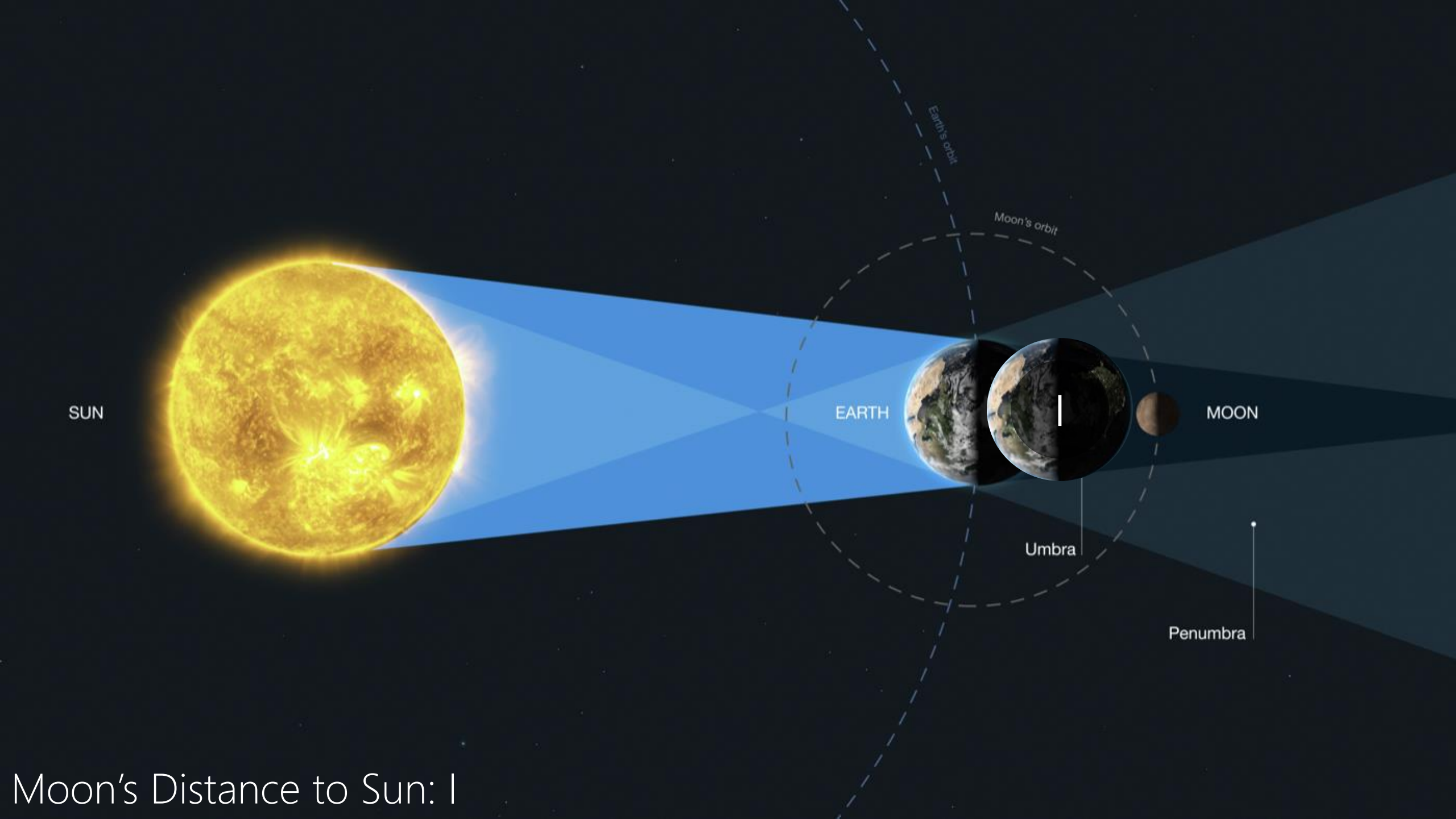
STAR		DISCRETE
TEMPERATURE	CONTINUOUS	
ELECTRON		DISCRETE
TIME	CONTINUOUS	
ATOM		DISCRETE
WEIGHT	CONTINUOUS	
SPEED	CONTINUOUS	
STUDENT		DISCRETE
SOUND	CONTINUOUS	
IMAGE	CONTINUOUS	
PAIN	CONTINUOUS	

“The world's first photograph—or at least the oldest surviving photo—was taken by Joseph Nicéphore Niépce in 1826 or 1827. Captured using a technique known as heliography, the shot was taken from an upstairs window at Niépce's estate in Burgundy. As heliography produces one-of-a-kind images, there are no duplicates of the piece, which is now part of the permanent collection at the University of Texas-Austin.” 18 Famous First Photographs in History: From the Oldest Photo Ever to the World’s First Instagram

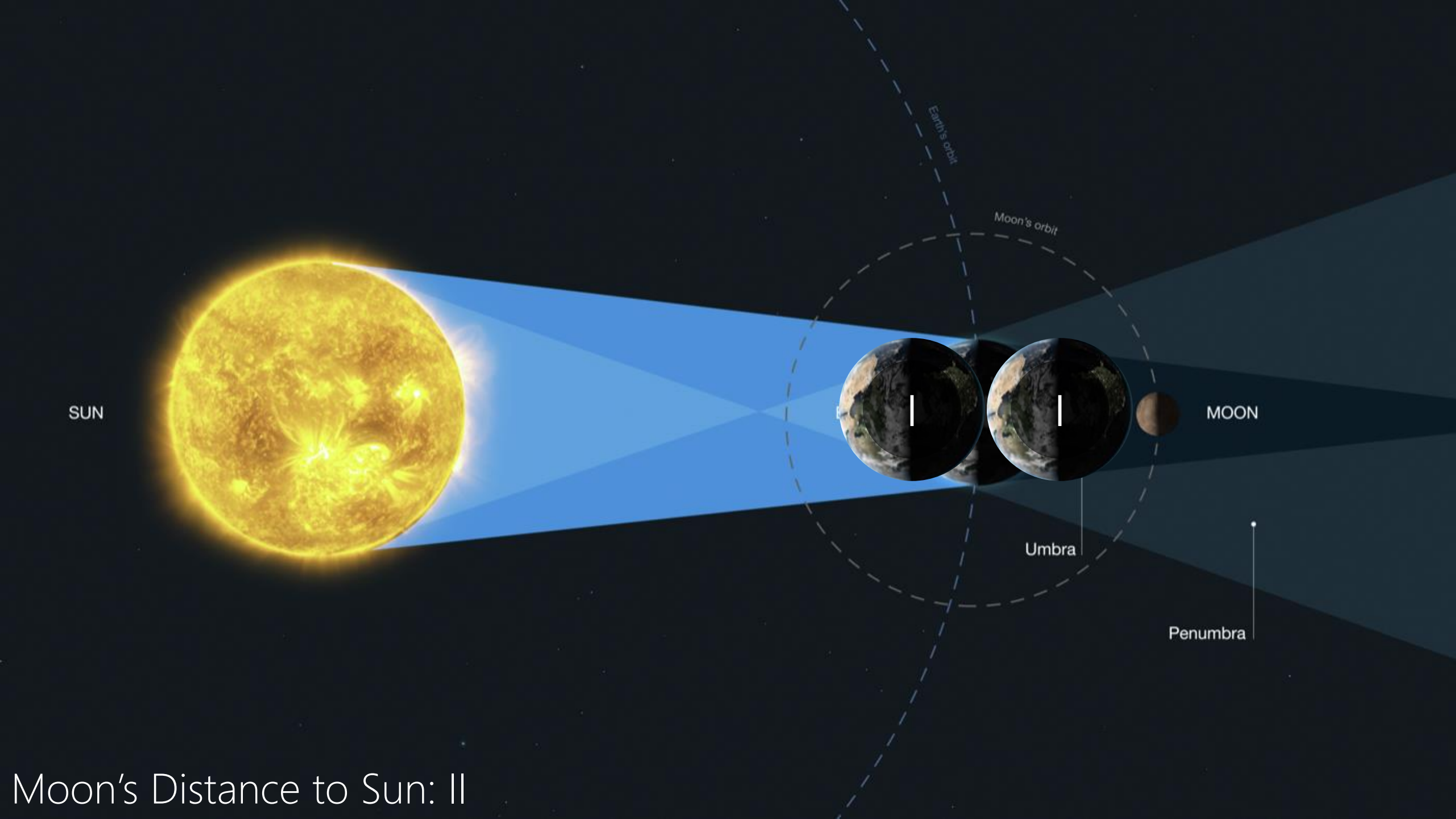
“We may be used to selfies now, but it's Robert Cornelius's 1839 image that lays claim to the first self-portrait. Taken in Philadelphia, Cornelius sat for a little over one minute before covering the lens.” 18 Famous First Photographs in History: From the Oldest Photo Ever to the World’s First Instagram



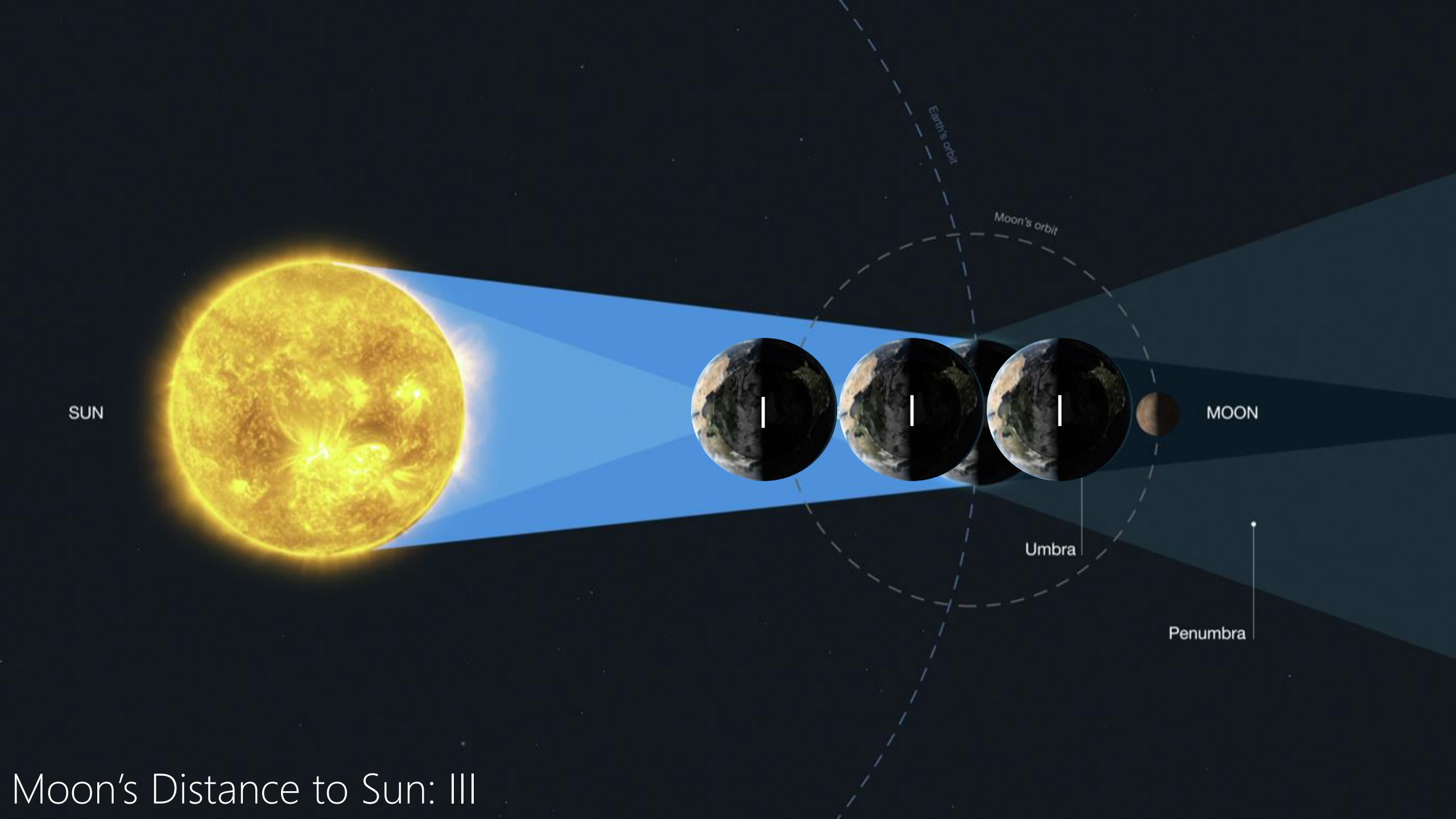
Moon’s Distance to Sun: [empty]



Moon's Distance to Sun: I



Moon's Distance to Sun: II



Moon's Distance to Sun: III

I II III IV V VI VII VIII IX X

Roman Numerals
Originated in Ancient Rome
8th Century BC

UNARY SYSTEM

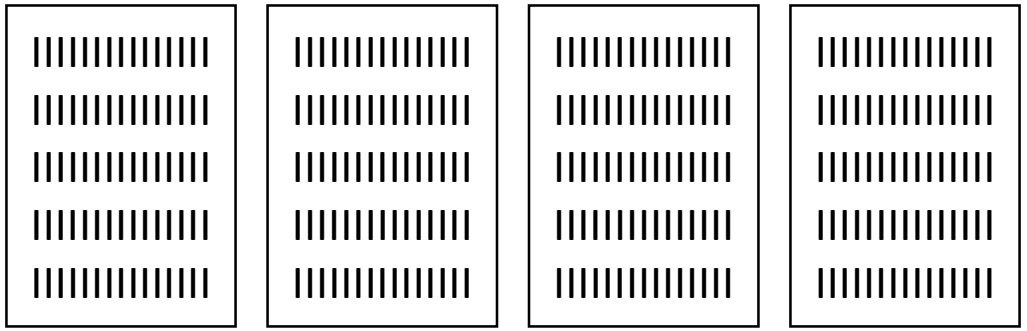
aka. Base-1

How many position is needed to represent the moon's distance to the sun if an Oracle said it is ~150 million km and Earth's diameter is ~13,000 km?

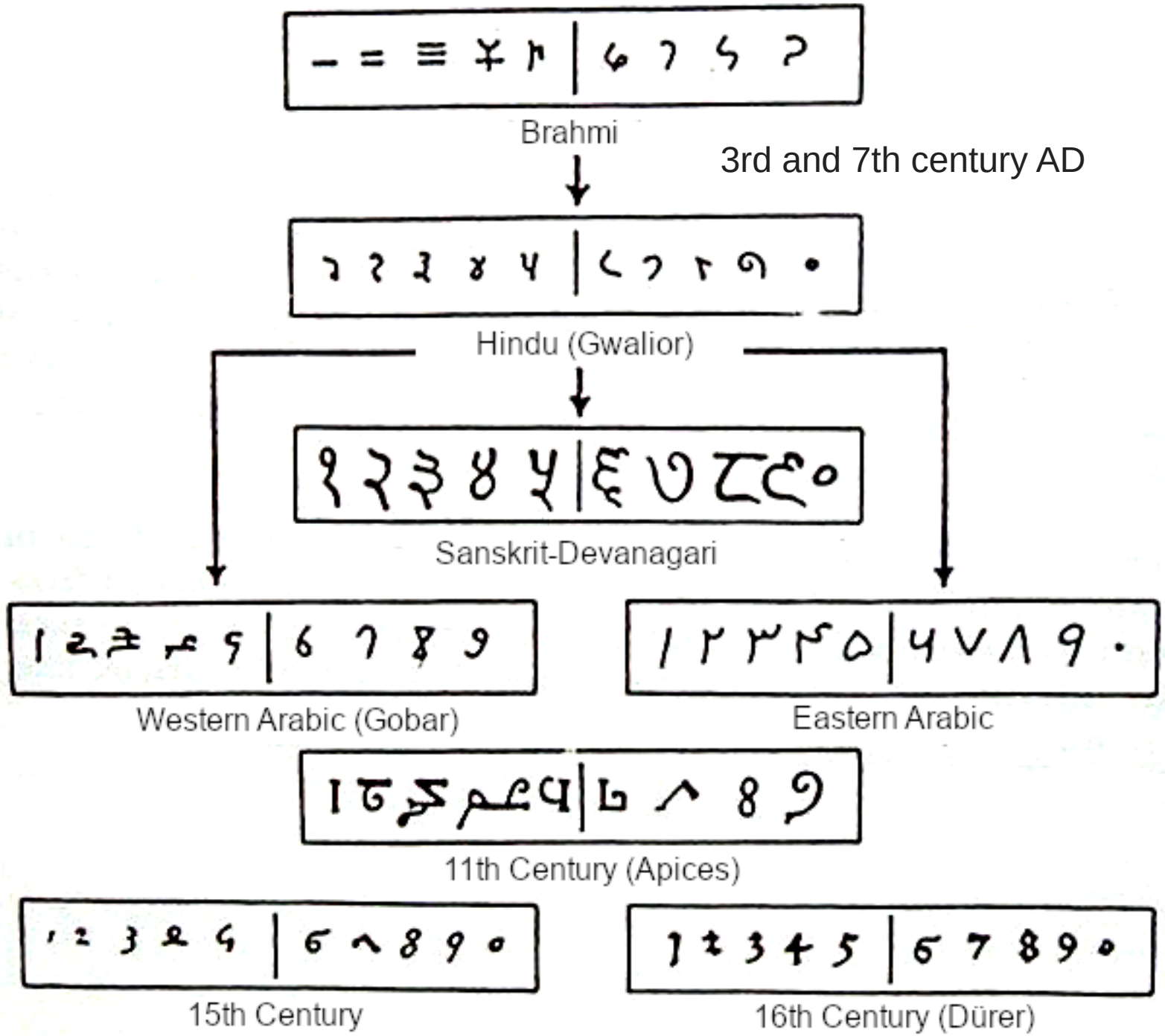
~150 million km / ~13,000 km = 12,000 Earth

Each A4 paper = ~3,000 positions

12,000 / 3,000 = 4 pages!

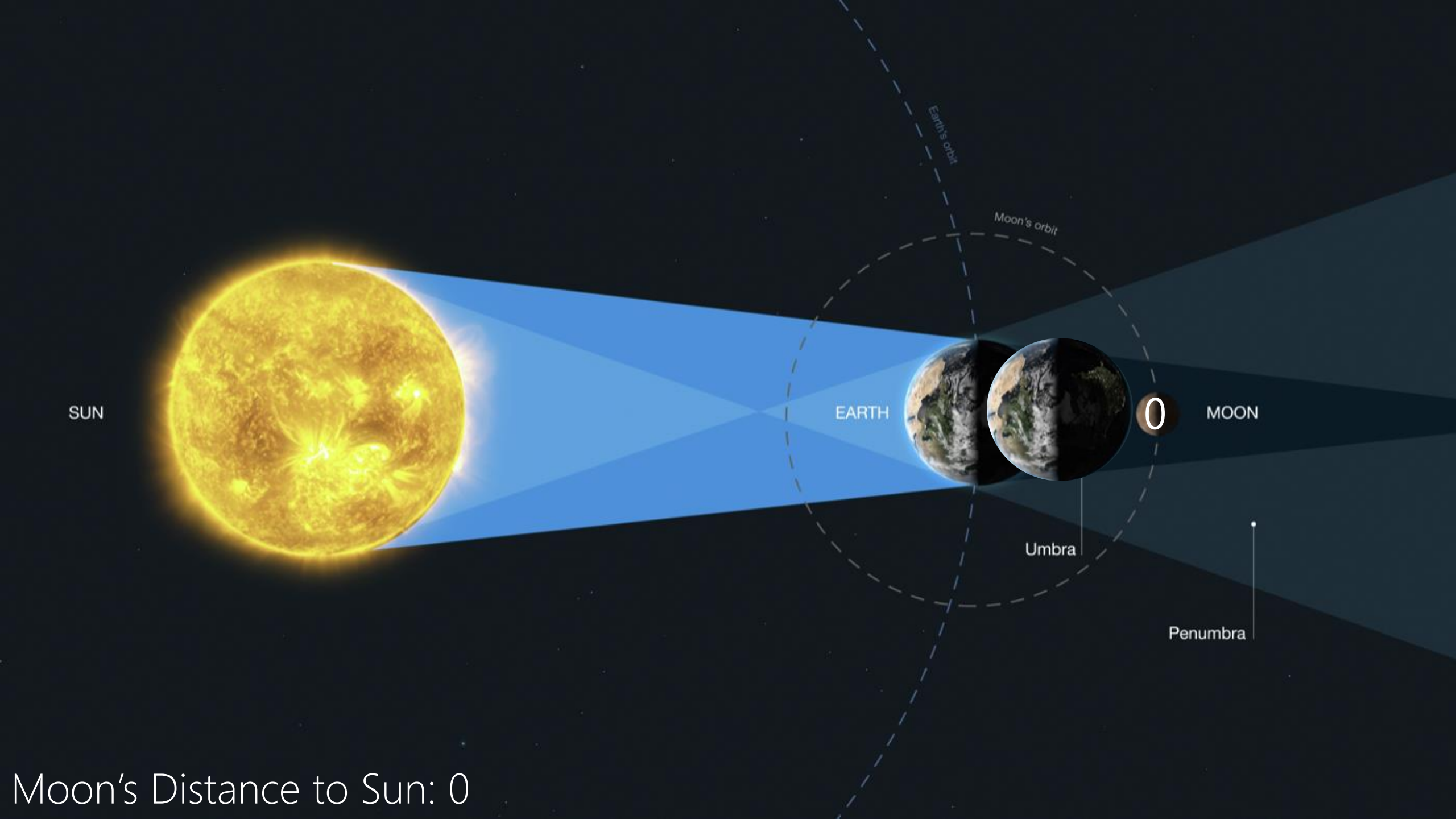


NUMBER SYSTEMS

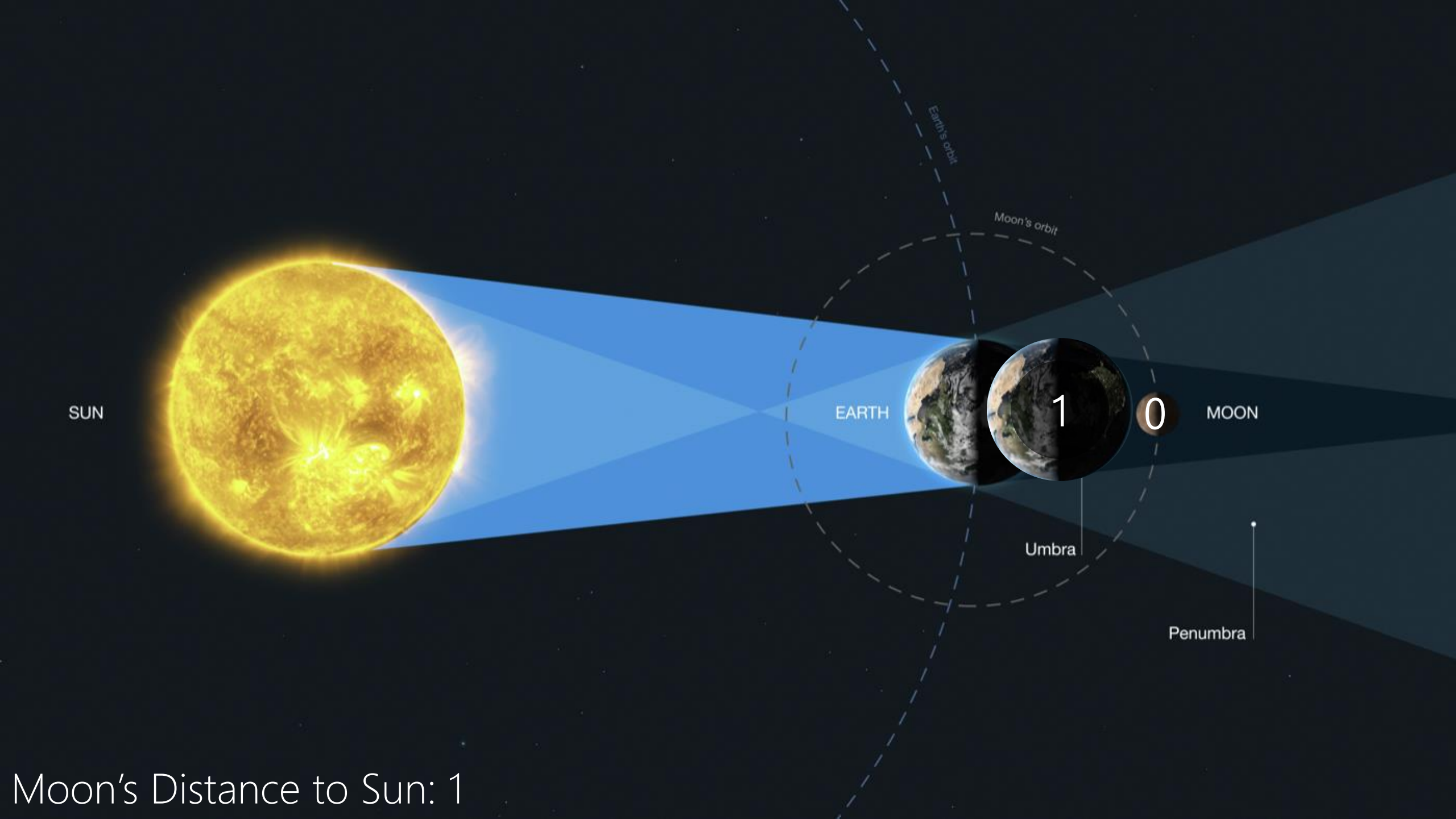

Brahmi
3rd and 7th century AD
Hindu (Gwalior)
Sanskrit-Devanagari
Western Arabic (Gobar)
Eastern Arabic
11th Century (Apices)
15th Century
16th Century (Dürer)

0123456789

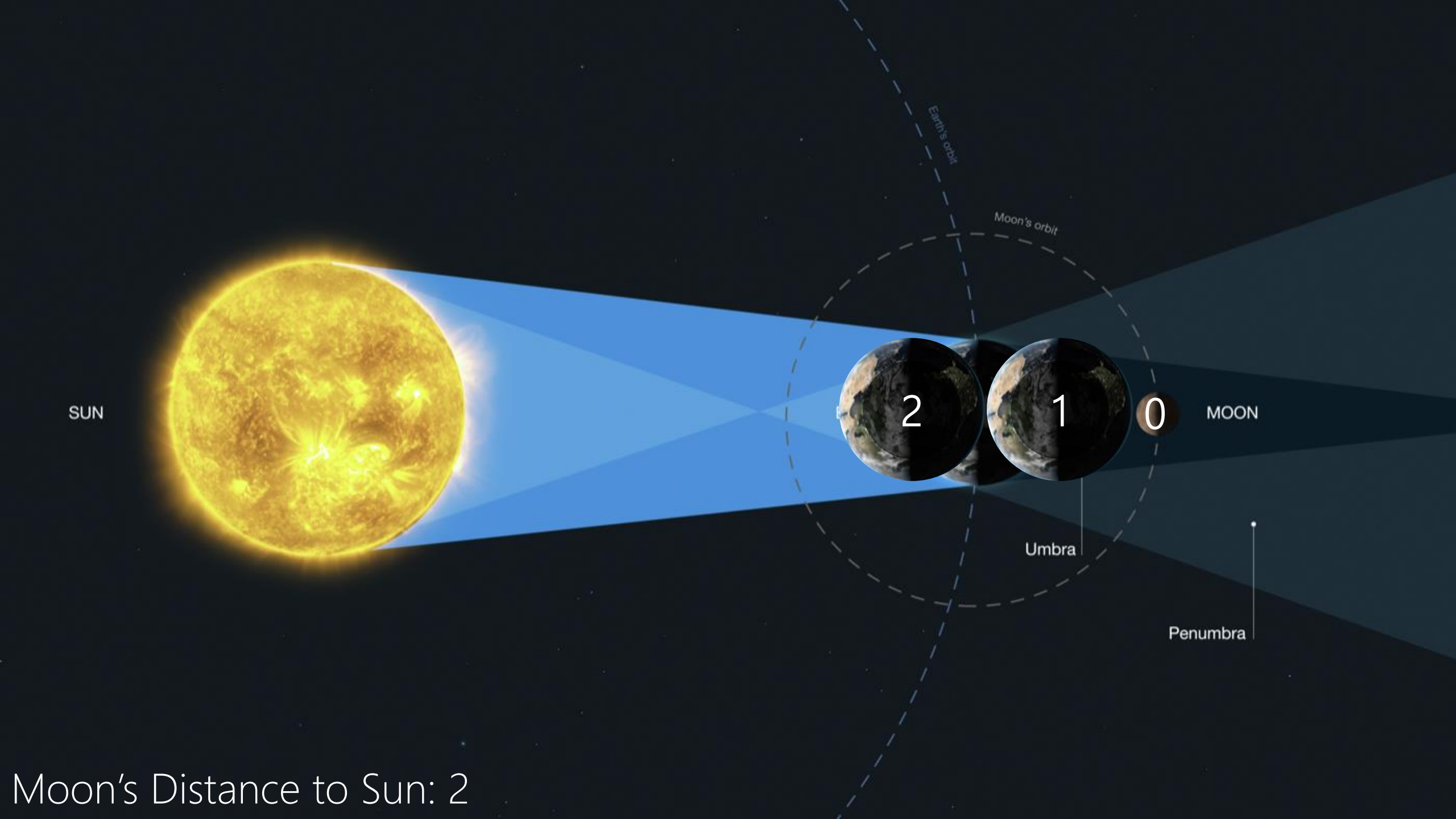
Hossein's Number System



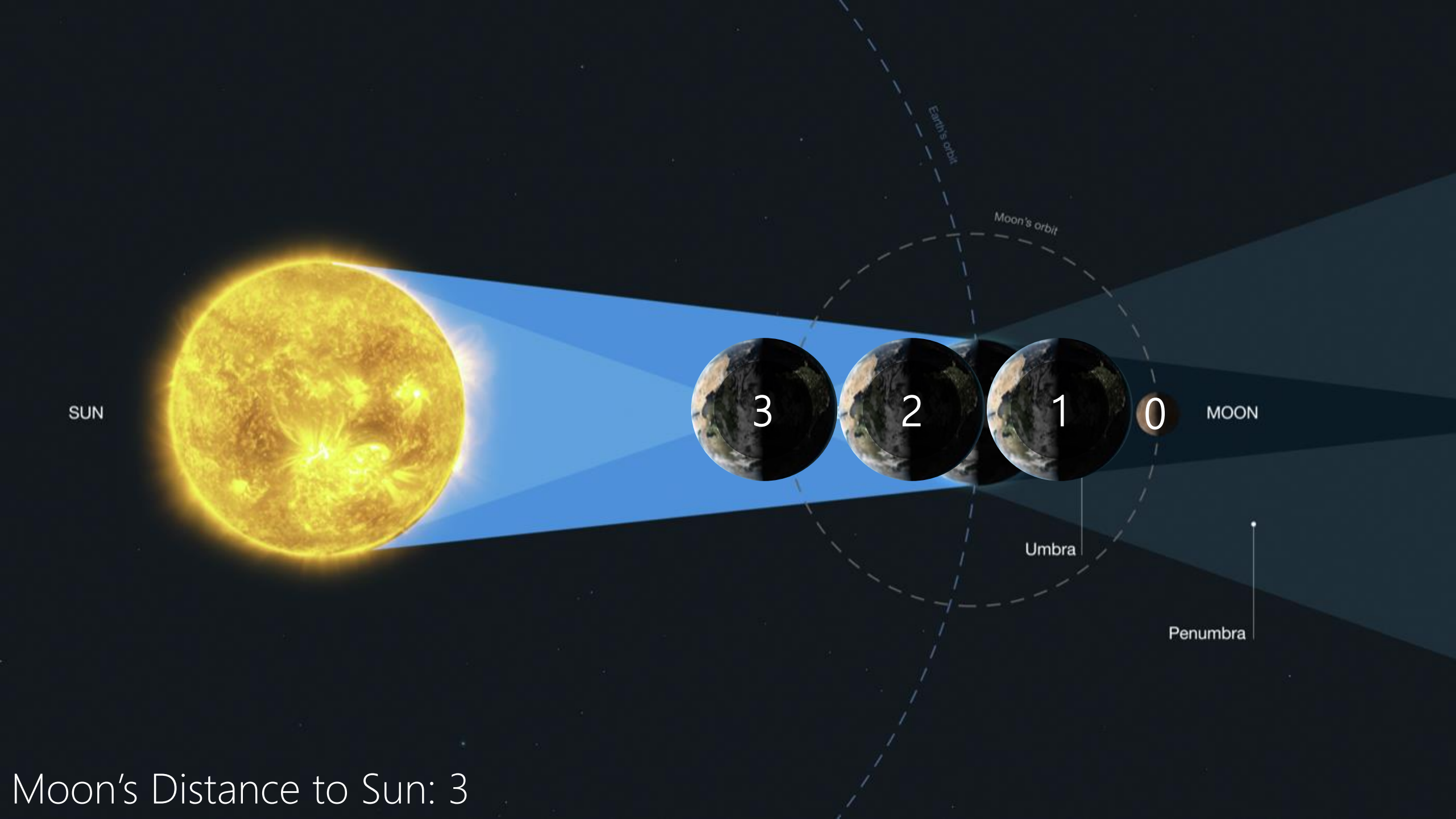
Moon’s Distance to Sun: 0



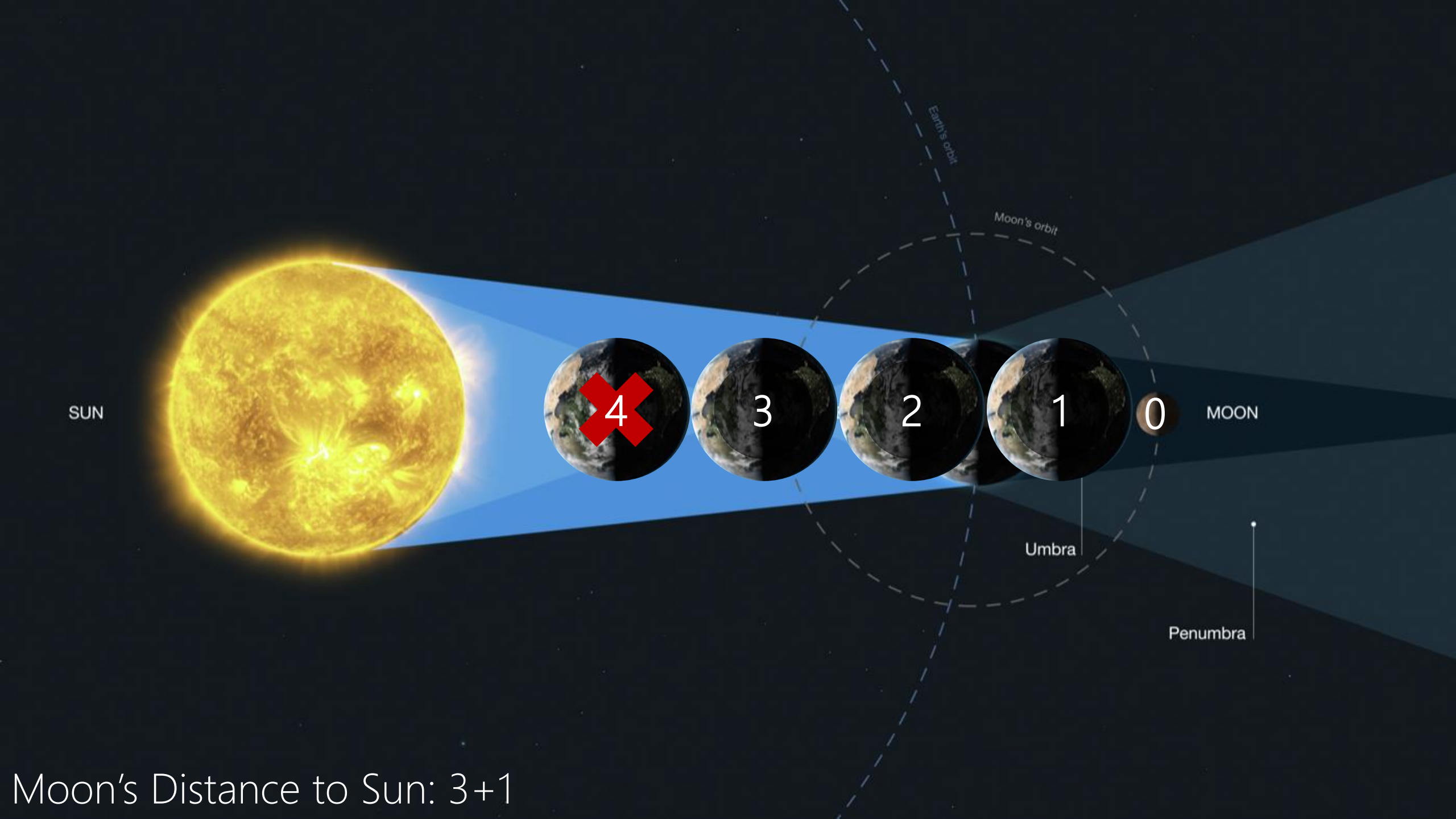
Moon’s Distance to Sun: 1



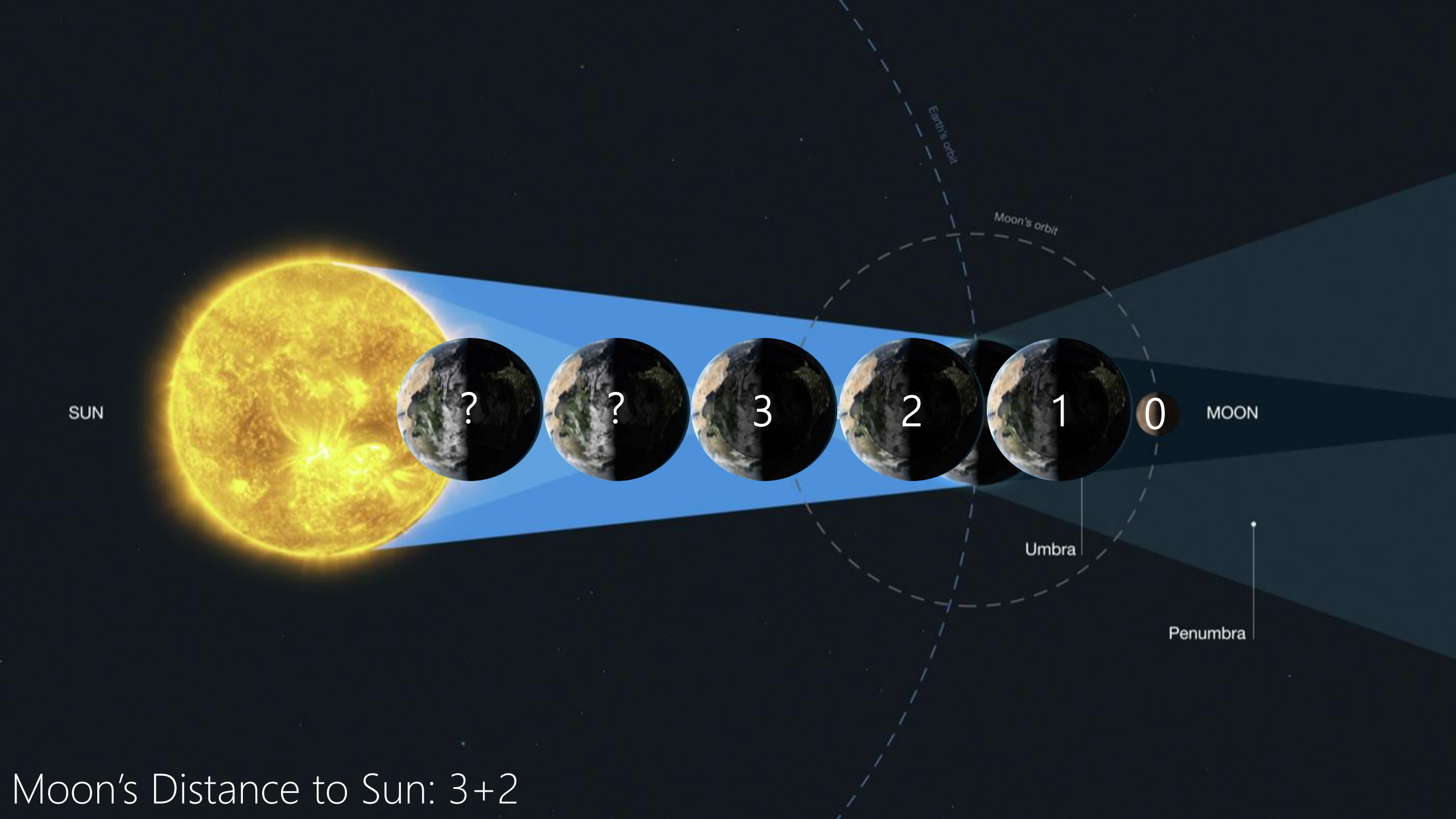
Moon’s Distance to Sun: 2



Moon’s Distance to Sun: 3

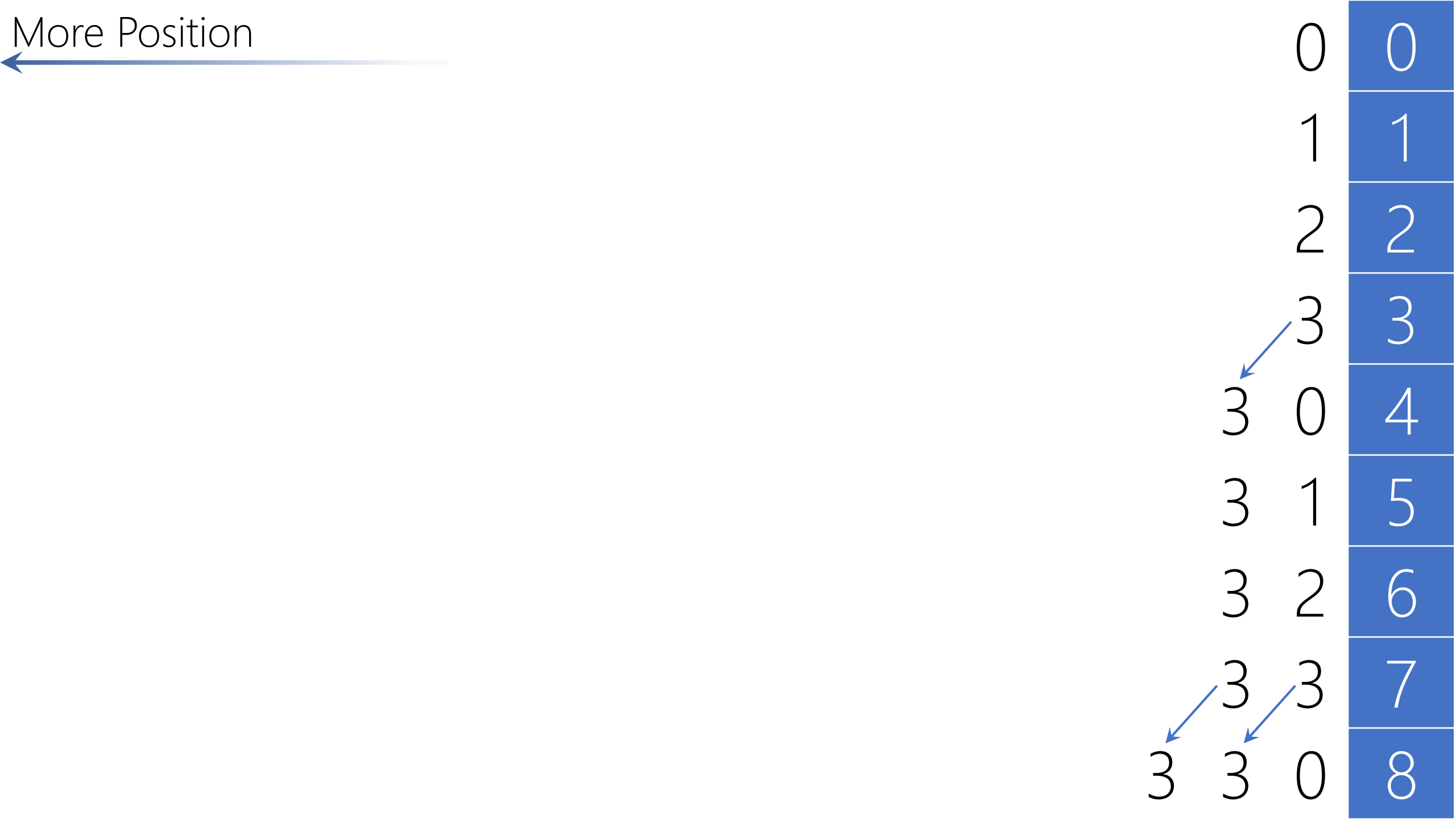


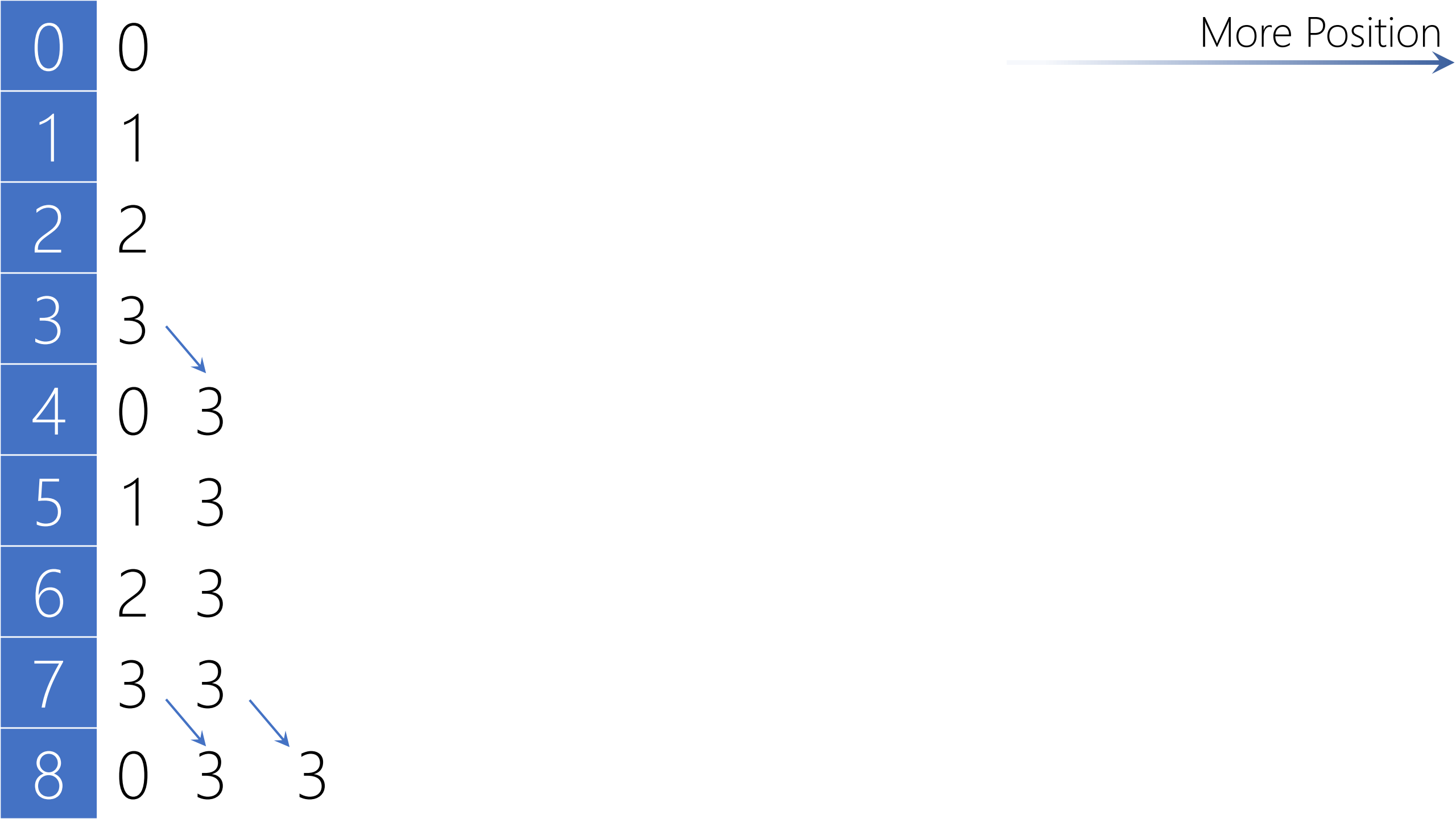
Moon's Distance to Sun: 3+1



Moon's Distance to Sun: 3+2

More Position




More Position
0
1
2
3
4
5
6
7
8
0
1
2
3
0
3
1
3
2
3
3
3
0
3
3

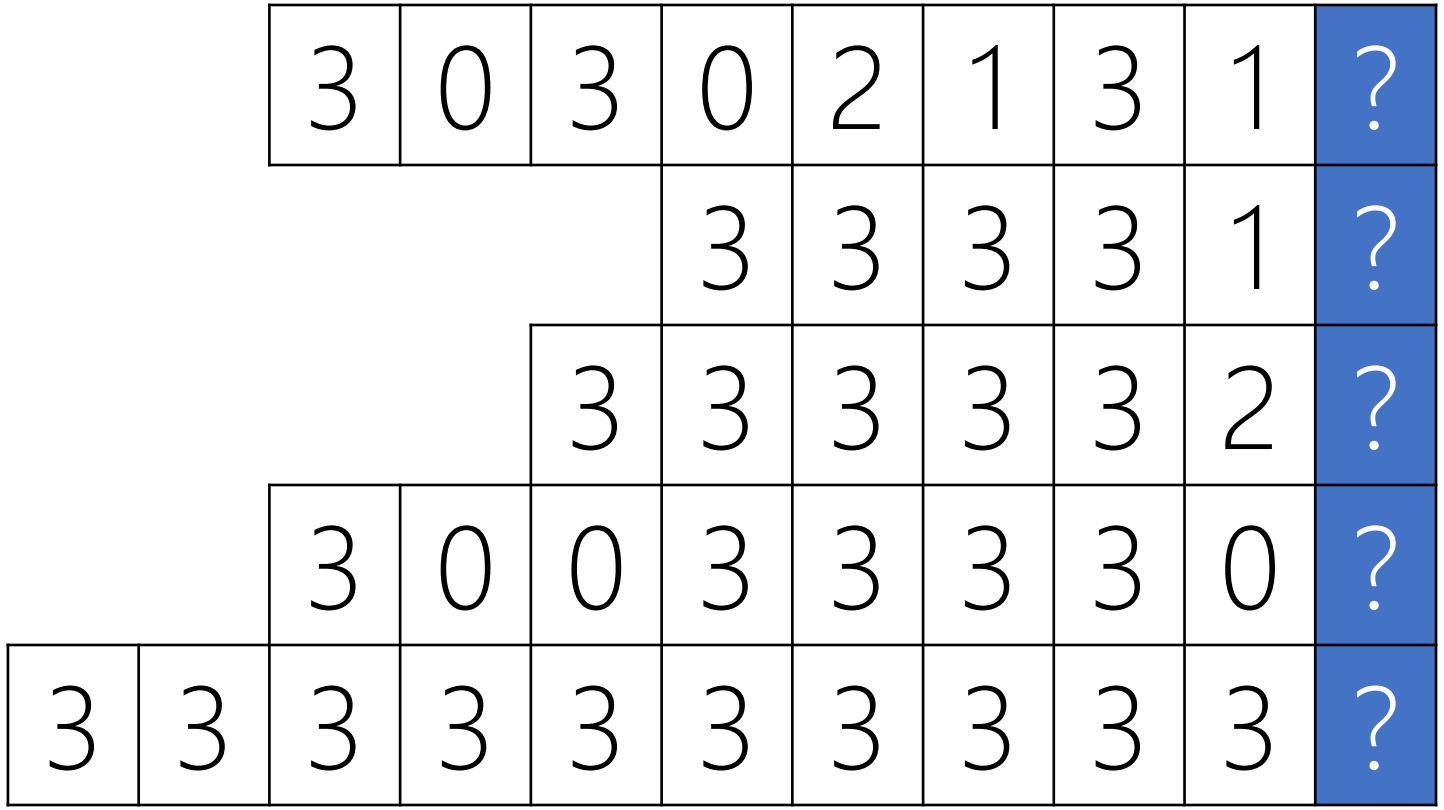
More Position

		3	3	0	8
		3	3	1	9
		3	3	2	10
		3	3	3	11
	3	3	3	0	12
	3	3	3	1	13
	3	3	3	2	14
	3	3	3	3	15
3	3	3	3	0	16

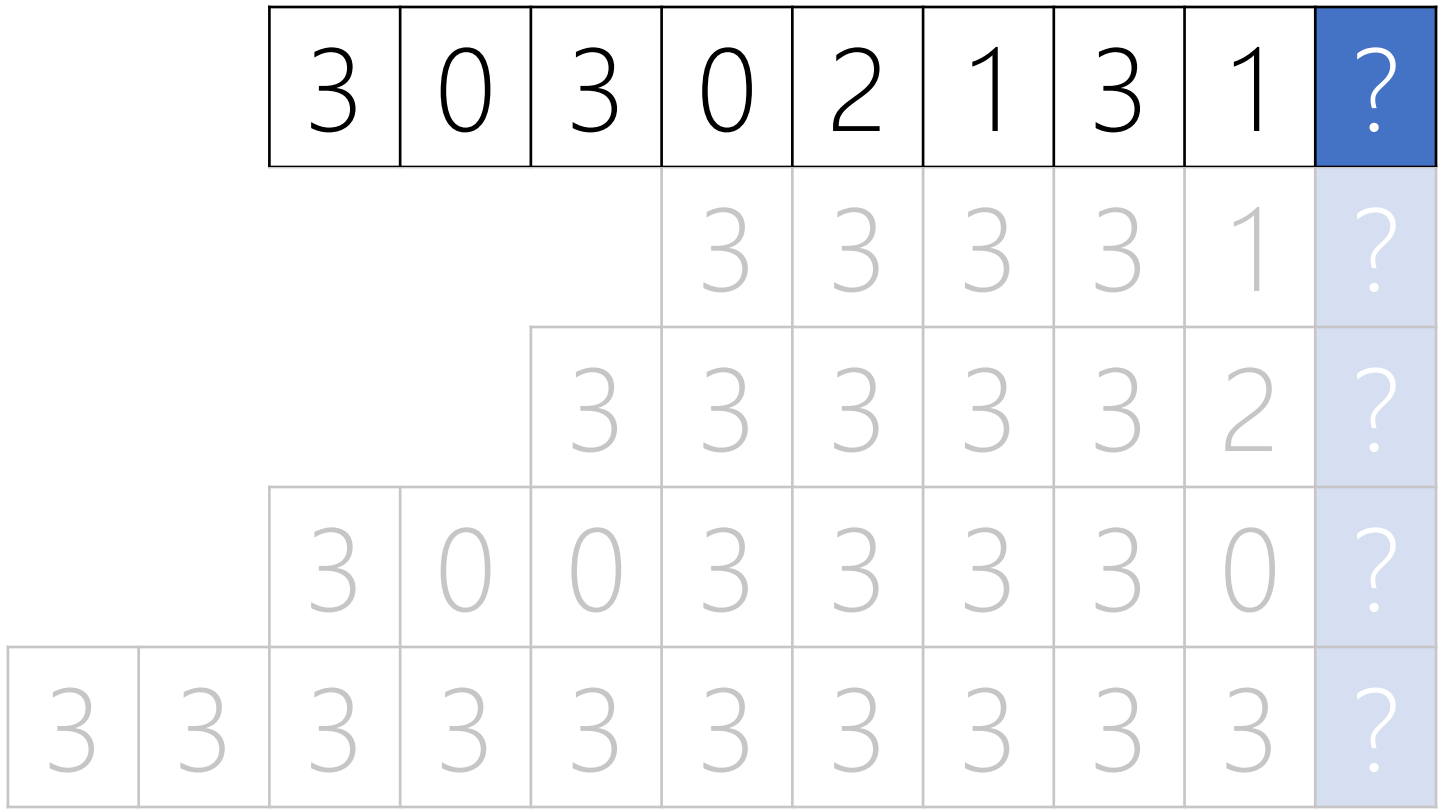
More Position

			3	3	0	3+3+0+(3-1)=8
			3	3	1	3+3+1+(3-1)=9
			3	3	2	3+3+2+(3-1)=10
			3	3	3	3+3+3+(3-1)=11
		3	3	3	0	3+3+3+0+(4-1)=12
		3	3	3	1	3+3+3+1+(4-1)=13
		3	3	3	2	3+3+3+2+(4-1)=14
		3	3	3	3	3+3+3+3+(4-1)=15
	3	3	3	3	0	3+3+3+3+(5-1)=16

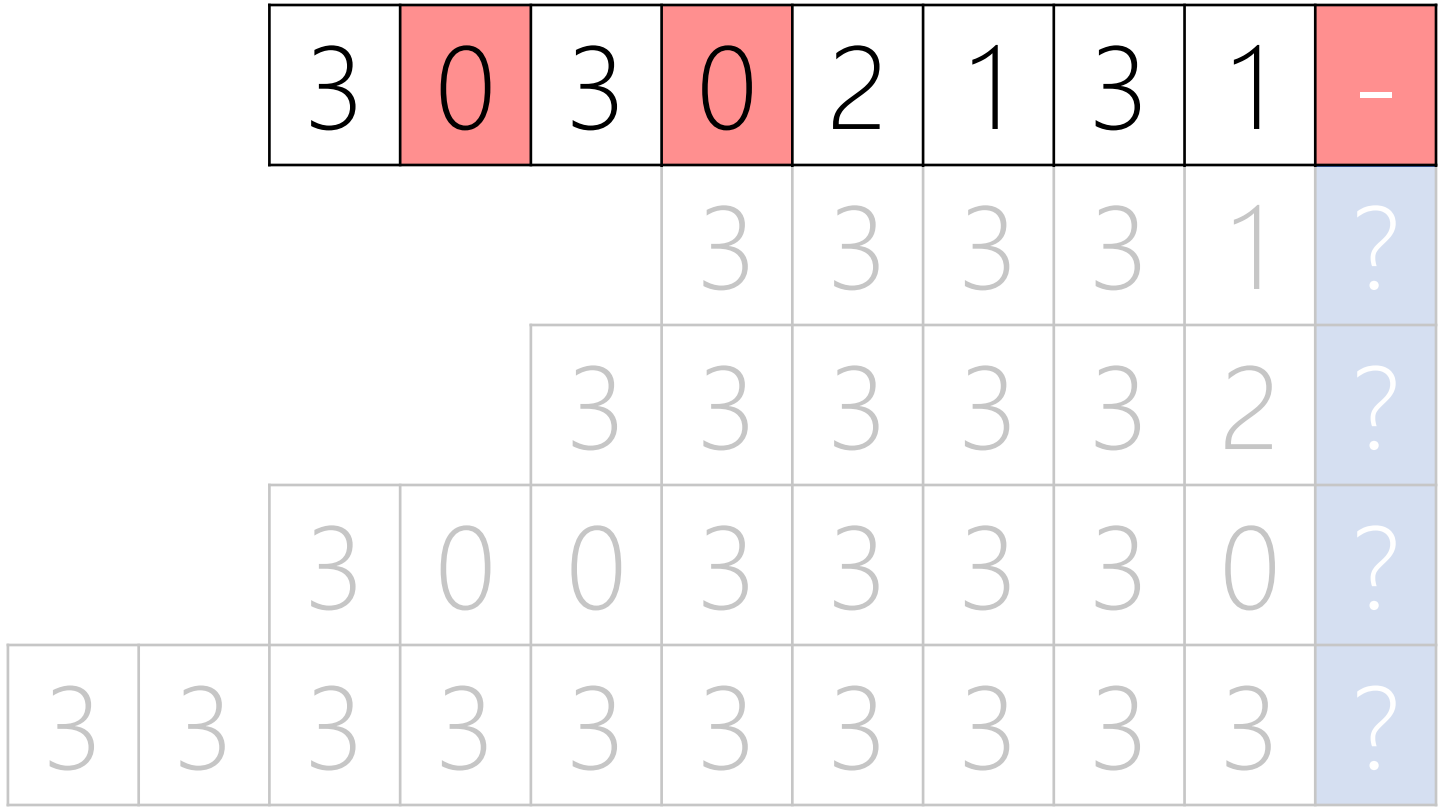
More Position



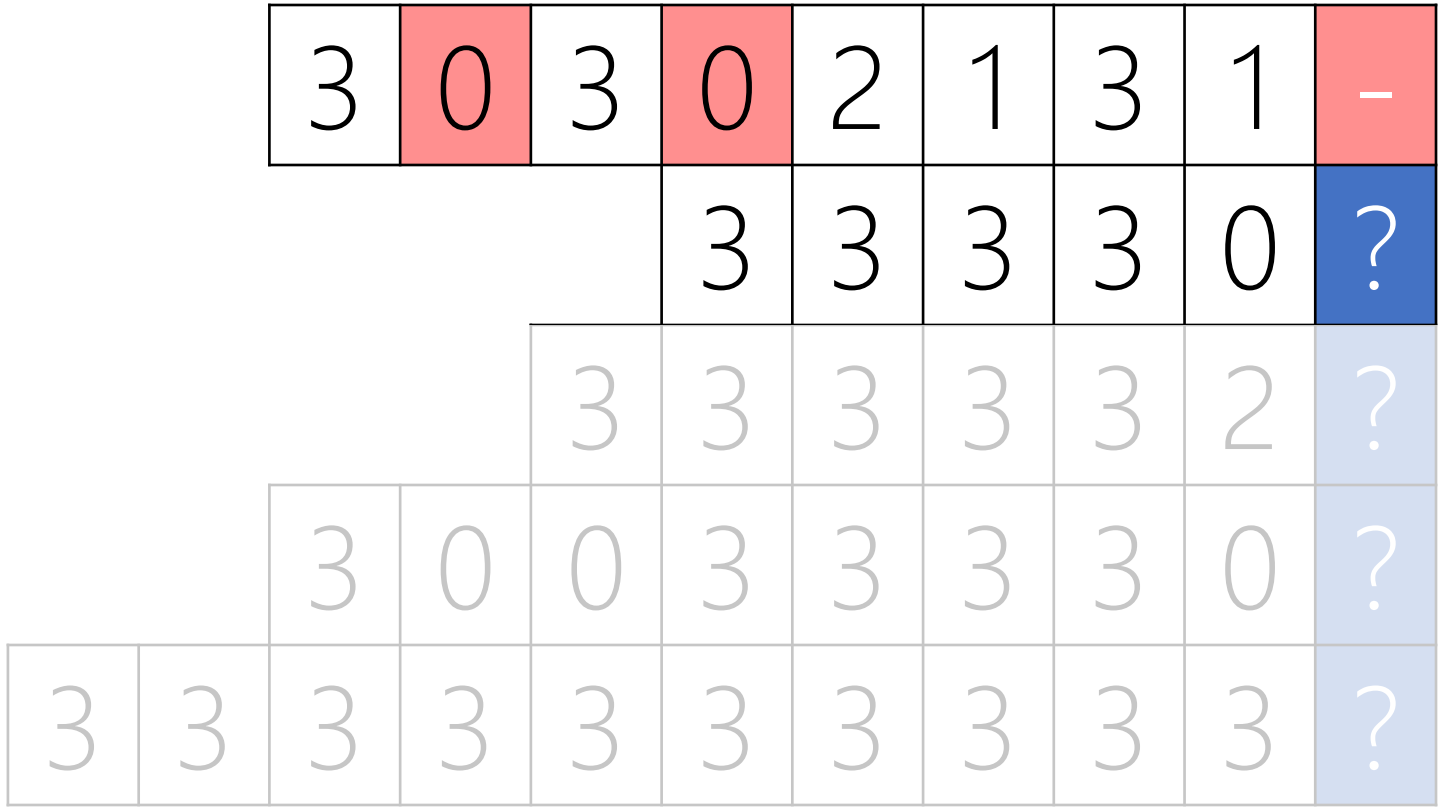
More Position



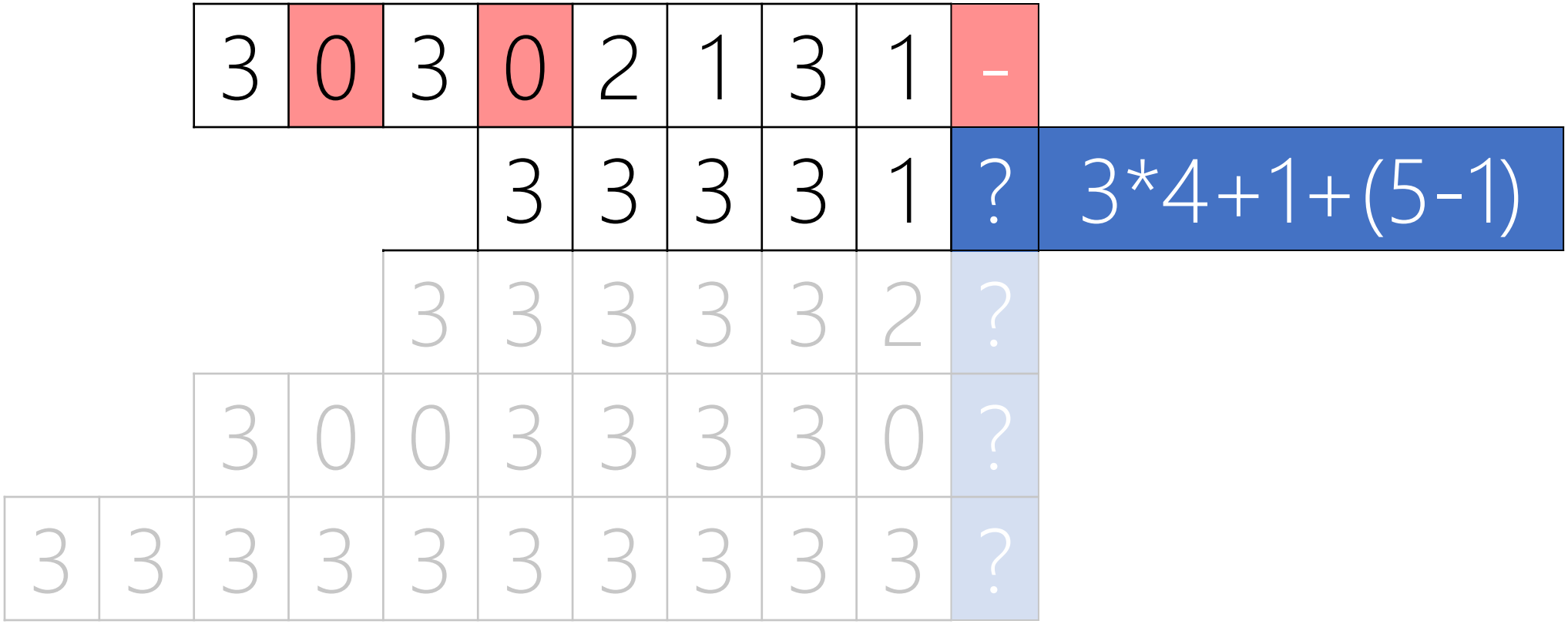
More Position



More Position

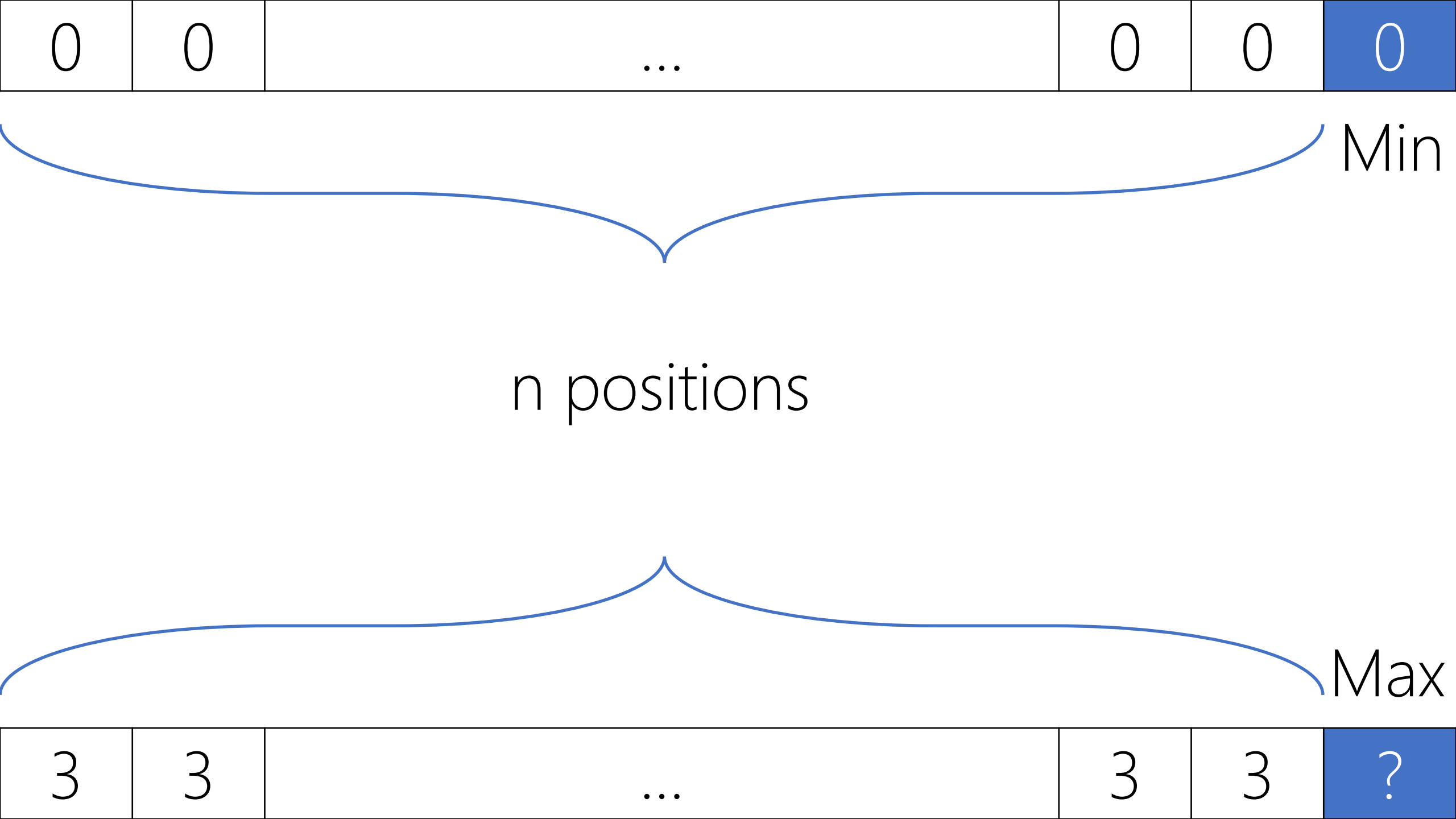


More Position



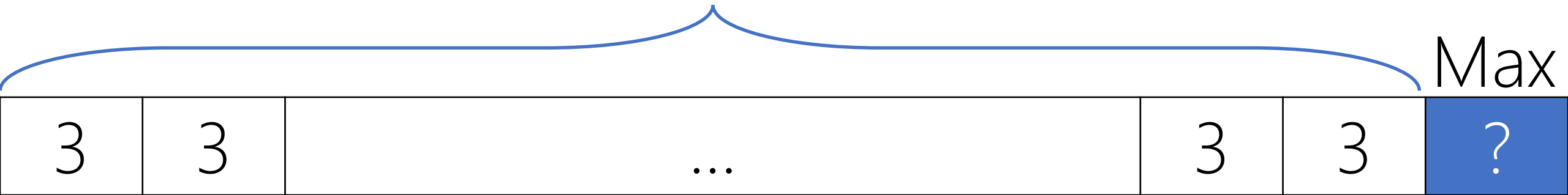
More Position

		3	0	3	0	2	1	3	1	-
					3	3	3	3	1	17
				3	3	3	3	3	2	22
		3	0	0	3	3	3	3	0	-
3	3	3	3	3	3	3	3	3	3	39


0
0
...
0
0
0
Min
n positions
Max
3
3
...
3
3
?

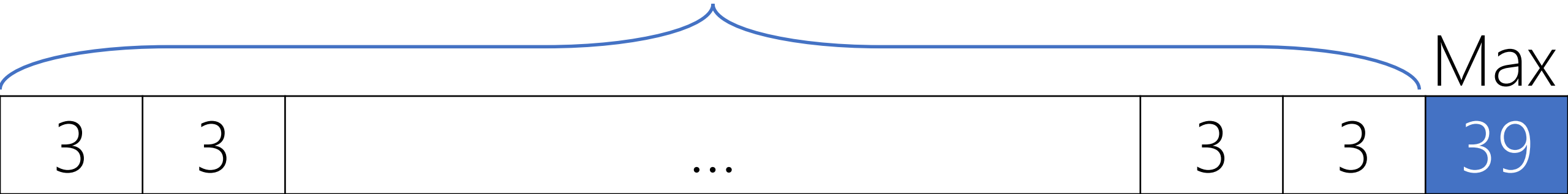
$$3 + 3 + 3 + \ \ldots + 3 + (n\ - 1) = 3\ \times n\ + (n\ - 1) = 4n - 1$$

n positions



$$3 + 3 + 3 + \ \ldots + 3 + (n\ - 1) = 3\ \times n\ + (n\ - 1) = 4n - 1$$
$$n = 10 \Rightarrow\ 4 \times 10 - 1 = 39$$

10 positions



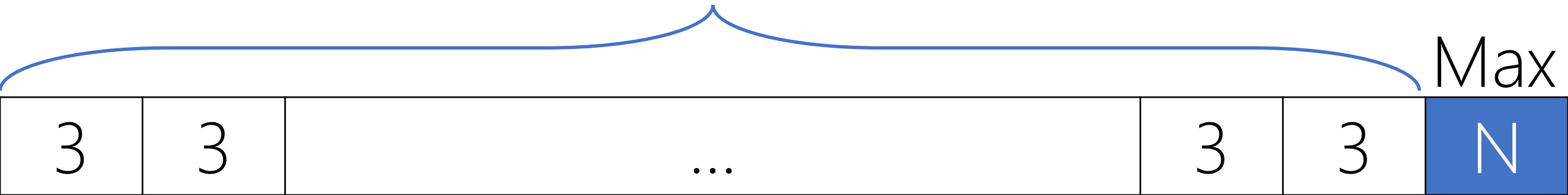
How many position needed to represent the moon's distance to the sun if an Oracle said it is ~150 million km and Earth's diameter is ~13,000 km?

$$3 + 3 + 3 + \ \ldots + 3 + (n\ - 1) = 3\ \times n\ + (n\ - 1) = 4n - 1$$

$$4n - 1 = N$$

$$n = \frac{N + 1}{4}$$

? positions



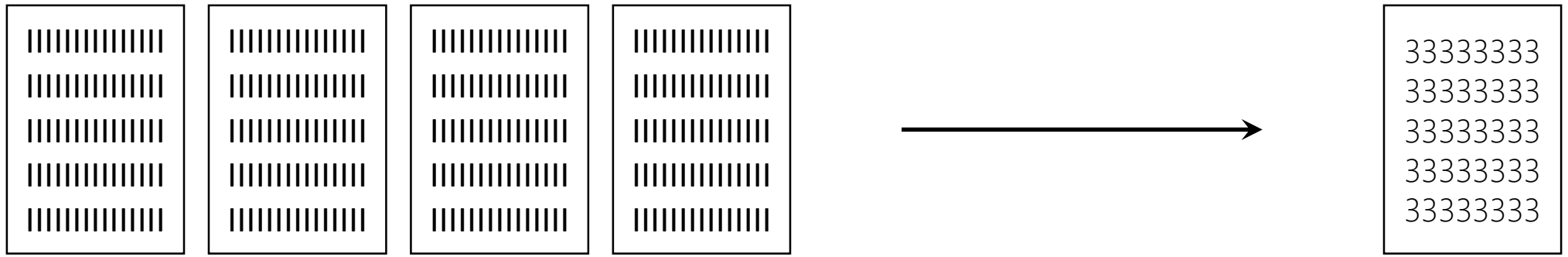
~150 million km / ~13,000 km = 12,000 * Earth

N = 12,000

n = (N + 1) / 4 = (12,000 + 1)/4 ~= 3,000 positions

Each A4 paper = ~3,000 positions

3,000 / 3,000 = 1 page

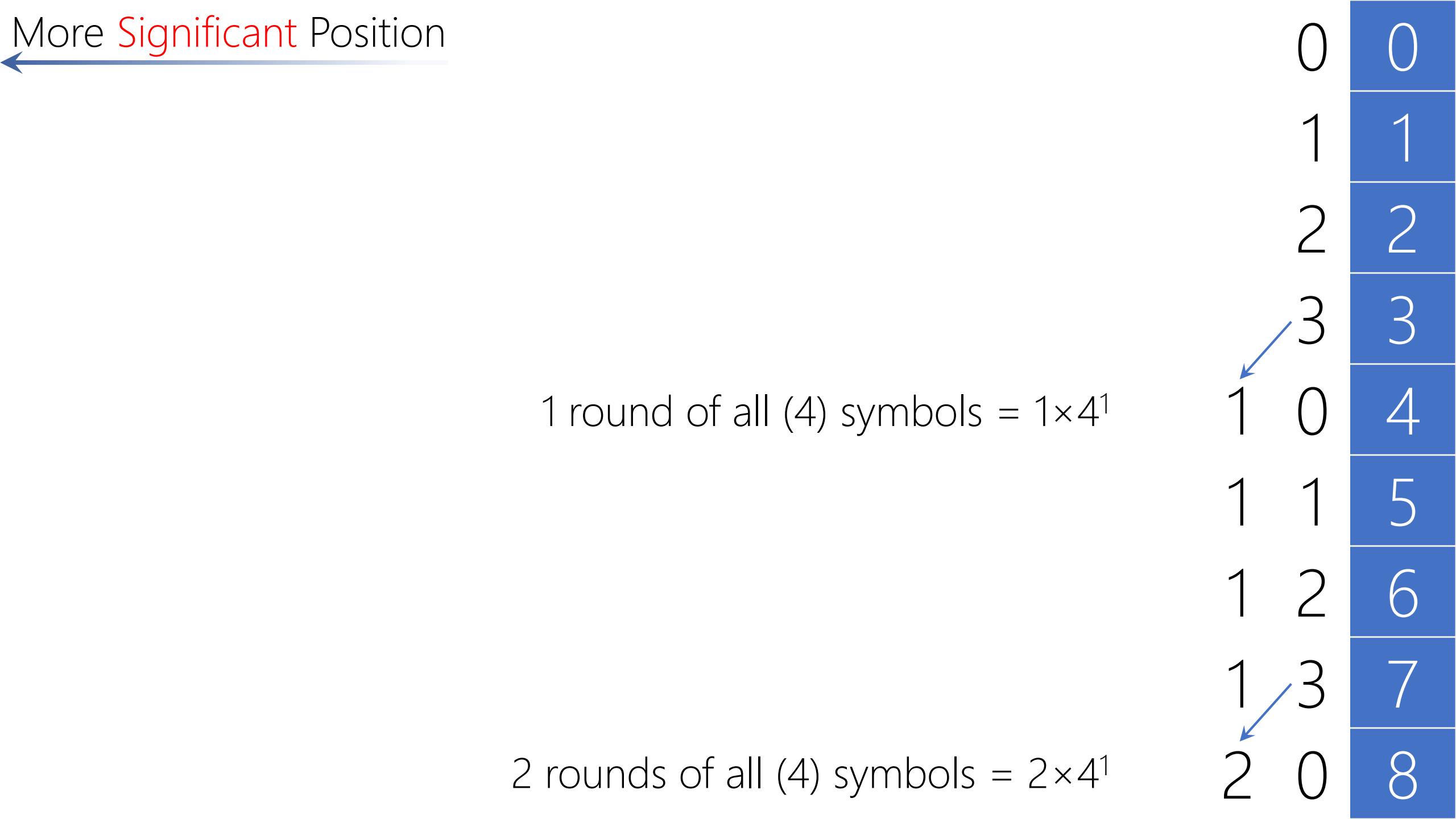


More Position

Same Significance

		3	3	0	8
		3	3	1	9
		3	3	2	10
		3	3	3	11
	3	3	3	0	12
	3	3	3	1	13
	3	3	3	2	14
	3	3	3	3	15
3	3	3	3	0	16

More Significant Position



More Significant Position

3 rounds of all (4) symbols = 3×4^1

1 rounds of all (4) symbols in the more significant position
= $4 \times 4^1 = 1 \times 4^2$

2	0	8
2	1	9
2	2	10
2	3	11
3	0	12
3	1	13
3	2	14
3	3	15
1 0	0	16

0123456789

Hindu-Arabic Numerals
Originated in India
7th Century AD
Quaternary
/kwaa·**tur**·neh·ree/

More Significant Position

#Symbols=4
Radix-4
Base-4

4^0	
0	$0\times4^0=0$
1	$1\times4^0=1$
2	$2\times4^0=2$
3	$3\times4^0=3$

More Significant Position

#Symbols=4
Radix-4
Base-4

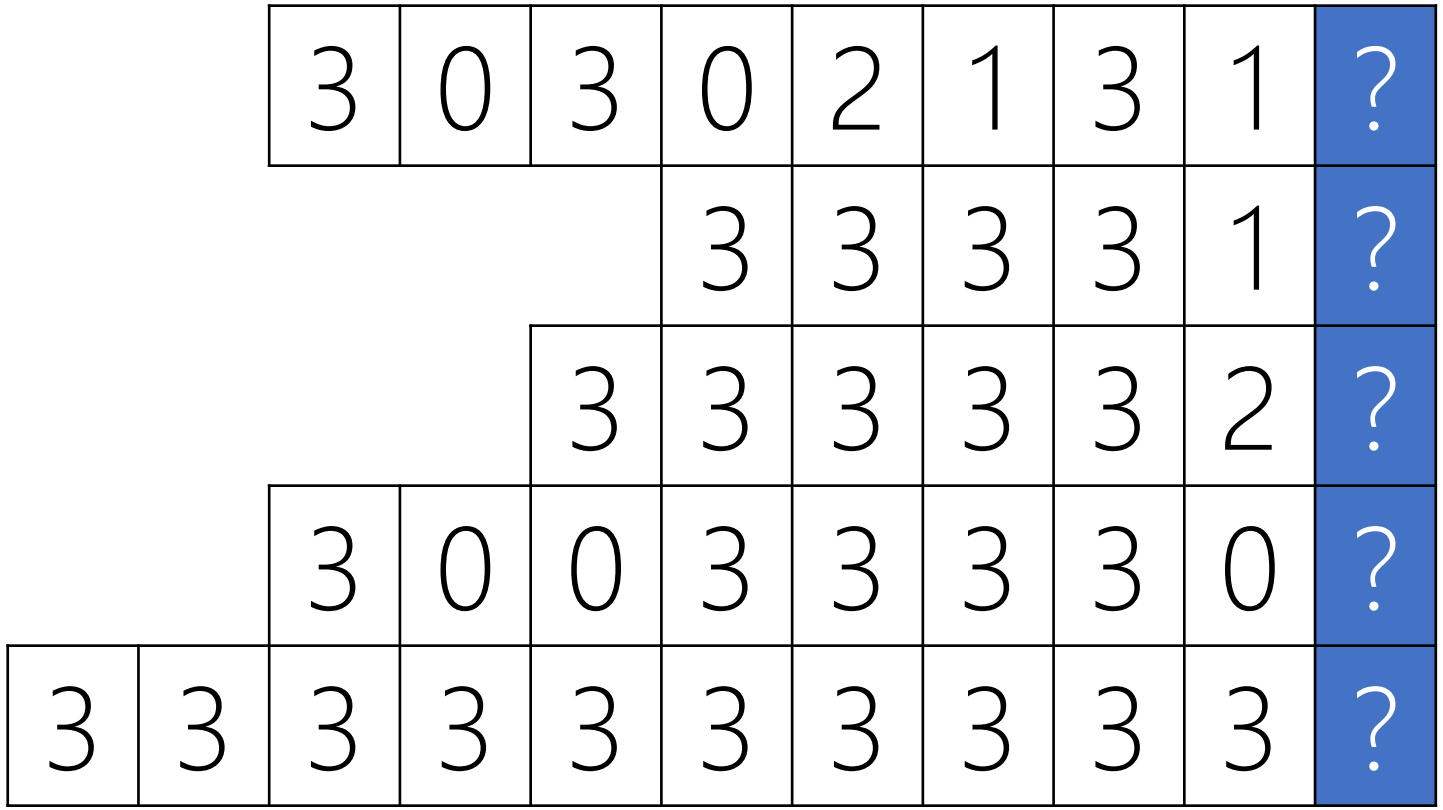
4^1	4^0	
	0	$0\times4^0=0$
	1	$1\times4^0=1$
	2	$2\times4^0=2$
	3	$3\times4^0=3$
1	0	$1\times4^1+0\times4^0=4$
1	1	$1\times4^1+1\times4^0=5$
1	2	$1\times4^1+2\times4^0=6$
1	3	$1\times4^1+3\times4^0=7$
2	0	$2\times4^1+0\times4^0=8$

More Significant Position

#Symbols=4
Radix-4
Base-4

4^2	4^1	4^0	
	2	0	$2\times4^1+0\times4^0=8$
	2	1	$2\times4^1+1\times4^0=9$
	2	2	$2\times4^1+2\times4^0=10$
	2	3	$2\times4^1+3\times4^0=11$
	3	0	$3\times4^1+0\times4^0=12$
	3	1	$3\times4^1+1\times4^0=13$
	3	2	$3\times4^1+2\times4^0=14$
	3	3	$3\times4^1+3\times4^0=15$
1	0	0	$1\times4^2+0\times4^1+0\times4^0=16$

More Significant Position



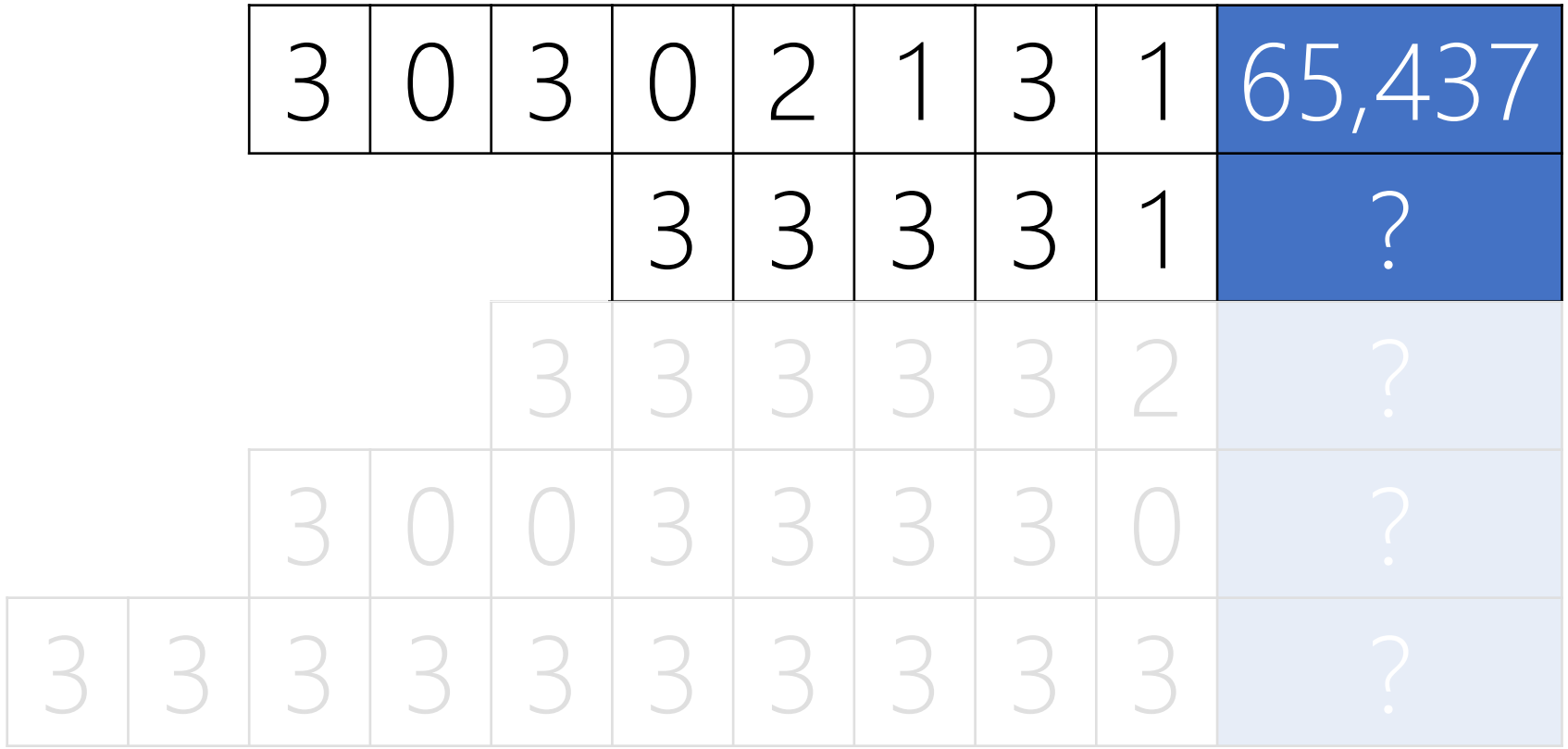
More Significant Position

3	0	3	0	2	1	3	1	?
			3	3	3	3	1	?
		3	3	3	3	3	2	?
	3	0	0	3	3	3	0	?
3	3	3	3	3	3	3	3	3	3	?

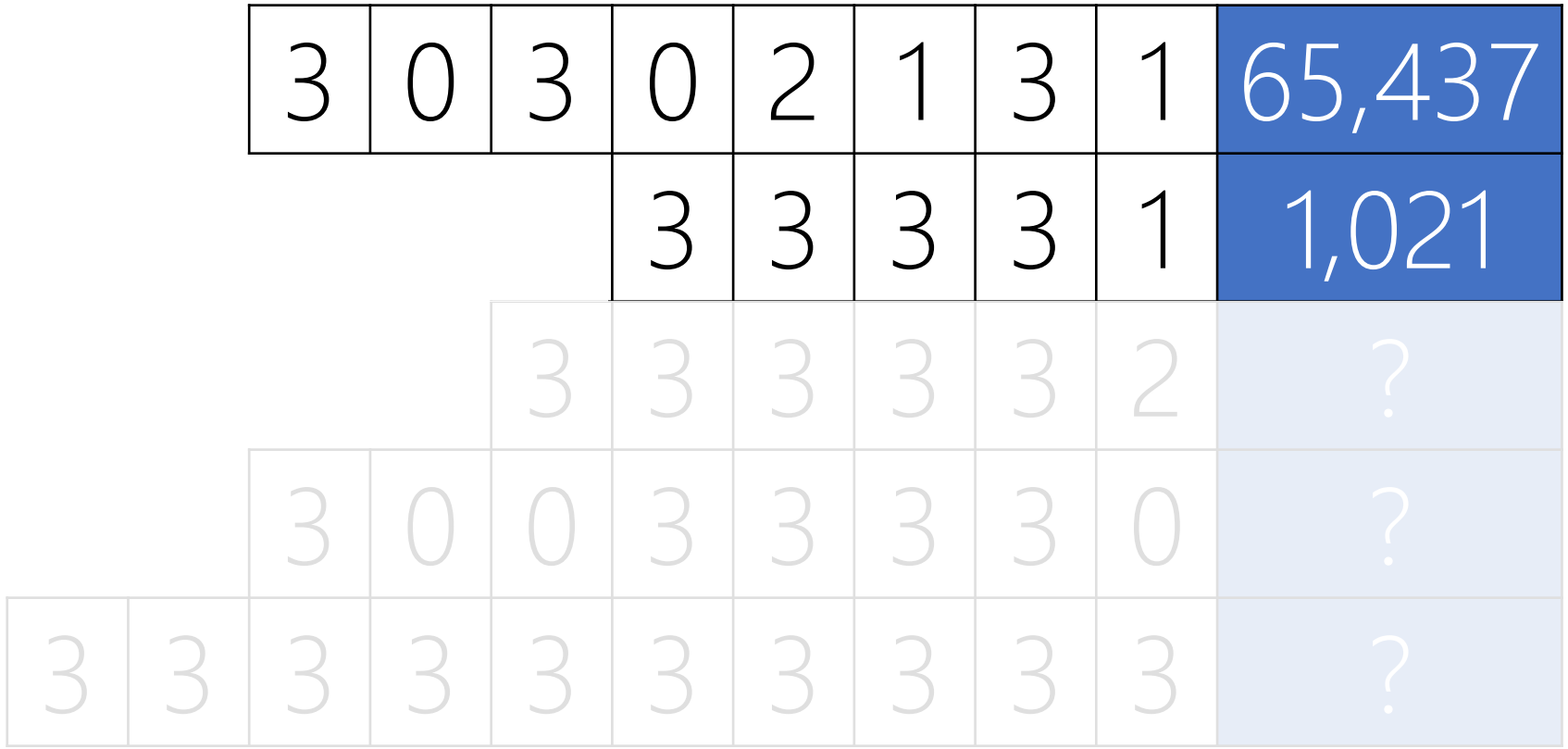
More Significant Position

4^7	4^6	4^5	4^4	4^3	4^2	4^1	4^0	
3	0	3	0	2	1	3	1	×
3×4^7	0×4^6	3×4^5	0×4^4	2×4^3	1×4^2	3×4^1	1×4^0	$\sum$
								65,437

More Significant Position



More Significant Position

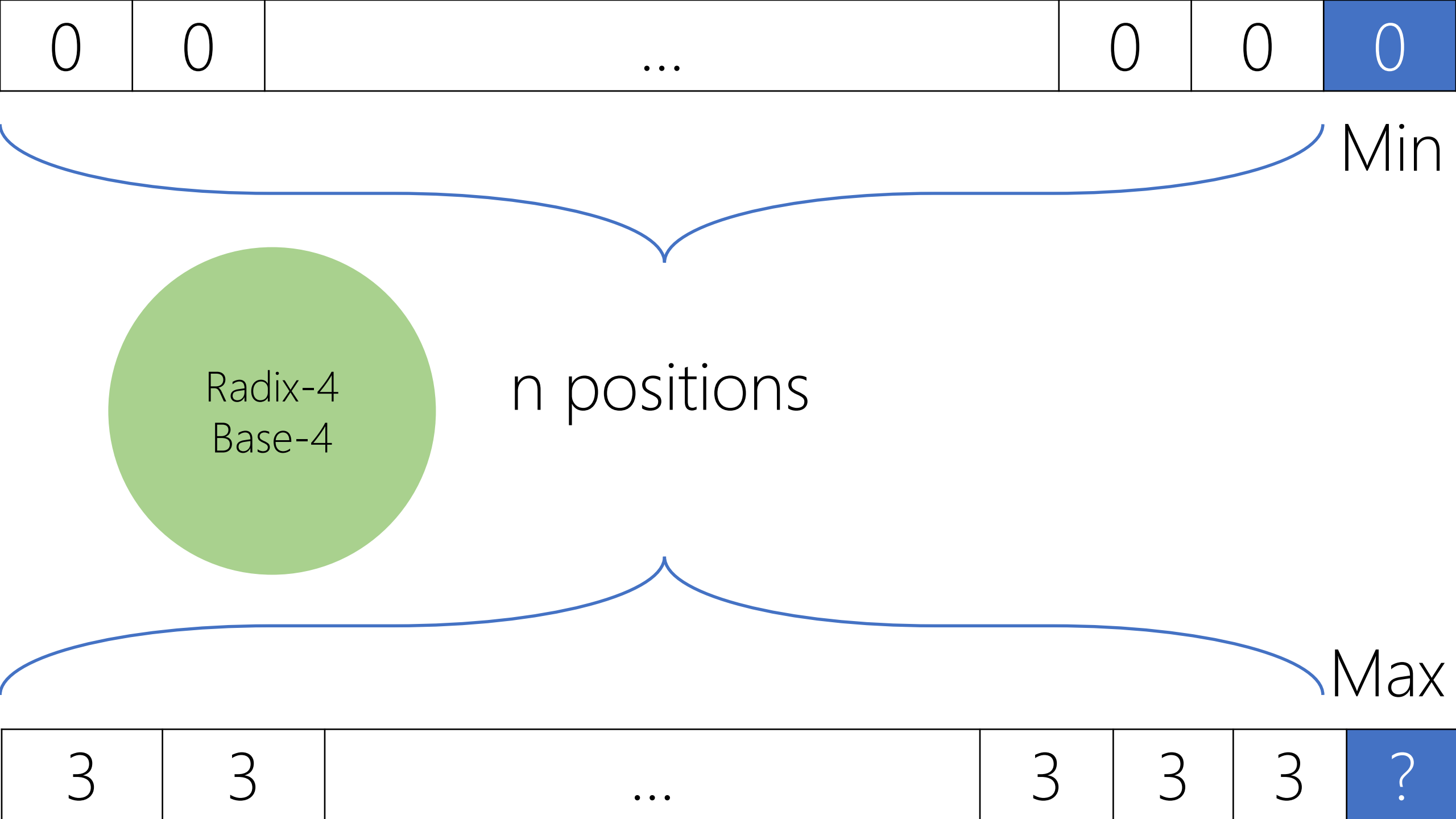


More Significant Position

		3	0	3	0	2	1	3	1	65,437
					3	3	3	3	1	1,021
				3	3	3	3	3	2	4,094
		3	0	0	3	3	3	3	0	50,172
3	3	3	3	3	3	3	3	3	3	1,048,575

More Significant Position

									Base-4	Hossein's Number System
		3	0	3	0	2	1	3	1	65,437	-
					3	3	3	3	1	1,021	17
				3	3	3	3	3	2	4,094	22
		3	0	0	3	3	3	3	0	50,172	-
3	3	3	3	3	3	3	3	3	3	1,048,575	39


0
0
...
0
0
0
Min
Radix-4
Base-4
n positions
Max
3
3
...
3
3
3
?

$$3 \times 4^{n-1} + 3 \times 4^{n-2} + \cdots + 3 \times 4^2 + 3 \times 4^1 + 3 \times 4^0 =$$
$$3 \times (4^{n-1} + 4^{n-2} + \cdots + 4^2 + 4^1 + 4^0) =$$
$$3 \times \left(\frac{4^n - 1}{4 - 1}\right) = 4^n - 1$$

$4n - 1$
Hossein's System

n positions

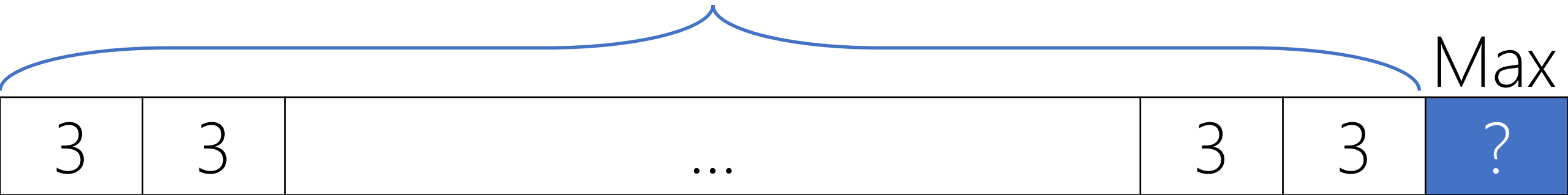
Max

4^{n-1}	4^{n-2}		4^2	4^1	4^0	
3	3	…	3	3	3	?

$$n = 10 \Rightarrow 410 - 1 = 1{,}048{,}575$$

39
Hossein's System

10 positions

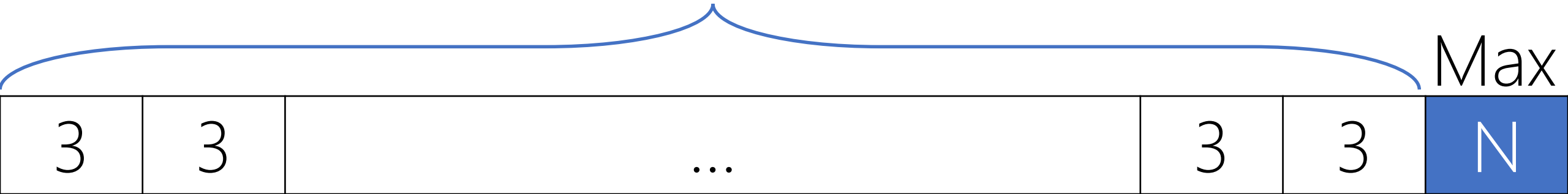


How many position needed to represent the moon's distance to the sun if an Oracle said it is ~150 million km and Earth's diameter is ~13,000 km?

$$4^n - 1 = N$$

$$n = \log_4(N + 1)$$

? positions

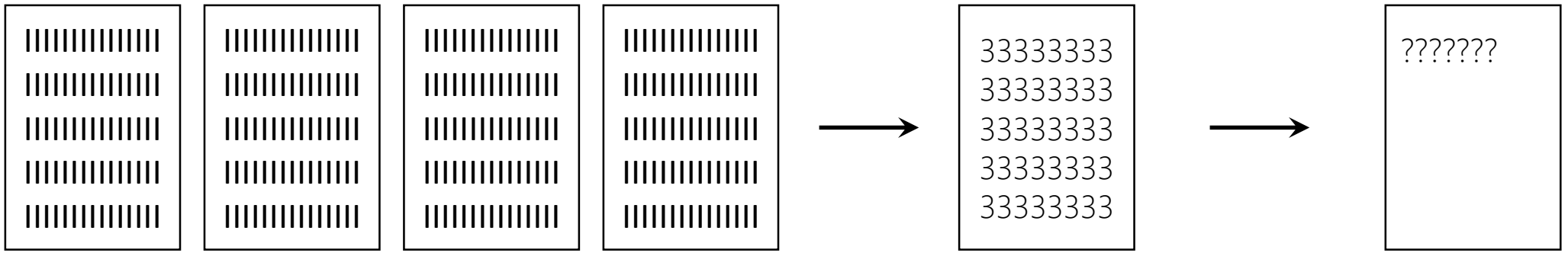


~150 million km / ~13,000 km = 12,000 * Earth

N = 12,000

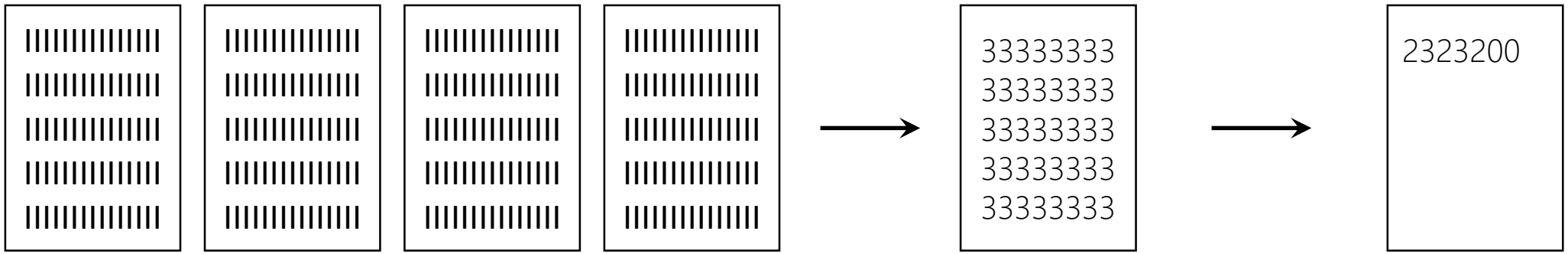
$n = \text{Log}_4 (12{,}000 + 1) = \text{Log}_{10}\ 12{,}001 / \text{Log}_{10}\ 4 = 4 / 0.6 = 6.79$

~ 7 positions



~150 million km / ~13,000 km = 12,000 * Earth

$N = 12{,}000 = (2323200)_{\text{Base-4}} = (2323200)_4$

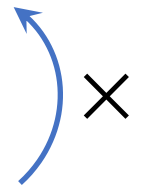


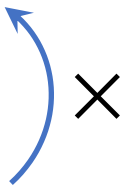
COMMON NUMBER SYSTEMS

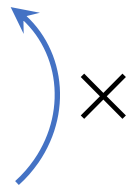
Binary
Base-2
Radix-2
$(0,1)_2$

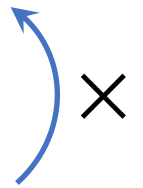
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
1	0	1	0	1	1	0	1	

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	0	1	0	1	1	0	1
1×2^7	0×2^6	1×2^5	0×2^4	1×2^3	1×2^2	0×2^1	1×2^0



2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
1	0	1	0	1	1	0	1	
1×2^7	0×2^6	1×2^5	0×2^4	1×2^3	1×2^2	0×2^1	1×2^0	$\sum$
								173

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
3	0	3	0	2	1	3	1	
								$\sum$

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
3	0	3	0	2	1	3	1	
3×2^7	0×2^6	3×2^5	0×2^4	2×2^3	1×2^2	3×2^1	1×2^0	$\sum$
								-

Octal

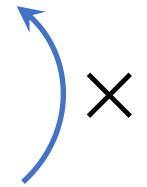
Base-8

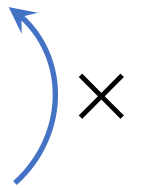
Radix-8

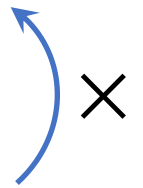
$(0,1,2,3,4,5,6,7)_8$

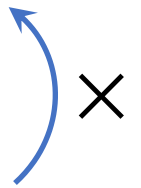
8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	
3	0	3	0	2	1	3	1	

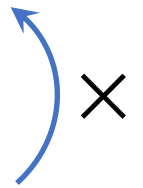
8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0
3	0	3	0	2	1	3	1
3×8^7	0×8^6	3×8^5	0×8^4	2×8^3	1×8^2	3×8^1	1×8^0



8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	
3	0	3	0	2	1	3	1	
3×8^7	0×8^6	3×8^5	0×8^4	2×8^3	1×8^2	3×8^1	1×8^0	$\sum$
								57,508,953

8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	
3	0	A	0	9	1	3	1	
								$\sum$
								-

8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	
3	0	A	0	9	1	3	1	
3×8^7	0×8^6	$A\times8^5$	0×8^4	9×8^3	1×8^2	3×8^1	1×8^0	$\sum$
								-

8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	
1	0	1	0	1	1	0	1	
1×8^7	0×8^6	1×8^5	0×8^4	1×8^3	1×8^2	0×8^1	1×8^0	$\sum$
								2,130,497

8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	
1	0	1	0	1	1	0	1	
1×8^7	0×8^6	1×8^5	0×8^4	1×8^3	1×8^2	0×8^1	1×8^0	
								2,130,497

Radix-8
vs.
Radix-2

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
1	0	1	0	1	1	0	1	
1×2^7	0×2^6	1×2^5	0×2^4	1×2^3	1×2^2	0×2^1	1×2^0	
								173

Decimal
Base-10
Radix-10
$(0,1,2,3,4,5,6,7,8,9)_{10}$

0123456789

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I II III IV V VI VII VIII IX X

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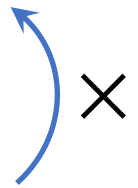
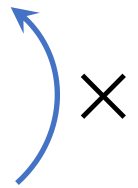
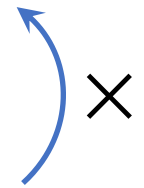
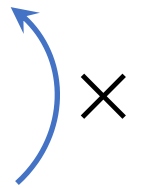
10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	
3	0	3	0	2	1	3	1	

10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	
3	0	3	0	2	1	3	1	
3×10^7	0×10^6	3×10^5	0×10^4	2×10^3	1×10^2	3×10^1	1×10^0	$\sum$
								30,302,131

10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	
3	0	3	0	2	1	3	1	
3×10^7	0×10^6	3×10^5	0×10^4	2×10^3	1×10^2	3×10^1	1×10^0	$\sum$
								30,302,131

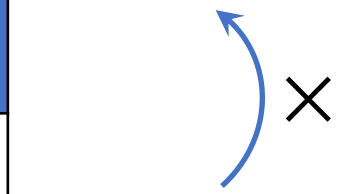
10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	
3	0	3	0	2	1	3	1	
3×10^7	0×10^6	3×10^5	0×10^4	2×10^3	1×10^2	3×10^1	1×10^0	$\sum$
								30,302,131

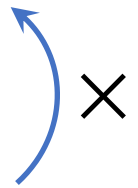
10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	
1	0	1	0	1	1	0	1	
1×10^7	0×10^6	1×10^5	0×10^4	1×10^3	1×10^2	0×10^1	1×10^0	$\sum$
								10,101,101

10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	
1	0	1	0	1	1	0	1	
1×10^7	0×10^6	1×10^5	0×10^4	1×10^3	1×10^2	0×10^1	1×10^0	
								10,101,101

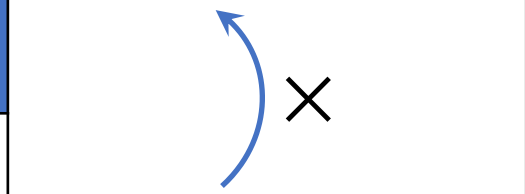
8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	
1	0	1	0	1	1	0	1	
1×8^7	0×8^6	1×8^5	0×8^4	1×8^3	1×8^2	0×8^1	1×8^0	
								2,130,497

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
1	0	1	0	1	1	0	1	
1×2^7	0×2^6	1×2^5	0×2^4	1×2^3	1×2^2	0×2^1	1×2^0	
								173

10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	
3	0	A	0	9	1	3	1	
								$\sum$

10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	
3	0	A	0	9	1	3	1	
3×10^7	0×10^6	$A\times10^5$	0×10^4	9×10^3	1×10^2	3×10^1	1×10^0	$\sum$
							Base-10	-

Hexadecimal
Base-16
Radix-16
$(0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F)_{16}$

16^7	16^6	16^5	16^4	16^3	16^2	16^1	16^0	
3	0	A	0	9	1	3	1	
								$\sum$

16^7	16^6	16^5	16^4	16^3	16^2	16^1	16^0	
3	0	A	0	9	1	3	1	×
3×16^7	0×16^6	$A\times16^5$	0×16^4	9×16^3	1×16^2	3×16^1	1×16^0	$\sum$

16^7	16^6	16^5	16^4	16^3	16^2	16^1	16^0	×
3	0	A	0	9	1	3	1	
3×16^7	0×16^6	$A\times16^5$	0×16^4	9×16^3	1×16^2	3×16^1	1×16^0	$\sum$

$$A = (9 + 1) = (10)_{10}$$

16^7	16^6	16^5	16^4	16^3	16^2	16^1	16^0	
3	0	A	0	9	1	3	1	×
3×16^7	0×16^6	$A\times16^5$	0×16^4	9×16^3	1×16^2	3×16^1	1×16^0	$\sum$
								815,829,297

$$A = (9 + 1) = (10)_{10}$$

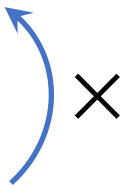
$$1,2,3,4,5,6,7,8,9,A = 9 + 1 = (10)_{10}$$
$$B = A + 1 = (11)_{10}$$
$$C = B + 1 = (12)_{10}$$
$$D = C + 1 = (13)_{10}$$
$$E = D + 1 = (14)_{10}$$
$$F = E + 1 = (15)_{10}$$

16^7	16^6	16^5	16^4	16^3	16^2	16^1	16^0	
3	0	3	0	2	1	3	1	
3×16^7	0×16^6	3×16^5	0×16^4	2×16^3	1×16^2	3×16^1	1×16^0	
								808,460,593

10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	
1	0	1	0	1	1	0	1	
1×10^7	0×10^6	1×10^5	0×10^4	1×10^3	1×10^2	0×10^1	1×10^0	
								10,101,101

8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	
1	0	1	0	1	1	0	1	
1×8^7	0×8^6	1×8^5	0×8^4	1×8^3	1×8^2	0×8^1	1×8^0	
								2,130,497

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
1	0	1	0	1	1	0	1	
1×2^7	0×2^6	1×2^5	0×2^4	1×2^3	1×2^2	0×2^1	1×2^0	
								173

16^7	16^6	16^5	16^4	16^3	16^2	16^1	16^0	
1	0	1	0	1	1	0	1	
1×16^7	0×16^6	1×16^5	0×16^4	1×16^3	1×16^2	0×16^1	1×16^0	$\sum$
							Base-10	269,488,385

Base-64

Radix-64

$(A,B,C,\ \ldots,Z,\ a,b,c,\ldots,z,0,1,2,\ldots,9,+,/)_{64}$

RADIX-R NUMBER SYSTEM

aka. Base-r Number System

Let $(N)_r$ be a radix-r (base-r) number in a positional weighting number system, then

$$(N)_r = (d_{n-1} r^{n-1} + d_{n-2} r^{n-2} + \cdots + d_i r^i + \cdots + d_2 r^2 + d_1 r^1 + d_0 r^0)_{10}$$

where:

- r = radix (base)
- d_i = digit at position i, $0 \le d_i \le r - 1$
- r^i = weight of position i
- n = number of digits in N

Let $(N)_r$ be a radix-r (base-r) number in a positional weighting number system, then

$$(N)_r = (d_{n-1} r^{n-1} + d_{n-2} r^{n-2} + \cdots + d_i r^i + \cdots + d_2 r^2 + d_1 r^1 + d_0 r^0)_{10}$$

where:

r = radix (base)

d_i = digit at position i, $0 \le d_i \le r - 1$

r^i = weight of position i

n = number of digits in N

Let $(N)_r$ be a radix-r (base-r) number in a positional weighting number system, then

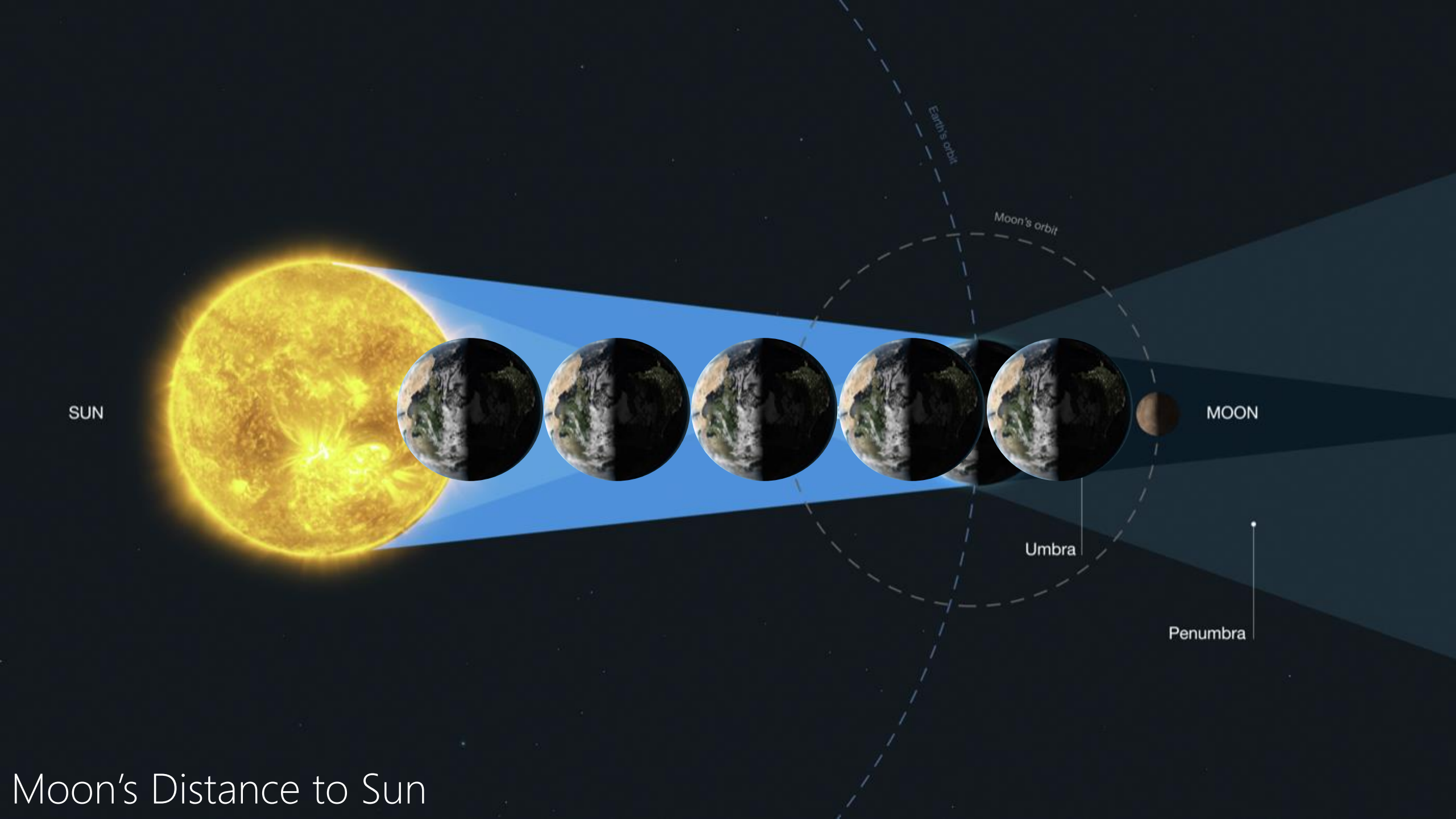
Min	$= (0_{n-1}0_{n-2} \cdots 0_1 0_0)_r$	$= (0)_{10}$
Max	$= ((r-1)_{n-1}(r-1)_{n-2} \cdots (r-1)_1(r-1)_0)_r$	$= (r^n-1)_{10}$
Unit	$= (0_{n-1}0_{n-2} \cdots 0_1 1_0)_r$	$= (1)_{10}$

where:

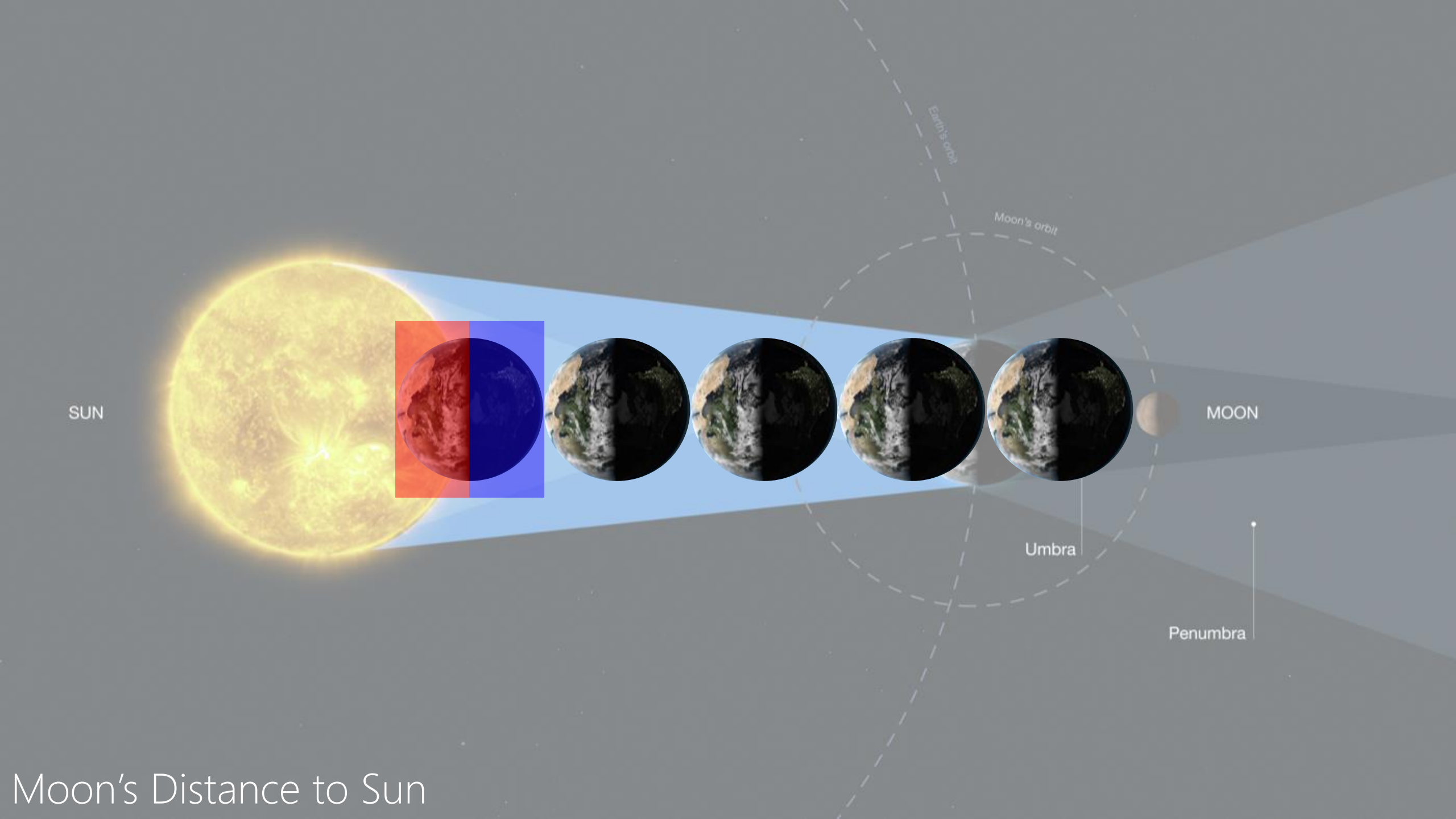
- r = radix (base)
- r^i = weight of position i
- n = number of digits in N

PRACTICE

FRACTION

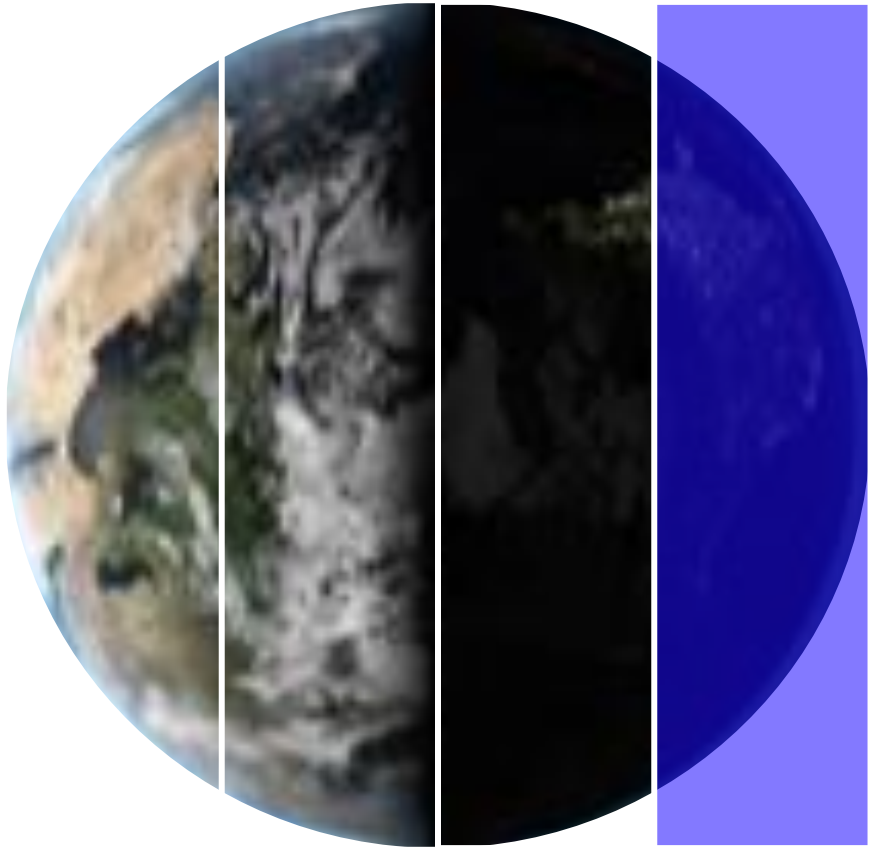


Moon's Distance to Sun



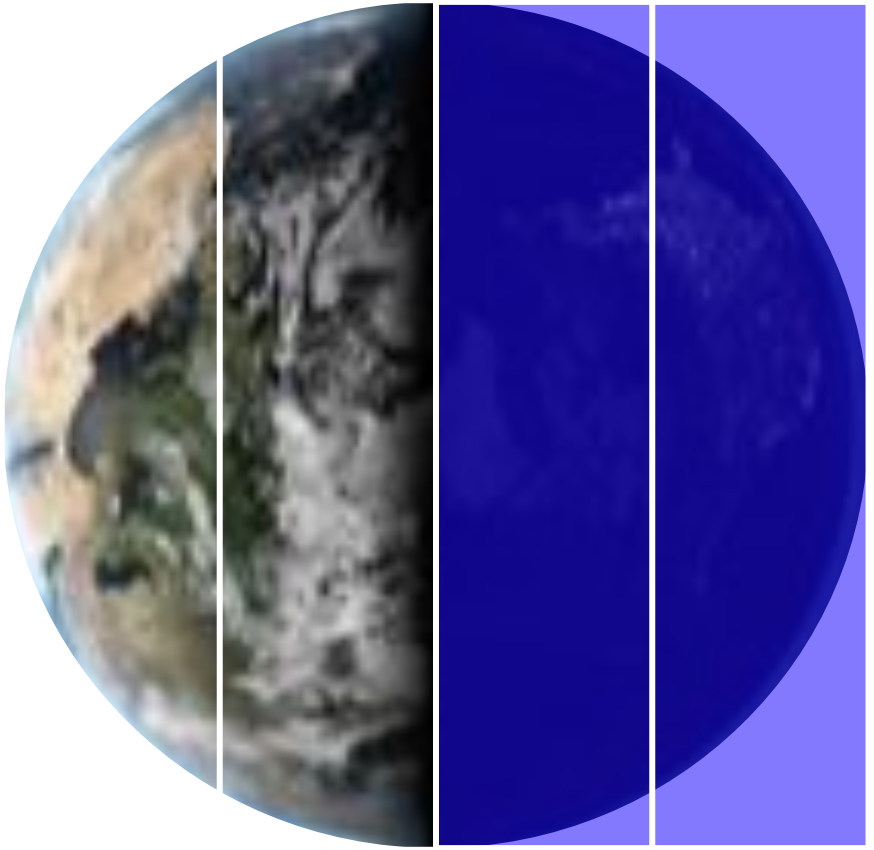
Moon's Distance to Sun

1 Earth

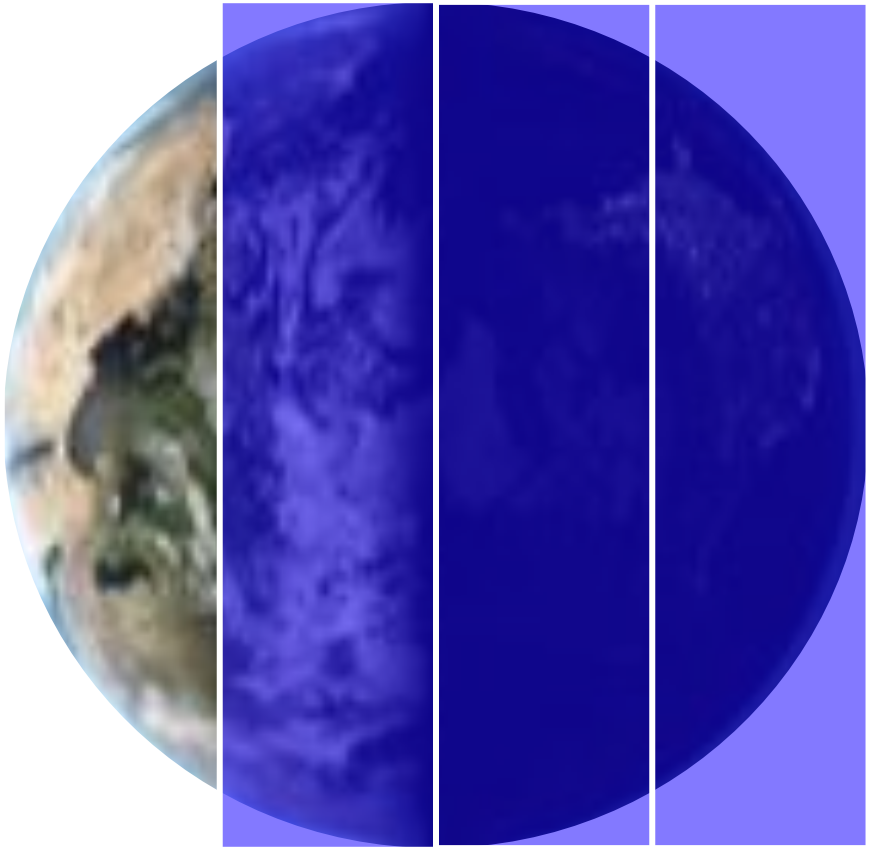


Fraction Point

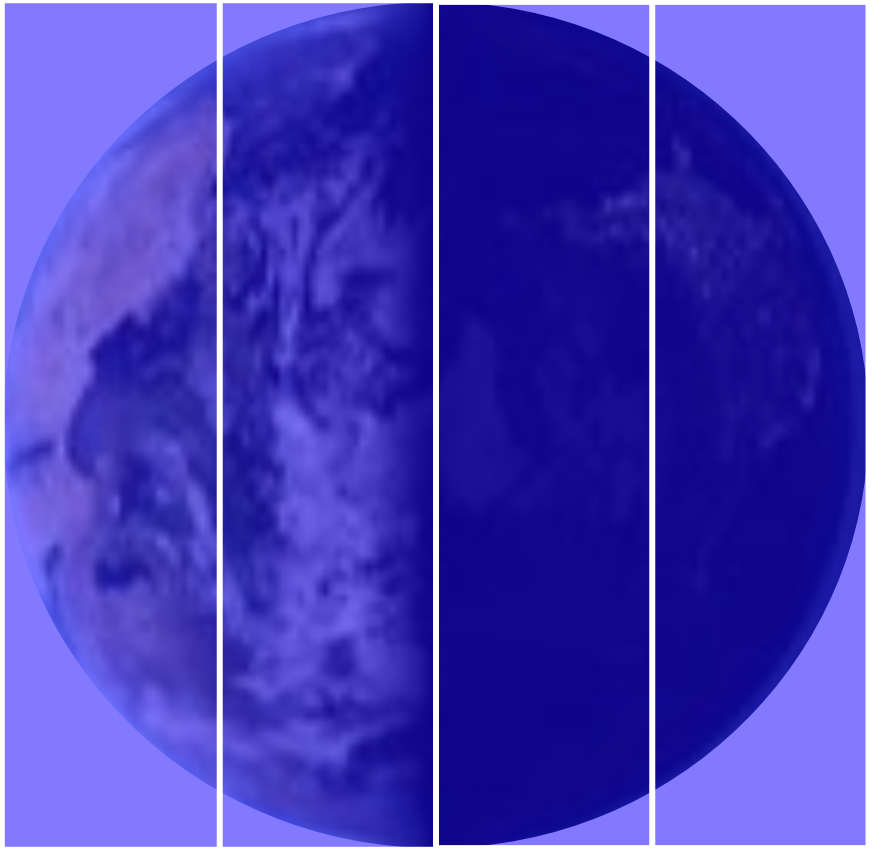
Radix-4 (Base-4) = 1/4 Earth = 4^{-1} Earth = $(.1)_4$



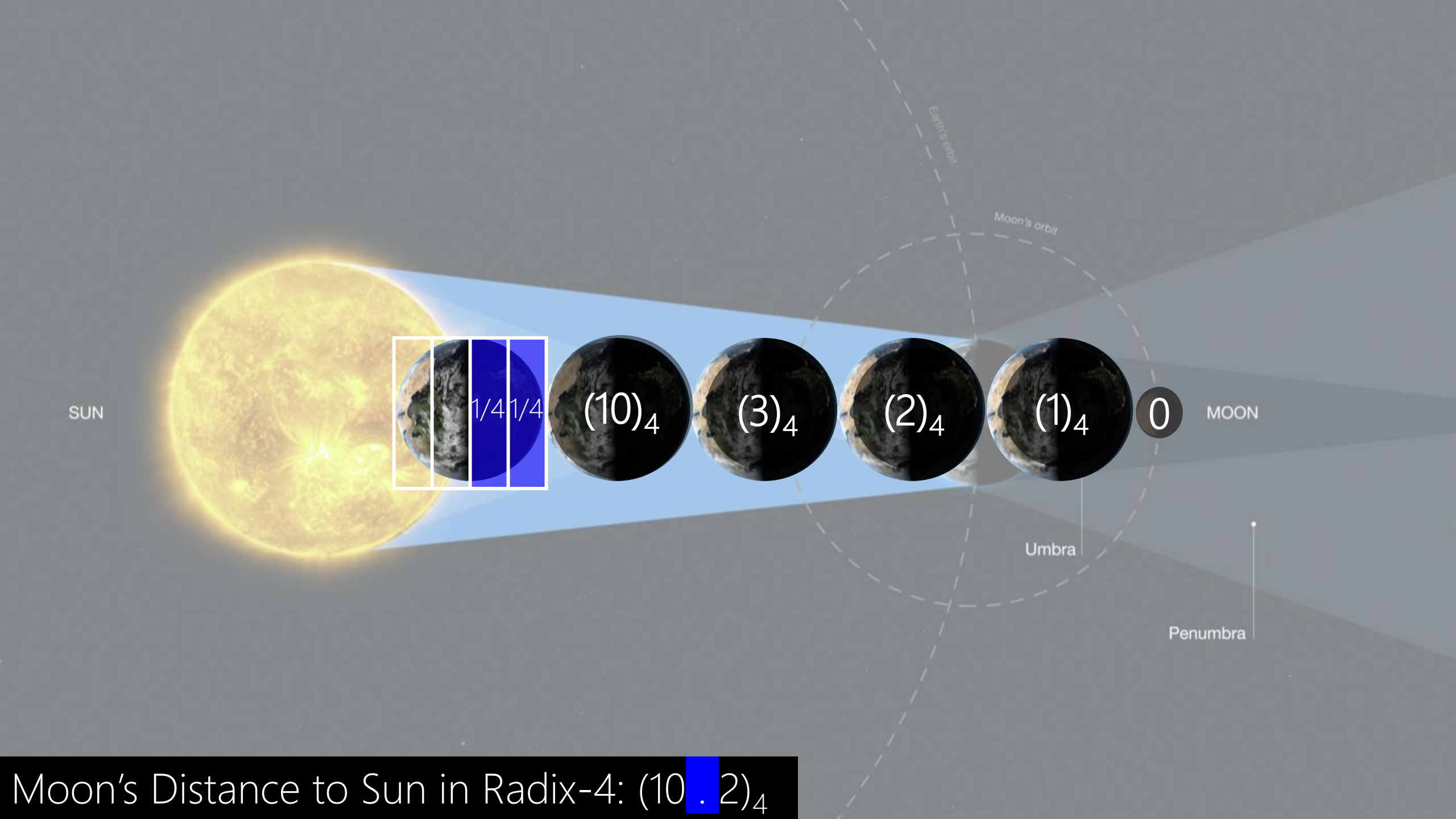
Radix-4 (Base-4) = 2 × 1/4 Earth = 2 × 4^{-1} Earth=$(.2)_4$



Radix-4 (Base-4) = 3 × 1/4 Earth = 3 × 4^{-1} Earth=$(.3)_4$

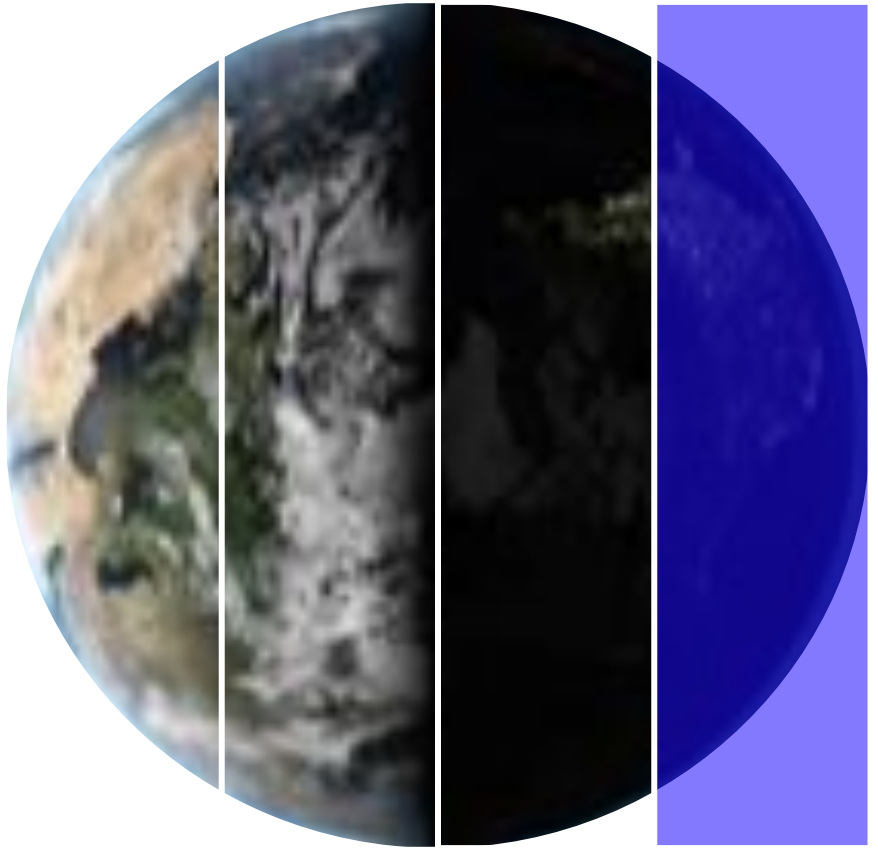


Radix-4 (Base-4) = 4 × 1/4 Earth = 4 × 4^{-1} Earth=$(1)_4$

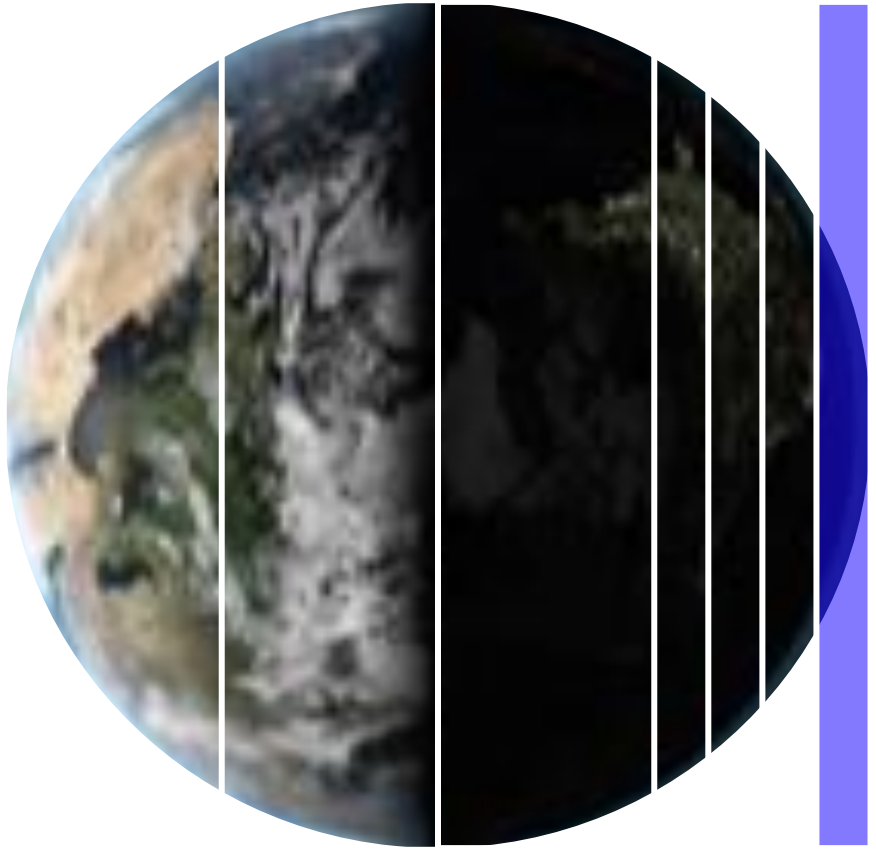


Moon's Distance to Sun in Radix-4: $(10.2)_4$

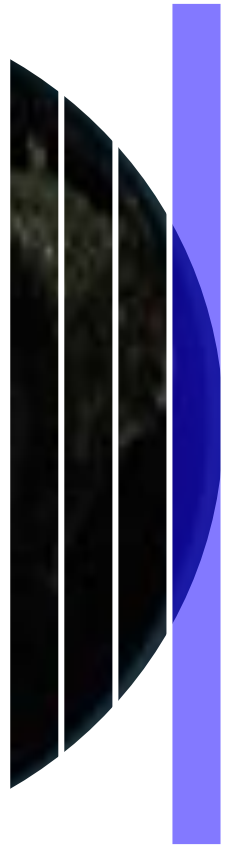
MORE PRECISION



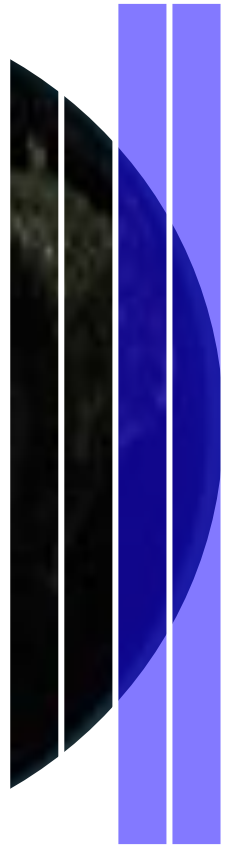
Radix-4 (Base-4) = 1/4 Earth = 4^{-1} Earth = $(.1)_4$



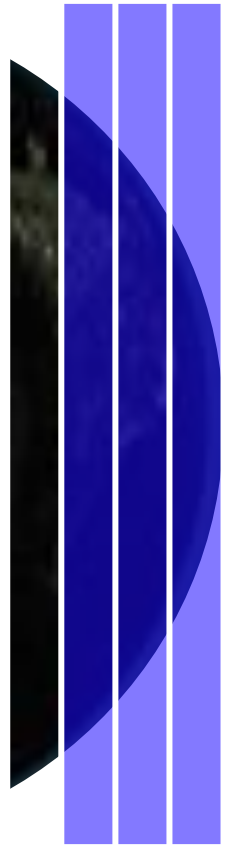
Radix-4 (Base-4) = (1/4)/4) Earth = 1/8 Earth = 4^{-2} Earth



Radix-4 (Base-4) = 1/8 = 4^{-2} Earth = $(.01)_4$

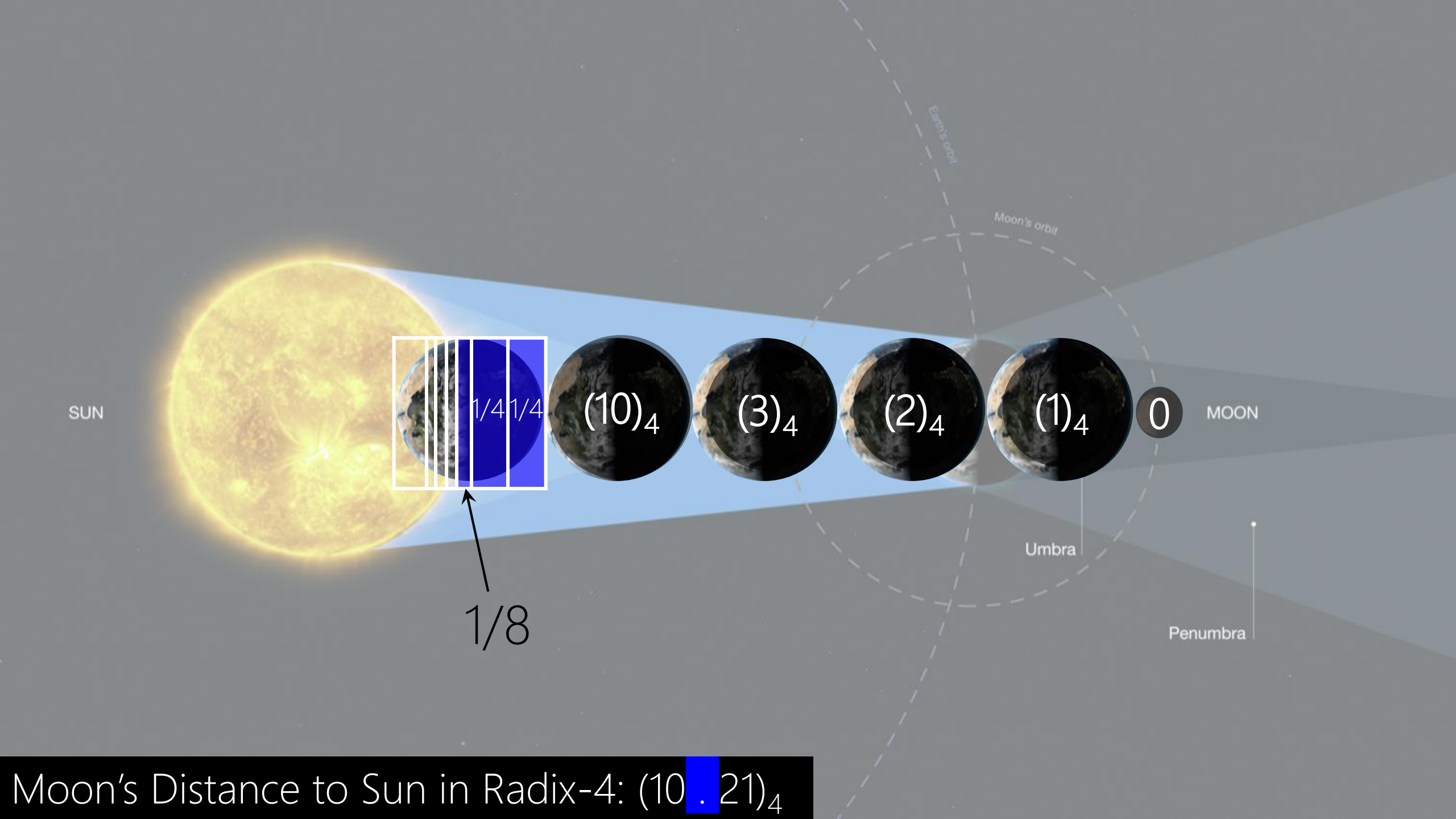


Radix-4 (Base-4) = 2 × 1/8 = 2 × 4^{-2} Earth = $(.02)_4$



Radix-4 (Base-4) = 3 × 1/8 = 3 × 4^{-2} Earth = $(.03)_4$

Radix-4 (Base-4) = $4 \times 1/8 = 4 \times 4^{-2}$ Earth = $(.1)_4$



Moon's Distance to Sun in Radix-4: $(10.21)_4$

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}
1	0	1	0	1	1	0	1	.	1	0	1	0	1

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}
1	0	1	0	1	1	0	1	×	1	0	1	0	1
128	0	32	0	8	4	0	1	.	1/2	0	1/8	0	1/32

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}
1	0	1	0	1	1	0	1	×	1	0	1	0	1
128	0	32	0	8	4	0	1	$\sum$	1/2	0	1/8	0	1/32
Base-10				173				.	65625				

8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0		8^{-1}	8^{-2}	8^{-3}	8^{-4}	8^{-5}
1	0	1	0	1	1	0	1	×	1	0	1	0	1
0	0	32,768	0	512	64	0	8	∑	1/8	0	1/512	0	1/32,768
				33,352				.	12698364257				

Let $(N)_r$ be a radix-r (base-r) number in a positional weighting number system, then

$$(N)_r = (d_{n-1} r^{n-1} + \cdots + d_0 r^0 . d_{-1} r^{-1} + d_{-2} r^{-2} + \cdots + d_{-m} r^{-m})_{10}$$

Fraction Point

where:

- r = radix (base)
- d_i = digit at position i, $0 \le d_i \le r - 1$
- r^i = weight of position i
- n = number of digits in integer part of N
- m = number of digits in fraction part of N

Let $(N)_r$ be a radix-r (base-r) number in a positional weighting number system, then

Min $= (0_{n-1} \cdots 0_1 0_0 \,.\, 0_{-1} 0_{-2} \cdots 0_{-m-1} 0_{-m})_r = (0 \,.\, 0)_{10}$

Max $= ((r\text{-}1)_{n-1} \cdots (r\text{-}1)_0 \,.\, (r\text{-}1)_{-1} (r\text{-}1)_{-2} \cdots (r\text{-}1)_{-m-1} (r\text{-}1)_{-m})_r = (r^n\text{-}1 \,.\, ?\,)_{10}$

Unit $= (0_{n-1} \cdots 0_1 0_0 \,.\, 0_{-1} 0_{-2} \cdots 0_{-m-1} 1_{-m})_r = (r^{-m})_{10}$

where:

- r = radix (base)
- r^i = weight of position i
- n = number of digits in integer part of N
- m = number of digits in fraction part of N

Lecture Assignment

PRACTICE RADIX-2

Radix-2								Radix-10
Integer (n=4)					Fraction (m=3)			
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*2^{-3} = 1/8 = 0.125$
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3} = 1/4 = 0.25$
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3} = ½ = 0.5$
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$
0	0	0	1	.	0	0	0	$1*2^{0} + 0*2^{-1} + 0*2^{-2} + 0*2^{-3} = 1$
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

Radix-2								Radix-10
Integer (n=4)					Fraction (m=3)			
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*2^{-3} = 1/8 = 0.125$
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3} = 1/4 = 0.25$
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3} = ½ = 0.5$
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$
0	0	0	1	.	0	0	0	$1*2^{0} + 0*2^{-1} + 0*2^{-2} + 0*2^{-3} = 1$
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

What is the max in this system with these spaces?

Radix-2								Radix-10
Integer (n=4)					Fraction (m=3)			
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*2^{-3} = 1/8 = 0.125$
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3} = 1/4 = 0.25$
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3} = ½ = 0.5$
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$
0	0	0	1	.	0	0	0	$1*2^{0} + 0*2^{-1} + 0*2^{-2} + 0*2^{-3} = 1$
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

What is the max in this system with these spaces?

$(1111.111)_2 = (15.875)_{10}$

Radix-2								Radix-10
Integer (n=4)					Fraction (m=3)			
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*2^{-3} = 1/8 = 0.125$
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3} = 1/4 = 0.25$
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3} = ½ = 0.5$
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$
0	0	0	1	.	0	0	0	$1*2^{0} + 0*2^{-1} + 0*2^{-2} + 0*2^{-3} = 1$
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

Radix-2								Radix-10
Integer (n=4)					Fraction (m=3)			
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*2^{-3} = 1/8 = 0.125$
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3} = 1/4 = 0.25$
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3} = ½ = 0.5$
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$
0	0	0	1	.	0	0	0	$1*2^{0} + 0*2^{-1} + 0*2^{-2} + 0*2^{-3} = 1$
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! The numbers in this system increments by 0.125 unit.

Radix-2								Radix-10
Integer (n=4)					Fraction (m=3)			
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*2^{-3} = 1/8 = 0.125$
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3} = 1/4 = 0.25$
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3} = ½ = 0.5$
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$
0	0	0	1	.	0	0	0	$1*2^{0} + 0*2^{-1} + 0*2^{-2} + 0*2^{-3} = 1$
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! The numbers in this system increments by 0.125 unit.

Solution?

Radix-2								Radix-10
Integer (n=4)					Fraction (m=3)			
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*2^{-3} = 1/8 = 0.125$
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3} = 1/4 = 0.25$
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3} = ½ = 0.5$
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$
0	0	0	1	.	0	0	0	$1*2^{0} + 0*2^{-1} + 0*2^{-2} + 0*2^{-3} = 1$
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! The numbers in this system increments by 0.125 unit.

Solution?
A. More precision.

Radix-2								Radix-10
Integer (n=4)					Fraction (m=3)			
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*2^{-3} = 1/8 = 0.125$
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3} = 1/4 = 0.25$
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3} = ½ = 0.5$
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$
0	0	0	1	.	0	0	0	$1*2^{0} + 0*2^{-1} + 0*2^{-2} + 0*2^{-3} = 1$
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! The numbers in this system increments by 0.125 unit.

Solution?

A. More precision.

A. More fraction positions.

Radix-2								Radix-10
Integer (n=4)					Fraction (m=3)			
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*2^{-3} = 1/8 = 0.125$
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3} = 1/4 = 0.25$
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3} = ½ = 0.5$
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$
0	0	0	1	.	0	0	0	$1*2^{0} + 0*2^{-1} + 0*2^{-2} + 0*2^{-3} = 1$
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! The numbers in this system increments by 0.125 unit.

Solution?

A. More precision.

A. More fraction positions.

A. More in m!

Radix-2								Radix-10
Integer (n=4)					Fraction (m=3)			
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*2^{-3} = 1/8 = 0.125$
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3} = 1/4 = 0.25$
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3} = ½ = 0.5$
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$
0	0	0	1	.	0	0	0	$1*2^{0} + 0*2^{-1} + 0*2^{-2} + 0*2^{-3} = 1$
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! The numbers in this system increments by 0.125 unit.

Solution?

A. More precision.

A. More fraction positions.

A. More in m! How much?

Radix-2								Radix-10
Integer (n=4)					Fraction (m=3)			
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*2^{-3} = 1/8 = 0.125$
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3} = 1/4 = 0.25$
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3} = ½ = 0.5$
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$
0	0	0	1	.	0	0	0	$1*2^{0} + 0*2^{-1} + 0*2^{-2} + 0*2^{-3} = 1$
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! The numbers in this system increments by 0.125 unit.

Solution?

B. Find the closest number

Radix-2								Radix-10
Integer (n=4)					Fraction (m=3)			
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*2^{-3} = 1/8 = 0.125$
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3} = 1/4 = 0.25$
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3} = ½ = 0.5$
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$
0	0	0	1	.	0	0	0	$1*2^{0} + 0*2^{-1} + 0*2^{-2} + 0*2^{-3} = 1$
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! The numbers in this system increments by 0.125 unit.

Solution?

B. Find the closest number

$(1.000)_2$ => Error = 0.02

$(1.001)_2$ => Error = 0.105

Radix-2								Radix-10
Integer (n=4)					Fraction (m=3)			
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*2^{-3} = 1/8 = 0.125$
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3} = 1/4 = 0.25$
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3} = ½ = 0.5$
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$
0	0	0	1	.	0	0	0	$1*2^{0} + 0*2^{-1} + 0*2^{-2} + 0*2^{-3} = 1$
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! The numbers in this system increments by 0.125 unit.

Solution?

B. Find the closest number

$(1.000)_2$ => Error = 0.02

$(1.001)_2$ => Error = 0.105

PRACTICE RADIX-4

Radix-4								Radix-10
Integer (n=4)					Fraction (m=3)			
4^3	4^2	4^1	4^0	.	4^{-1}	4^{-2}	4^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*4^{-3} = 1/64 = 0.015625$
0	0	0	0	.	0	0	2	$2*4^{-3} = 2/64 = 0.03125$
0	0	0	0	.	0	0	3	$3*4^{-3} = 3/64 = 0.046875$
0	0	0	0	.	0	1	0	$1*4^{-2} + 0*4^{-2} = 1/16 = 0.0625$
0	0	0	0	.	0	1	1	$1*4^{-2} + 1*4^{-2} = 1/16 + 1/64 = 0.078125$
0	0	0	0	.	0	1	2	$1*4^{-2} + 2*4^{-2} = 1/16 + 2/64 = 0.09375$
0	0	0	0	.	0	1	3	$1*4^{-2} + 3*4^{-2} = 1/16 + 3/64 = 0.109375$
0	0	0	0	.	0	2	0	$2*4^{-2} + 0*4^{-2} = 2/16 = 0.125$
								…
0	0	0	0	.	3	3	3	$3*4^{-1} + 3*4^{-2} + 3*4^{-3} = 0.984375$
0	0	0	1	.	0	0	0	1
								…
3	3	3	3	.	3	3	0	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 0*4^{-3} = ?$
3	3	3	3	.	3	3	1	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 1*4^{-3} = ?$
3	3	3	3	.	3	3	2	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 2*4^{-3} = ?$
3	3	3	3	.	3	3	3	255.984375

Radix-4								Radix-10
Integer (n=4)					Fraction (m=3)			
4^3	4^2	4^1	4^0	.	4^{-1}	4^{-2}	4^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*4^{-3} = 1/64 = 0.015625$
0	0	0	0	.	0	0	2	$2*4^{-3} = 2/64 = 0.03125$
0	0	0	0	.	0	0	3	$3*4^{-3} = 3/64 = 0.046875$
0	0	0	0	.	0	1	0	$1*4^{-2} + 0*4^{-2} = 1/16 = 0.0625$
0	0	0	0	.	0	1	1	$1*4^{-2} + 1*4^{-2} = 1/16 + 1/64 = 0.078125$
0	0	0	0	.	0	1	2	$1*4^{-2} + 2*4^{-2} = 1/16 + 2/64 = 0.09375$
0	0	0	0	.	0	1	3	$1*4^{-2} + 3*4^{-2} = 1/16 + 3/64 = 0.109375$
0	0	0	0	.	0	2	0	$2*4^{-2} + 0*4^{-2} = 2/16 = 0.125$
								…
0	0	0	0	.	3	3	3	$3*4^{-1} + 3*4^{-2} + 3*4^{-3} = 0.984375$
0	0	0	1	.	0	0	0	1
								…
3	3	3	3	.	3	3	0	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 0*4^{-3} = ?$
3	3	3	3	.	3	3	1	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 1*4^{-3} = ?$
3	3	3	3	.	3	3	2	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 2*4^{-3} = ?$
3	3	3	3	.	3	3	3	255.984375

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

Radix-4								Radix-10
Integer (n=4)					Fraction (m=3)			
4^3	4^2	4^1	4^0	.	4^{-1}	4^{-2}	4^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*4^{-3} = 1/64 = 0.015625$
0	0	0	0	.	0	0	2	$2*4^{-3} = 2/64 = 0.03125$
0	0	0	0	.	0	0	3	$3*4^{-3} = 3/64 = 0.046875$
0	0	0	0	.	0	1	0	$1*4^{-2} + 0*4^{-2} = 1/16 = 0.0625$
0	0	0	0	.	0	1	1	$1*4^{-2} + 1*4^{-2} = 1/16 + 1/64 = 0.078125$
0	0	0	0	.	0	1	2	$1*4^{-2} + 2*4^{-2} = 1/16 + 2/64 = 0.09375$
0	0	0	0	.	0	1	3	$1*4^{-2} + 3*4^{-2} = 1/16 + 3/64 = 0.109375$
0	0	0	0	.	0	2	0	$2*4^{-2} + 0*4^{-2} = 2/16 = 0.125$
								…
0	0	0	0	.	3	3	3	$3*4^{-1} + 3*4^{-2} + 3*4^{-3} = 0.984375$
0	0	0	1	.	0	0	0	1
								…
3	3	3	3	.	3	3	0	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 0*4^{-3} = ?$
3	3	3	3	.	3	3	1	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 1*4^{-3} = ?$
3	3	3	3	.	3	3	2	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 2*4^{-3} = ?$
3	3	3	3	.	3	3	3	255.984375

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! Why?

Radix-4								Radix-10
Integer (n=4)					Fraction (m=3)			
4^3	4^2	4^1	4^0	.	4^{-1}	4^{-2}	4^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*4^{-3} = 1/64 = 0.015625$
0	0	0	0	.	0	0	2	$2*4^{-3} = 2/64 = 0.03125$
0	0	0	0	.	0	0	3	$3*4^{-3} = 3/64 = 0.046875$
0	0	0	0	.	0	1	0	$1*4^{-2} + 0*4^{-2} = 1/16 = 0.0625$
0	0	0	0	.	0	1	1	$1*4^{-2} + 1*4^{-2} = 1/16 + 1/64 = 0.078125$
0	0	0	0	.	0	1	2	$1*4^{-2} + 2*4^{-2} = 1/16 + 2/64 = 0.09375$
0	0	0	0	.	0	1	3	$1*4^{-2} + 3*4^{-2} = 1/16 + 3/64 = 0.109375$
0	0	0	0	.	0	2	0	$2*4^{-2} + 0*4^{-2} = 2/16 = 0.125$
								…
0	0	0	0	.	3	3	3	$3*4^{-1} + 3*4^{-2} + 3*4^{-3} = 0.984375$
0	0	0	1	.	0	0	0	1
								…
3	3	3	3	.	3	3	0	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 0*4^{-3} = ?$
3	3	3	3	.	3	3	1	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 1*4^{-3} = ?$
3	3	3	3	.	3	3	2	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 2*4^{-3} = ?$
3	3	3	3	.	3	3	3	255.984375

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! Why?

Solution:

A. More in m

B. Find the closest number

Radix-4								Radix-10
Integer (n=4)					Fraction (m=3)			
4^3	4^2	4^1	4^0	.	4^{-1}	4^{-2}	4^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*4^{-3} = 1/64 = 0.015625$
0	0	0	0	.	0	0	2	$2*4^{-3} = 2/64 = 0.03125$
0	0	0	0	.	0	0	3	$3*4^{-3} = 3/64 = 0.046875$
0	0	0	0	.	0	1	0	$1*4^{-2} + 0*4^{-2} = 1/16 = 0.0625$
0	0	0	0	.	0	1	1	$1*4^{-2} + 1*4^{-2} = 1/16 + 1/64 = 0.078125$
0	0	0	0	.	0	1	2	$1*4^{-2} + 2*4^{-2} = 1/16 + 2/64 = 0.09375$
0	0	0	0	.	0	1	3	$1*4^{-2} + 3*4^{-2} = 1/16 + 3/64 = 0.109375$
0	0	0	0	.	0	2	0	$2*4^{-2} + 0*4^{-2} = 2/16 = 0.125$
								…
0	0	0	0	.	3	3	3	$3*4^{-1} + 3*4^{-2} + 3*4^{-3} = 0.984375$
0	0	0	1	.	0	0	0	1
								…
3	3	3	3	.	3	3	0	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 0*4^{-3} = ?$
3	3	3	3	.	3	3	1	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 1*4^{-3} = ?$
3	3	3	3	.	3	3	2	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 2*4^{-3} = ?$
3	3	3	3	.	3	3	3	255.984375

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! Why?

Solution:

A. More in m

B. Find the closest number

$(1.001)_4 = (1.015625)_{10}$ => Error = 0.004375

$(1.002)_4 = (1.03125)_{10}$ => Error = 0.01125

Radix-4								Radix-10
Integer (n=4)					Fraction (m=3)			
4^3	4^2	4^1	4^0	.	4^{-1}	4^{-2}	4^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*4^{-3} = 1/64 = 0.015625$
0	0	0	0	.	0	0	2	$2*4^{-3} = 2/64 = 0.03125$
0	0	0	0	.	0	0	3	$3*4^{-3} = 3/64 = 0.046875$
0	0	0	0	.	0	1	0	$1*4^{-2} + 0*4^{-2} = 1/16 = 0.0625$
0	0	0	0	.	0	1	1	$1*4^{-2} + 1*4^{-2} = 1/16 + 1/64 = 0.078125$
0	0	0	0	.	0	1	2	$1*4^{-2} + 2*4^{-2} = 1/16 + 2/64 = 0.09375$
0	0	0	0	.	0	1	3	$1*4^{-2} + 3*4^{-2} = 1/16 + 3/64 = 0.109375$
0	0	0	0	.	0	2	0	$2*4^{-2} + 0*4^{-2} = 2/16 = 0.125$
								…
0	0	0	0	.	3	3	3	$3*4^{-1} + 3*4^{-2} + 3*4^{-3} = 0.984375$
0	0	0	1	.	0	0	0	1
								…
3	3	3	3	.	3	3	0	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 0*4^{-3} = ?$
3	3	3	3	.	3	3	1	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 1*4^{-3} = ?$
3	3	3	3	.	3	3	2	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 2*4^{-3} = ?$
3	3	3	3	.	3	3	3	255.984375

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! Why?

Solution:

A. More in m

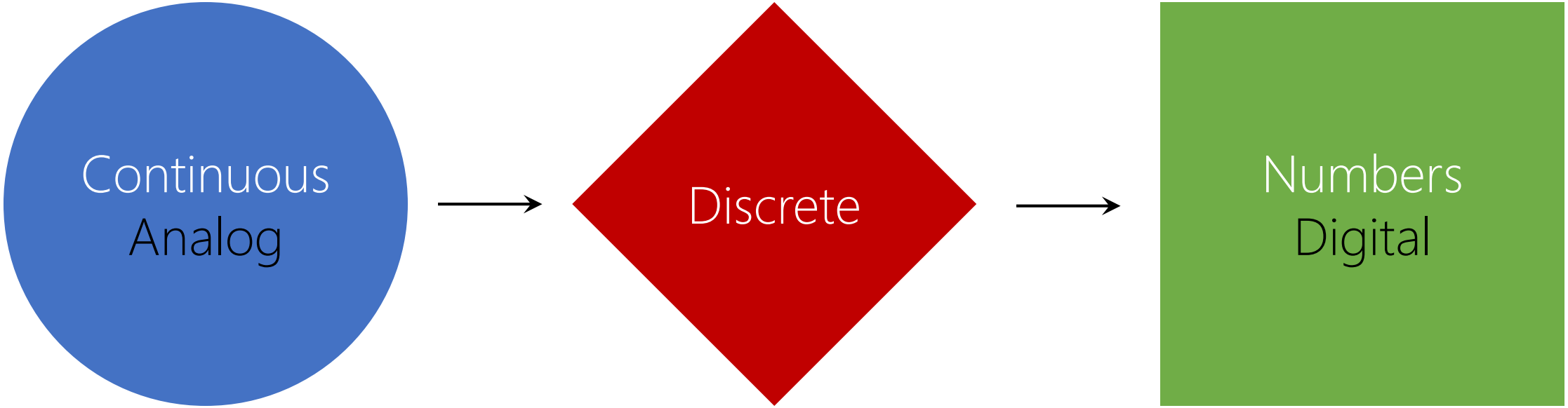
B. Find the closest number

$(1.001)_4 = (1.015625)_{10}$ => Error = 0.004375

$(1.002)_4 = (1.03125)_{10}$ => Error = 0.01125

PRACTICE RADIX-[8,10,16]

At Home



Quantization

DIGITAL SYSTEMS

CONVERSION OF FRACTIONAL NUMBERS BETWEEN NUMERAL SYSTEMS

https://planetcalc.com/862/