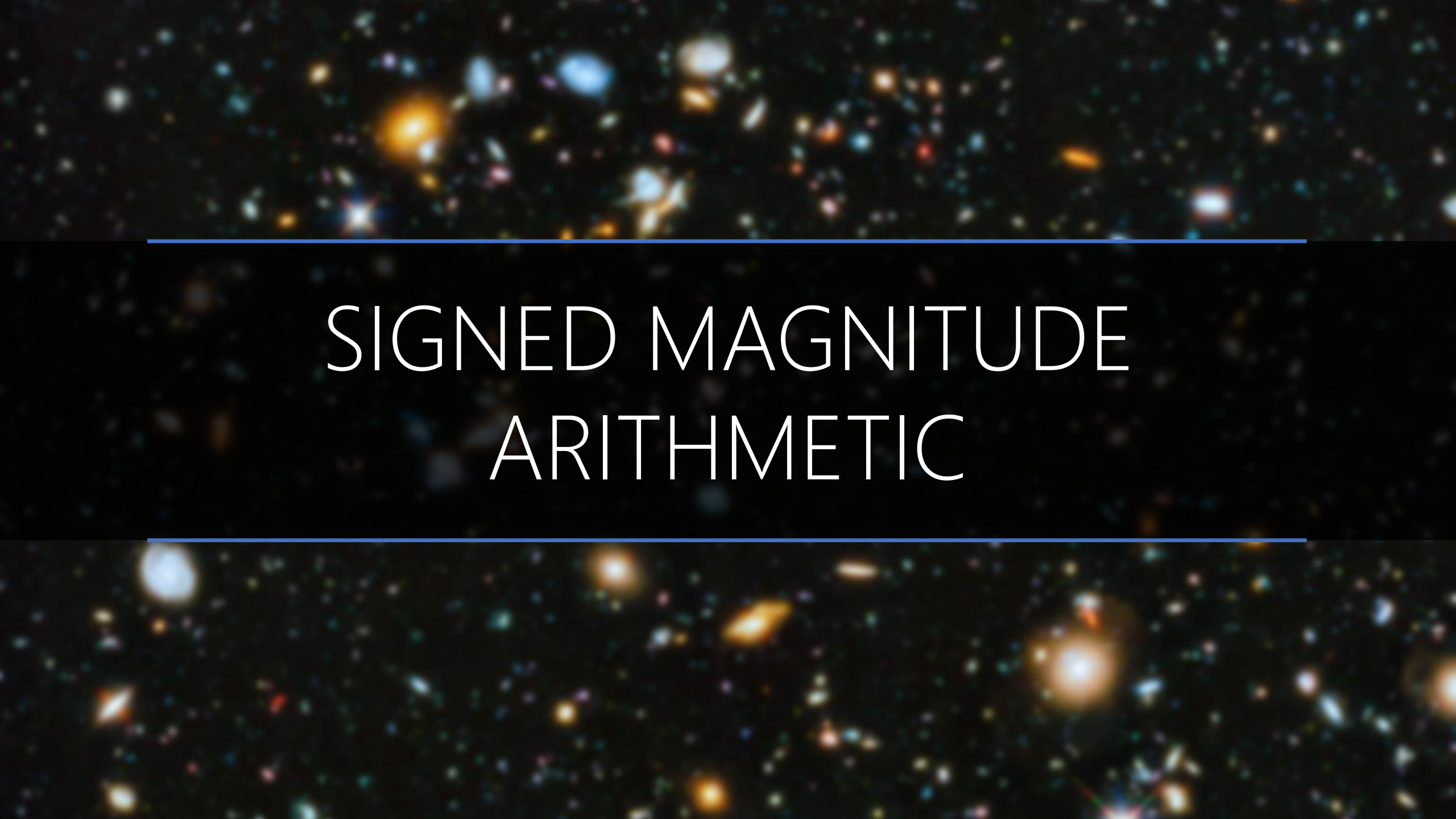
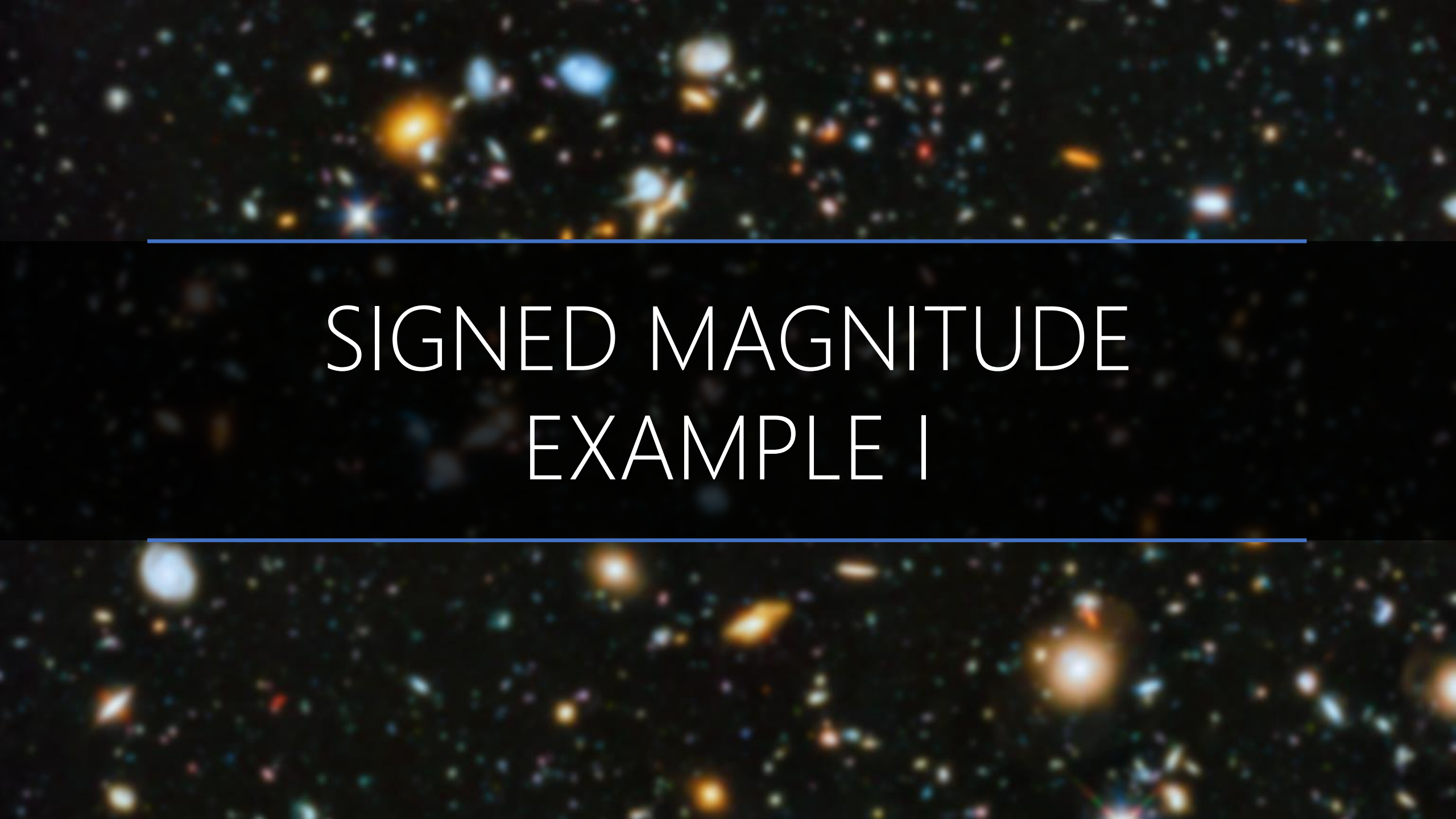


SIGNED MAGNITUDE ARITHMETIC

0	X	+	0	Y	=	0	X+Y
0	X	+	1	Y	=	X<Y (if borrow)	X–Y (if borrow, apply it)
1	X	+	1	Y	=	1	X+Y

0	X	–	0	Y	=	X<Y (if borrow)	X–Y (if borrow, apply it)
0	X	–	1	Y	=	0	X+Y
1	X	–	1	Y	=	X>Y (if borrow)	–X+Y=Y–X (if borrow, apply it)



SIGNED MAGNITUDE EXAMPLE I

+	1	2	A	.	E	5	4
Base-16	1	B	F	.	2	B	
				.			

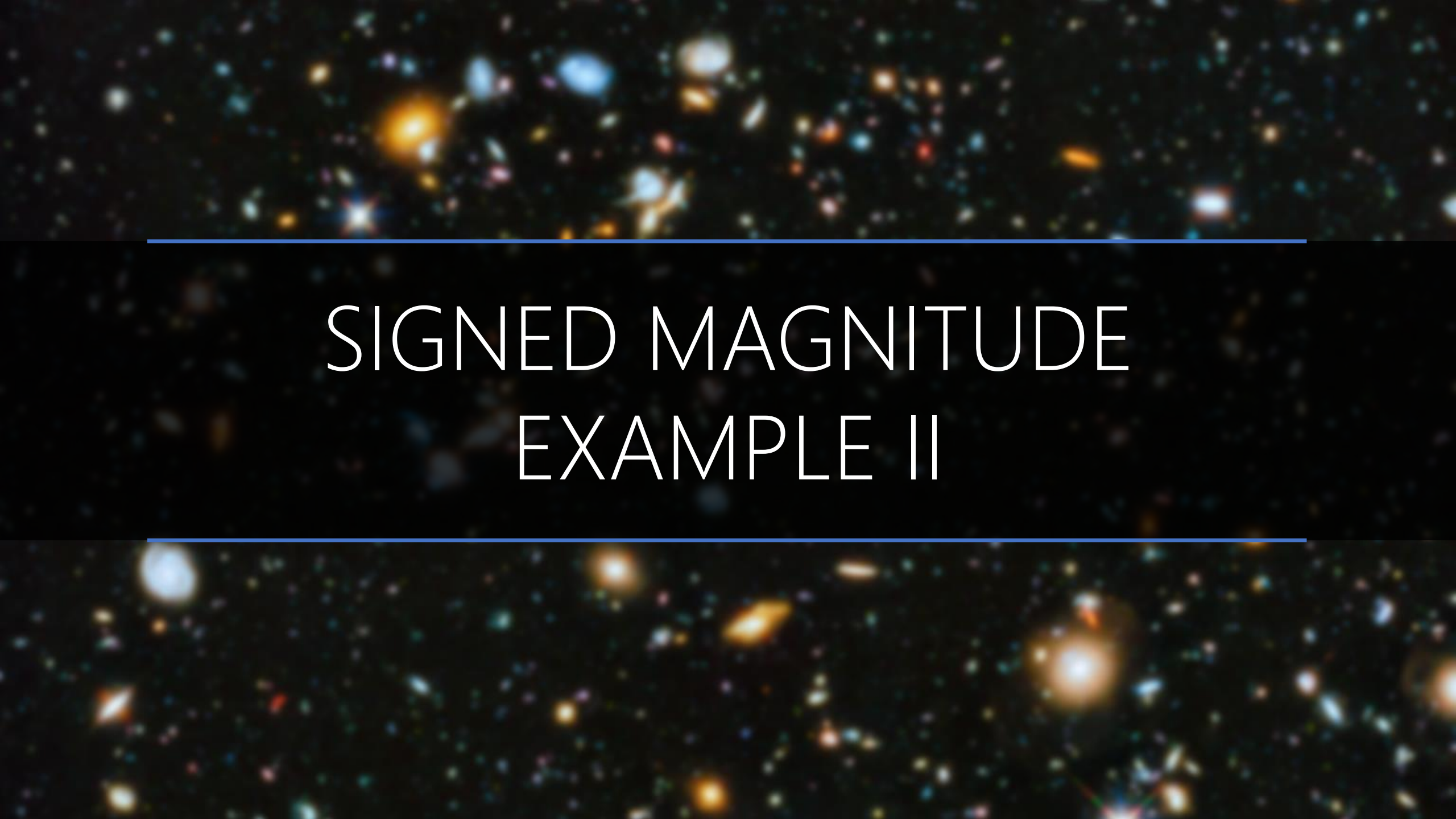
PADDING

+							
Base-16	1	2	A	.	E	5	4
	1	B	F	.	2	B	0
				.			

SIGNED: $(-X)+(-Y)=-(X+Y)$

+ Base-16	1	2	A	.	E	5	4
	1	B	F	.	2	B	0
	1			.			

			1	1		1		
+ Base-16	1	2	A=10	.	E=14	5	4
	1	B	F=15	.	2	B=11	0
	1	E=14	A=10	.	1	0	4



SIGNED MAGNITUDE EXAMPLE II

+	0	2	A	.	E	5	4
Base-16	1	B	F	.	2	B	
				.			

PADDING

+	0	2	A	.	E	5	4
Base-16	1	B	F	.	2	B	0
				.			

SIGNED: (+X)+(–Y)=?(X–Y)

–	0	2	A	.	E	5	4
Base-16	1	B	F	.	2	B	0
	?			.			

	–1 →	+16					
		–1	+16		–1	+16	
–	0	2	A=10	.	E=14	5	4
Base-16	1	B=11	F=15	.	2	B=11	0
	1	6	B=11	.	B=11	A=10	4

2A.E54 < BF.2B0

Last Borrow ⟶ Negative Result

		+16 −1	+16 −1		+16 −1	+16 −1	+16
−	1	0	0	.	0	0	0
Base-16		6	$B_{=11}$	.	$B_{=11}$	$A_{=10}$	4
		9	4	.	4	5	$C_{=12}$

$= (2A.E54)_{16} - (BF.2B0)_{16} = (6B.BA4)_{16}$ and last borrow!

$= (100.000)_{16} - (6B.BA4)_{16}$

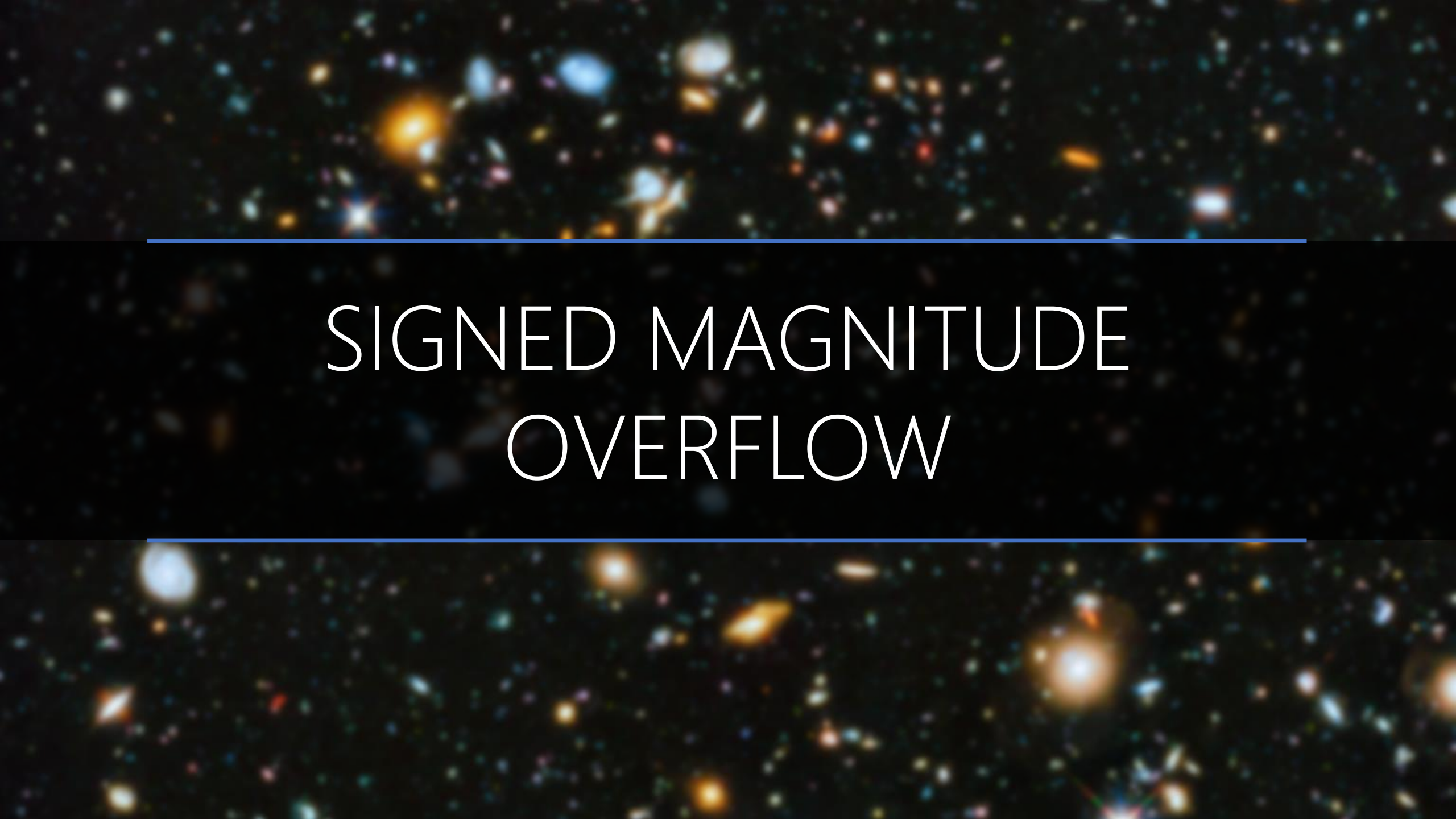
$= (94.45C)_{16}$

	−1 →	+16					
		−1	+16		−1	+16	
−	0	2	A=10	.	E=14	5	4
Base-16	1	B=11	F=15	.	2	B=11	0
	1	6	B=11	.	B=11	A=10	4

2A.E54 < BF.2B0

Last Borrow ⟶ Negative Result

1	9	4	.	4	5	C



SIGNED MAGNITUDE OVERFLOW

Overflow!

0	X	+	0	Y	=	0 → 1	X+Y
0	X	+	1	Y	=	X<Y	X–Y
1	X	+	1	Y	=	1 → 0	X+Y

0	X	–	0	Y	=	X<Y	X–Y
0	X	–	1	Y	=	0 → 1	X+Y
1	X	–	1	Y	=	X>Y	–X+Y=Y–X

–	$+(27)_{10}$	0	1	1	0	1	1
Base-2		1	0	1	1	1	0

— Base-2	$+(27)_{10}$	0	1	1	0	1	1
	$-(14)_{10}$	1	0	1	1	1	0

$+X - (-Y)$

– Base-2	$+(27)_{10}$	0	1	1	0	1	1
	$-(14)_{10}$	1	0	1	1	1	0

$$+X - (-Y) = +(X+Y)$$

+	$+(27)_{10}$	0	1	1	0	1	1
Base-2	$+(14)_{10}$	0	0	1	1	1	0

$$+X - (-Y) = +(X+Y)$$

+	$+(27)_{10}$	0	1	1	0	1	1
Base-2	$+(14)_{10}$	0	0	1	1	1	0
		0					

$$+X - (-Y) = +(X+Y)$$

		1	1	1	1		
+	$+(27)_{10}$	0	1	1	0	1	1
Base-2	$+(14)_{10}$	0	0	1	1	1	0
		0	0	1	0	0	1

$$+X - (-Y) = +(X + Y)$$

		1	1	1	1		
+	$+(27)_{10}$	0	1	1	0	1	1
Base-2	$+(14)_{10}$	0	0	1	1	1	0
		1	0	1	0	0	1
	If you consider the last carry → $-(9)_{10}$ → Negative!						

$$+X - (-Y) = +(X+Y)$$

		1	1	1	1		
+ Base-2	$+(27)_{10}$	0	1	1	0	1	1
	$+(14)_{10}$	0	0	1	1	1	0
		0	0	1	0	0	1
	If you ignore it → $+(9)_{10}$ → Result is not correct!						

$$+X - (-Y) = +(X+Y)$$

		1	1	1	1		
+ Base-2	$+(27)_{10}$	0	1	1	0	1	1
	$+(14)_{10}$	0	0	1	1	1	0
		▒	▒	▒	▒	▒	▒
	Overflow: The result is not reliable!						

SIGNED COMPLEMENT

Give up left most position for sign! What are the wastes?

r^{n-1}	r^{n-2}	r^{n-3}	...	r^2	r^1	r^0
0	Positive Numbers					
Nonzero	Negative Numbers					

$+0 \longrightarrow \text{Max} = r^{n-1} - 1 = r^n - 1$

$\text{Min} = -(r^{n-1} - 1) \longleftarrow -0$

DIMINISHED RADIX COMPLEMENT

Given $(N)_r$ with n digits, the (r – 1)'s complement of N, i.e., its *diminished radix complement*, is defined as $(r^n – 1) – N$.


1's COMP. BASE-2

Base-2		2^4	2^3	2^2	2^1	2^0
$2^5=$	1	0	0	0	0	0

Base-2		2^4	2^3	2^2	2^1	2^0
2^5=	1	0	0	0	0	0
1=		0	0	0	0	1

Base-2		2^4	2^3	2^2	2^1	2^0
		+2	+2	+2	+2	
	–1	–1	–1	–1	–1	+2
2^5=	1	0	0	0	0	0
1=		0	0	0	0	1
2^5–1=		1	1	1	1	1
	5 digits of 1					

Base-2	2^5	2^4	2^3	2^2	2^1	2^0
$2^5-1=$		1	1	1	1	1
$N=$		1	0	1	0	1
$(2^5-1)-N=$		0	1	0	1	0

1's complement of $(10101)_2=(01010)_2=$ NOT on each digit


3’s COMP. BASE-4

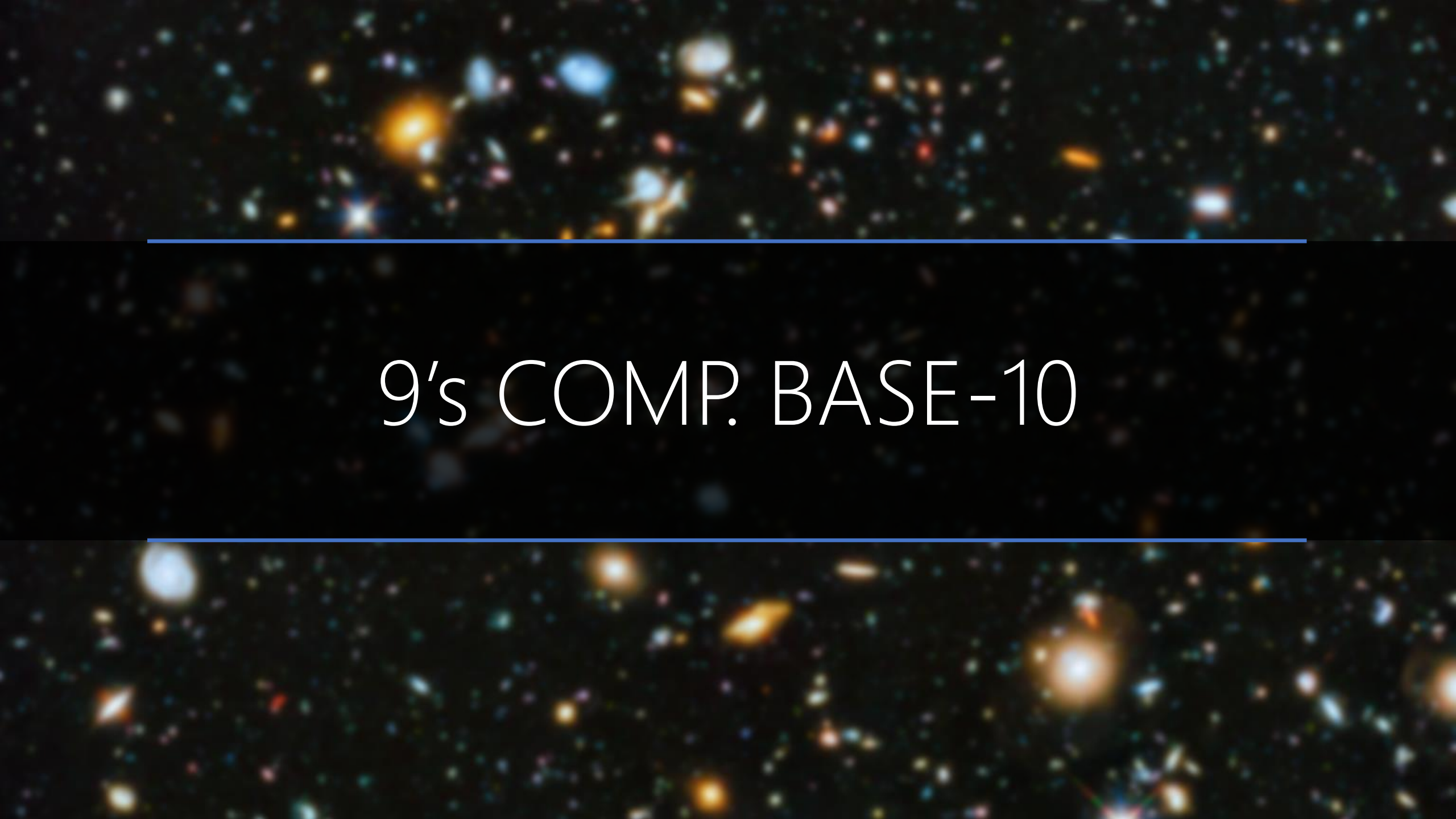
Base-4		4^4	4^3	4^2	4^1	4^0
$4^5=$	1	0	0	0	0	0

Base-4		4^4	4^3	4^2	4^1	4^0
$4^5=$	1	0	0	0	0	0
$1=$	0	0	0	0	0	1

Base-4		4^4	4^3	4^2	4^1	4^0
		+4	+4	+4	+4	
	–1	–1	–1	–1	–1	+4
4^5=	1	0	0	0	0	0
1=		0	0	0	0	1
4^5–1=		3	3	3	3	3
	5 digits of 3					

Base-4		4^4	4^3	4^2	4^1	4^0
$4^5-1=$		3	3	3	3	3
$N=$		1	2	1	3	0
$(4^5-1)-N=$		2	1	2	0	3

3's complement of $(12130)_4=(21203)_4=$ 3 – Each digit

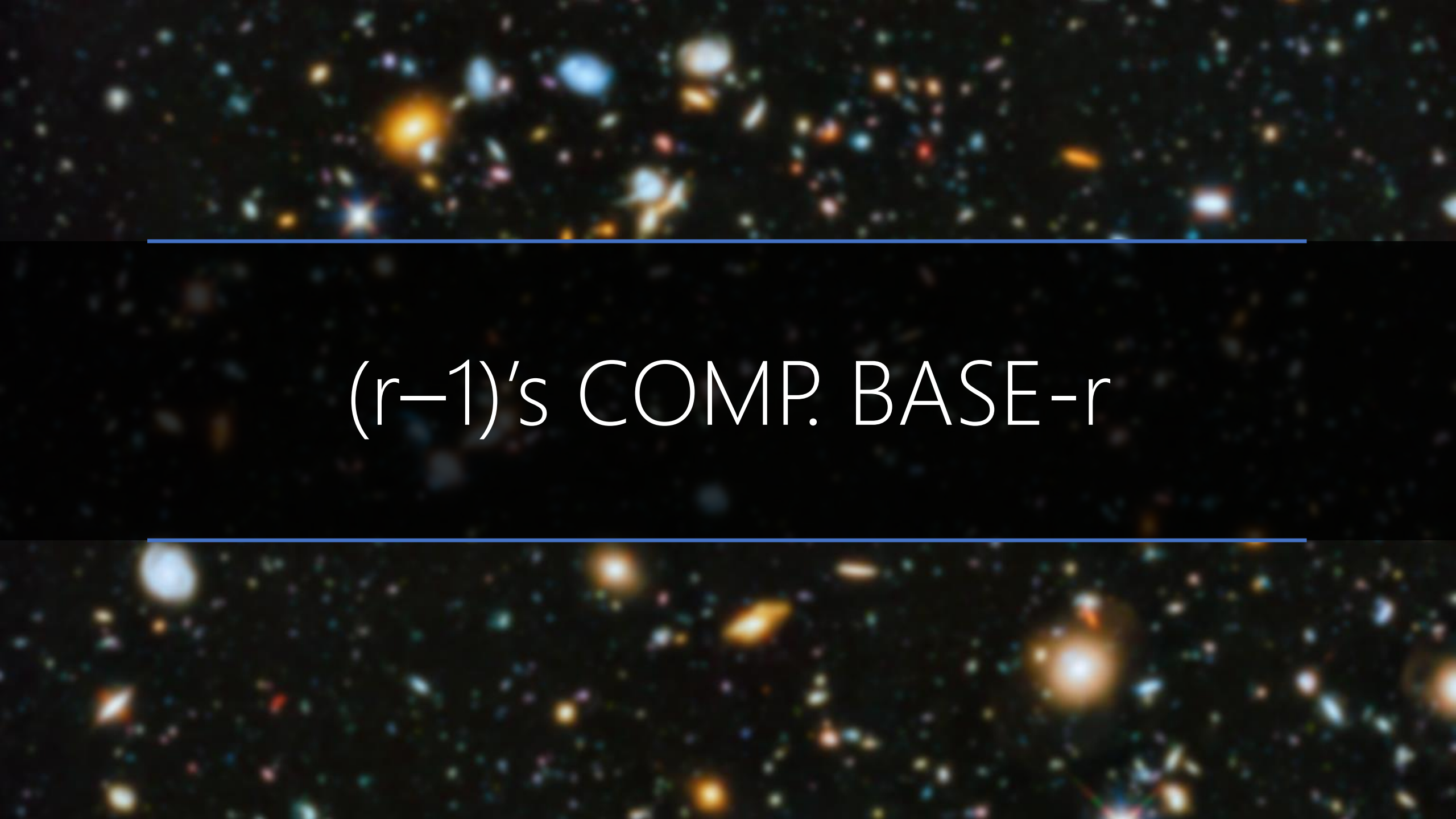


9's COMP. BASE-10

Base-10		10^4	10^3	10^2	10^1	10^0
		+10	+10	+10	+10	
	–1	–1	–1	–1	–1	+10
10^5=	1	0	0	0	0	0
1=		0	0	0	0	1
10^5–1=		9	9	9	9	9
	5 digits of 9					

Base-10		10^4	10^3	10^2	10^1	10^0
$10^5-1=$		9	9	9	9	9
$N=$		1	2	1	3	0
$(10^5-1)-N=$		8	7	8	6	9

9's complement of $(12130)_{10}=(87869)_{10}=$ 9 – Each digit


(r–1)'s COMP. BASE-r

Base-r		r^{n-1}	…	r^2	r^1	r^0
$r^n-1=$		$r-1$	…	$r-1$	$r-1$	$r-1$
$N=$		d_{n-1}	…	d_2	d_1	d_0
$(r^n-1)-N=$		$r-1-d_{n-1}$	…	$r-1-d_{n-1}$	$r-1-d_{n-1}$	$r-1-d_{n-1}$

(r–1)'s complement of $(N)_r$= (r–1) – Each digit

RADIX COMPLEMENT

Given $(N)_r$ with n digits, the r's complement of N, i.e., its *radix complement*, is defined as $r^n - N$.

Equivalently,

(r–1)'s complement + 1 = $[(r^n - 1) - N] + 1 = r^n - N$

2’s COMP. BASE-2

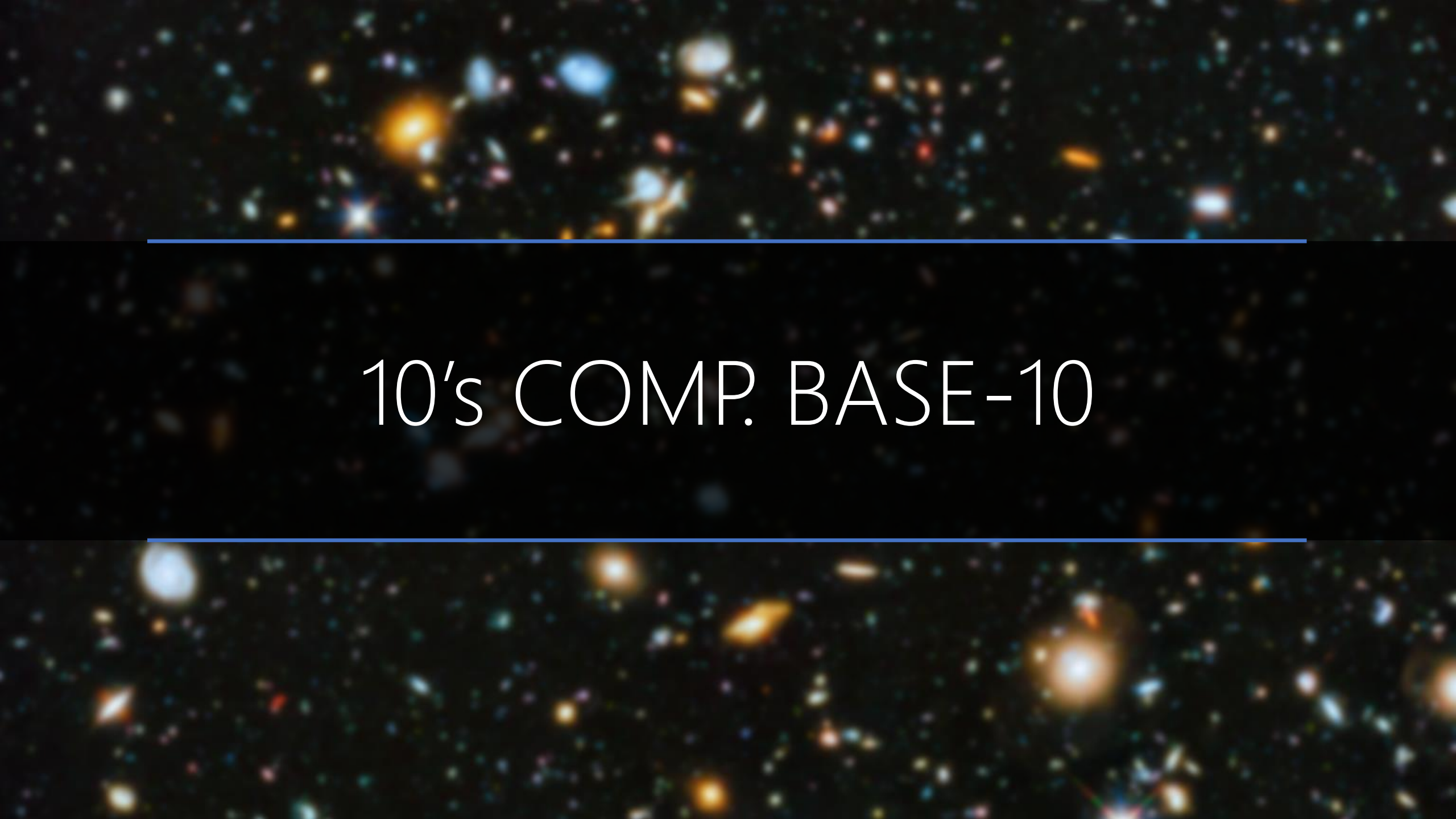
Base-2		2^4	2^3	2^2	2^1	2^0
$2^5-1=$		1	1	1	1	1
N=		1	0	1	0	1
$(2^5-1)-N=$		0	1	0	1	0
NOT on each digit						
+1=		0	0	0	0	1
$2^5-N=$		0	1	0	1	1

2's complement of $(10101)_2=(01010+1)_2=(01011)_2$


4’s COMP. BASE-4

Base-4		4^4	4^3	4^2	4^1	4^0
$4^5-1=$		3	3	3	3	3
$N=$		1	2	1	3	0
$(4^5-1)-N=$		2	1	2	0	3
Each digit – 3						
$+1=$		0	0	0	0	1
$4^5-N=$		2	1	2	1	0

4's complement of $(12130)_4=(21203+1)_4=(21210)_4$



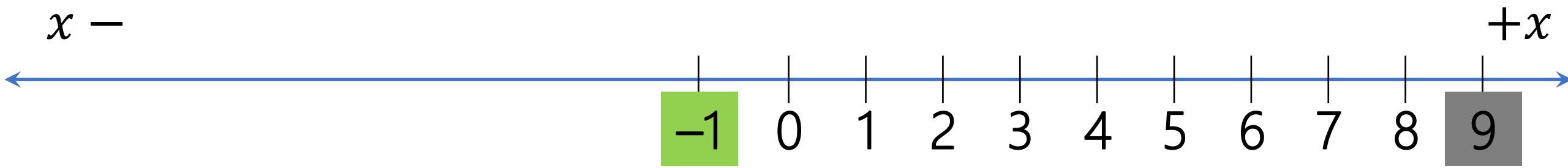
10’s COMP. BASE-10

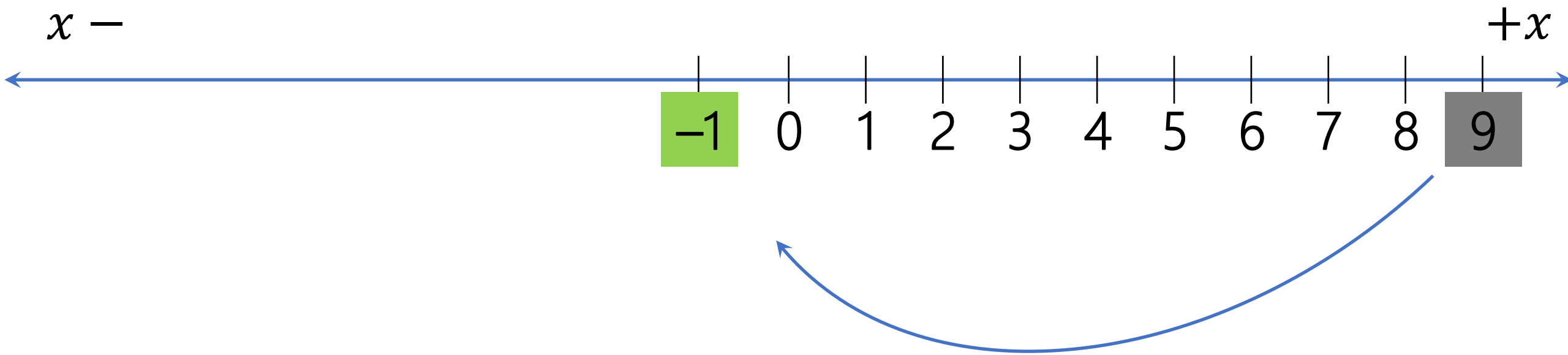
Base-10		10^4	10^3	10^2	10^1	10^0
$10^5-1=$		9	9	9	9	9
N=		1	2	1	3	0
$(10^5-1)-N=$		8	7	8	6	9
Each digit – 9						
+1=		0	0	0	0	1
$10^5-N=$		8	7	8	7	0

10's complement of $(12130)_{10}=(87869+1)_{10}=(87870)_{10}$

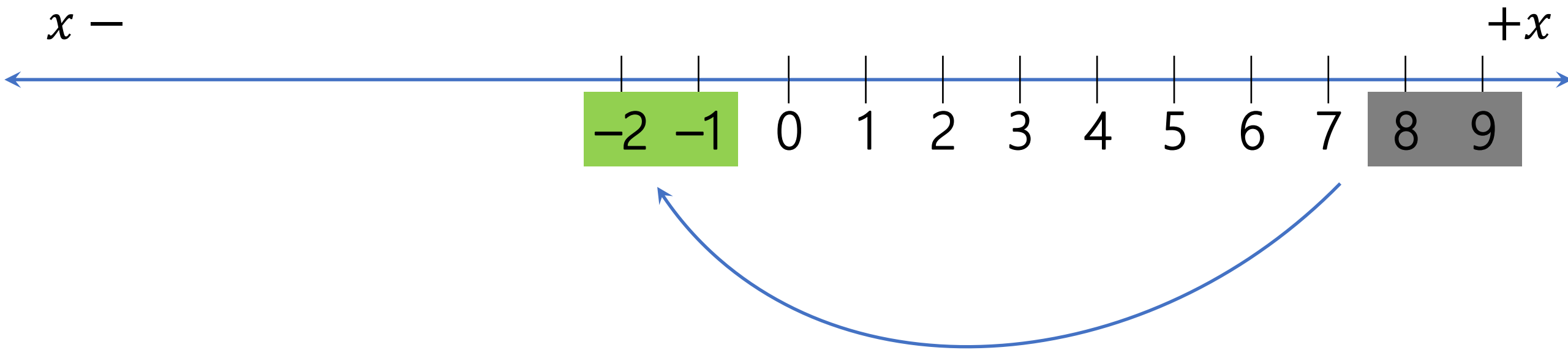

r’s COMP. BASE-r

SIGNED COMPLEMENT

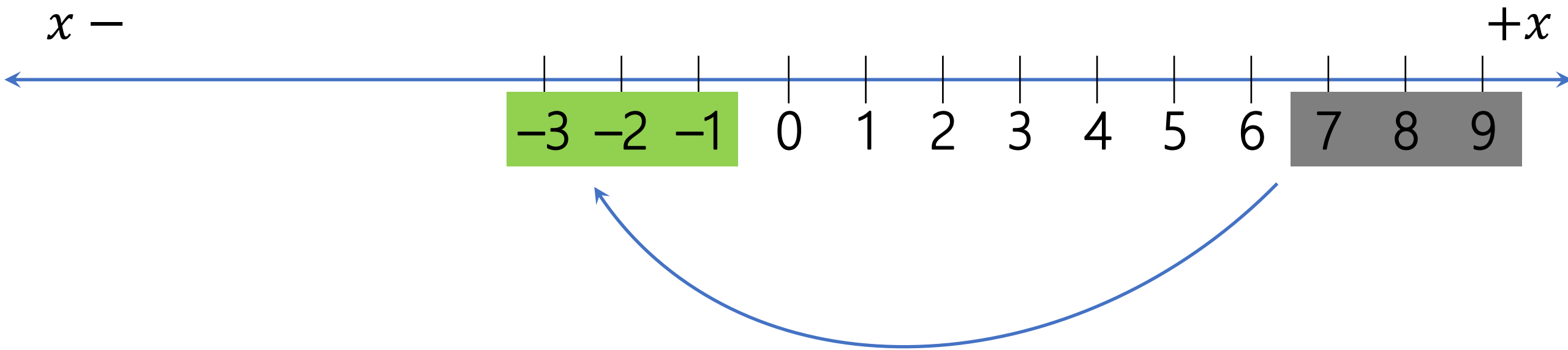

$x-$
$+x$
−1
0
1
2
3
4
5
6
7
8
9



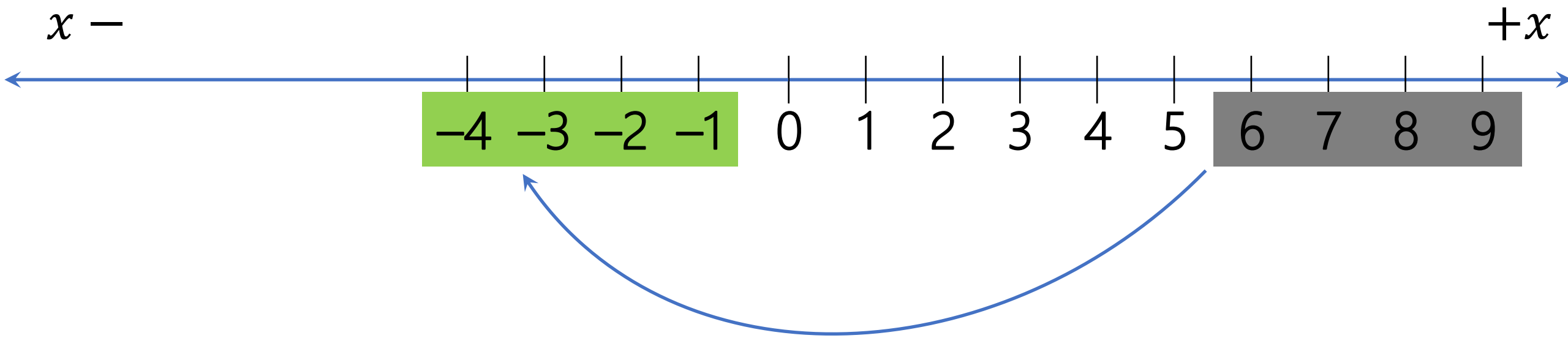
$-1 = -(10^{1}-9) = -(10\text{'s comp. } 9) = -(9-9+1) = -(9\text{'s comp. } 9 + 1)$



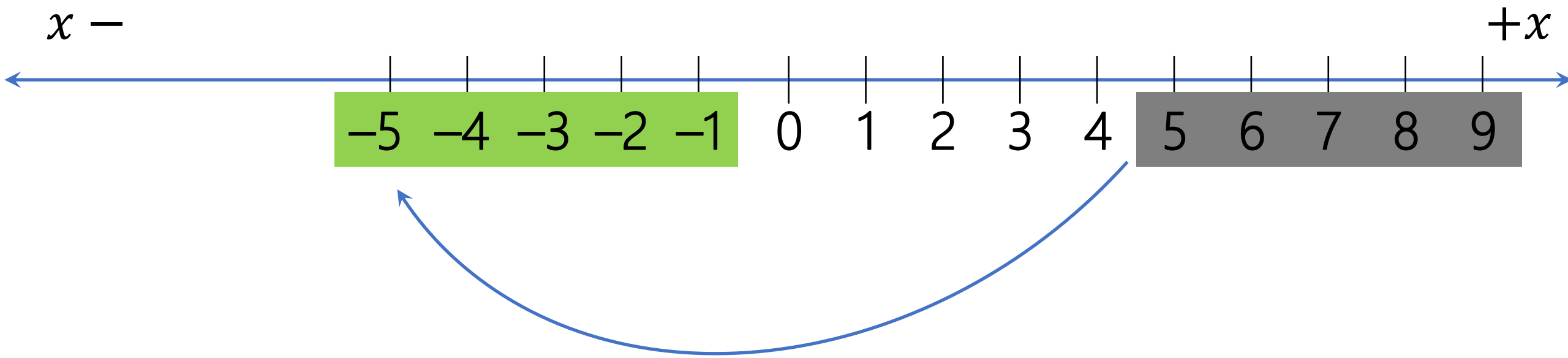
$-2 = -(10^{1} - 8) = -(10\text{'s comp. } 8) = -(9 - 8 + 1) = -(9\text{'s comp. } 8 + 1)$



$-3 = -(10^{1} - 7) = -(10\text{'s comp. } 7) = -(9 - 7 + 1) = -(9\text{'s comp. } 7 + 1)$



$-4 = -(10^{1}-6) = -(10\text{'s comp. } 6) = -(9-6+1) = -(9\text{'s comp. } 6 + 1)$



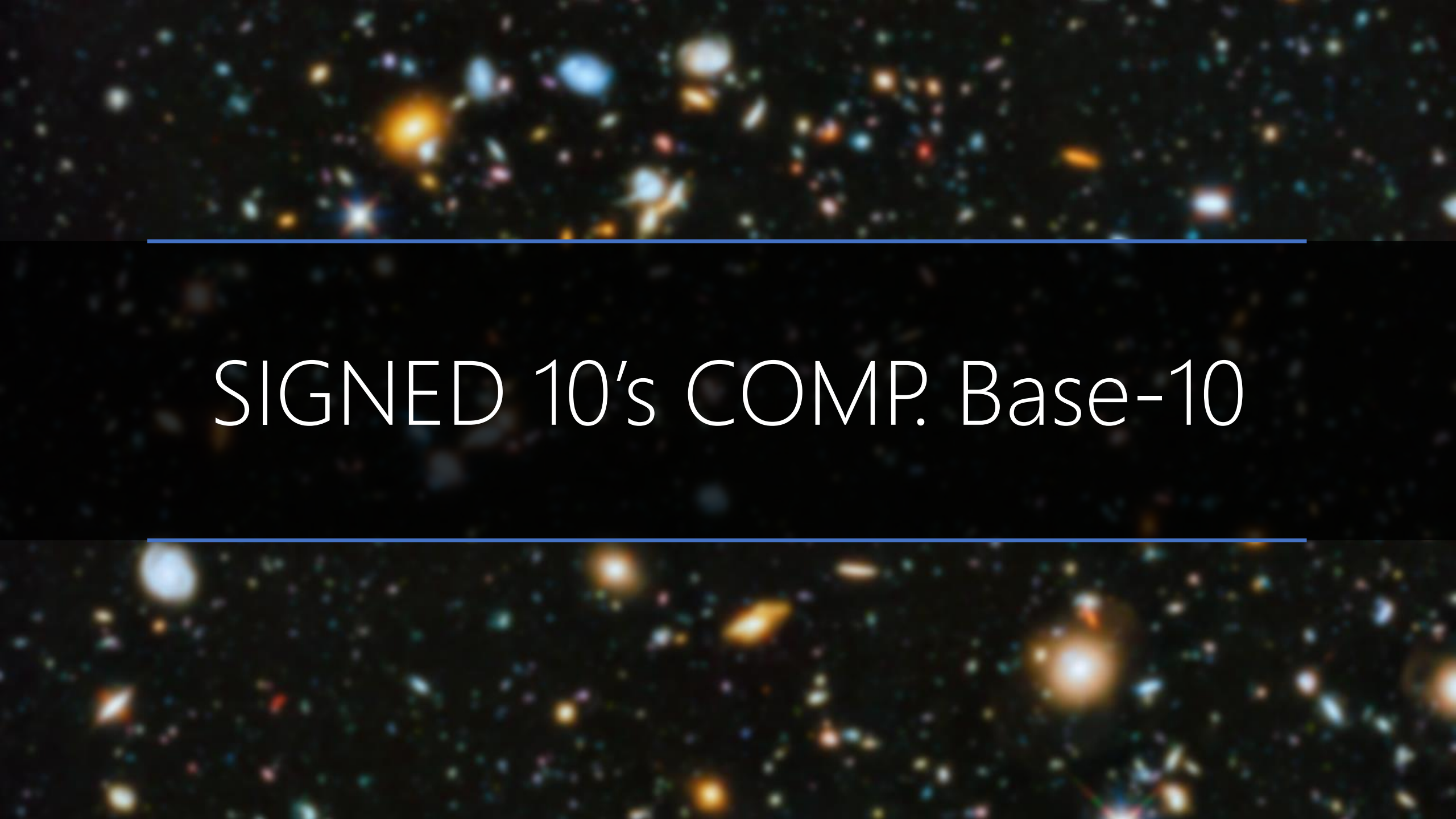
$-5 = -(10^{1} - 5) = -(10\text{'s comp. } 5) = -(9 - 5 + 1) = -(9\text{'s comp. } 5 + 1)$

10^0	–(10's comp.)	10^0
0	→	0
1		–9
2		–8
3		–7
4		–6
5		–5
6		–4
7		–3
8		–2
9		–1

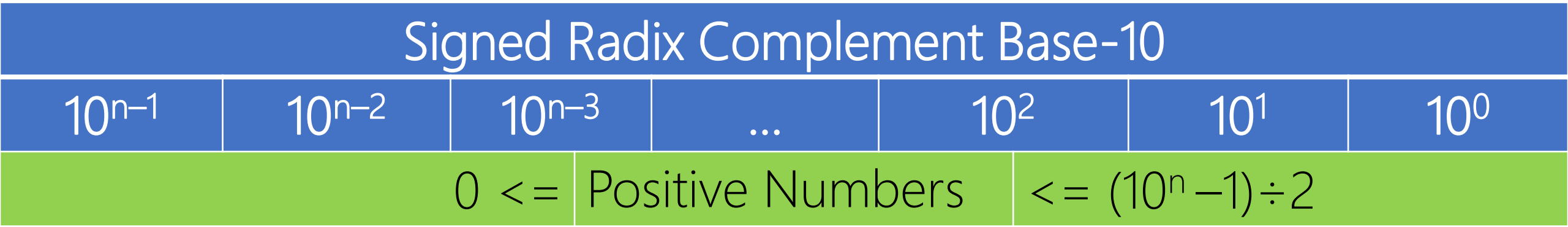
10^0	–(10's comp.)	10^0
0		0
1		–9
2		–8
3		–7
4		–6
5	←	–5
6		–4
7		–3
8		–2
9		–1

10^0		10^0
0	Base-10 Signed 10's comp.	0
1		–9
2		–8
3		–7
4		–6
5		–5
6		–4
7		–3
8		–2
9		–1

10^0		10^0
0	Base-10 Signed 10's comp.	see 0 interpret 0
1		see 1 interpret 1
2		see 2 interpret 2
3		see 3 interpret 3
4		see 4 interpret 4
5→–5		see 5 interpret –5
6→–4		see 6 interpret –4
7→–3		see 7 interpret –3
8→–2		see 8 interpret –2
9→–1		see 9 interpret –1



SIGNED 10's COMP. Base-10



Nothing to do!

Signed Radix Complement Base-10						
10^{n-1}	10^{n-2}	10^{n-3}	…	10^{2}	10^{1}	10^{0}
$(10^{n}-1)\div 2 <$	Negative Numbers	$<= (10^{n}-1)$				

Although we see positive numbers, we must interpret them negative!

How?

Signed Radix Complement Base-10						
10^{n-1}	10^{n-2}	10^{n-3}	…	10^{2}	10^{1}	10^{0}
$(10^{n}-1)\div 2 <$	Negative Numbers	$<= (10^{n}-1)$				

– (10's comp. of the number we see)

Signed Radix Complement Base-10						
10^6	10^5	10^4	10^3	10^2	10^1	10^0
5	8	0	5	0	7	4

$$n = 7$$

$$0 <= \text{Positive Numbers} <= (10^7 - 1) \div 2$$

$$(10^7 - 1) \div 2 < \text{Negative Numbers} <= (10^7 - 1)$$

Signed Radix Complement Base-10						
10^6	10^5	10^4	10^3	10^2	10^1	10^0
5	8	0	5	0	7	4

$$n = 7$$

$$(10^7 - 1) \div 2 = 9{,}999{,}999 \div 2 = 4{,}999{,}999$$

$$5{,}805{,}074 > 4{,}999{,}999$$

This number must be interpreted negative!

Signed Radix Complement Base-10						
10^6	10^5	10^4	10^3	10^2	10^1	10^0
5	8	0	5	0	7	4

$$= -(10\text{'s comp. } (5{,}805{,}074))$$
$$= -(9\text{'s comp. } (5{,}805{,}074) + 1)$$
$$= -(9{,}999{,}999 - 5{,}805{,}074 + 1)$$
$$= -(4{,}194{,}925 + 1)$$
$$= -(4{,}194{,}926)$$

Signed Radix Complement Base-10						
10^6	10^5	10^4	10^3	10^2	10^1	10^0
X	X	X	X	X	X	X

– 3,450,256

Signed Radix Complement Base-10						
10^6	10^5	10^4	10^3	10^2	10^1	10^0
X	X	X	X	X	X	X

– 3,450,256

We must find its positive complement to represent it!

How?

Signed Radix Complement Base-10						
10^6	10^5	10^4	10^3	10^2	10^1	10^0
6	5	4	9	7	4	4

– 3,450,256

= 10's comp (3,450,256)

= 9's comp (3,450,256) + 1

= 9,999,999 – 3,450,256 + 1

= 6,549,743 + 1

= 6,549,744

SIGNED 16's COMP. Base-16

Base-16	1	B	F	2	B	4
	Is this a negative number or positive?					

Base-16	1	B	F	2	B	4
	How many positions? n = 6					

Base-16	1	B	F	2	B	4
	What is the maximum positive number? $(16^6 - 1) \div 2 = (8{,}388{,}607)_{10} = (7FF{,}FFF)_{16}$					

Base-16	1	B	F	2	B	4
	This number is less than $(7FF,FFF)_{16}$ $(+1BF,2B4)_{16}$					

Base-16	E	B	F	2	B	4
	Is this a negative number or positive?					

Base-16	E	B	F	2	B	4
	How many positions? n = 6					

Base-16	E	B	F	2	B	4
	What is the maximum positive number? $(16^6-1)\div 2 = (8{,}388{,}608)_{10} = (7FF{,}FFF)_{16}$					

Base-16	E	B	F	2	B	4
	This number is greater than $(7FF,FFF)_{16}$ The number is negative, but what is the number?					

Base-16	E	B	F	2	B	4
	= – (16's comp. (EBF,2B4)) = – (15's comp. (EBF,2B4) + 1) = – (FFF,FFF – EBF2B4 + 1) = – (140,D4B + 1) = – (140,D4C)					

Base-16	X	X	X	X	X	X
	– (140,D4C)					

Base-16	X	X	X	X	X	X
	– 140,D4C = 16's comp(140,D4C) = 15's comp(140,D4C) + 1 = FFF,FFF – 140,D4C + 1 = EBF,2B3 + 1 = EBF,2B4					

Base-16	E	B	F	2	B	4
	– 140,D4C = 16's comp(140,D4C) = 15's comp(140,D4C) + 1 = FFF,FFF – 140,D4C + 1 = EBF,2B3 + 1 = EBF,2B4					

SIGNED r's COMP. Base-r

Base-r in Radix Complement

r^{n-1}	r^{n-2}	r^{n-3}	...	r^2	r^1	r^0

$0 <= \text{Positive Numbers} <= (r^n - 1) \div 2$

Nothing to do!

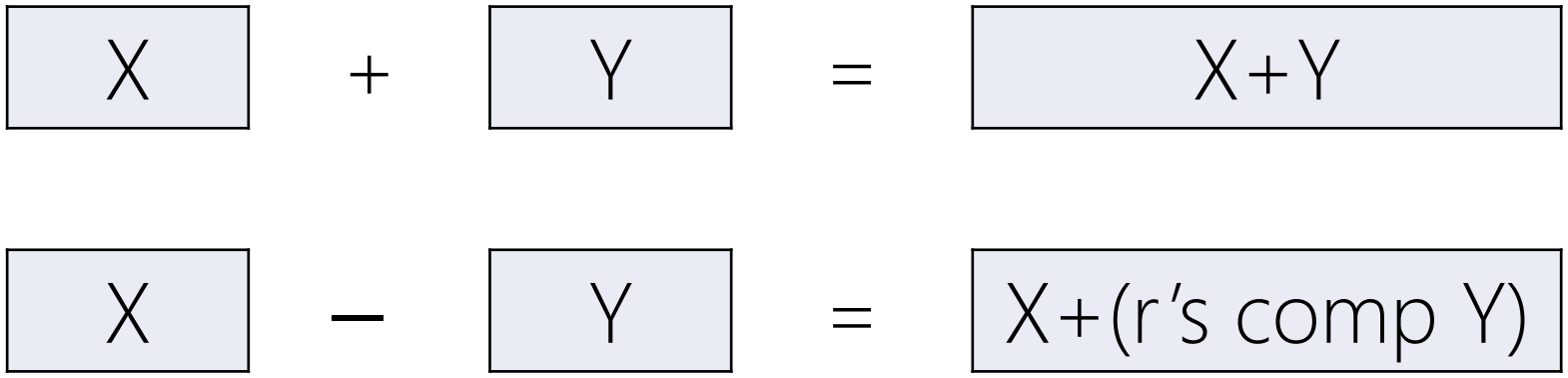
Base-2: 0,111,...,111
Base-4: 1,333,...,333
Base-8: 3,777,...,777
Base-10: 4,999,...,999
Base-16: 7,FFF,...,FFF

Base-r in Radix Complement						
r^{n-1}	r^{n-2}	r^{n-3}	...	r^2	r^1	r^0
$(r^n-1) \div 2 + 1 <=$			Negative Numbers		$<= (r^n-1)$	
Base-2: 1,000,...,000 Base-4: 2,000,...,000 Base-8: 4,000,...,000 Base-10: 5,000,...,000 Base-16: 8,000,...,000						

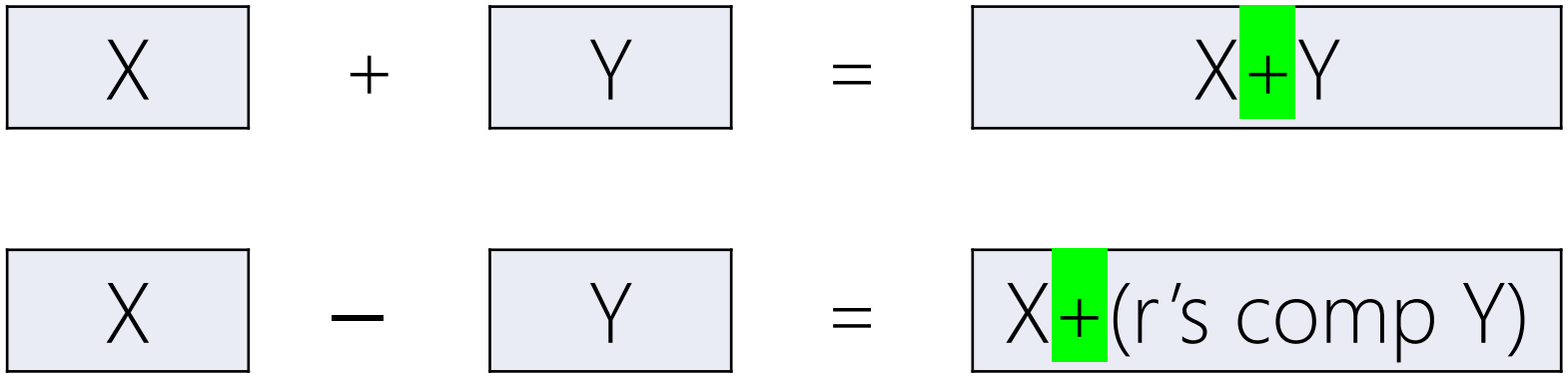
We see positive number, but we interpret negative!

= – (r's comp. (#)) = – ((r–1)'s comp. (#) + 1)

SIGNED r'S COMPLEMENT ARITHMETIC



Last Carry → Ignore



One adder for both addition and subtraction!

SIGNED r'S COMPLEMENT ADDITION

+		1	0	1	0	1	1
Base-2				1	1	1	0

+ Base-2		1	0	1	0	1	1
		> $(2^6-1)\div 2 = (011,111)_2$ $= -(2\text{'s comp}(\#))$ $= -(010101)_2$ $= -(21)_{10}$					

Padding

+	$-(21)_{10}$	1	0	1	0	1	1
Base-2		0	0	1	1	1	0

+							
Base-2		0	0	1	1	1	0
		$< (2^6 - 1) \div 2 = (011,111)_2$					
		$= +(001110)_2$					

			1	1	1		
+ Base-2	$-(21)_{10}$	1	0	1	0	1	1
	$+(14)_{10}$	0	0	1	1	1	0
		1	1	1	0	0	1

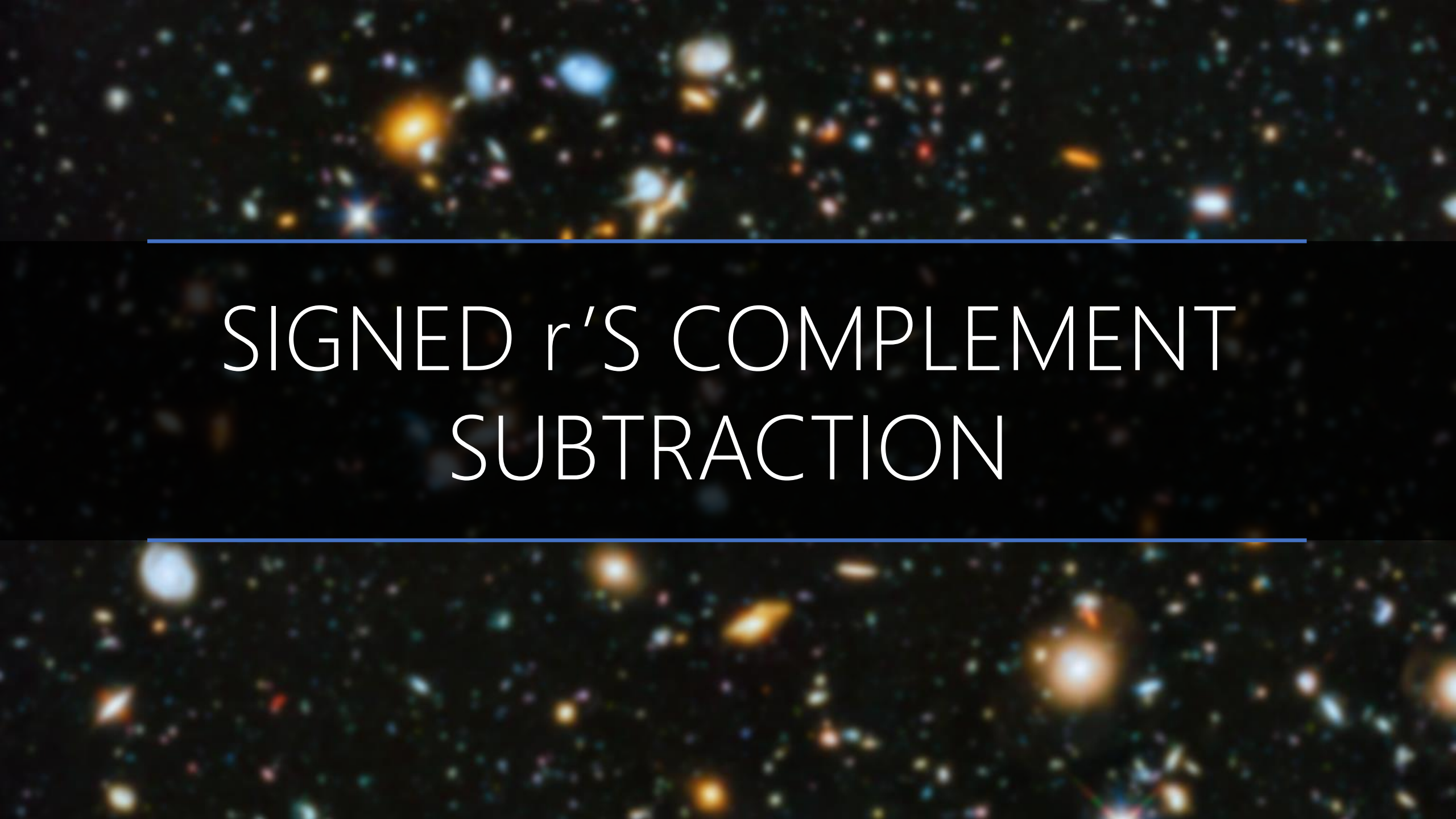
			1	1	1		
+ Base-2	$-(21)_{10}$	1	0	1	0	1	1
	$+(14)_{10}$	0	0	1	1	1	0
		1	1	1	0	0	1

$> (2^6-1) \div 2 = (011,111)_{10}$

$= -(2\text{'s comp}(\#))$

$= -(000111)_2$

$= -(7)_{10}$



SIGNED r'S COMPLEMENT SUBTRACTION

– Base-2	$-(21)_{10}$	1	0	1	0	1	1
		0	0	1	1	1	0

– Base-2	$-(21)_{10}$	1	0	1	0	1	1
	$+(8)_{10}$	0	0	1	0	0	0
		We must convert the subtraction to addition. Eq., addition with $-(8)_{10}$!					

–	$-(21)_{10}$	1	0	1	0	1	1
Base-2	$+(8)_{10}$	0	0	1	0	0	0

Make the number negative:

= – (2's comp. (001000))

= – (1's comp(001000) + 1)

= – (NOT(001000) + 1)

= – (110111 + 1)

= – (111000)

+	$-(21)_{10}$	1	0	1	0	1	1
Base-2	$-(8)_{10}$	1	1	1	0	0	0

Last Carry → Ignore

		1	1	1			
	+	$-(21)_{10}$	1	0	1	0	1	1
Base-2		$-(8)_{10}$	1	1	1	0	0	0
			1	0	0	0	1	1

		1	1				
+	$-(21)_{10}$	1	0	1	0	1	1
Base-2	$-(8)_{10}$	1	1	1	0	0	0
		1	0	0	0	1	1
		Done! This is the Result.					

		1	1				
+	$-(21)_{10}$	1	0	1	0	1	1
Base-2	$-(8)_{10}$	1	1	1	0	0	0
		1	0	0	0	1	1

$> 2^6 - 1 \div 2 = (011,111)_{10}$

The number is negative:

= – (2's comp. (100011))

= –(011101)

		1	1				
+	$-(21)_{10}$	1	0	1	0	1	1
Base-2	$-(8)_{10}$	1	1	1	0	0	0
	$-(29)_{10}$	1	0	0	0	1	1

SIGNED r'S COMPLEMENT OVERFLOW

X	+	Y	=	X+Y
X	−	Y	=	X+(r's comp Y)

The + result of two negative numbers → positive

The + result of two positive numbers → negative

– Base-2	$-(21)_{10}$	1	0	1	0	1	1
		0	0	1	1	1	0

– Base-2	$-(21)_{10}$	1	0	1	0	1	1
	$+(14)_{10}$	0	0	1	1	1	0
		We must convert the subtraction to addition. Eq., addition with $-(14)_{10}$!					

–	$-(21)_{10}$	1	0	1	0	1	1
Base-2	$+(14)_{10}$	0	0	1	1	1	0

Make the number negative:

= – (2's comp. (001110))

= – (1's comp(001110) + 1)

= – (NOT(001110) + 1)

= – (110001 + 1)

= – (110010)

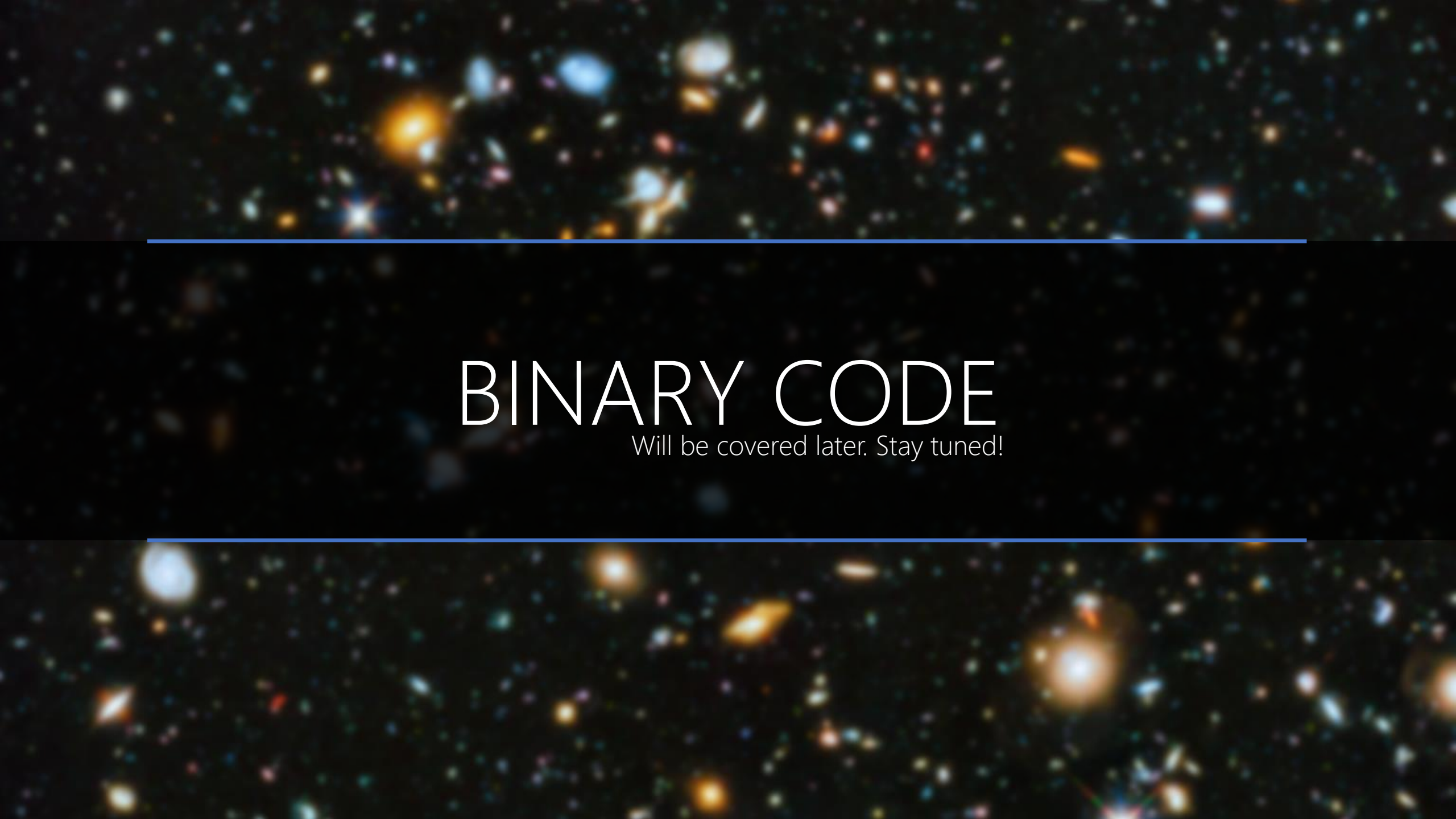
+	$-(21)_{10}$	1	0	1	0	1	1
Base-2	$-(14)_{10}$	1	1	0	0	1	0

Last Carry → Ignore

	1				1		
+	$-(21)_{10}$	1	0	1	0	1	1
Base-2	$-(14)_{10}$	1	1	0	0	1	0
		0	1	1	1	0	1

					1		
+	$-(21)_{10}$	1	0	1	0	1	1
Base-2	$-(14)_{10}$	1	1	0	0	1	0
		0	1	1	1	0	1
		$< 2^6 - 1 \div 2 = (011,111)_{10}$ $= +(011101)_2$					

					1		
+	$-(21)_{10}$	1	0	1	0	1	1
Base-2	$-(14)_{10}$	1	1	0	0	1	0
		▒	▒	▒	▒	▒	▒
		Overflow! Don't rely on the result!					



BINARY CODE

Will be covered later. Stay tuned!