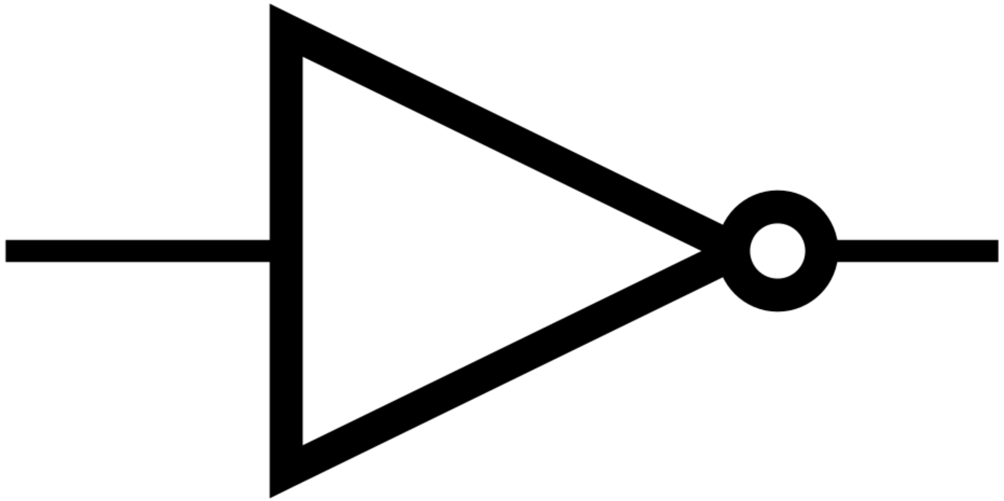
DIGITAL SYSTEMS

Discrete

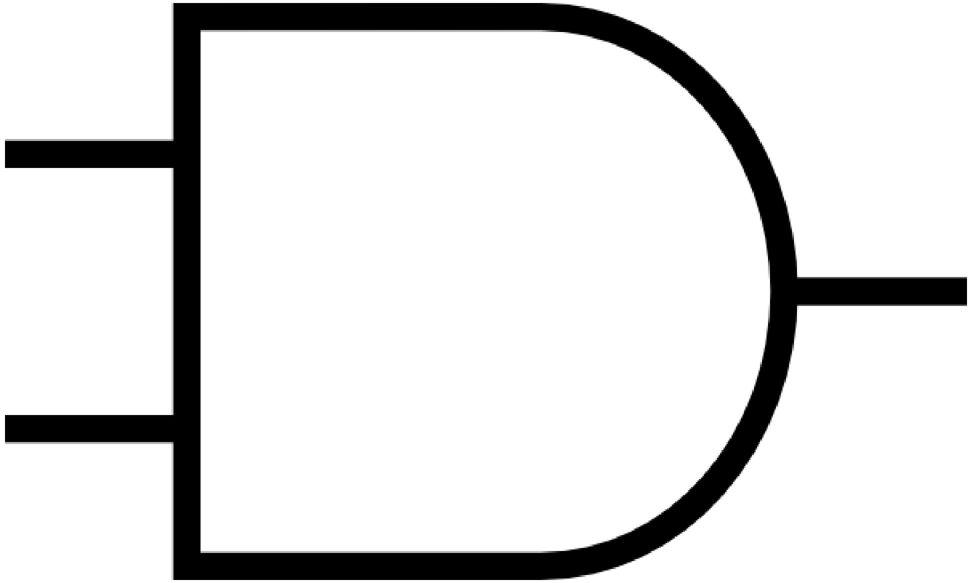
DESIGN COMPUTER

Positive Logic
Button-Up Approach

LOGIC GATES



NOT



AND

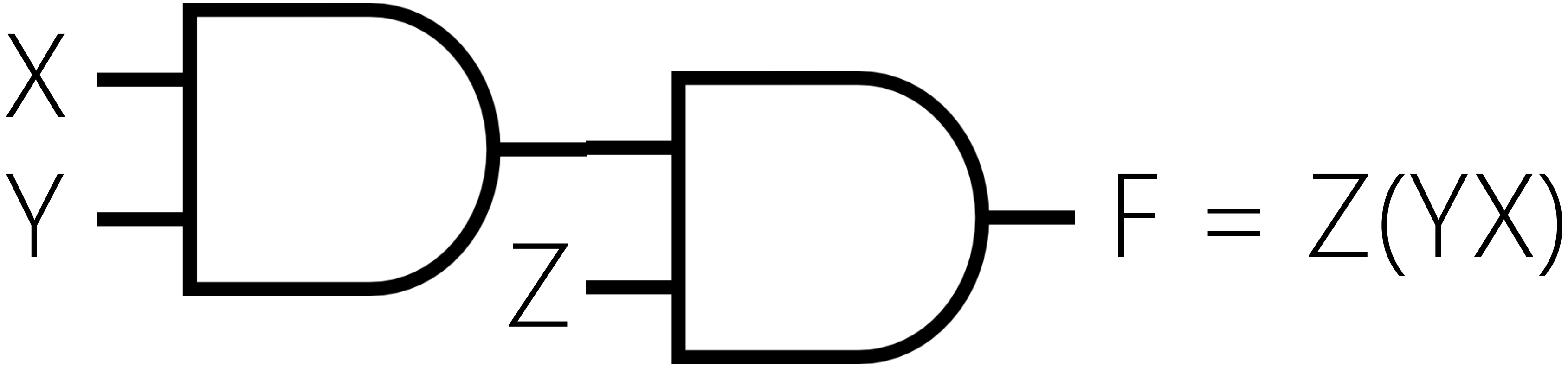
DESIGN

System design is to construct structure at *logical level* that:

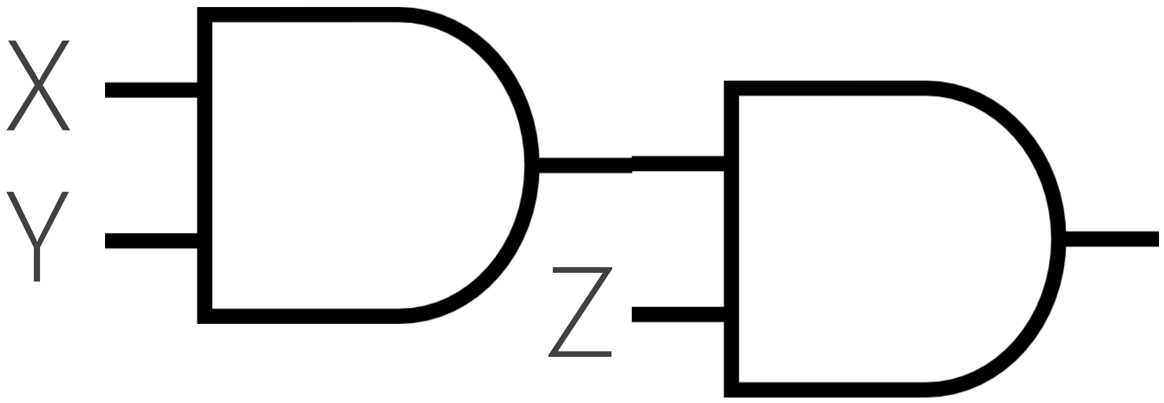
- Effective (true): provides a desired functionality
- Efficient (fast)
- Optimum Cost

3-INPUT AND

Z	Y	X	ZYX
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1



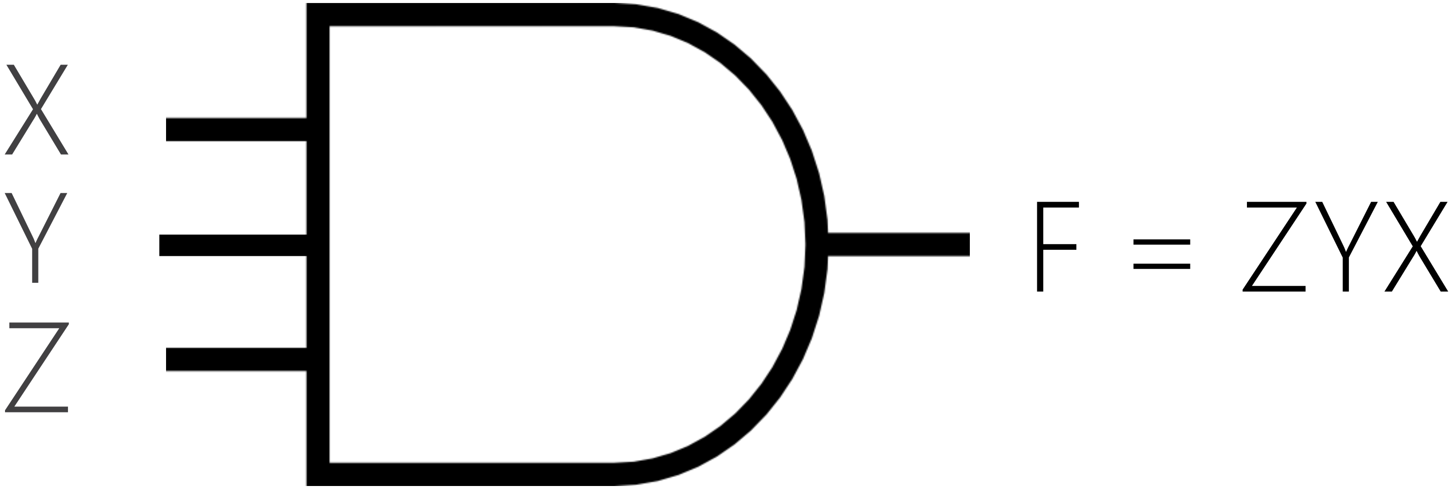
Z	Y	X	Z(YX)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1



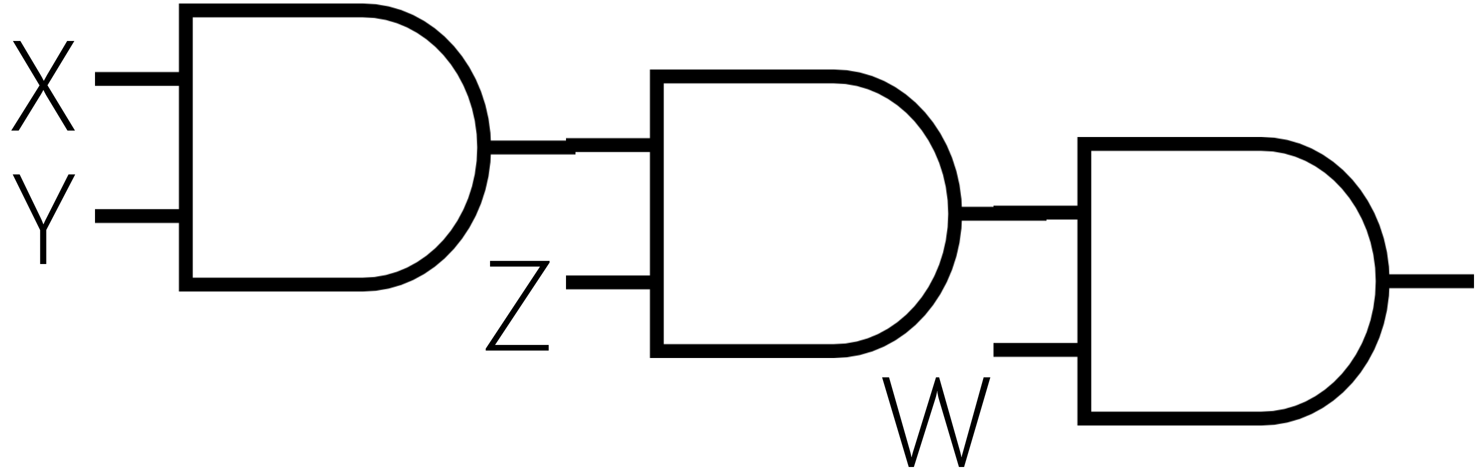
F=Z(YX)=Z(XY)=(ZX)Y=(XZ)Y=XZY

Associative

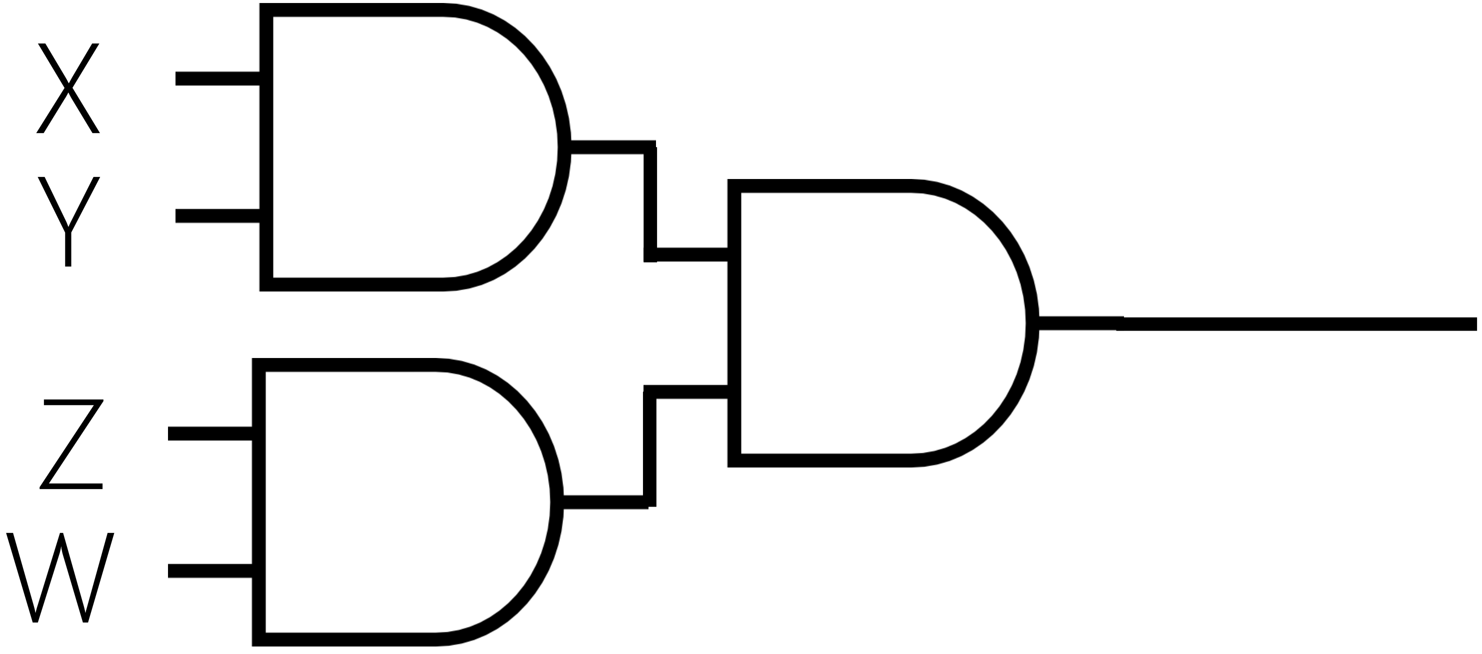
Z	Y	X	Z(YX)	Z(XY)	(ZX)Y	XZY
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	0	0	0
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	0	0	0	0
1	1	1	1	1	1	1



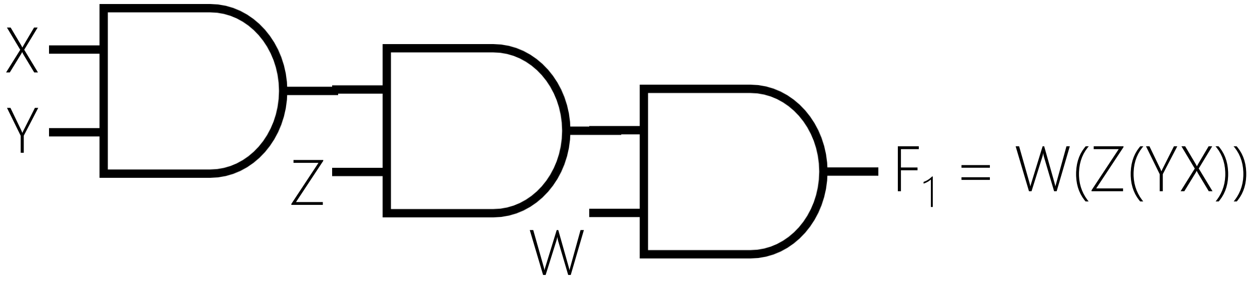
Z	Y	X	ZYX
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

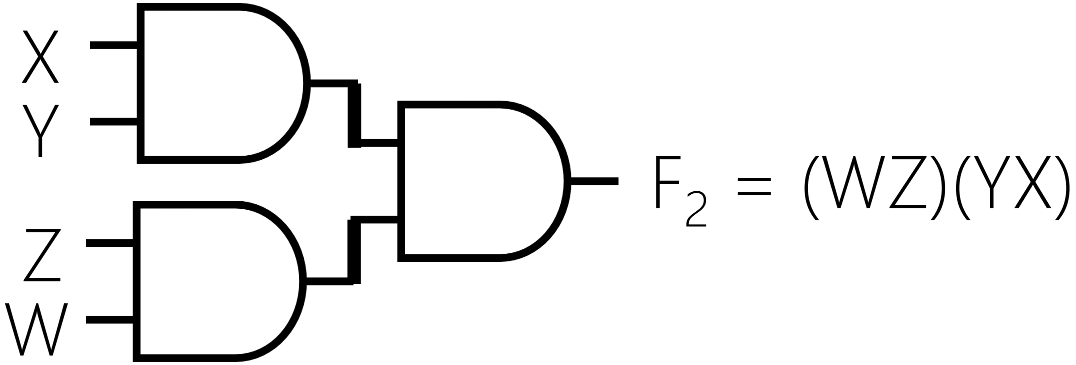


$F_1 = W(Z(YX))$

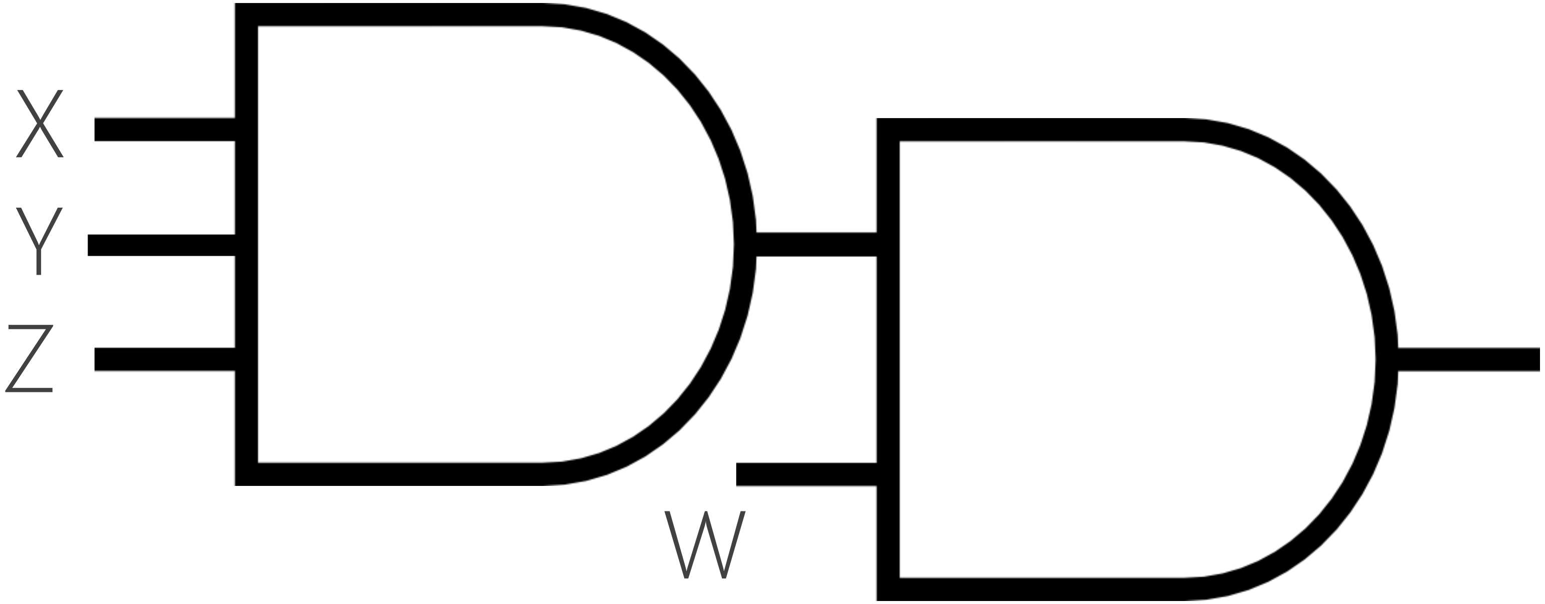


$F_2 = (WZ)(YX)$

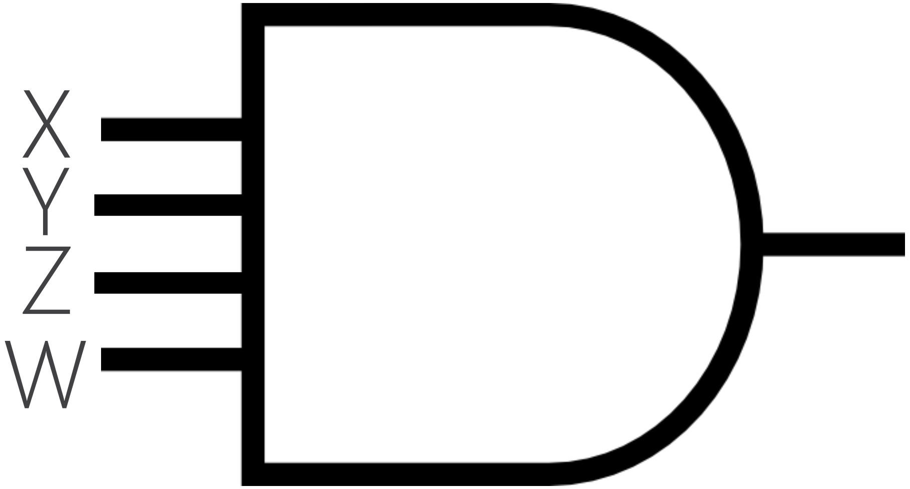




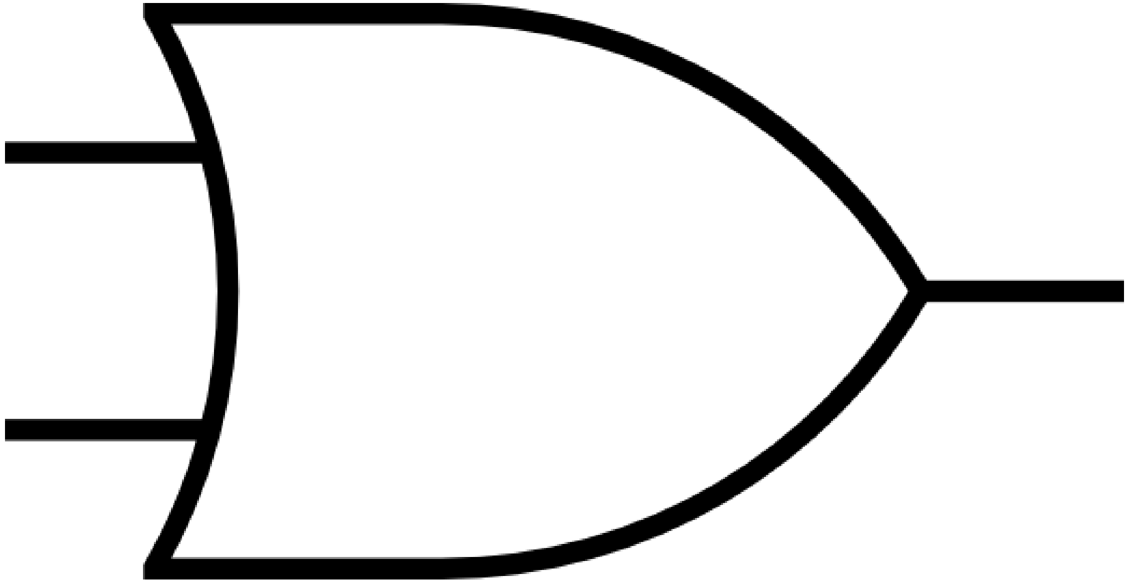
F = WZYX	F_1	F_2
Effective (True)	Yes	Yes
Efficient (Fast)	Hmm, 3 levels, No!	Yes! 2 levels
Min. Cost	3 gates, Yes	3 gates, Yes



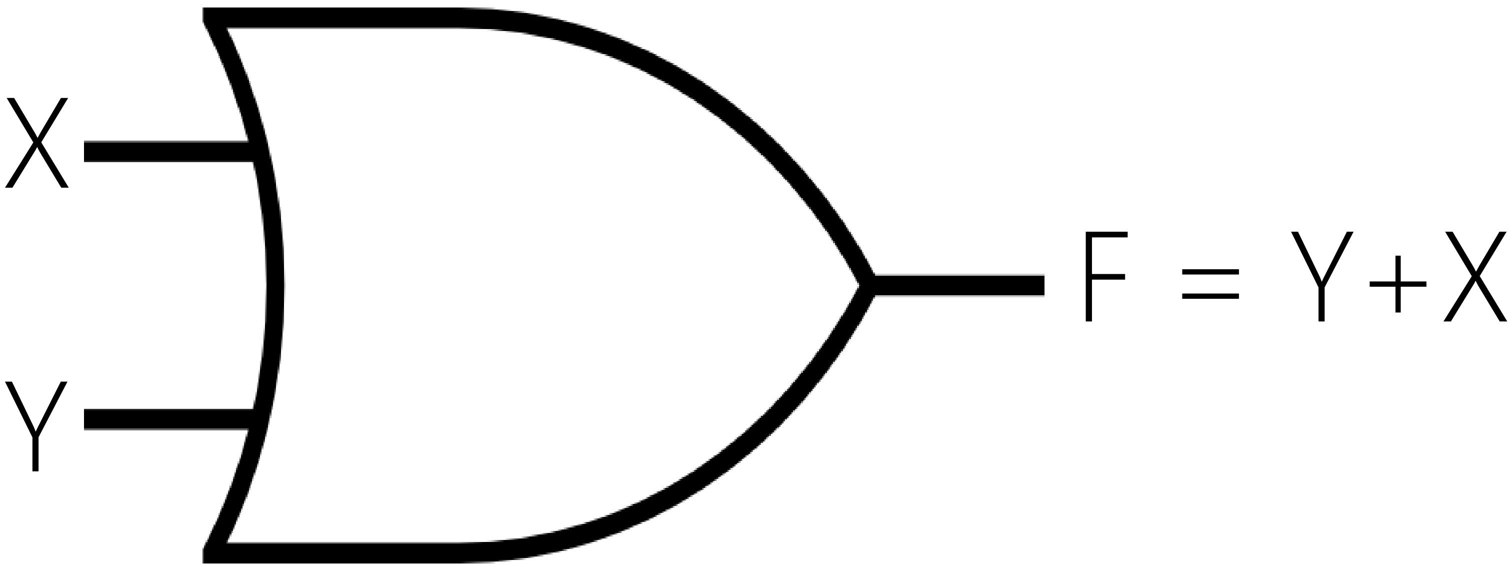
F = WZYX



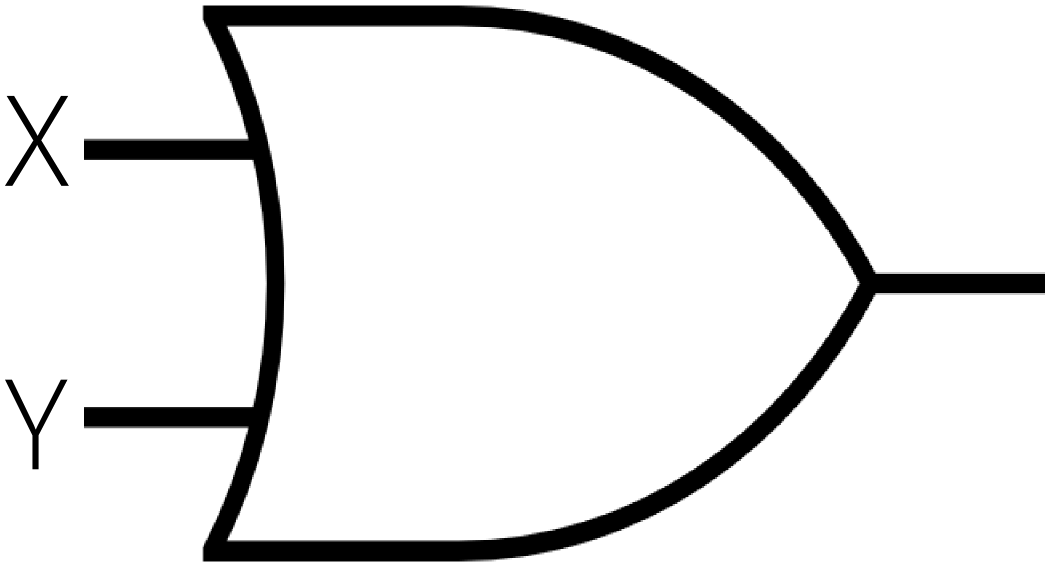
F = WZYX



OR



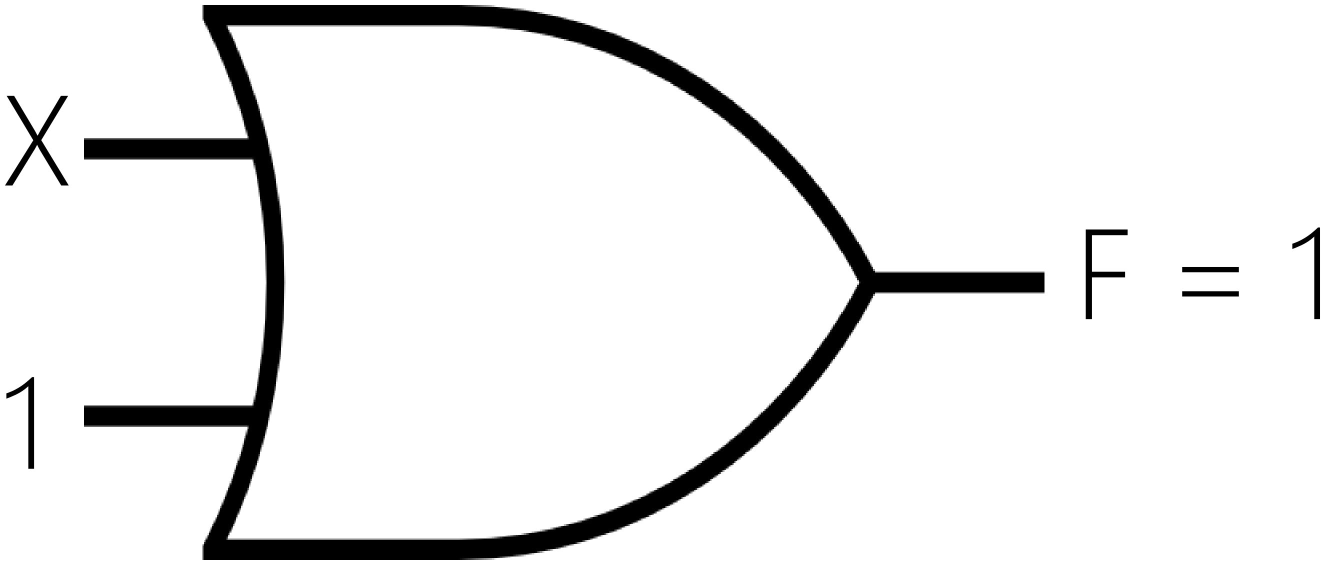
Y	X	Y OR X	Y+X
0	0	0	
0	1	1	
1	0	1	
1	1	1	



$F = Y + X = X + Y$

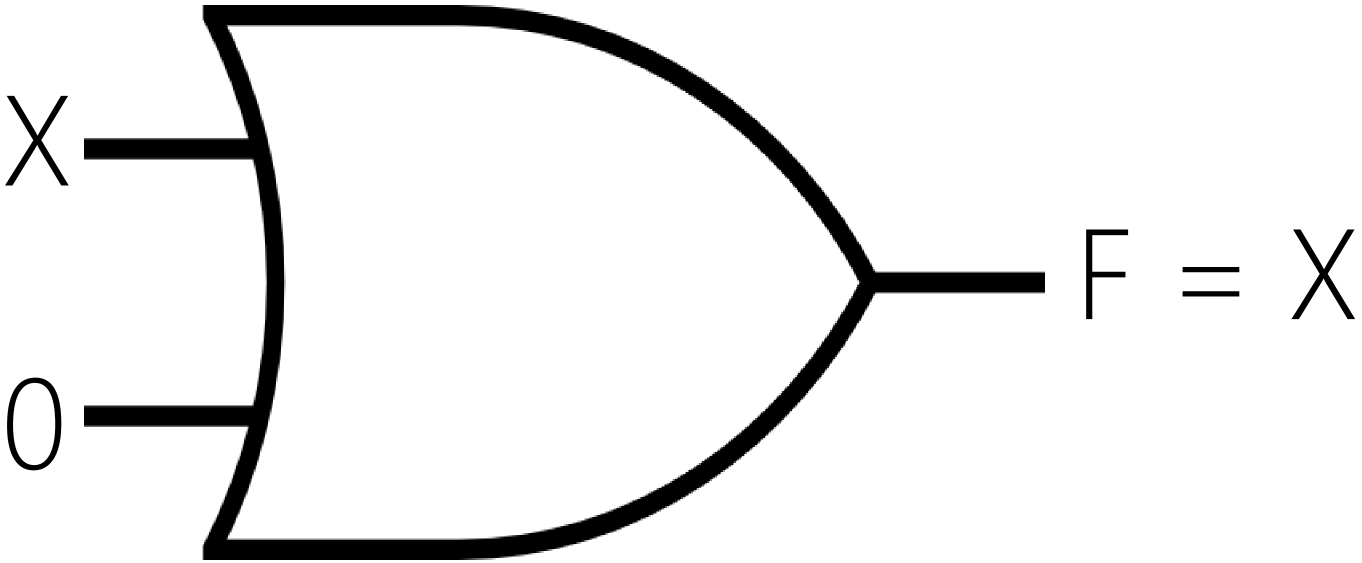
Commutative

X	Y	X OR Y	X+Y
0	0	0	
0	1	1	
1	0	1	
1	1	1	



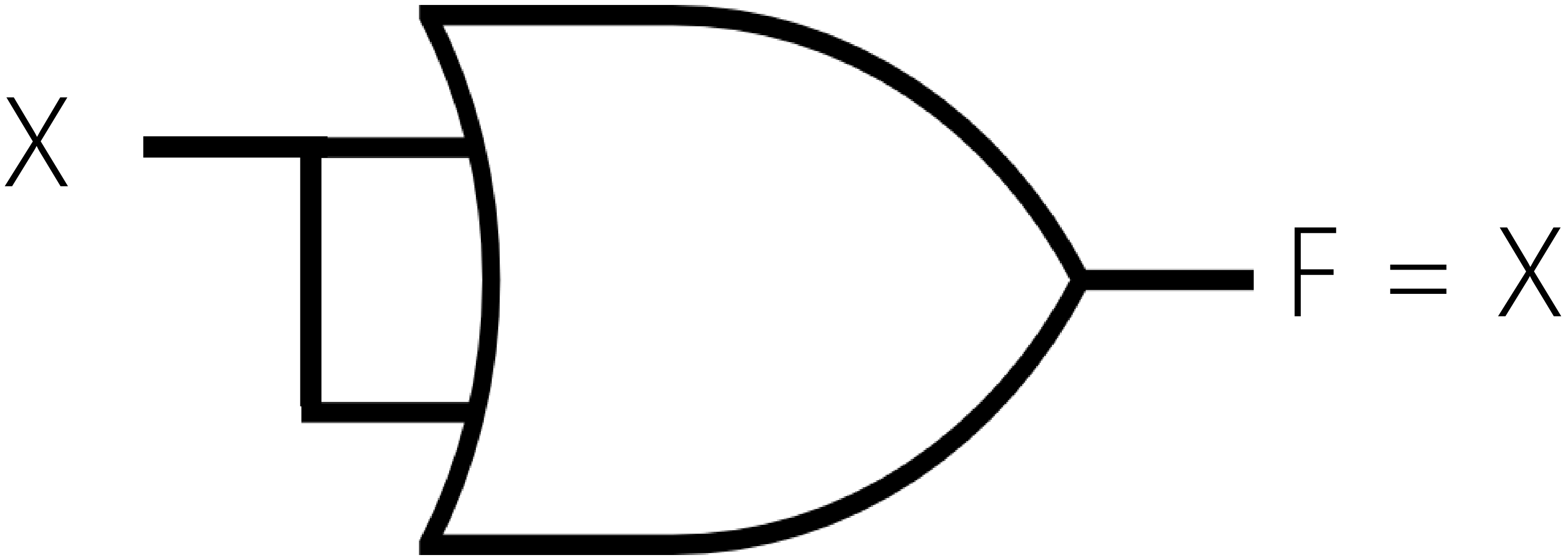
Y	X	Y+X
1	0	1
1	1	1

$F = X+1 = 1$



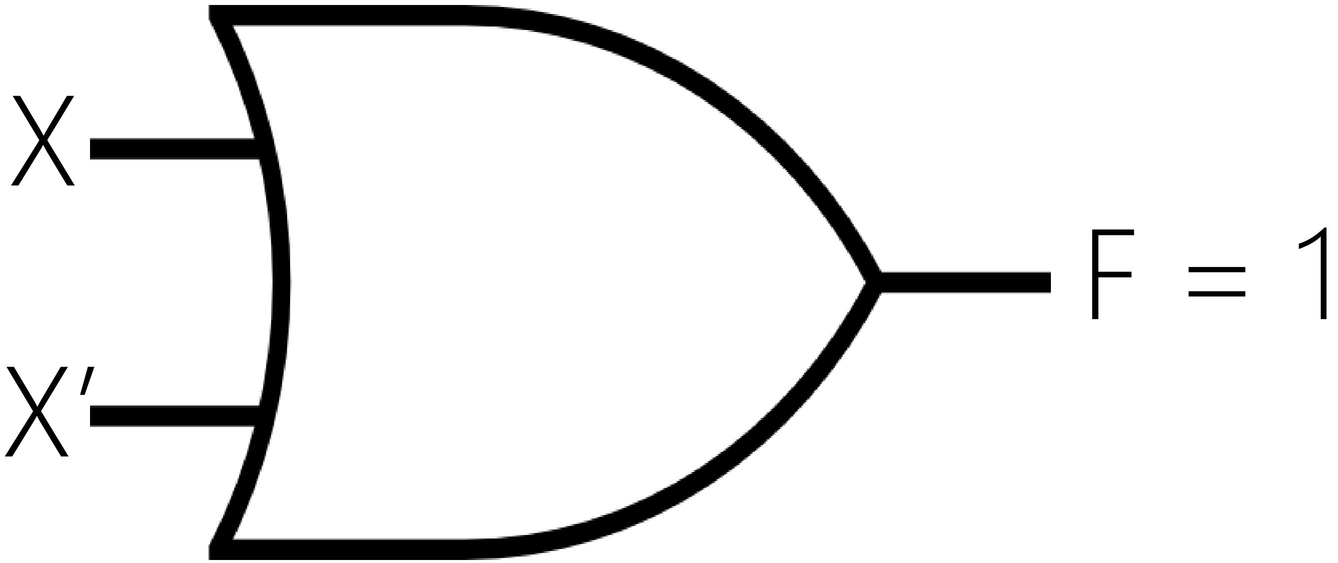
Y	X	Y+X
0	0	0
0	1	1

$$F = X + 0 = X$$



X	X	X+X
0	0	0
1	1	1

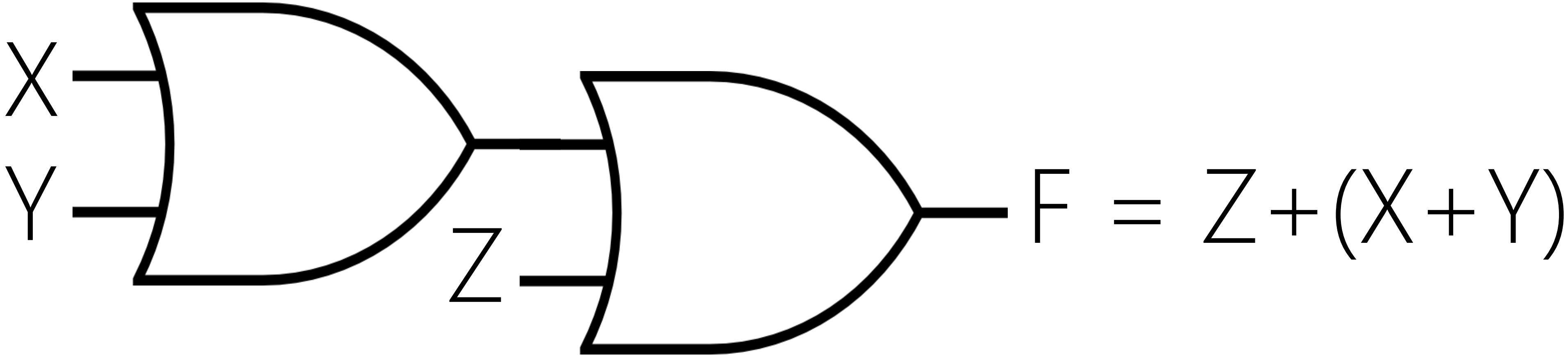
$$F = X + X = X$$



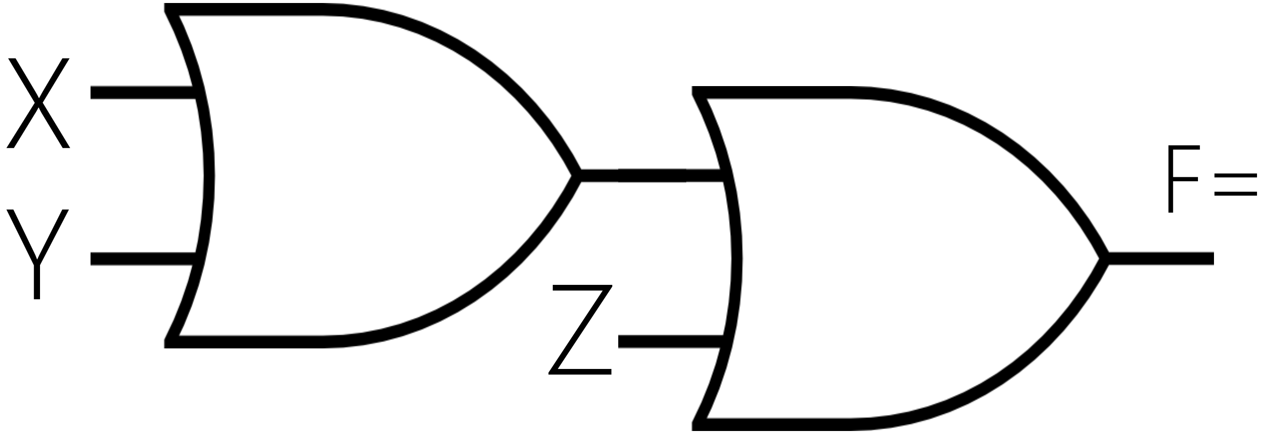
X′	X	X′+X
1	0	1
0	1	1

$$F = X + X' = 1$$

3-INPUT OR



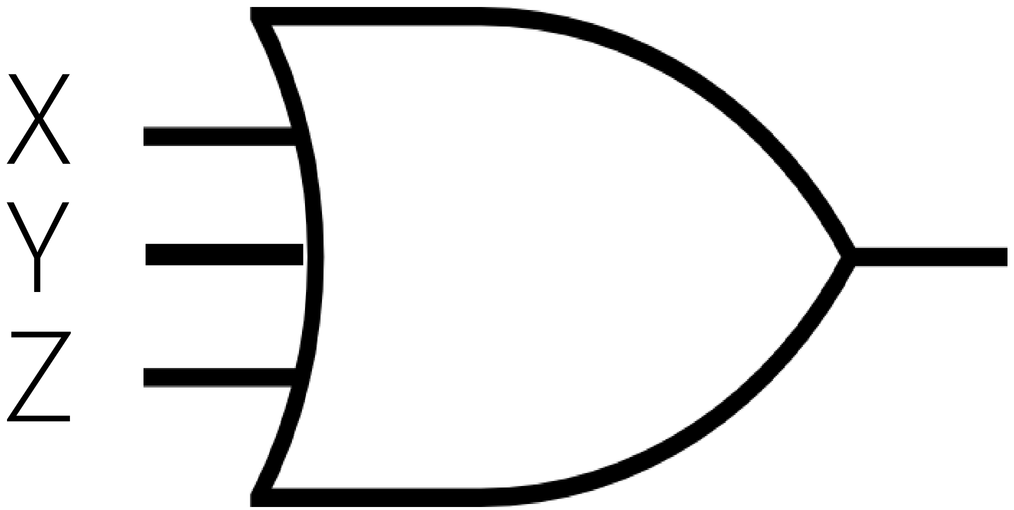
Z	Y	X	Z+(X+Y)
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1



$$F=Z+(Y+X)=Z+(X+Y)=(Z+X)+Y$$
$$= Z+Y+X$$

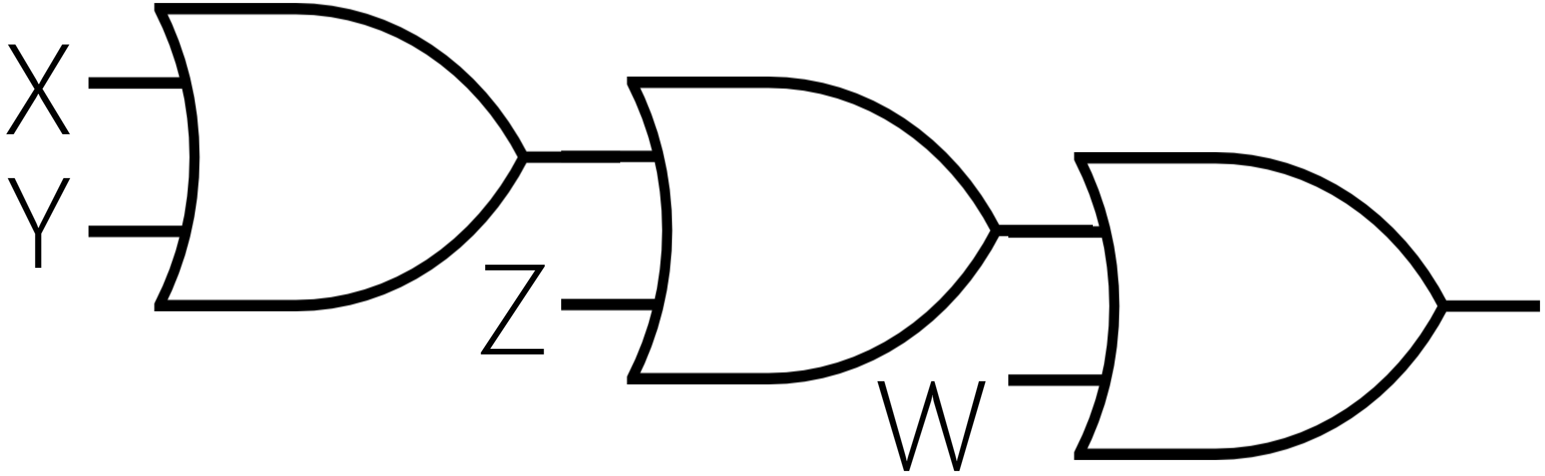
Associative

Z	Y	X	Z+(Y+X)	Z+(X+Y)	(Z+X)+Y	ZXY
0	0	0	0			
0	0	1	1			
0	1	0	1			
0	1	1	1			
1	0	0	1			
1	0	1	1			
1	1	0	1			
1	1	1	1			

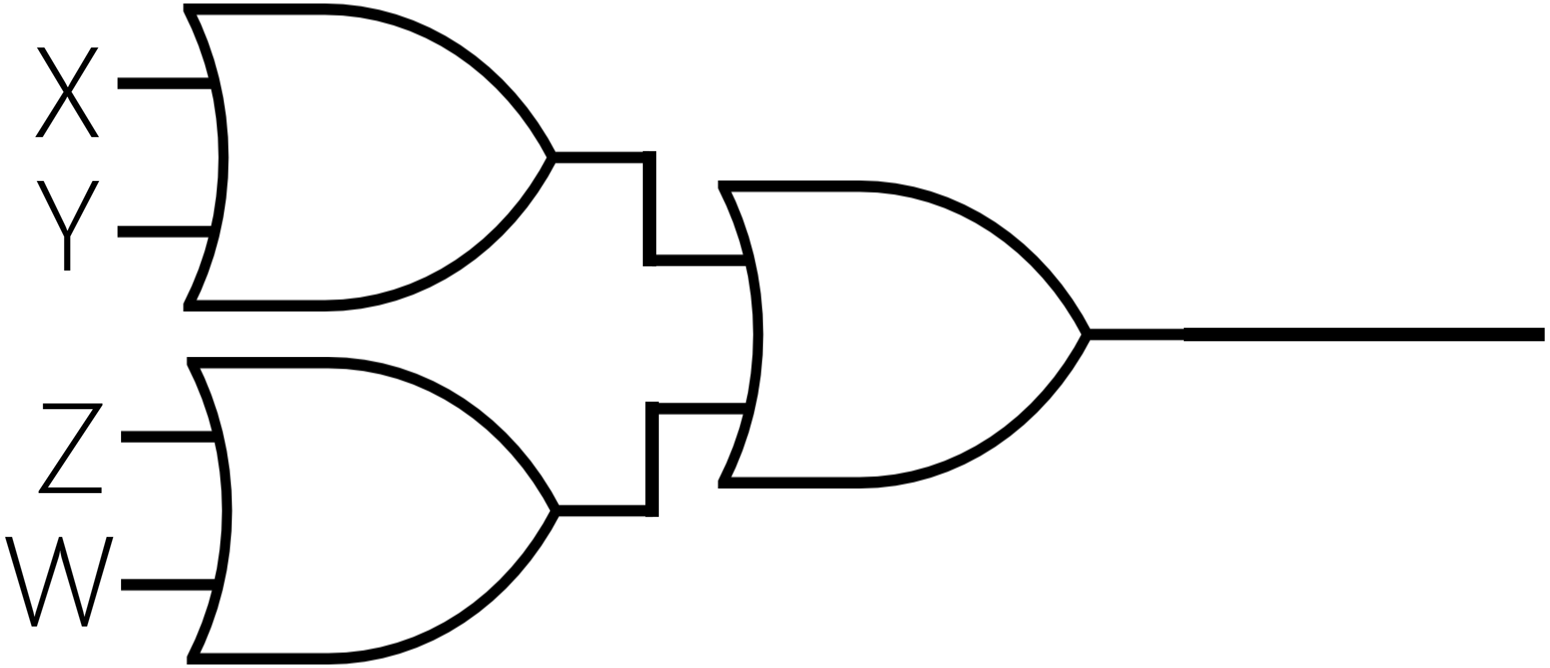


$F = Z+Y+X$

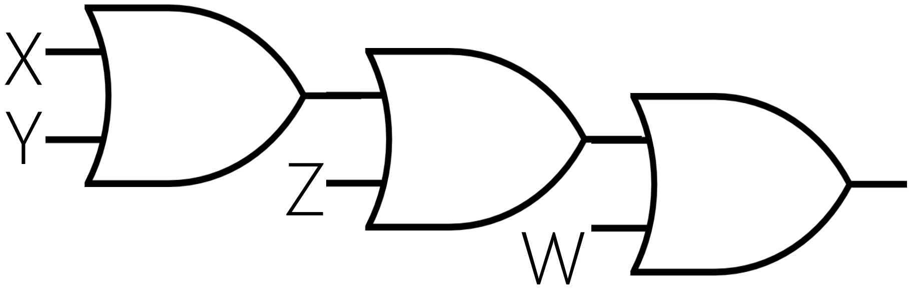
Z	Y	X	Z+(Y+X)	Z+(X+Y)	(Z+X)+Y	ZXY
0	0	0	0			
0	0	1	1			
0	1	0	1			
0	1	1	1			
1	0	0	1			
1	0	1	1			
1	1	0	1			
1	1	1	1			



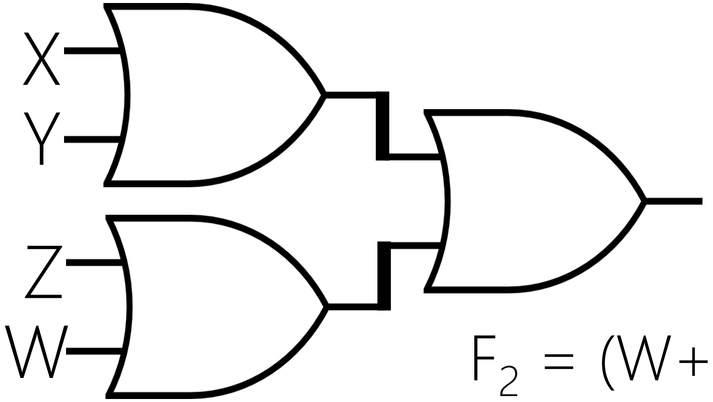
$F_1 = W+(Z+(Y+X))$



$F_2 = (W+Z)+(Y+X)$



$F_1 = W+(Z+(Y+X))$



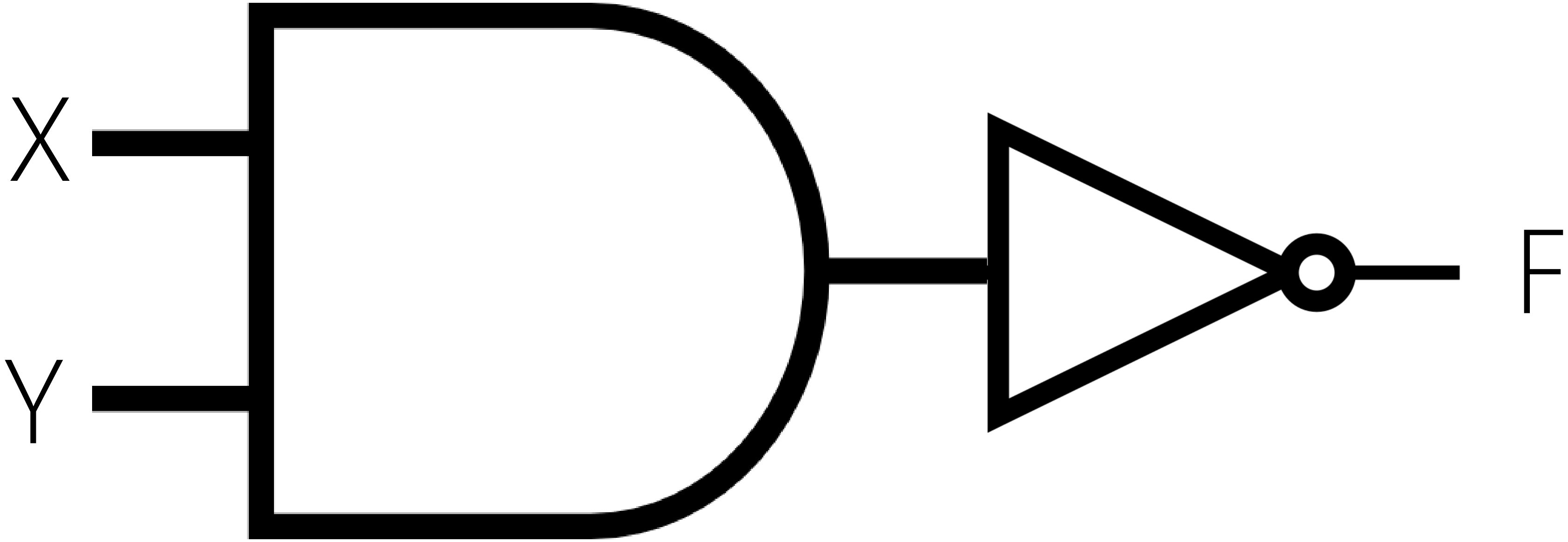
$F_2 = (W+Z)+(Y+X)$

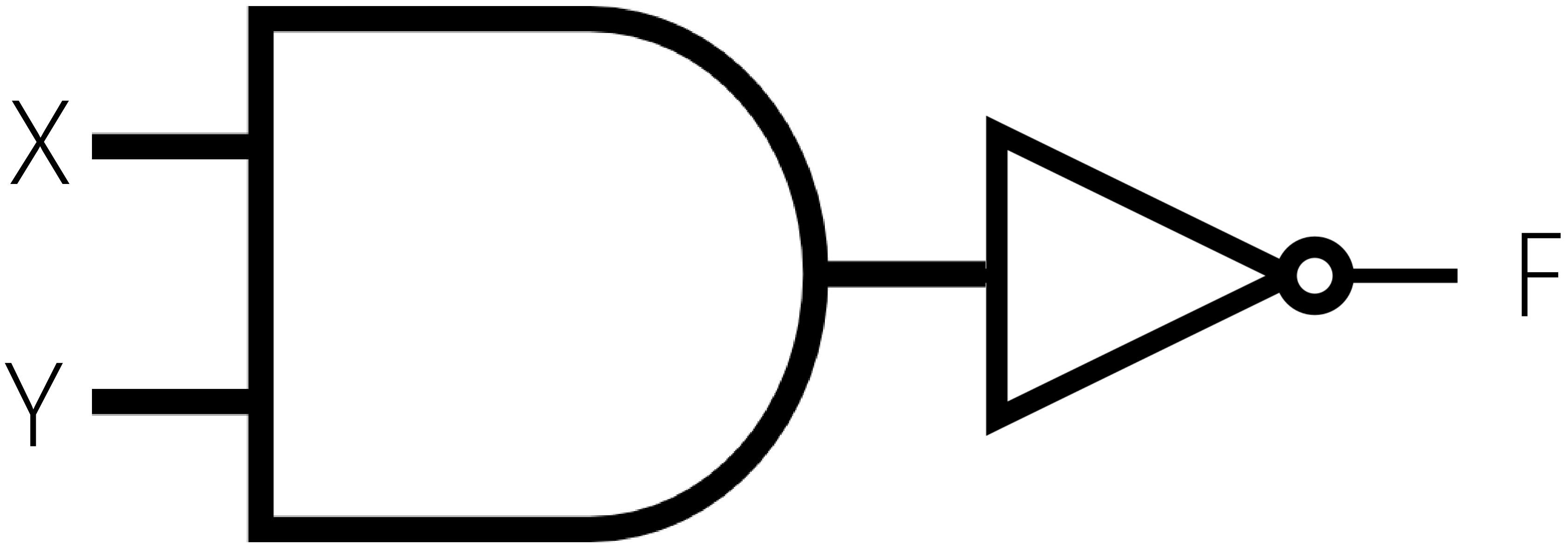
F = W+Z+Y+X	F_1	F_2
Effective (True)	Yes	Yes
Efficient (Fast)	Hmm, 3 levels, No!	Yes! 2 levels
Min. Cost	3 gates, Yes	3 gates, Yes

ANALYSIS I

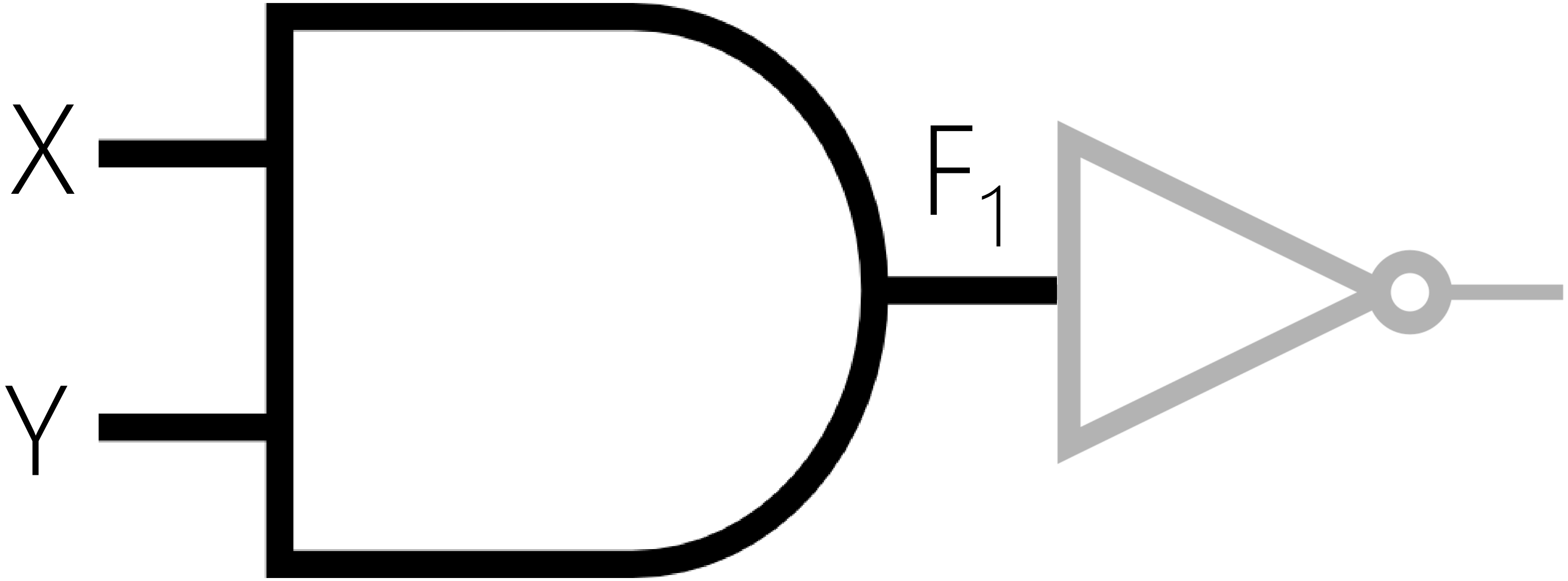
System analysis is given the structure of a system, find its functionality.

Determine the functionality exhibited by a structure.

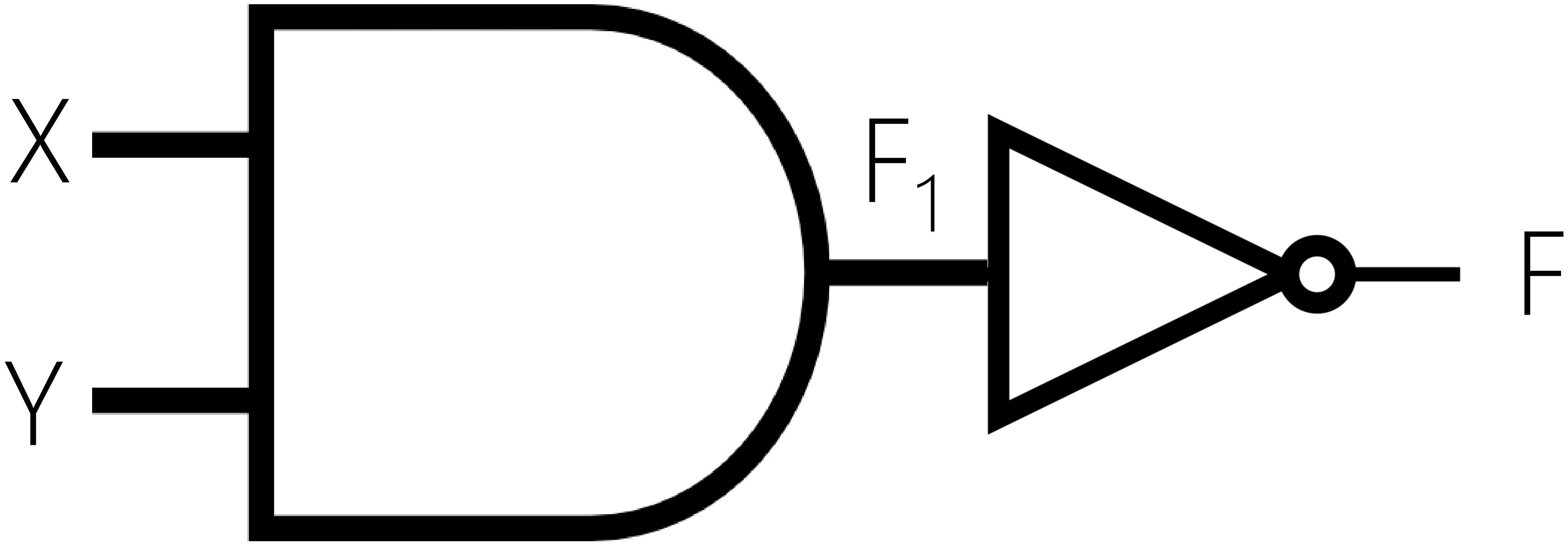

X
Y
F



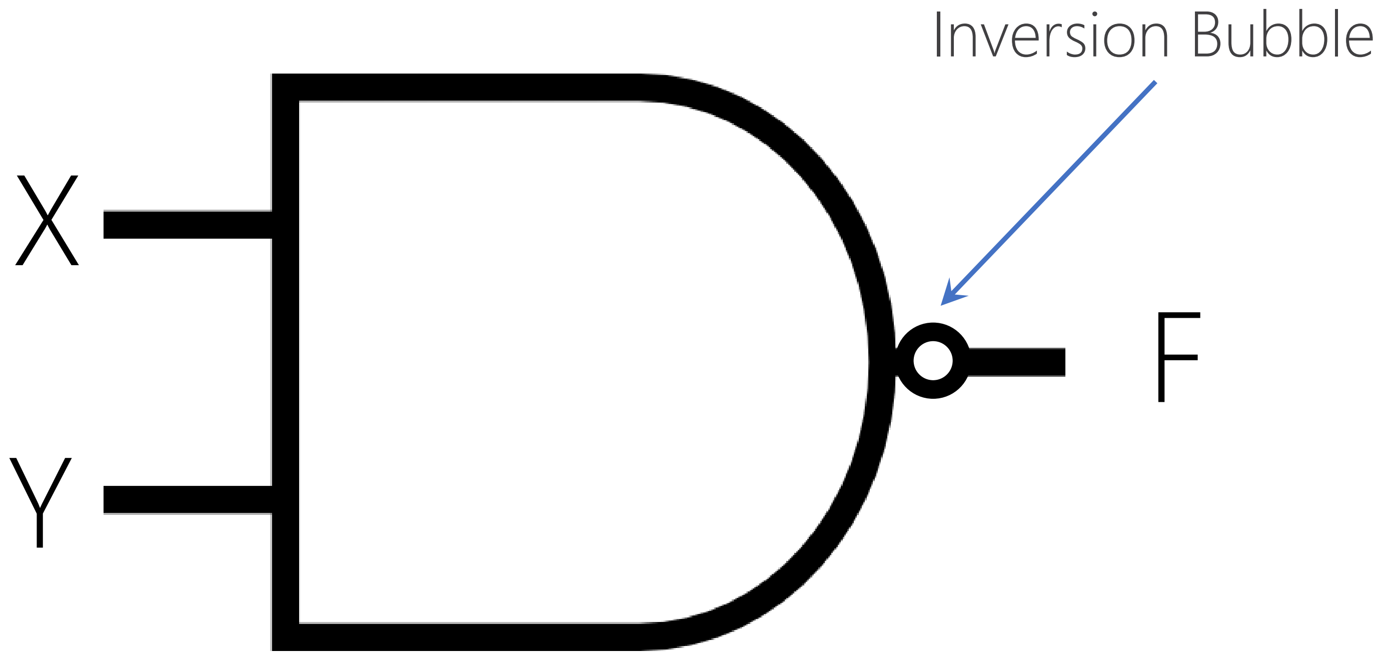
Y	X	F = ?
0	0	?
0	1	?
1	0	?
1	1	?



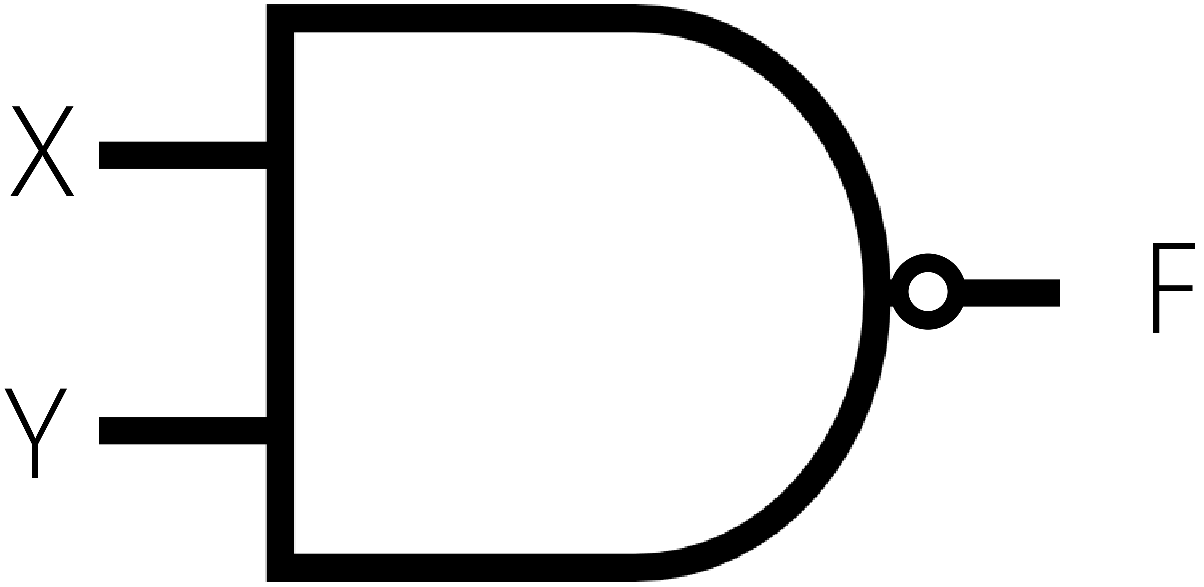
Y	X	$F_1 = YX$
0	0	0
0	1	0
1	0	0
1	1	1



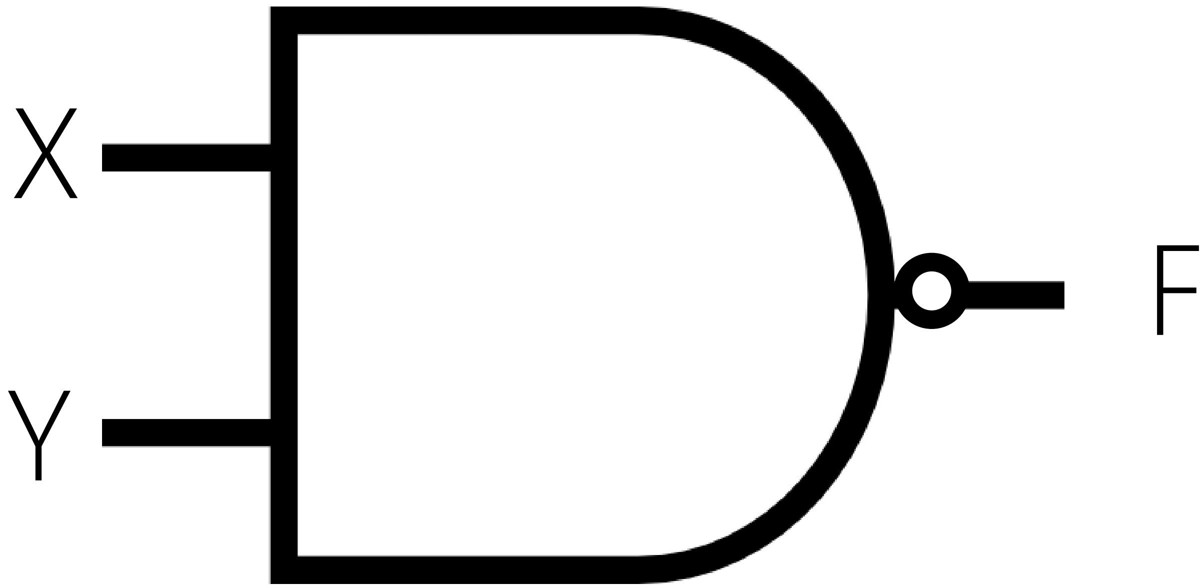
Y	X	$F_1 = YX$	$F = (YX)'$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0



NAND (Not – AND)

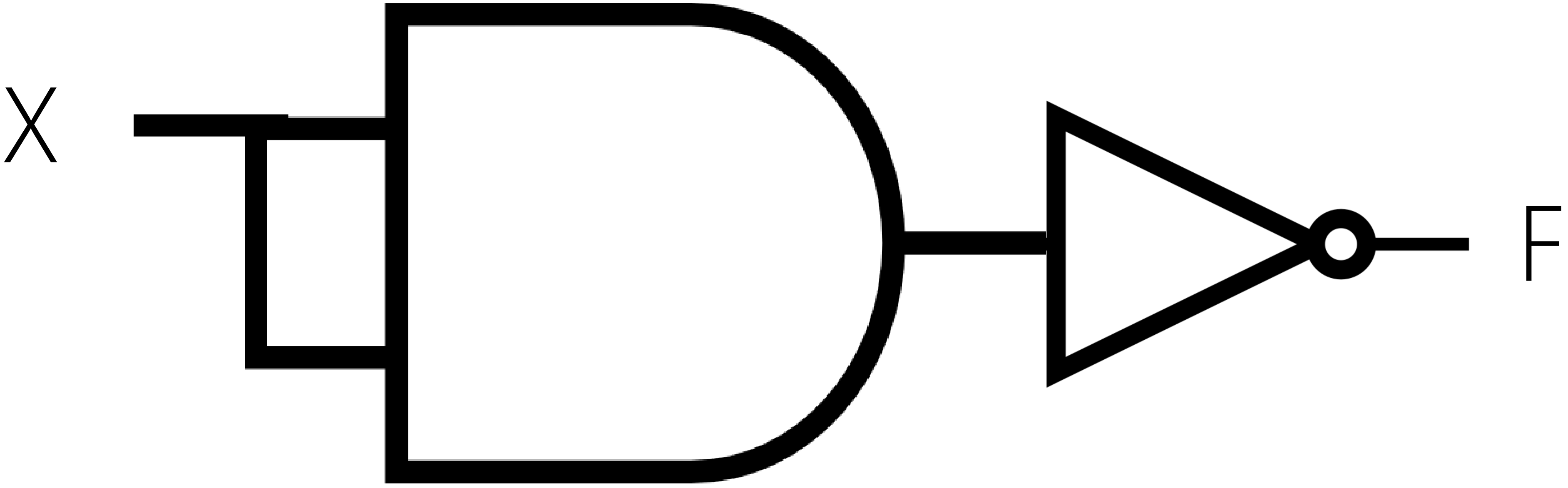


Y	X	F = (YX)′	F=Y ↑ X
0	0	1	
0	1	1	
1	0	1	
1	1	0	

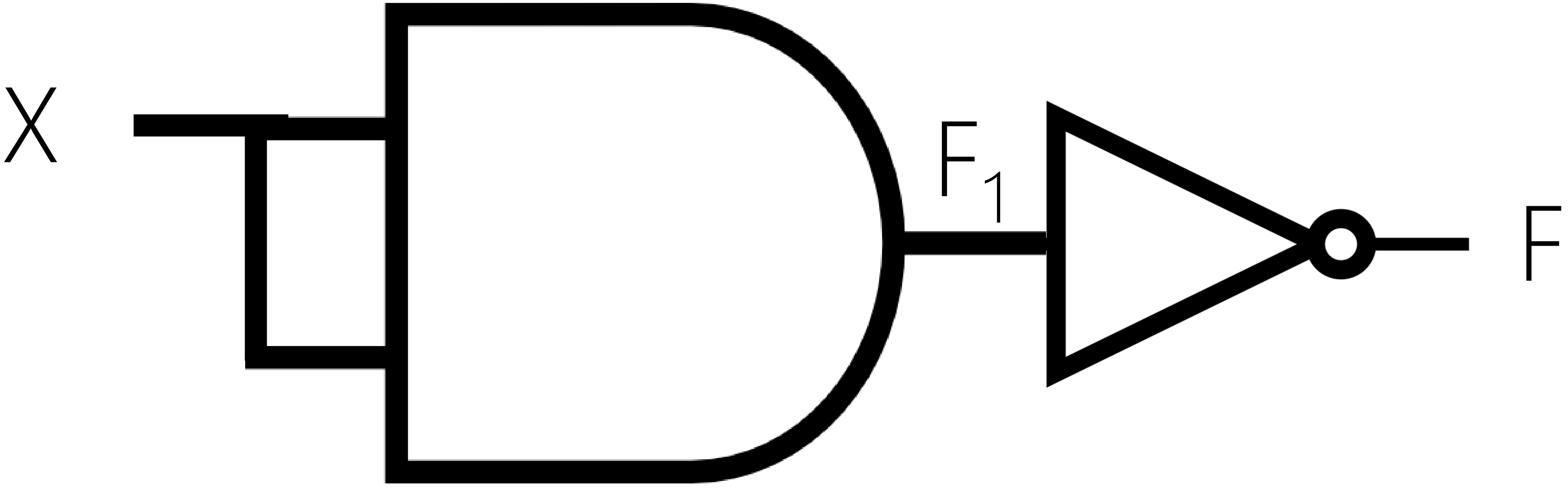


$F= (YX)' = (XY)' = Y \uparrow X = X \uparrow Y$

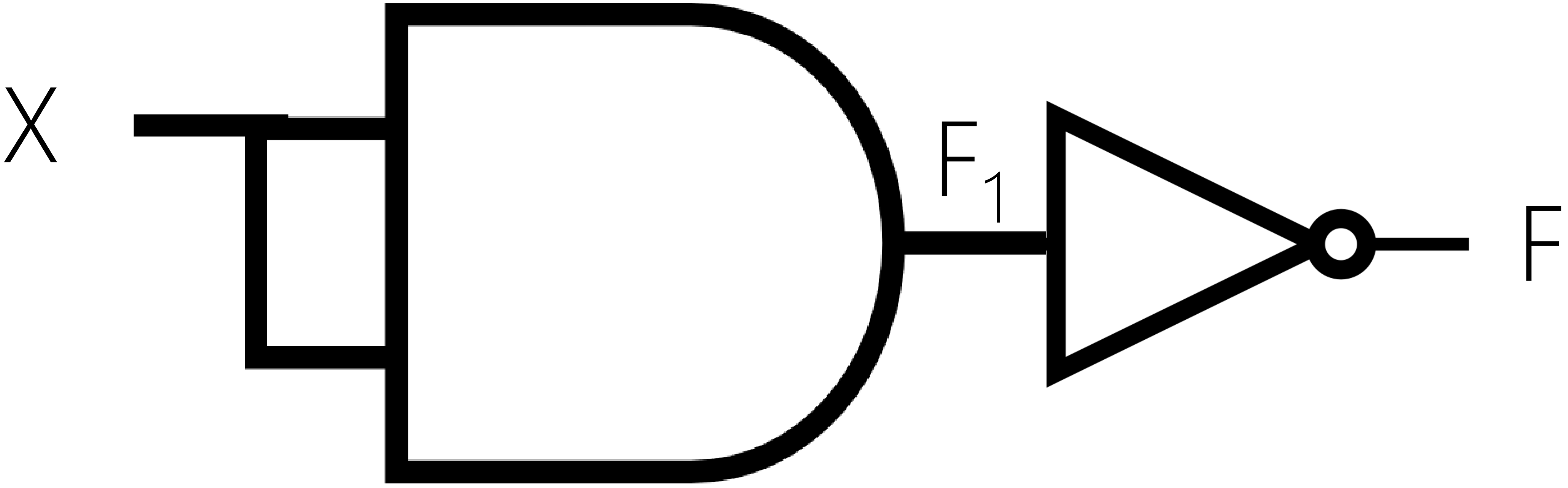
Commutative



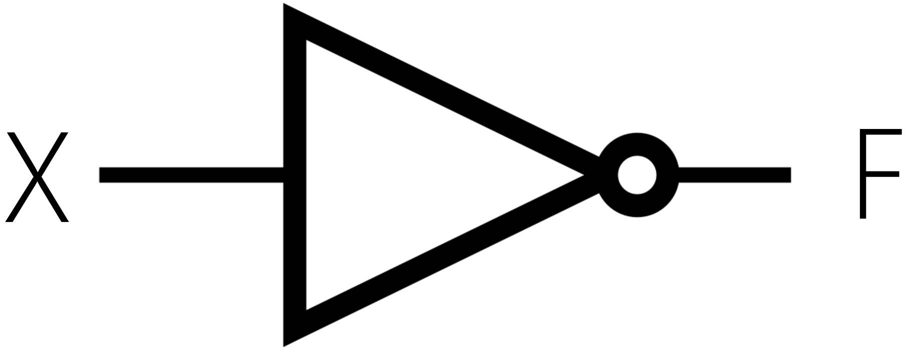
X	F = ?
0	
1	



X	$F_1 = XX$	$F = ?$
0	0	
1	1	



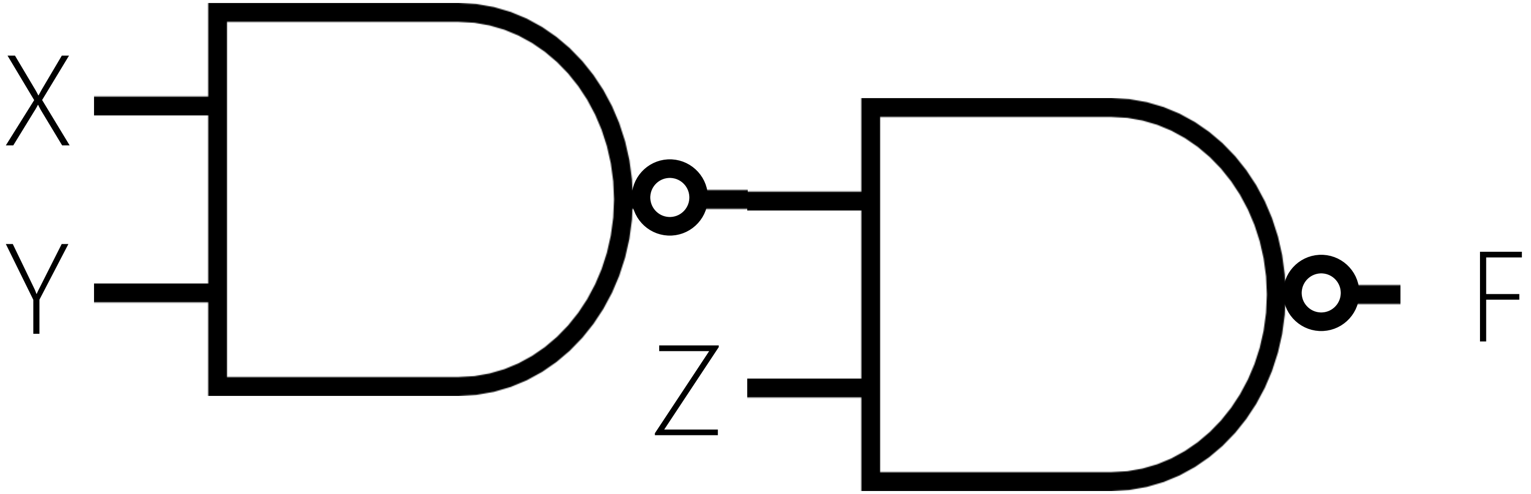
X	$F_1 = XX$	$F = (XX)'$
0	0	1
1	1	0



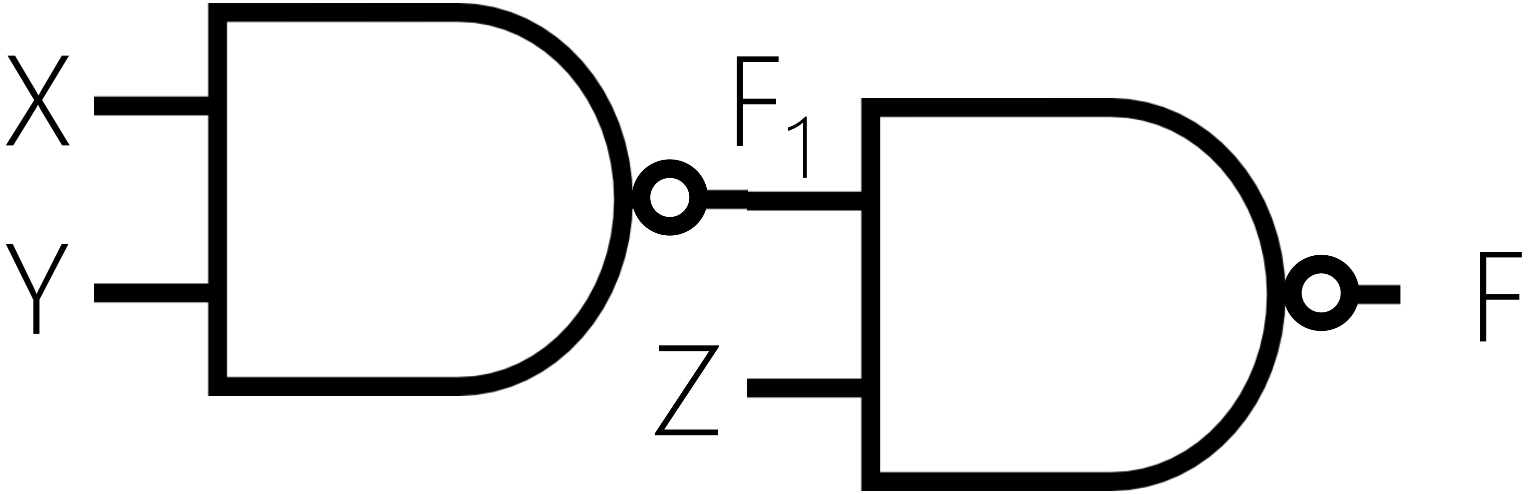
X	$F = (XX)' = X \uparrow X = X'$
0	1
1	0

3-INPUT NAND

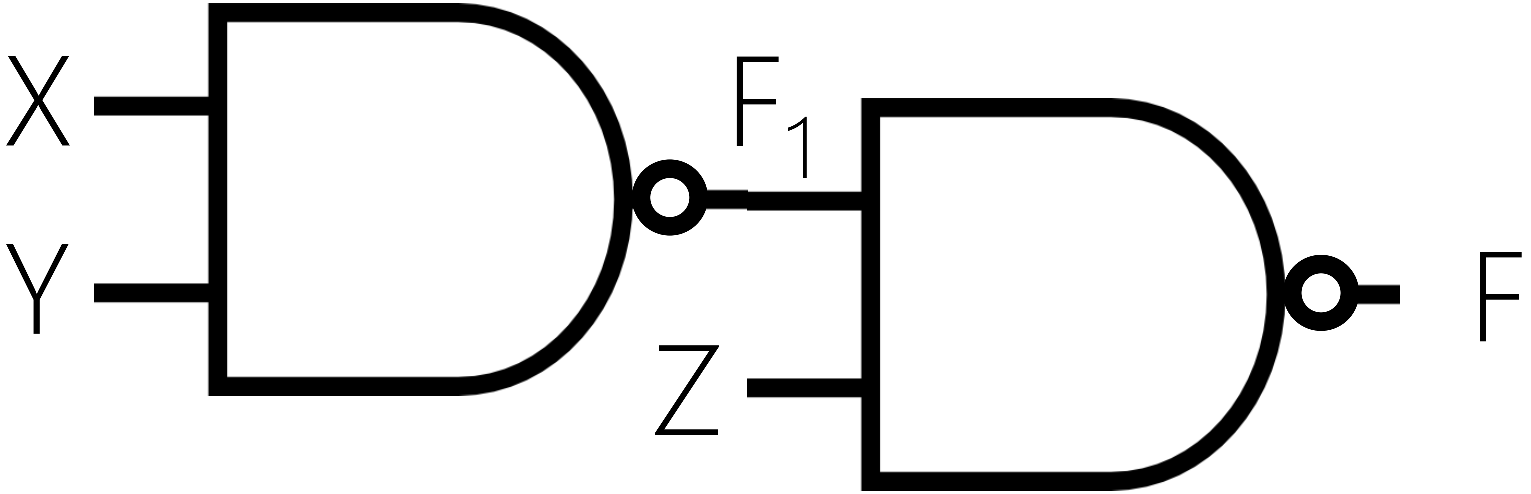
Z	Y	X	$F=(ZYX)'$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0



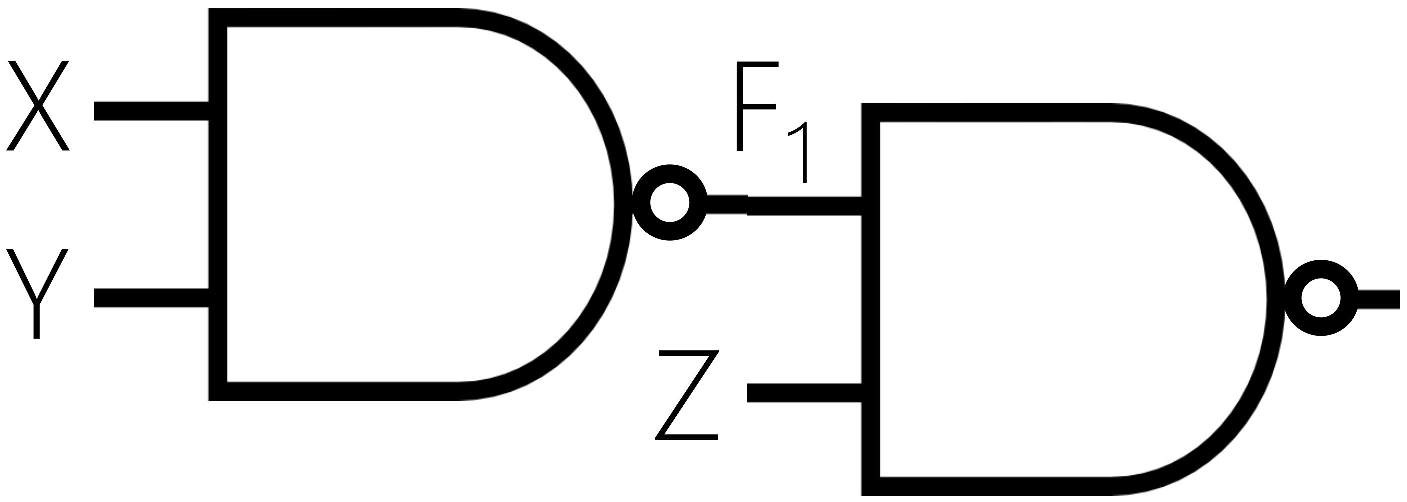
Z	Y	X	F = ?
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	



Z	Y	X	$F_1 = (YX)'$	F = ?
0	0	0	1	
0	0	1	1	
0	1	0	1	
0	1	1	0	
1	0	0	1	
1	0	1	1	
1	1	0	1	
1	1	1	0	



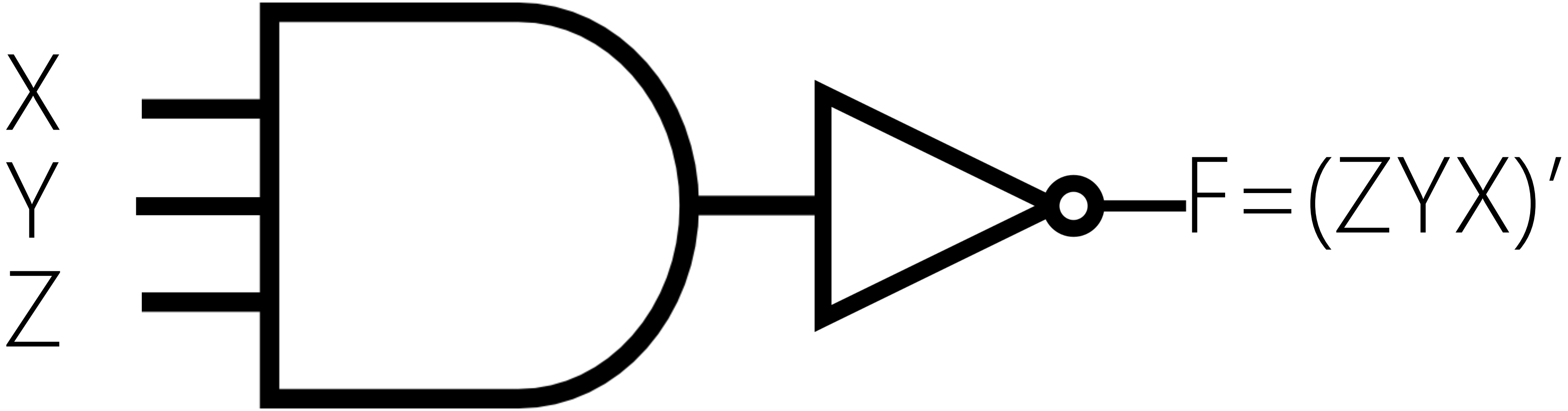
Z	Y	X	$F_1 = (YX)'$	$F = (ZF_1)' = (Z(YX)')'$
0	0	0	1	1
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	0	1



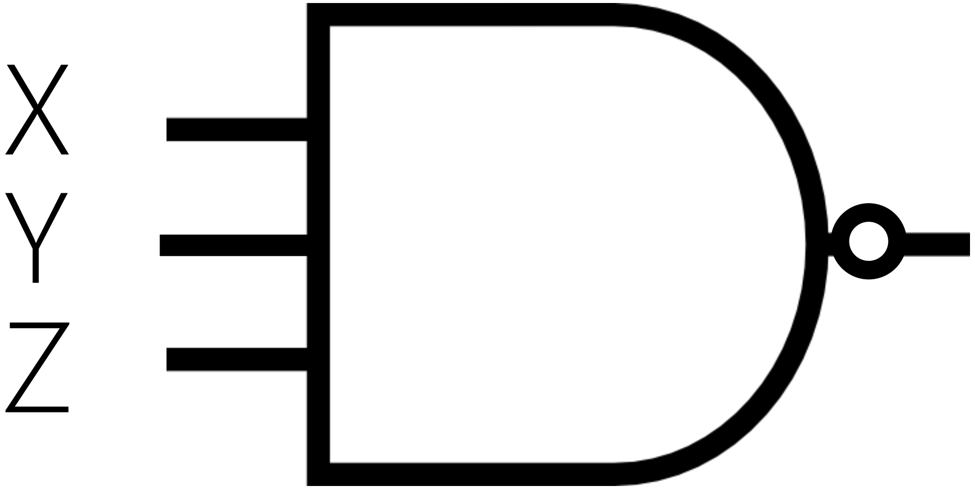
$F=(Z(YX)')' \neq (ZYX)'$

Z	Y	X	$F_1 = (YX)'$	$F = (ZF_1)' = (Z(YX)')'$	$F=(ZYX)'$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	1	1
0	1	1	0	1	1
1	0	0	1	0	1
1	0	1	1	0	1
1	1	0	1	0	1
1	1	1	0	1	0

NOT (3-INPUT AND)



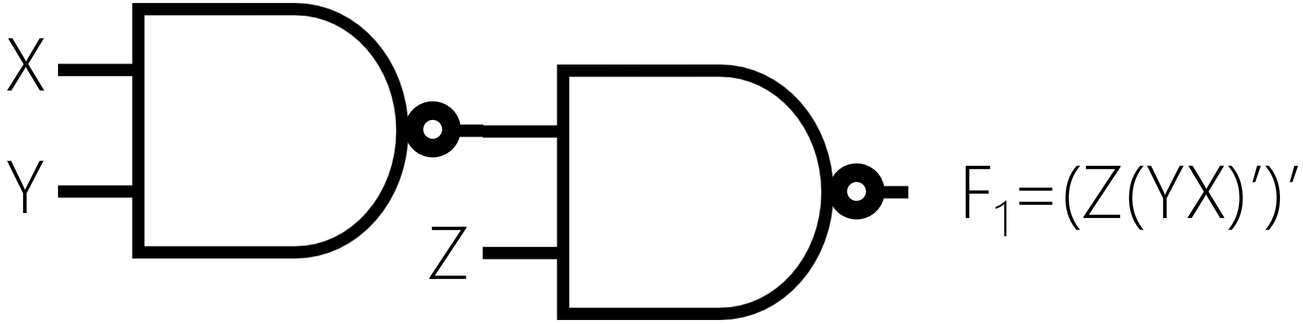
Z	Y	X	F=(ZYX)′	F=(ZYX)′
0	0	0	1	1
0	0	1	1	1
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	0	0

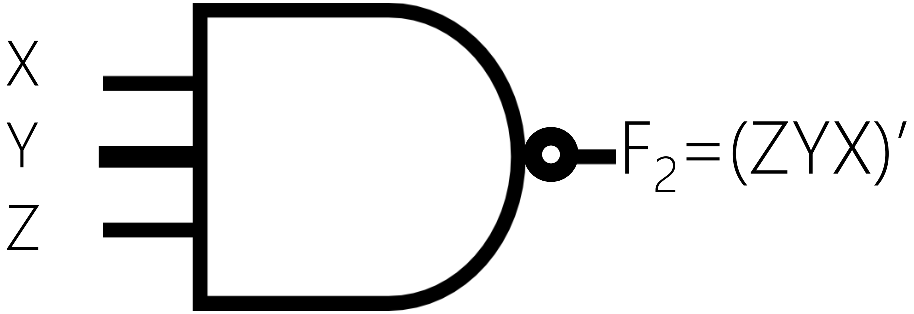


$F=(ZYX)'=(XZY)'$

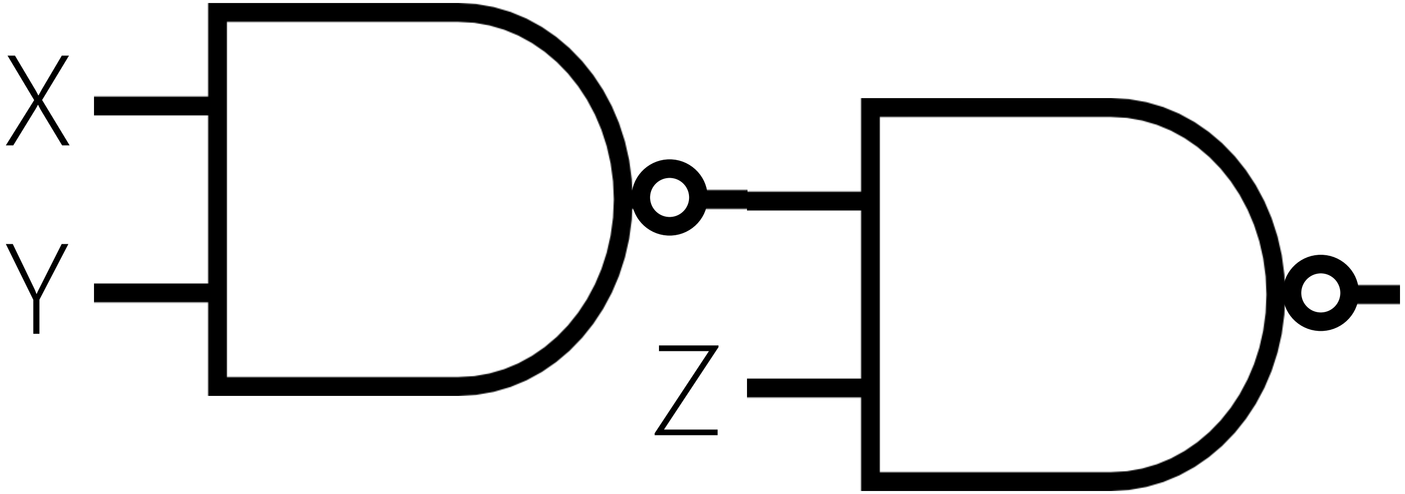
Associative

Z	Y	X	F=(ZYX)′	F=(ZYX)′
0	0	0	1	1
0	0	1	1	1
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	0	0





F = (ZYX)′	F_1	F_2
Effective (True)	No!	Yes
Efficient (Fast)	░	░
Min. Cost	░	░



F = Z ↑ (Y ↑ X)

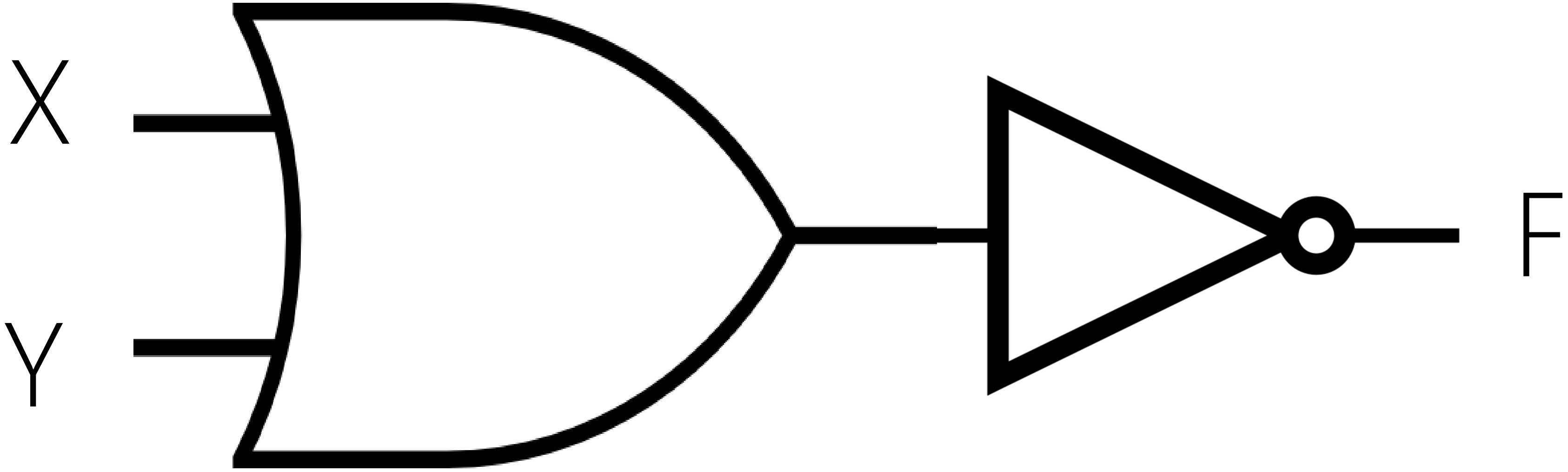
Associative?

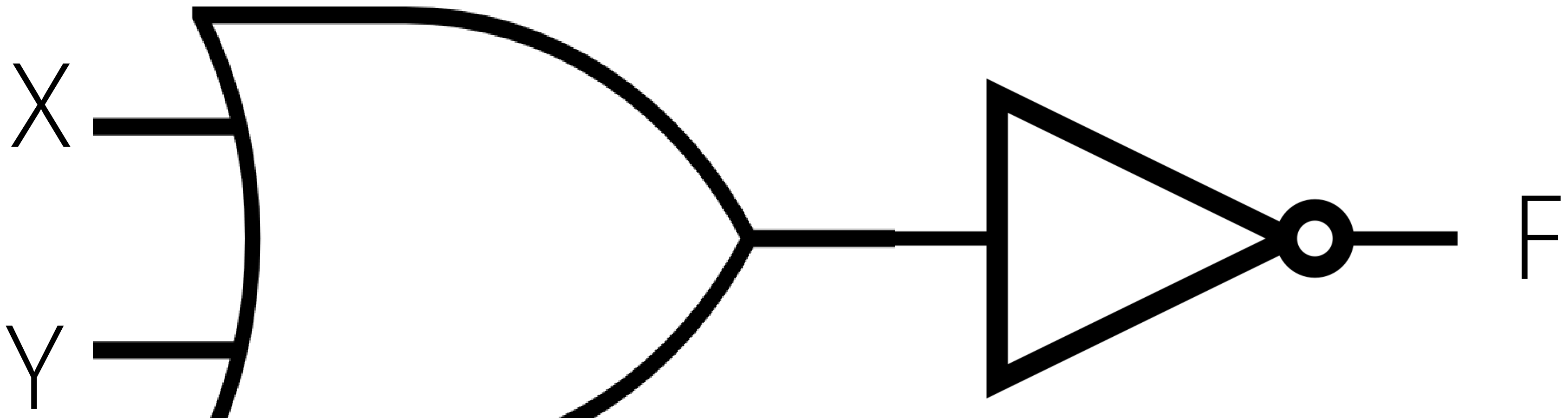
Z	Y	X	F = (Z(YX)’)’ = Z ↑ (Y ↑ X)
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

ANALYSIS II

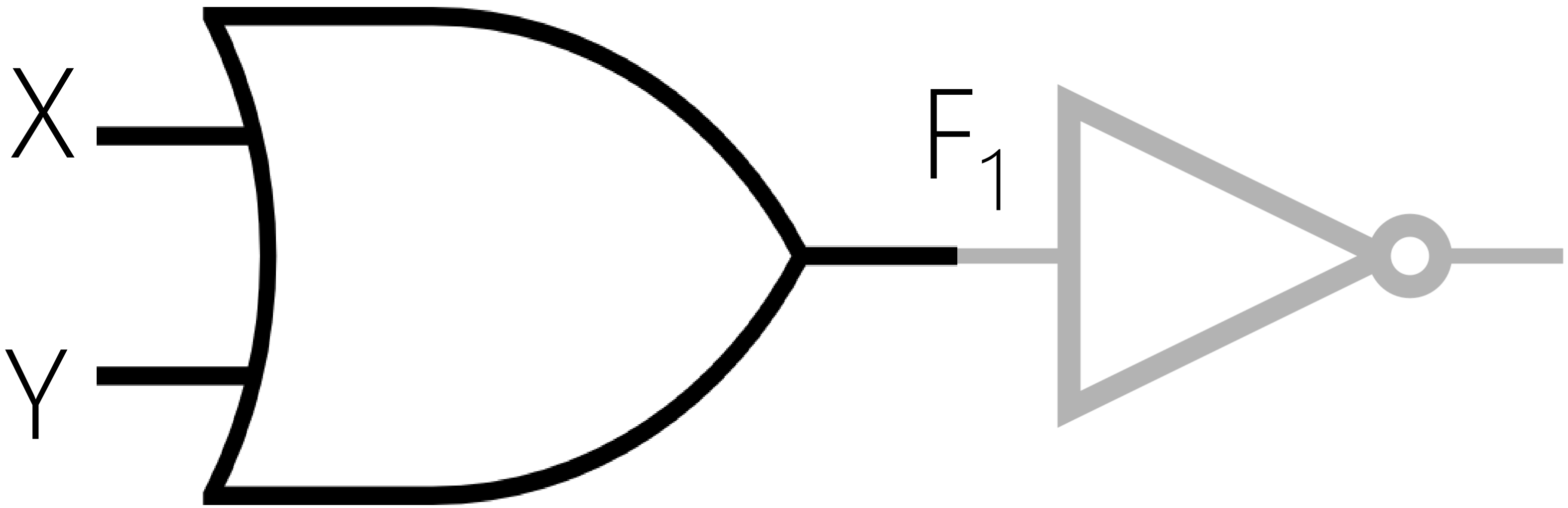
System analysis is given the structure of a system, find its functionality.

Determine the functionality exhibited by a structure.

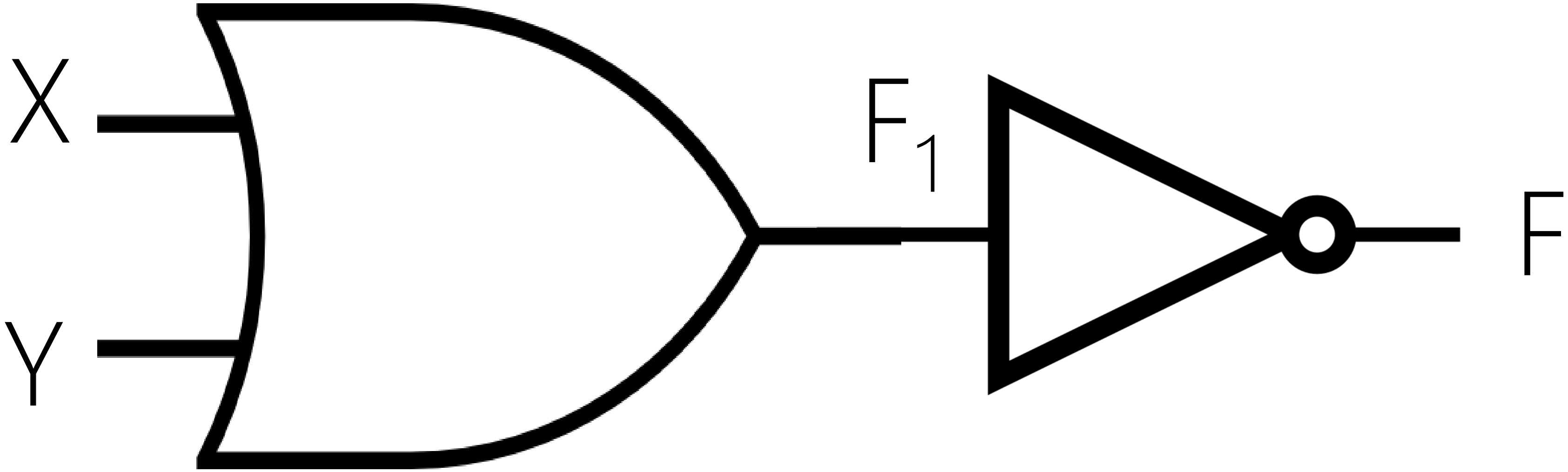

X
Y
F



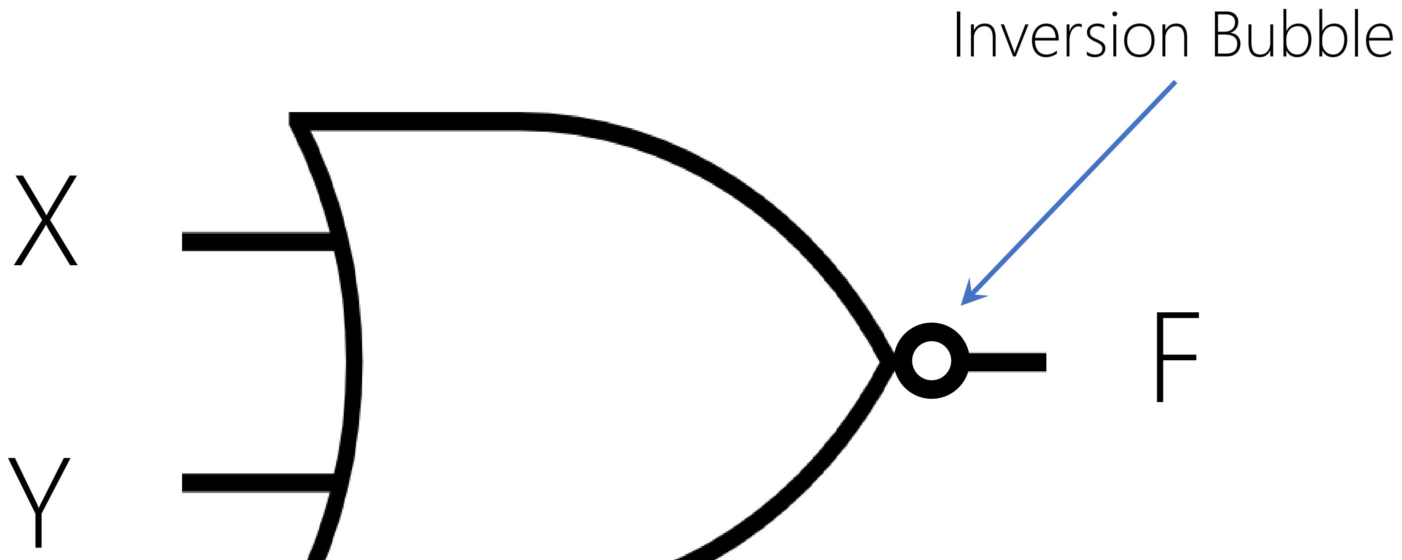
Y	X	F = ?
0	0	?
0	1	?
1	0	?
1	1	?



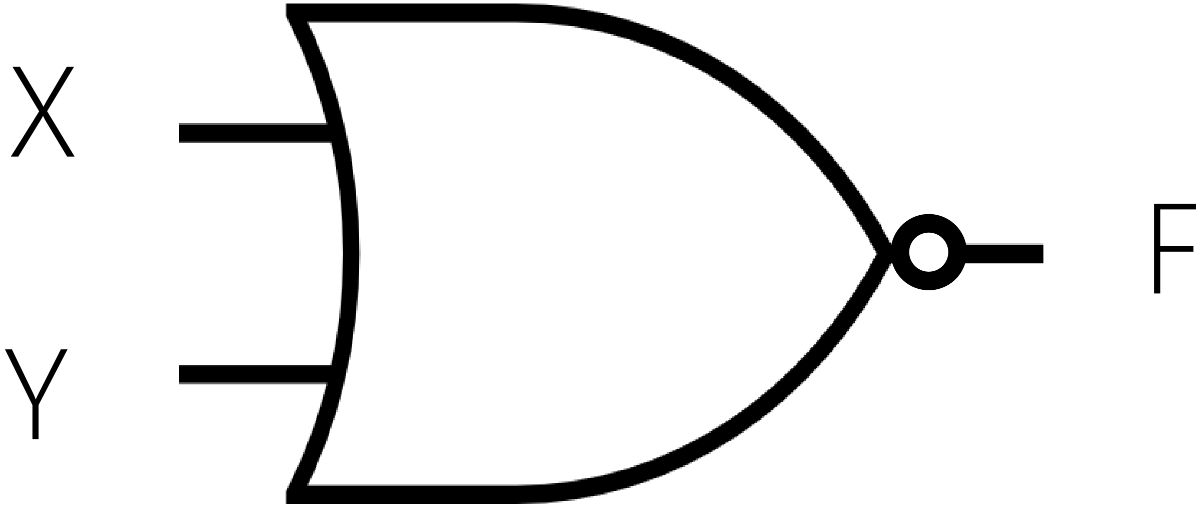
Y	X	$F_1 = Y+X$
0	0	0
0	1	1
1	0	1
1	1	1



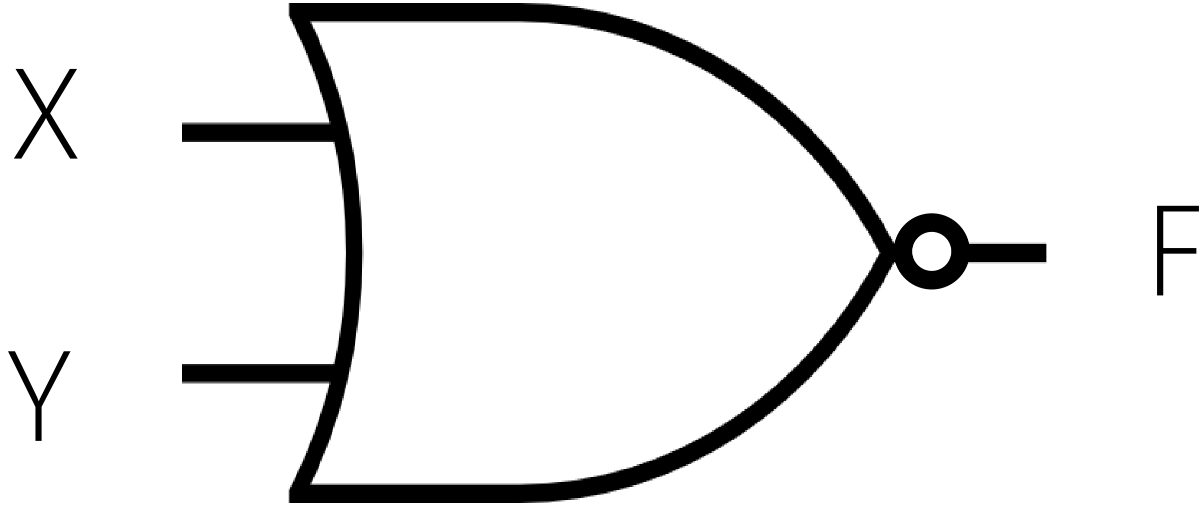
Y	X	$F_1 = Y+X$	$F = (Y+X)'$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0



NOR (Not – OR)

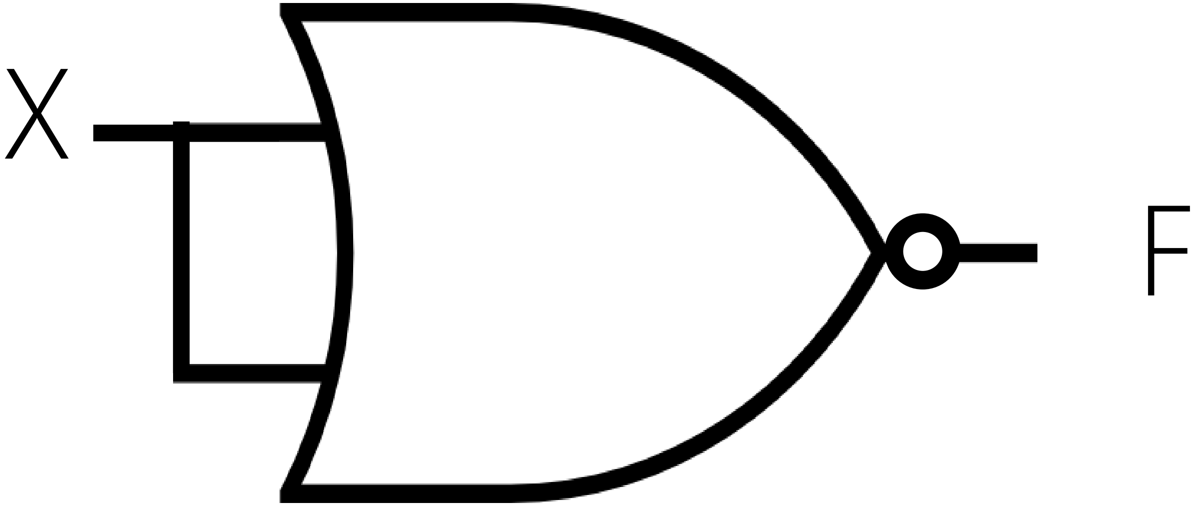


<table>
<tr><th>Y</th><th>X</th><th>$F = (Y+X)'$</th><th>$F = Y \downarrow X$</th></tr>
<tr><td>0</td><td>0</td><td colspan="2">1</td></tr>
<tr><td>0</td><td>1</td><td colspan="2">0</td></tr>
<tr><td>1</td><td>0</td><td colspan="2">0</td></tr>
<tr><td>1</td><td>1</td><td colspan="2">0</td></tr>
</table>

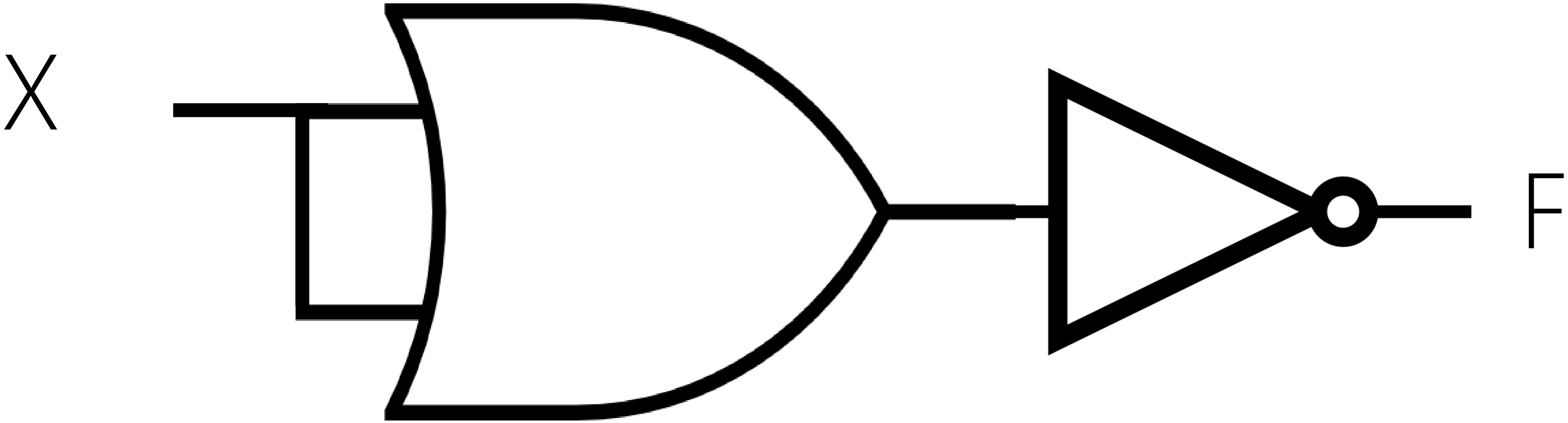


F= (Y+X)′ = (X+Y)′ = Y ↓ X = X ↓ Y

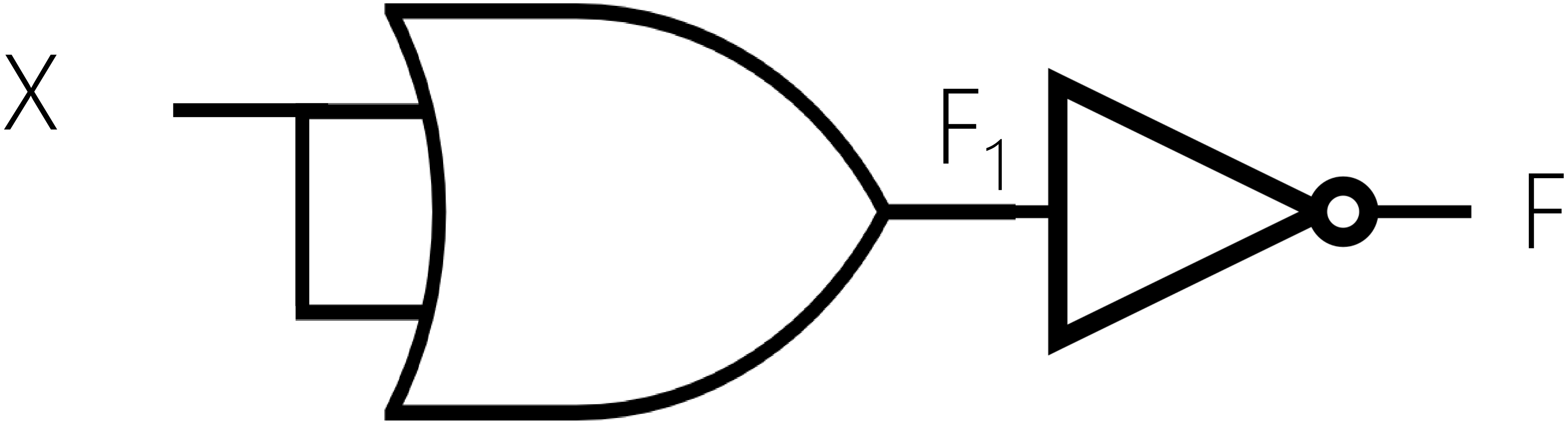
Commutative



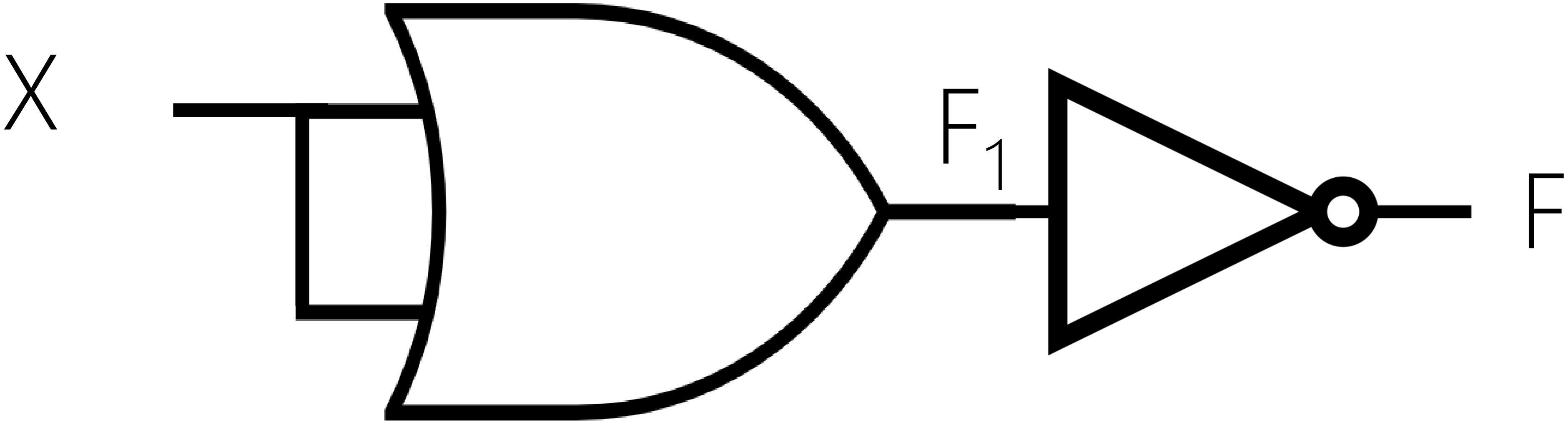
F=?



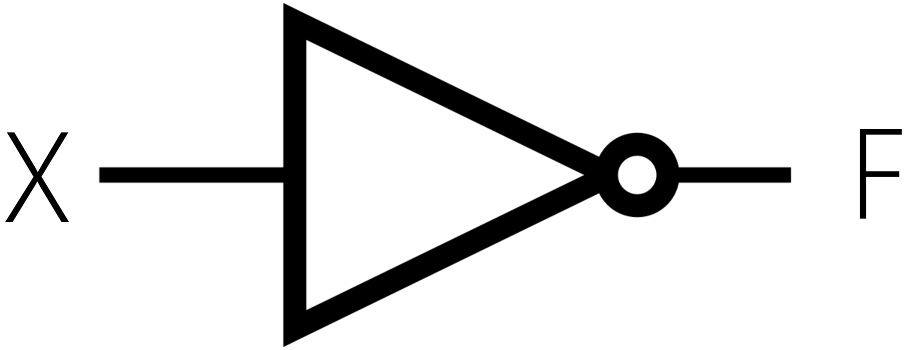
X	F = ?
0	
1	



X	$F_1 = X+X$	F = ?
0	0	
1	1	



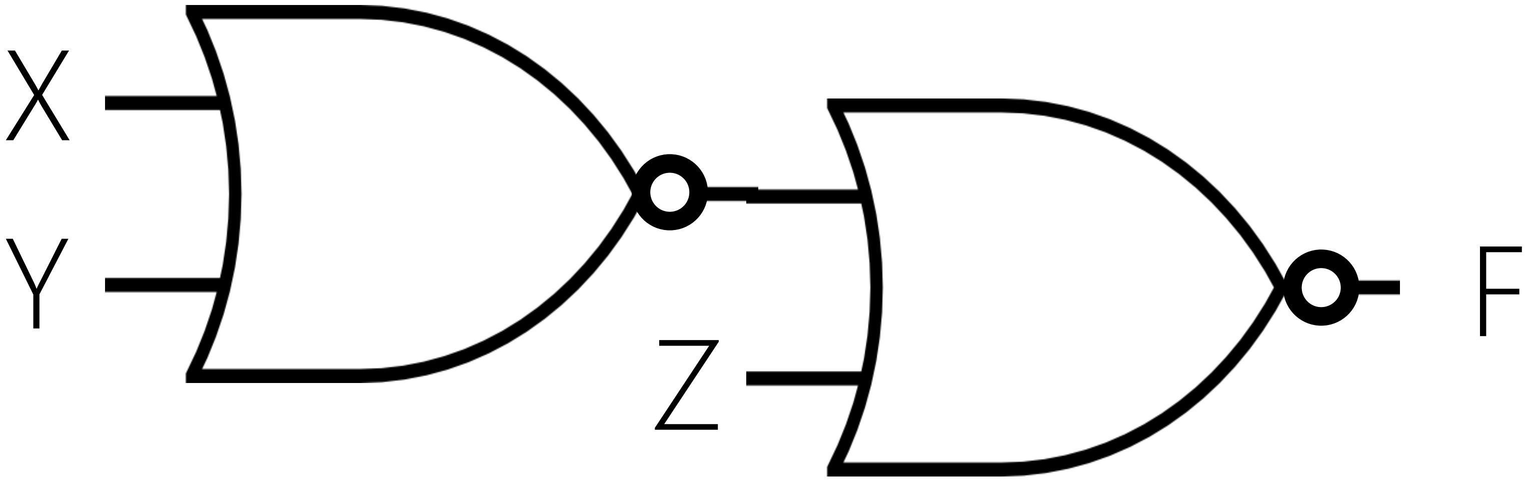
X	$F_1 = X+X$	$F = (X+X)'$
0	0	1
1	1	0



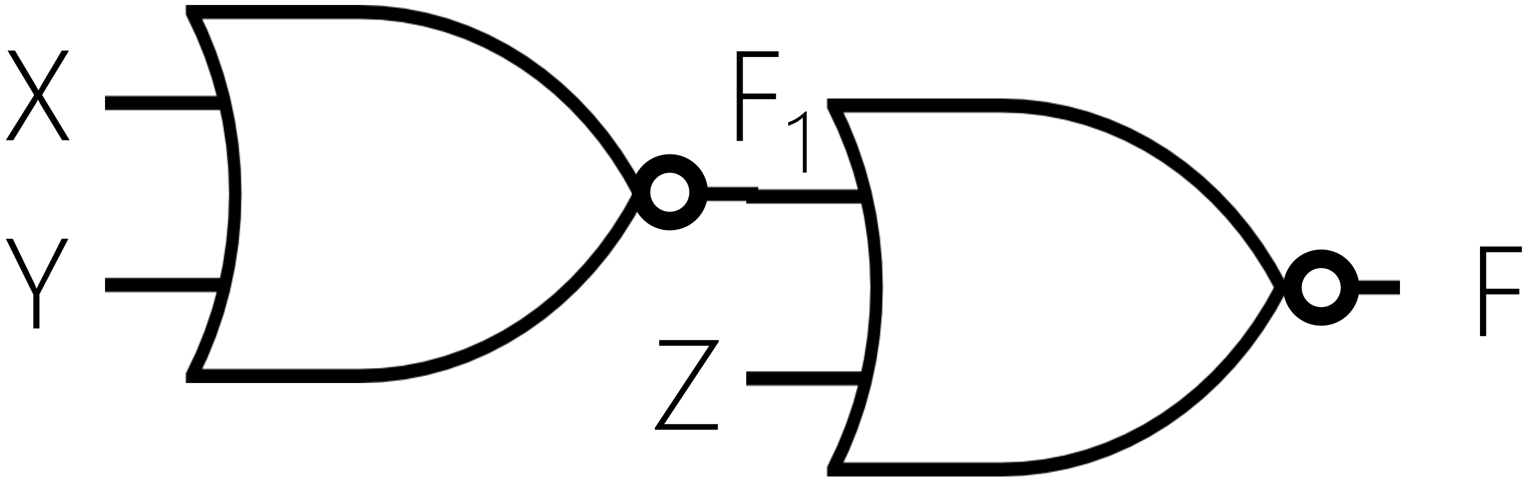
X	$F = (X+X)' = X \downarrow X = X'$
0	1
1	0

3-INPUT NOR

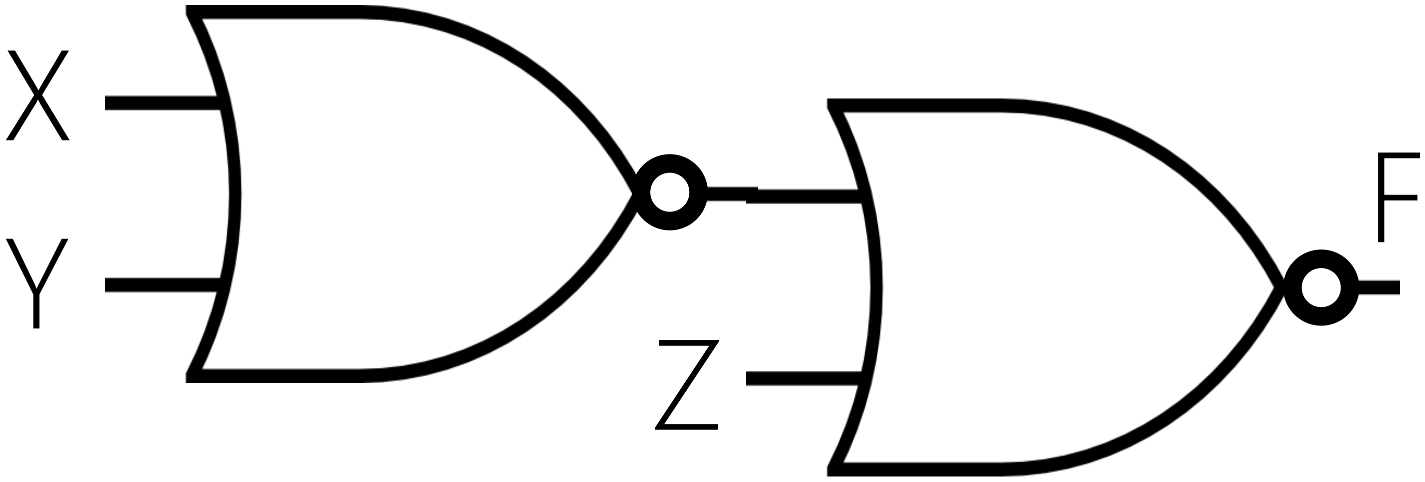
Z	Y	X	F=(Z+Y+X)′
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0



Z	Y	X	F = ?
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	



Z	Y	X	$F_1 = (Y+X)'$	$F = (Z+F_1)' = (Z+(Y+X)')'$
0	0	0	1	0
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	0	0

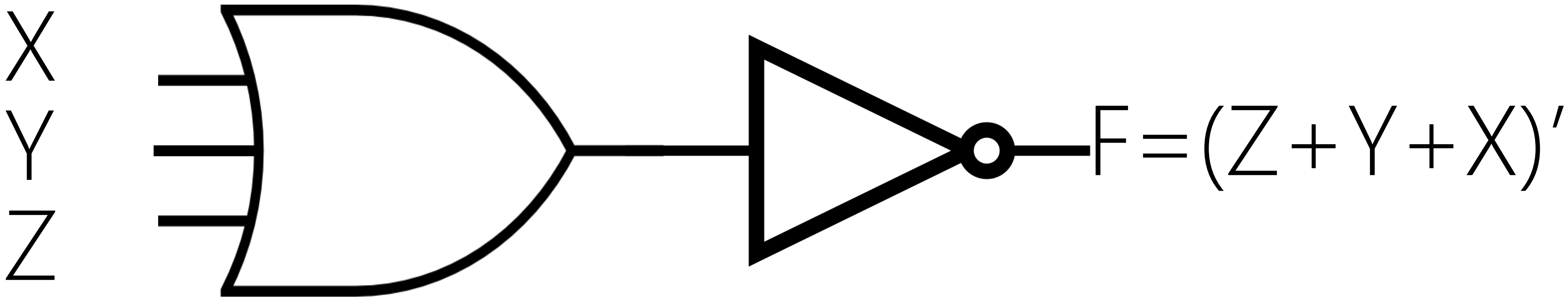


$F=(Z+(Y+X)')'$

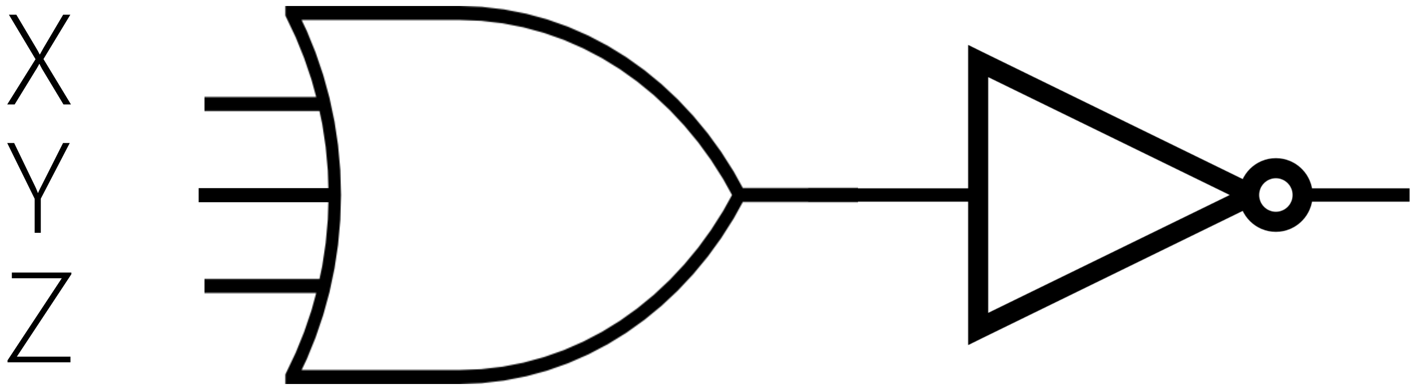
$\neq(Z+Y+X)'$

Z	Y	X	$F = (Z+F_1)' = (Z+(Y+X)')'$	$F=(Z+Y+X)'$
0	0	0	0	1
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	0	0
1	0	1	0	0
1	1	0	0	0
1	1	1	0	0

NOT (3-INPUT OR)



Z	Y	X	F=(Z+Y+X)′	F=(Z+Y+X)′
0	0	0	1	1
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	0	0
1	0	1	0	0
1	1	0	0	0
1	1	1	0	0

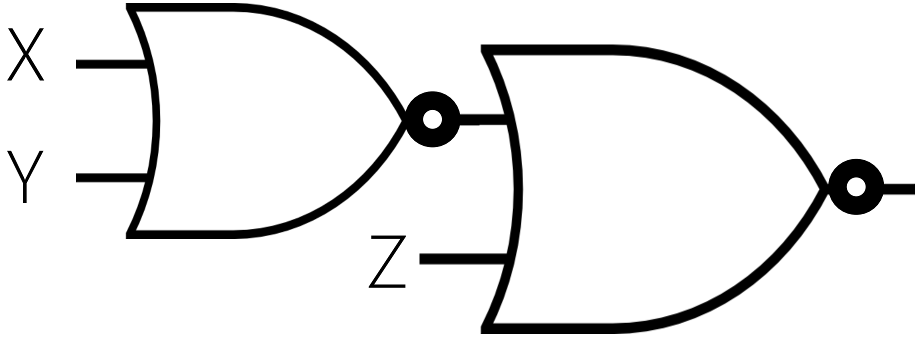


$F=(Z+Y+X)'$

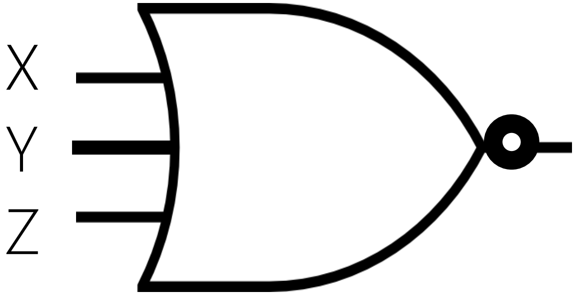
$F=(X+Y+Z)'$

Associative

Z	Y	X	$F=(Z+Y+X)'$	$F=(Z+Y+X)'$
0	0	0	1	1
0	0	1	1	1
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	0	0

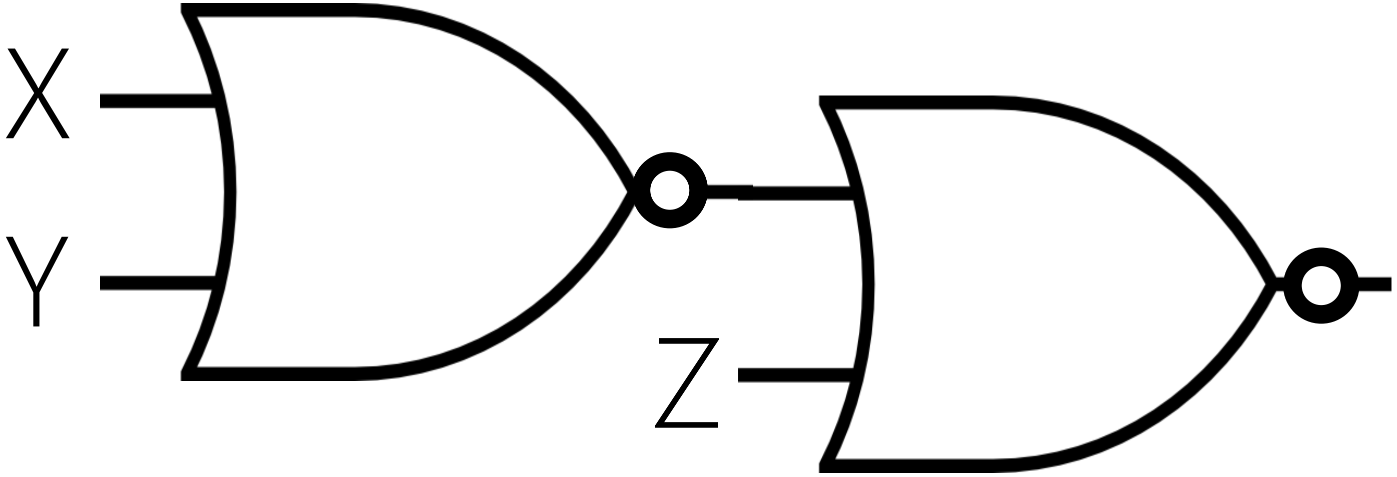


$F_1=(Z+(Y+X)')'$



$F_2=(Z+Y+X)'$

F = (Z+Y+X)′	F_1	F_2
Effective (True)	No!	Yes
Efficient (Fast)	░	░
Min. Cost	░	░



$F=Z \downarrow (Y \downarrow X)$

Associative?

Z	Y	X	$F = (Z(YX)')' = Z \downarrow (Y \downarrow X)$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

RECAP

GATE	WHEN F=1
NOT	The input is 0
AND	All the inputs are 1
OR	At least one input is 1
NAND	At least one input is 0
NOR	All the inputs are 0