DESIGN

an algorithm for designing any digital units (logic circuits), given truth table

minterm

aka. Standard Product

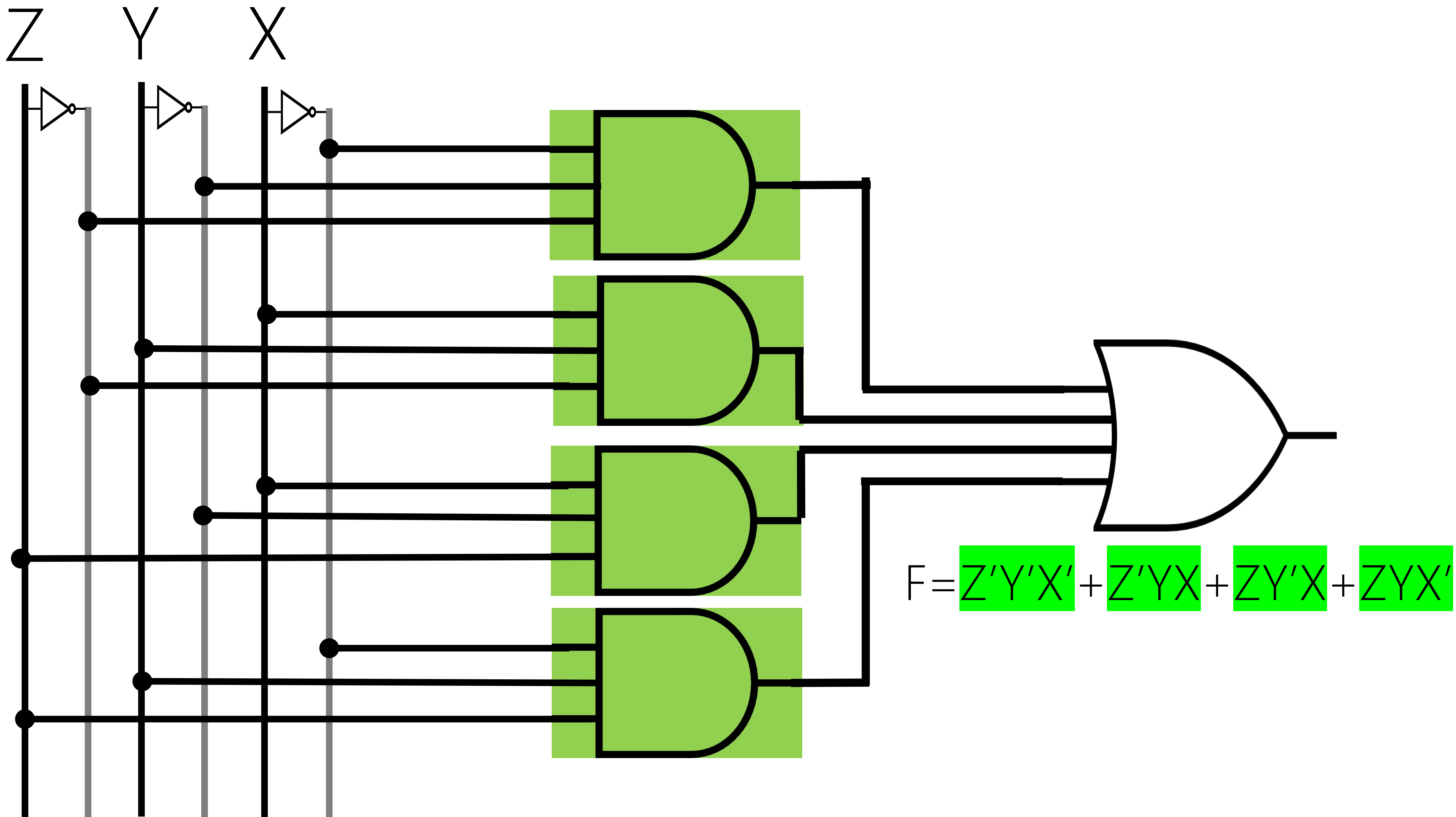
TRUTH TABLE

en.wikipedia.org/wiki/Truth_table

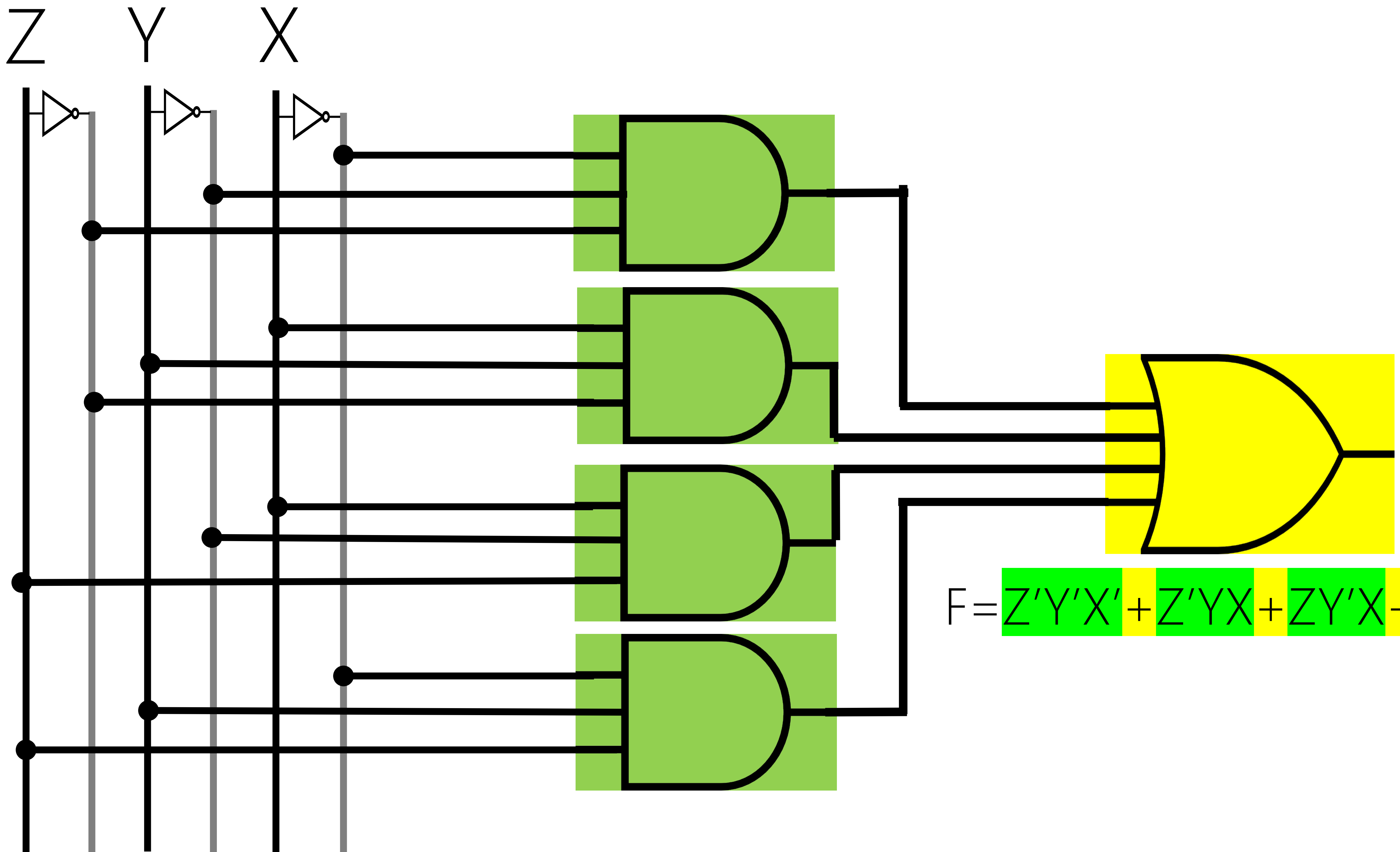
SUM OF PRODUCTS (SOP)
2 LEVELS AND-OR

with *product* meaning the ANDing
with *sum* meaning the ORing

Z	Y	X	$F(Z,Y,X)=m_0+m_3+m_5+m_6=\sum m(0,3,5,6)$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0



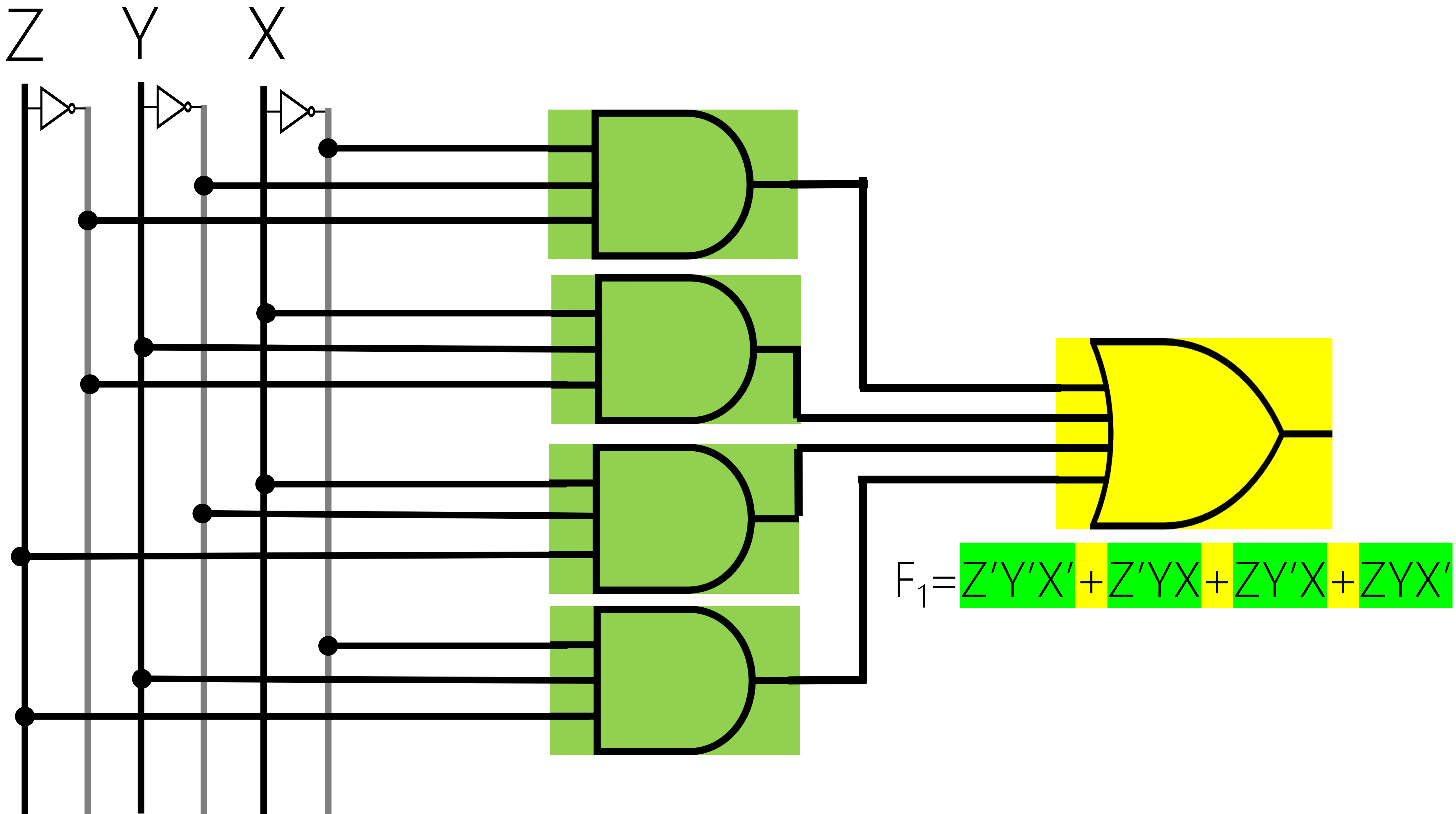
Z
Y
X
F=Z'Y'X'+Z'YX+ZY'X+ZYX'



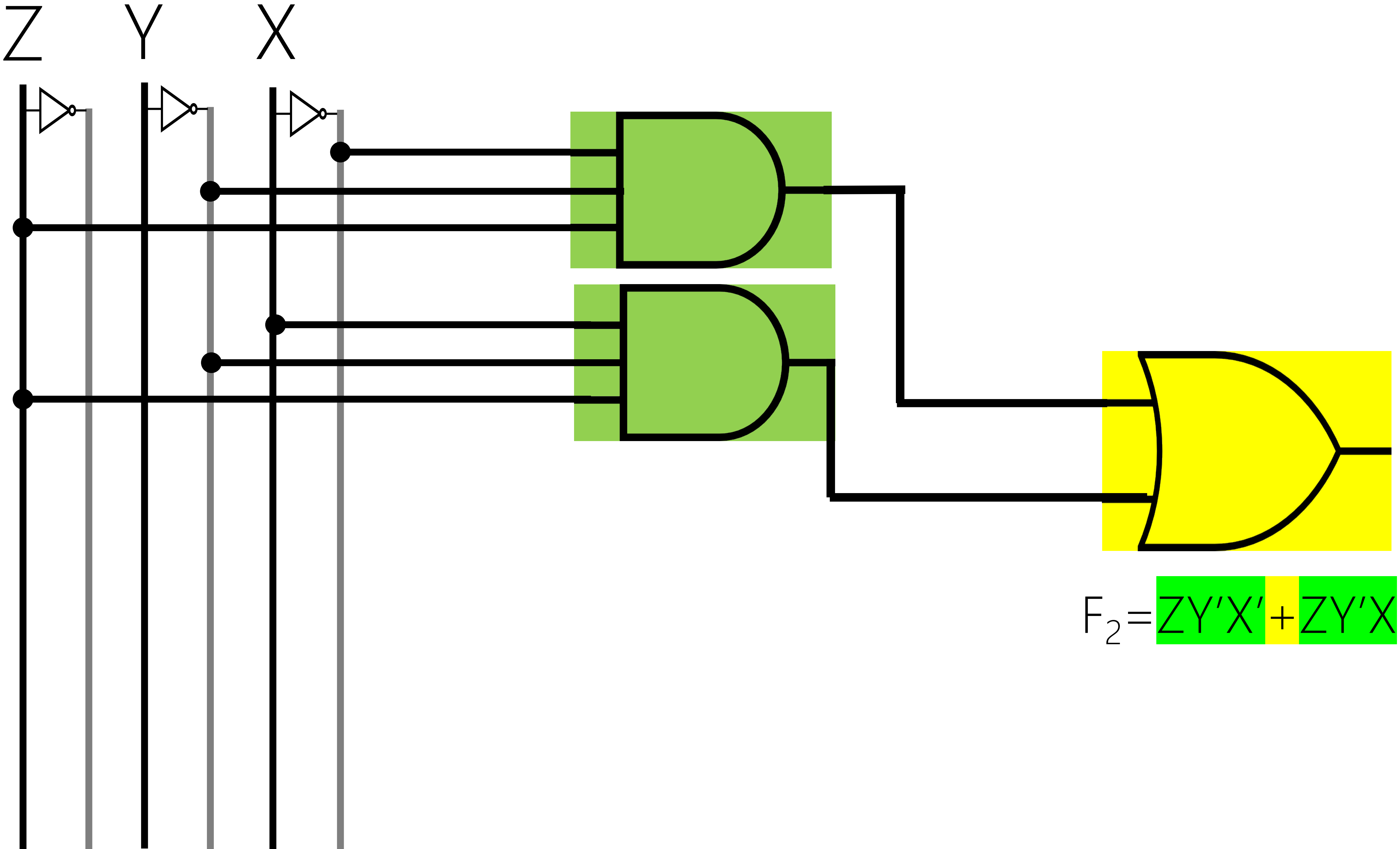
$$F = Z'Y'X' + Z'YX + ZY'X + ZYX'$$

MULTIPLE BOOLEAN FUNCTIONS F_1, F_2, …

Z	Y	X	$F_1(Z,Y,X)=\sum m(0,3,5,6)$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

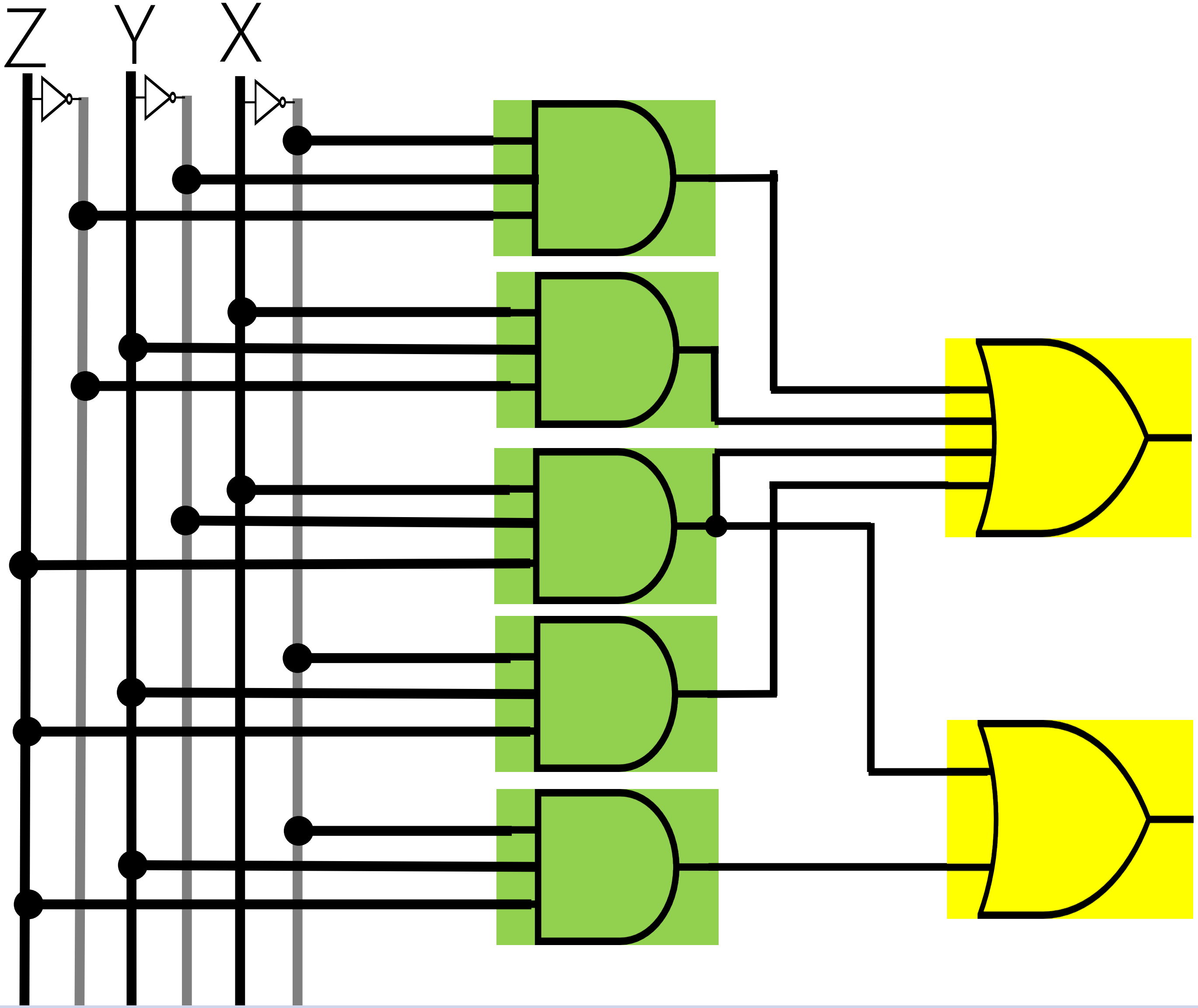

Z
Y
X
$F_1 = Z'Y'X' + Z'YX + ZY'X + ZYX'$

Z	Y	X	$F_2(Z,Y,X)=\sum m(4,5)$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0



$F_2 = ZY'X' + ZY'X$

Z	Y	X	$F_1(Z,Y,X)=\sum m(0,3,5,6)$	$F_2(Z,Y,X)=\sum m(4,5)$
0	0	0	1	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	0
1	0	0	0	1
1	0	1	1	1
1	1	0	1	0
1	1	1	0	0



$F_1 = Z'Y'X' + Z'YX + ZY'X + ZYX'$

$F_2 = ZY'X' + ZY'X$

RE-USE minterms

SHOW THE REMAINDER (MOD)
NUMBER % 3 = ?

WHAT IS THE RANGE OF NUMBERS?

WHAT IS THE RANGE OF NUMBERS?

$[0, 15]_{10}$

HOW MANY INPUT BINARY VARIABLES?

$$[0, 15]_{10} = [0, 1111]_2 = [0000, 1111]_2$$

W	Z	Y	X
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

WHAT IS THE RANGE OF OUTPUT?

WHAT IS THE RANGE OF OUTPUT?

The remainder of any number divided by 3 is 0, 1, 2

WHAT IS THE RANGE OF OUTPUT?

$[0, 2]_{10}$

HOW MANY BOOLEAN FUNCTION?

$$[0, 2]_{10} = [0, 10]_2 = [00, 10]_2$$

W	Z	Y	X	F_1	F_2
0	0	0	0		
0	0	0	1		
0	0	1	0		
0	0	1	1		
0	1	0	0		
0	1	0	1		
0	1	1	0		
0	1	1	1		
1	0	0	0		
1	0	0	1		
1	0	1	0		
1	0	1	1		
1	1	0	0		
1	1	0	1		
1	1	1	0		
1	1	1	1		

W	Z	Y	X	F_1	F_2
0	0	0	0	0	0
0	0	0	1		
0	0	1	0		
0	0	1	1		
0	1	0	0		
0	1	0	1		
0	1	1	0		
0	1	1	1		
1	0	0	0		
1	0	0	1		
1	0	1	0		
1	0	1	1		
1	1	0	0		
1	1	0	1		
1	1	1	0		
1	1	1	1		

W	Z	Y	X	F_1	F_2
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0		
0	0	1	1		
0	1	0	0		
0	1	0	1		
0	1	1	0		
0	1	1	1		
1	0	0	0		
1	0	0	1		
1	0	1	0		
1	0	1	1		
1	1	0	0		
1	1	0	1		
1	1	1	0		
1	1	1	1		

W	Z	Y	X	F_1	F_2
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1		
0	1	0	0		
0	1	0	1		
0	1	1	0		
0	1	1	1		
1	0	0	0		
1	0	0	1		
1	0	1	0		
1	0	1	1		
1	1	0	0		
1	1	0	1		
1	1	1	0		
1	1	1	1		

W	Z	Y	X	F_1	F_2
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1	0	0
0	1	0	0		
0	1	0	1		
0	1	1	0		
0	1	1	1		
1	0	0	0		
1	0	0	1		
1	0	1	0		
1	0	1	1		
1	1	0	0		
1	1	0	1		
1	1	1	0		
1	1	1	1		

W	Z	Y	X	F_1	F_2
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1	0	0
0	1	0	0	0	1
0	1	0	1	1	0
0	1	1	0	0	0
0	1	1	1	0	1
1	0	0	0	1	0
1	0	0	1	0	0
1	0	1	0	0	1
1	0	1	1	1	0
1	1	0	0	0	0
1	1	0	1	0	1
1	1	1	0	1	0
1	1	1	1	0	0

minterms

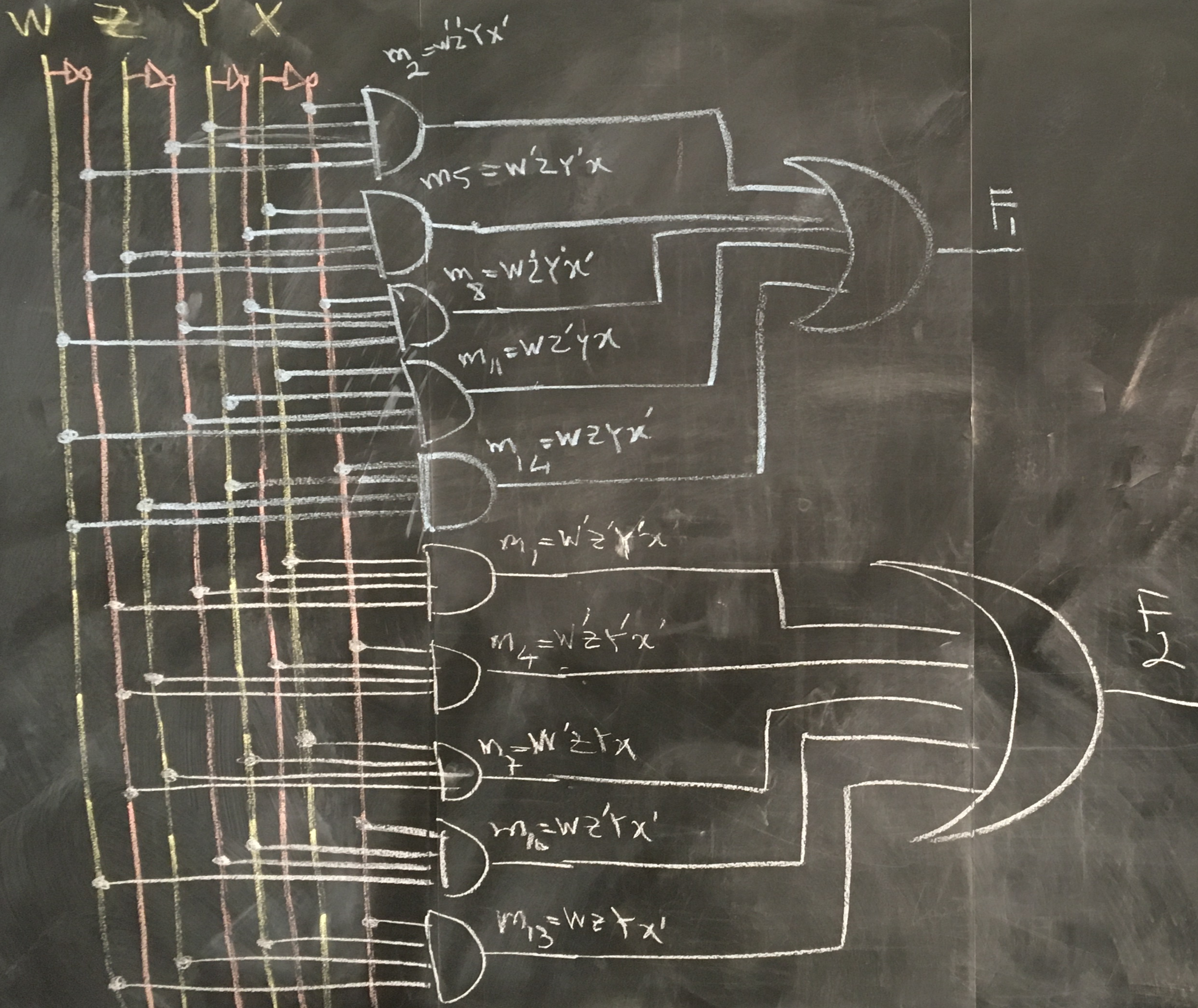
W	Z	Y	X	$F_1=m_2+m_5+m_8+m_{11}+m_{14}$	F_2
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1	0	0
0	1	0	0	0	1
0	1	0	1	1	0
0	1	1	0	0	0
0	1	1	1	0	1
1	0	0	0	1	0
1	0	0	1	0	0
1	0	1	0	0	1
1	0	1	1	1	0
1	1	0	0	0	0
1	1	0	1	0	1
1	1	1	0	1	0
1	1	1	1	0	0

W	Z	Y	X	$F_1=\sum m(2,5,8,11,14)$	$F_2=m_1+m_4+m_7+m_{10}+m_{13}$
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1	0	0
0	1	0	0	0	1
0	1	0	1	1	0
0	1	1	0	0	0
0	1	1	1	0	1
1	0	0	0	1	0
1	0	0	1	0	0
1	0	1	0	0	1
1	0	1	1	1	0
1	1	0	0	0	0
1	1	0	1	0	1
1	1	1	0	1	0
1	1	1	1	0	0

W	Z	Y	X	$F_1=\sum m(2,5,8,11,14)$	$F_2=\sum m(1,4,7,10,13)$
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1	0	0
0	1	0	0	0	1
0	1	0	1	1	0
0	1	1	0	0	0
0	1	1	1	0	1
1	0	0	0	1	0
1	0	0	1	0	0
1	0	1	0	0	1
1	0	1	1	1	0
1	1	0	0	0	0
1	1	0	1	0	1
1	1	1	0	1	0
1	1	1	1	0	0

$F_1 = \sum m(2,5,8,11,14)$

$F_2 = \sum m(1,4,7,10,13)$



no reuse for minterms

UNIVERSALITY

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Augustus De Morgan (1806–1871) Mathematician Logician

DE MORGAN'S LAWS

- $(Y+X)' = Y'X'$

Augustus De Morgan (1806–1871) Mathematician Logician

DE MORGAN'S LAWS

- $((Y+X)')' = (Y'X')'$

Augustus De Morgan (1806–1871) Mathematician Logician

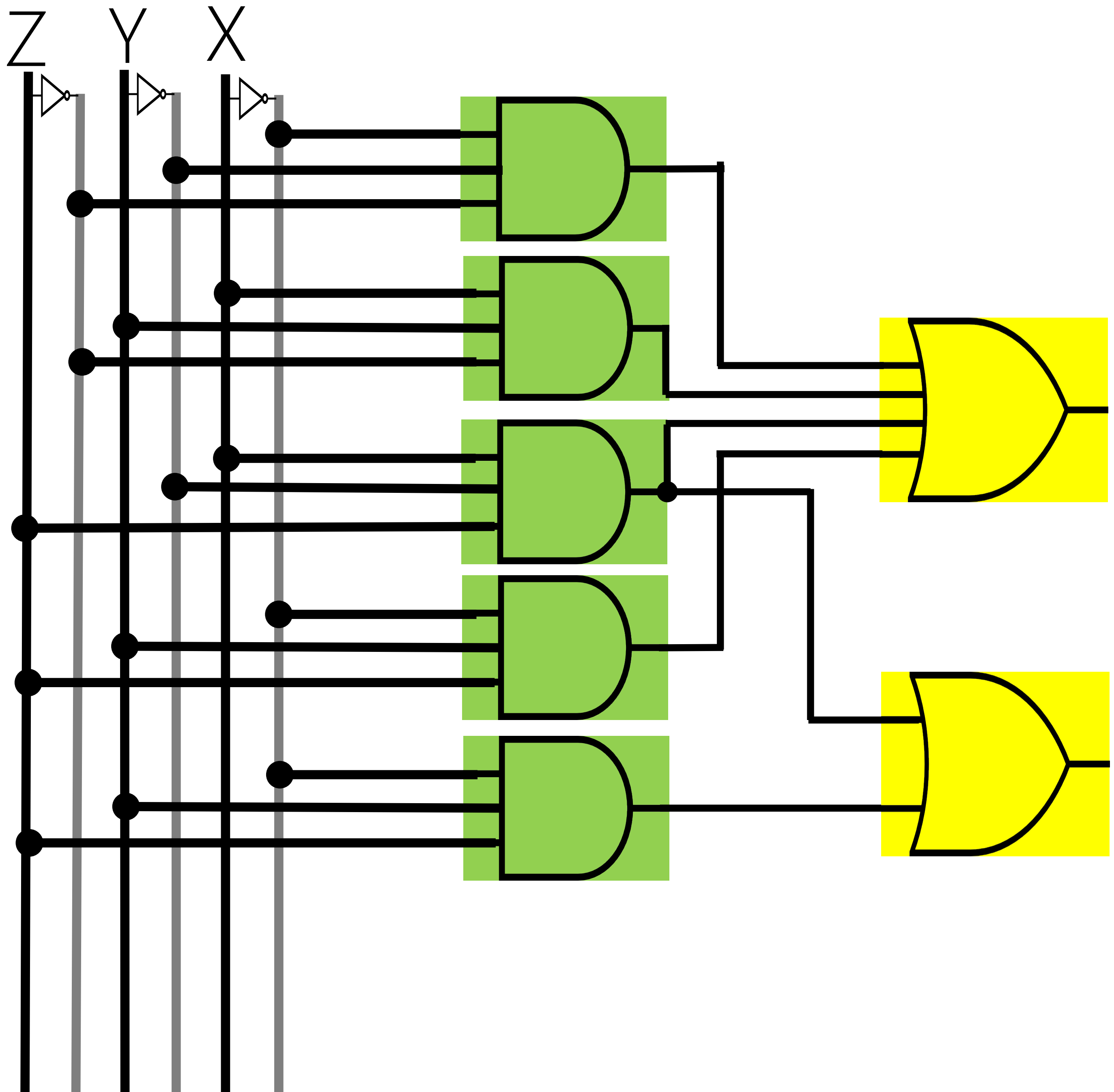
DE MORGAN'S LAWS

- $Y+X = (Y'X')'$

Augustus De Morgan
(1806–1871)
Mathematician
Logician

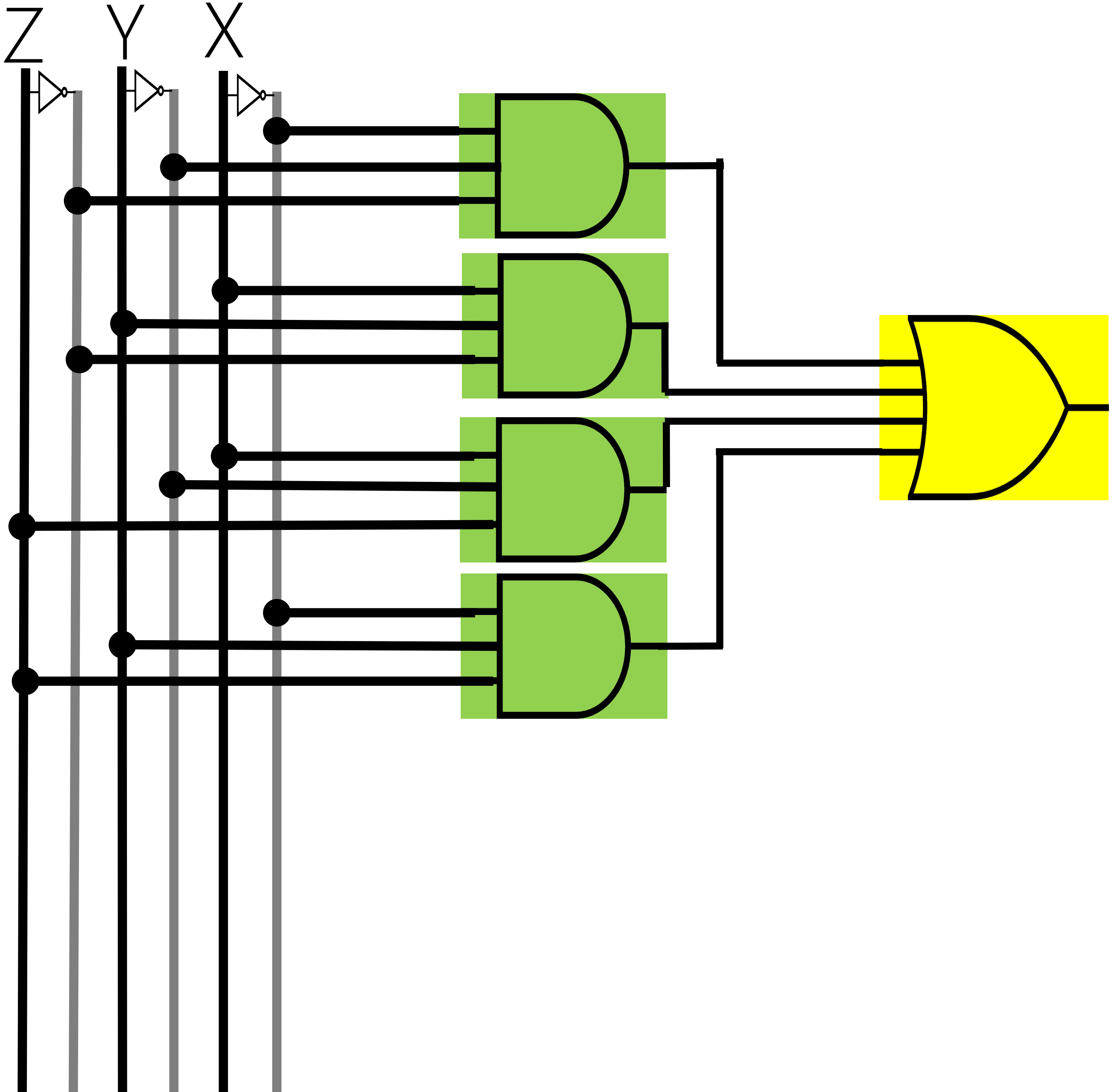
DE MORGAN'S LAWS

- $Y+X = Y' \uparrow X'$

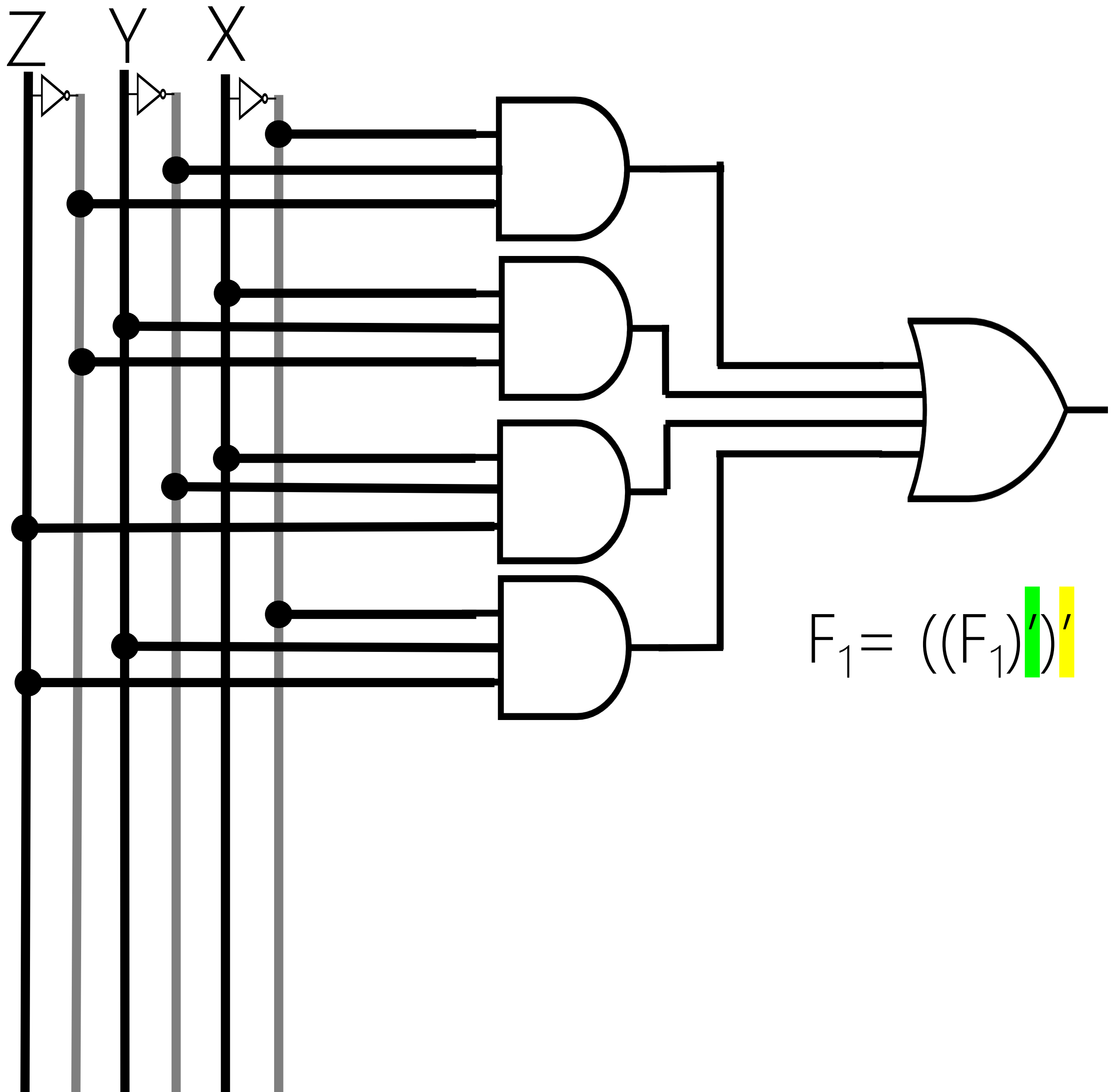


$$F_1 = Z'Y'X' + Z'YX + ZY'X + ZYX'$$

$$F_2 = ZY'X' + ZY'X$$

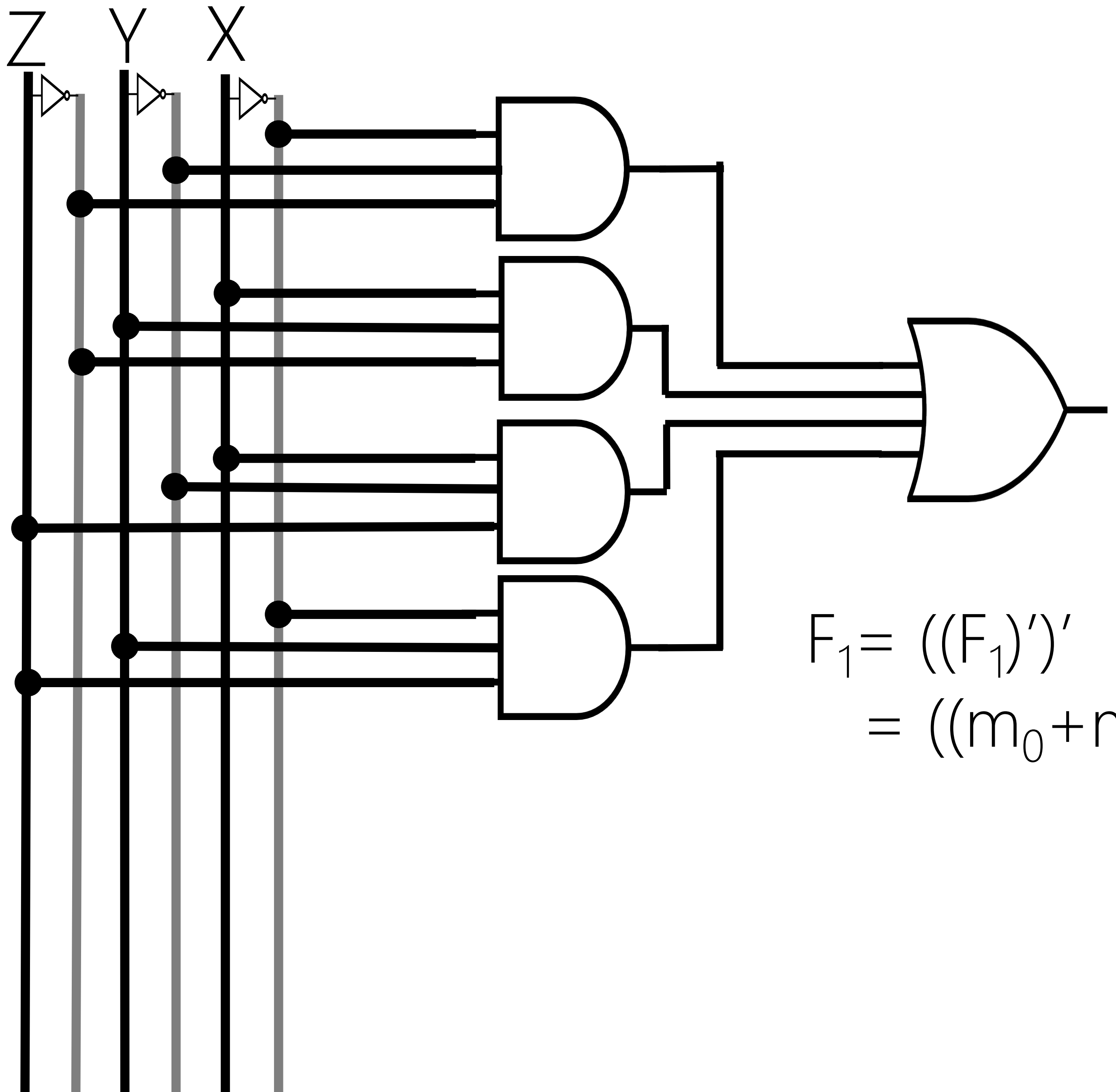


$$F_1 = Z'Y'X' + Z'YX + ZY'X + ZYX'$$



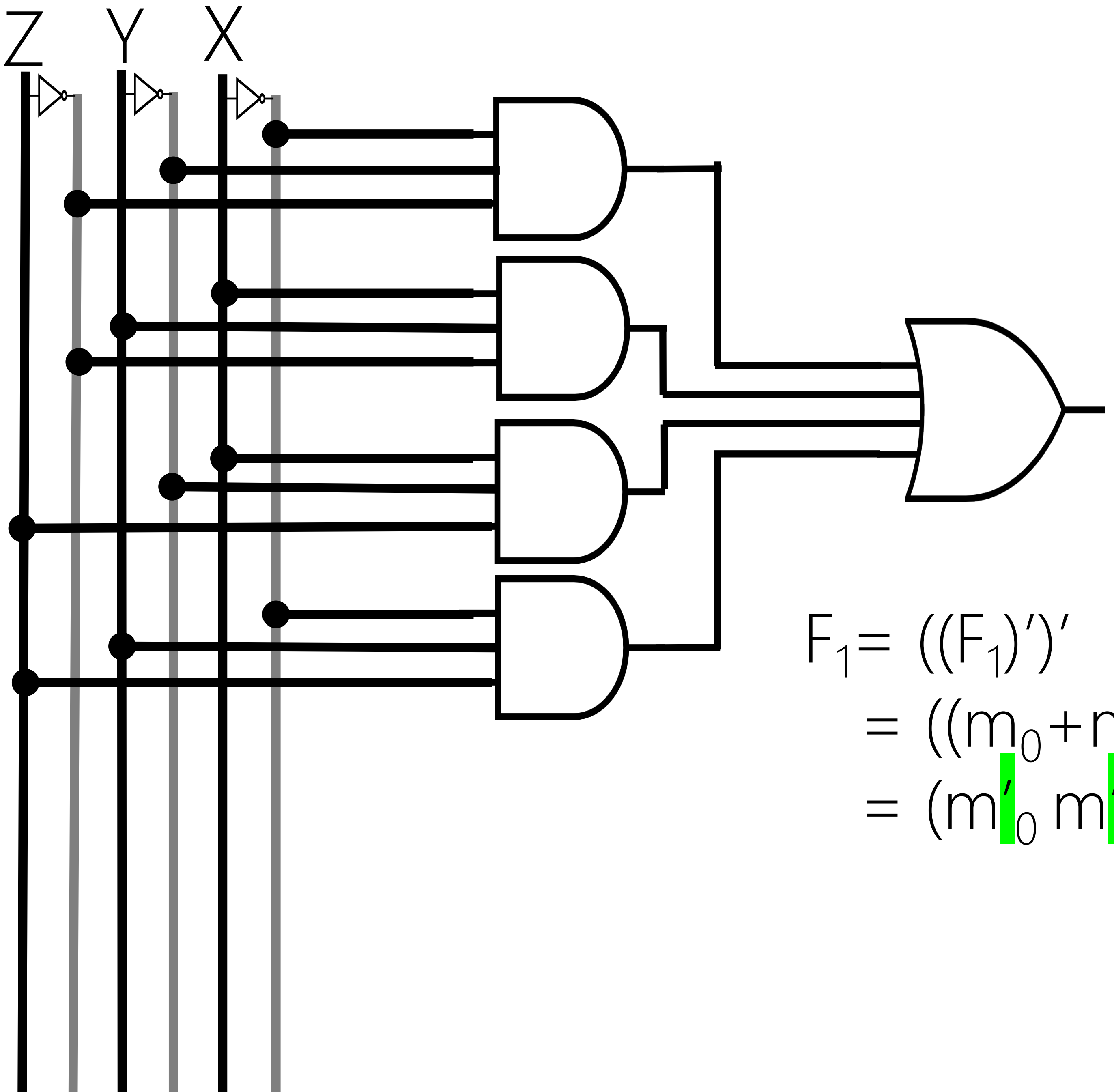
$$F_1 = Z'Y'X' + Z'YX + ZY'X + ZYX'$$

$$F_1 = ((F_1)')'$$



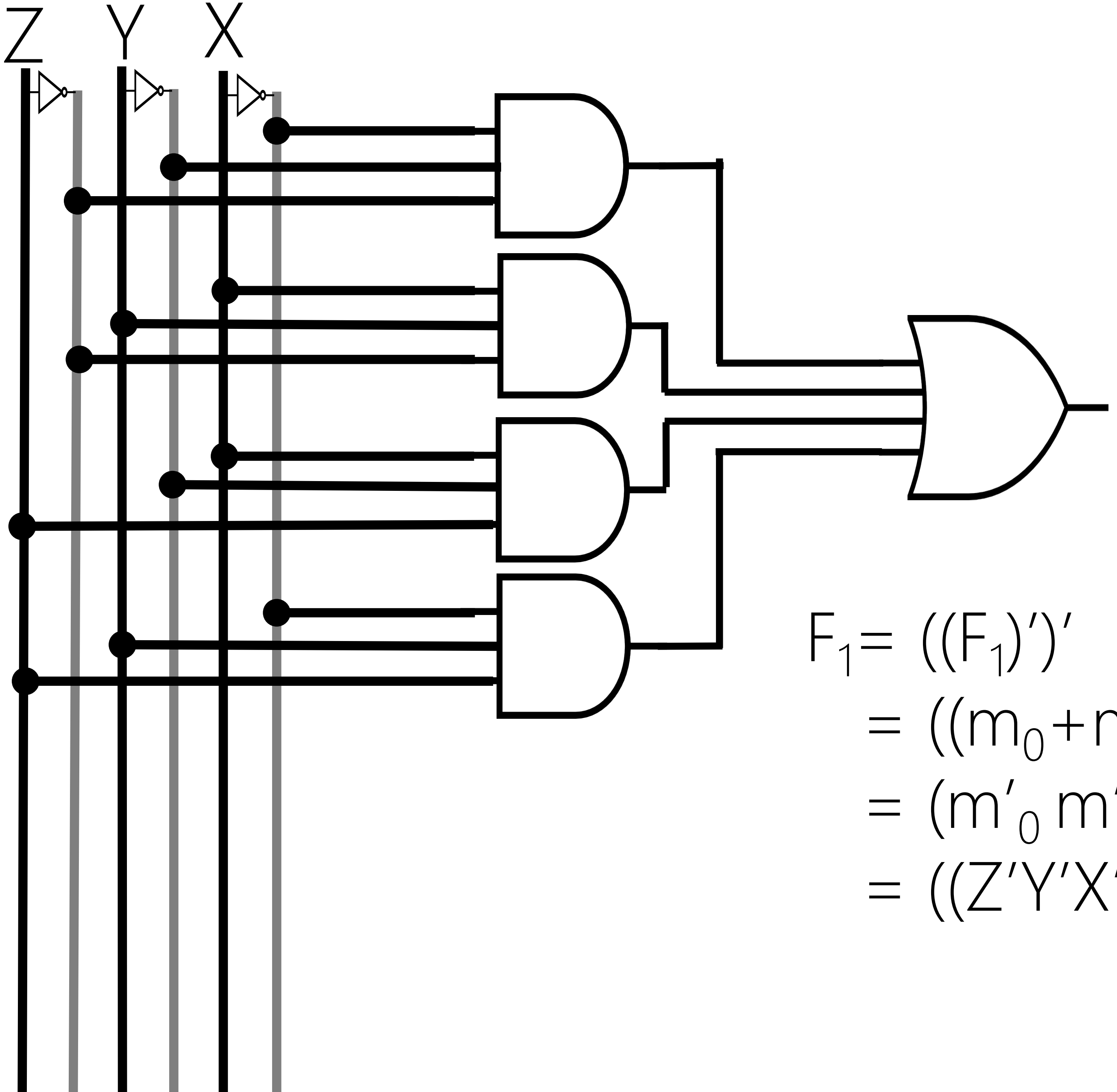
$$F_1 = Z'Y'X' + Z'YX + ZY'X + ZYX'$$

$$F_1 = ((F_1)')'$$
$$= ((m_0 + m_3 + m_5 + m_6)')'$$



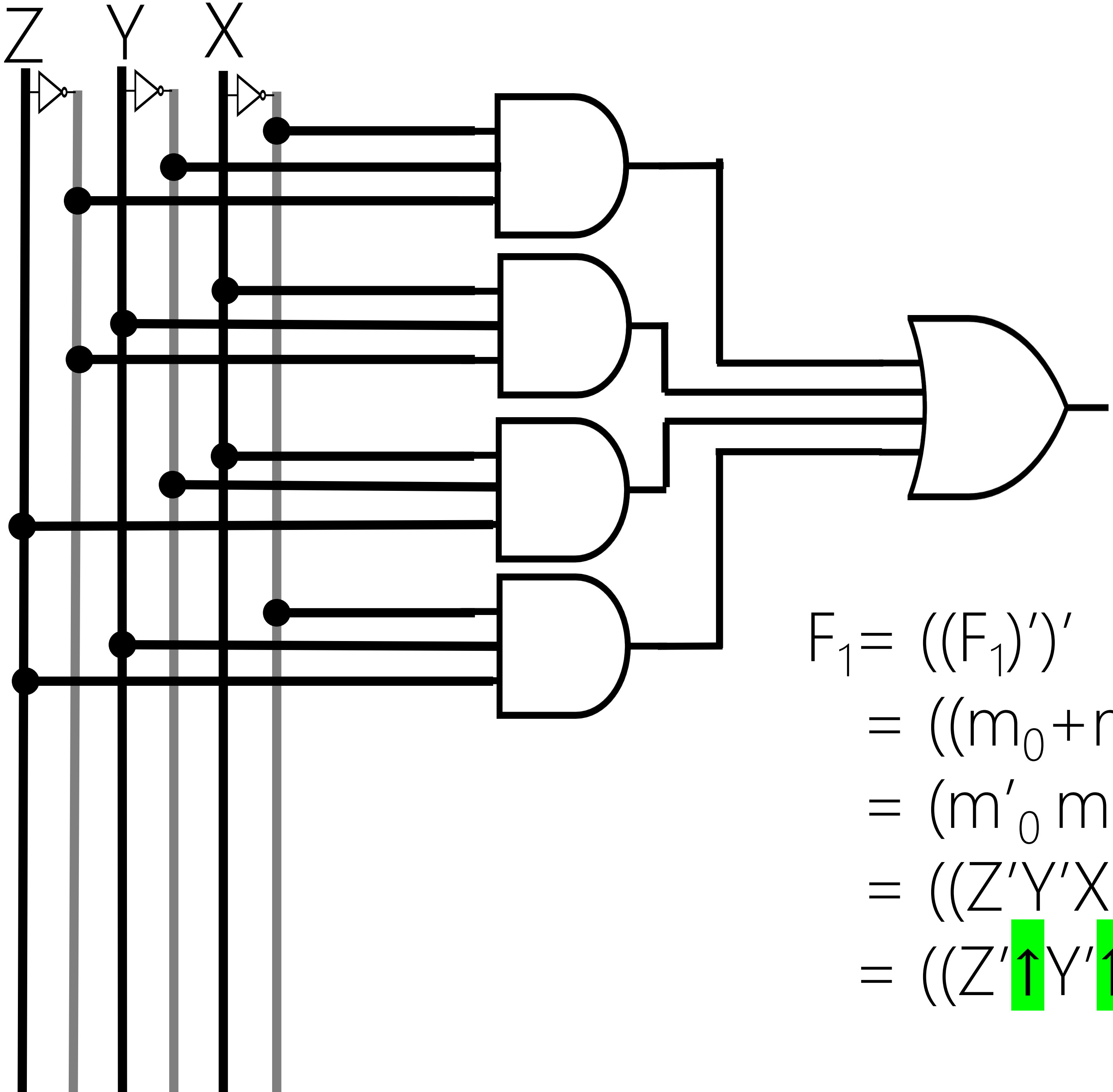
$$F_1 = Z'Y'X' + Z'YX + ZY'X + ZYX'$$

$$\begin{aligned} F_1 &= ((F_1)')' \\ &= ((m_0 + m_3 + m_5 + m_6)')' \\ &= (m'_0\, m'_3\, m'_5\, m'_6)' \end{aligned}$$



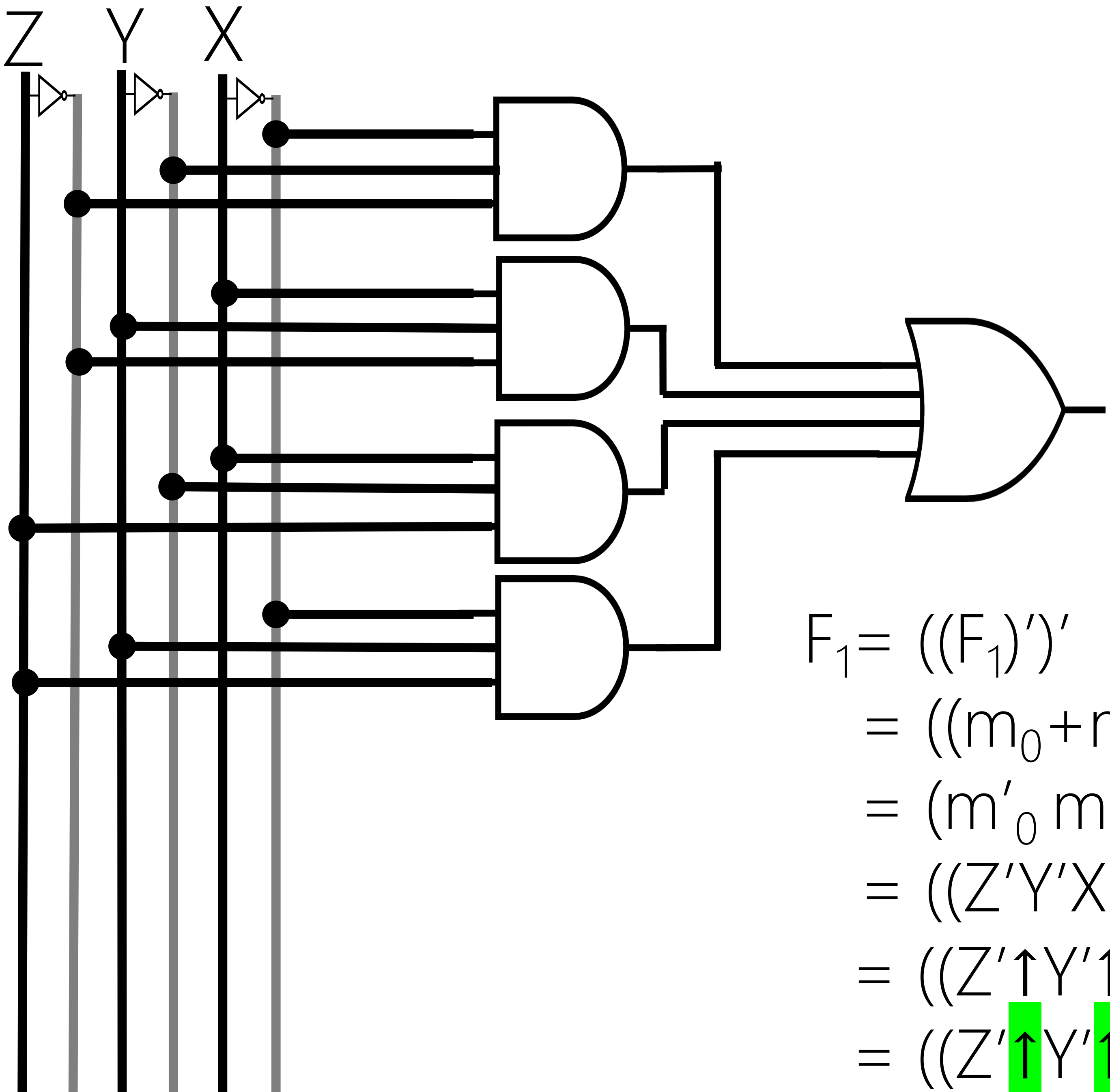
$$F_1 = Z'Y'X' + Z'YX + ZY'X + ZYX'$$

$$\begin{aligned} F_1 &= ((F_1)')' \\ &= ((m_0 + m_3 + m_5 + m_6)')' \\ &= (m'_0\, m'_3\, m'_5\, m'_6)' \\ &= ((Z'Y'X')'\,(Z'YX)'\,(ZY'X)'\,(ZYX'))' \end{aligned}$$



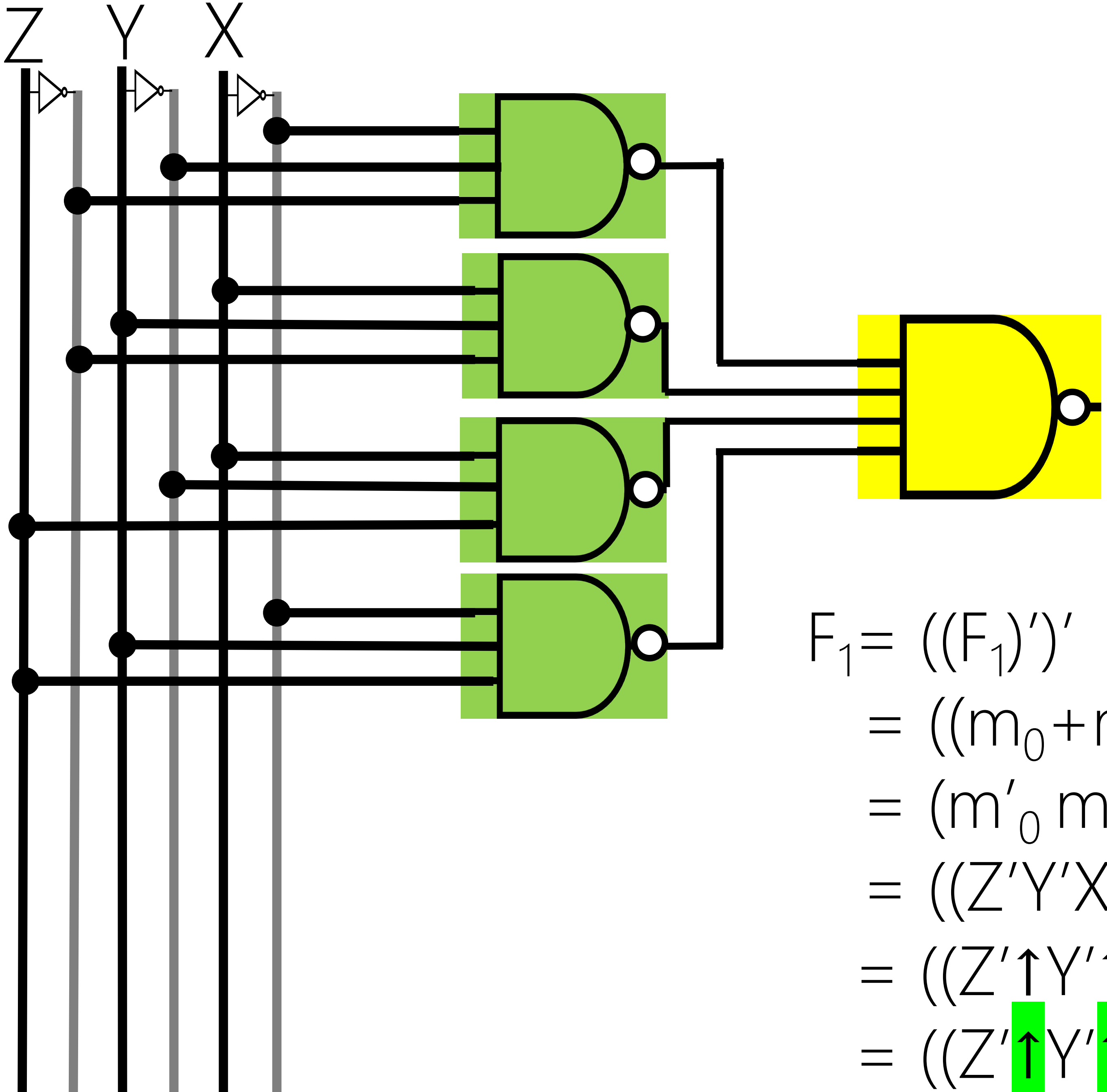
$$F_1 = Z'Y'X' + Z'YX + ZY'X + ZYX'$$

$$\begin{aligned}
F_1 &= ((F_1)')' \\
&= ((m_0 + m_3 + m_5 + m_6)')' \\
&= (m'_0\, m'_3\, m'_5\, m'_6)' \\
&= ((Z'Y'X')'\ (Z'YX)'\ (ZY'X)'\ (ZYX'))' \\
&= ((Z' \uparrow Y' \uparrow X')\ (Z' \uparrow Y \uparrow X)\ (Z \uparrow Y' \uparrow X)\ (Z \uparrow Y \uparrow X'))'
\end{aligned}$$

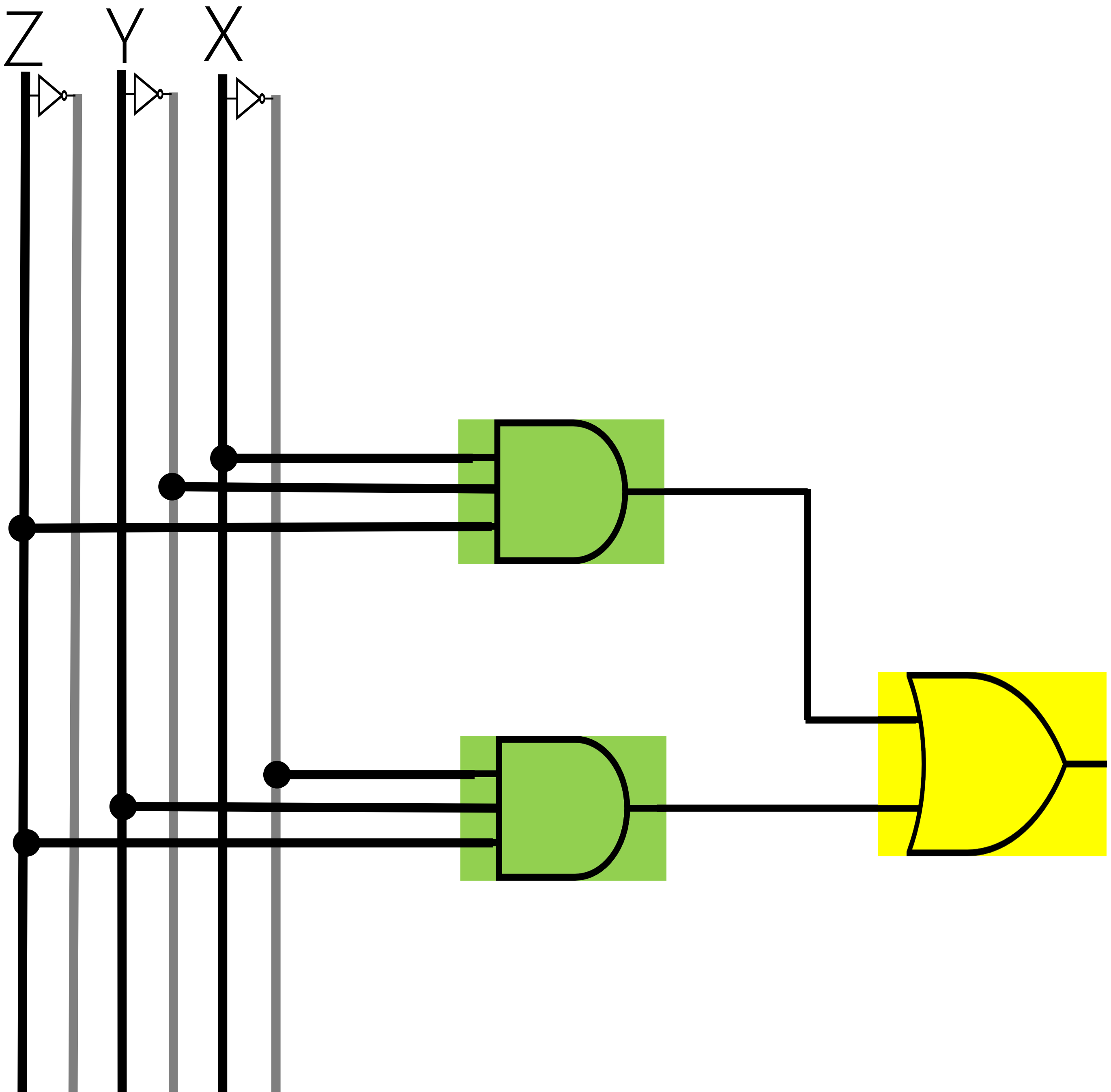


$$F_1 = Z'Y'X' + Z'YX + ZY'X + ZYX'$$

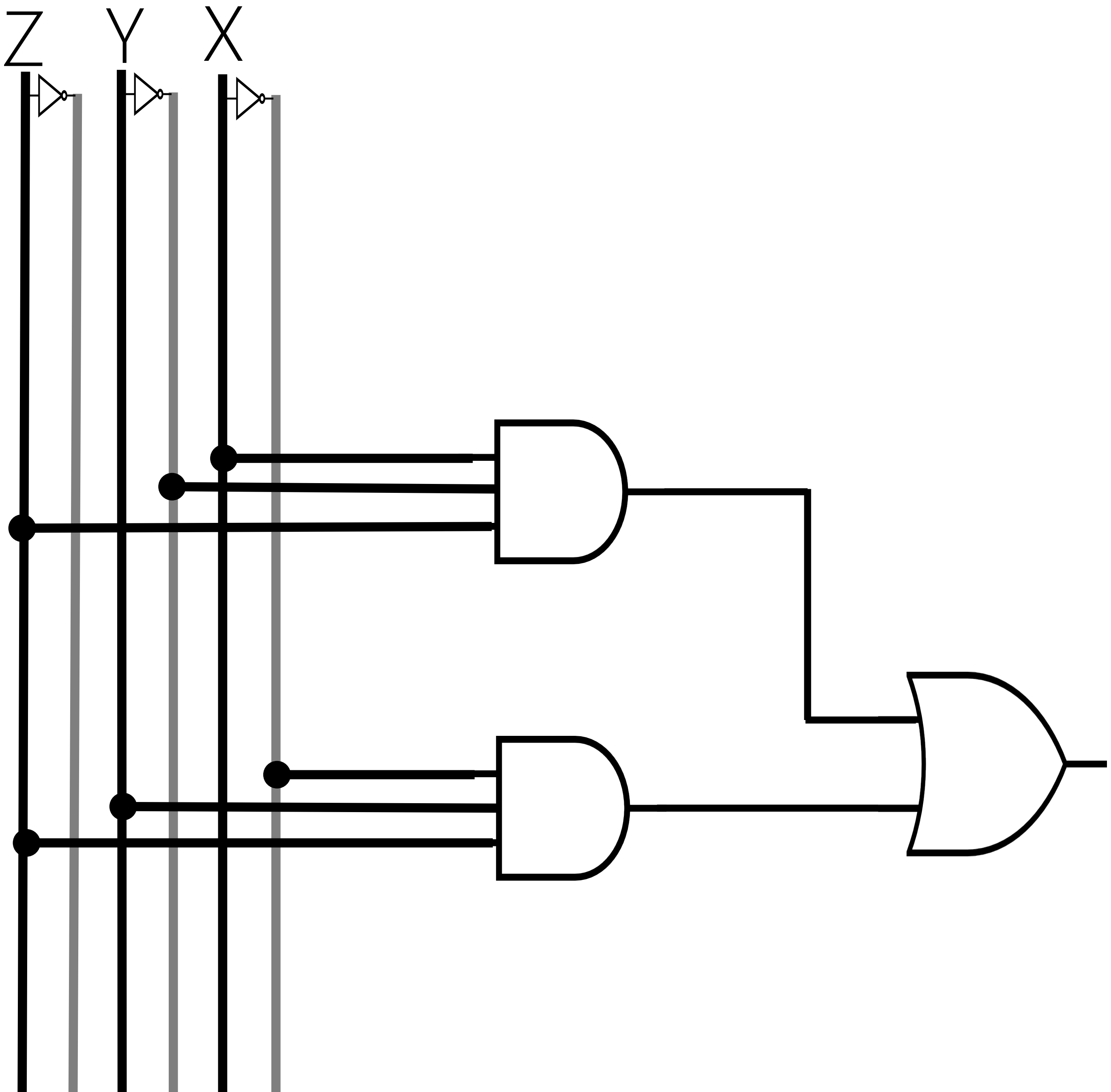
$$\begin{aligned}
F_1 &= ((F_1)')' \\
&= ((m_0 + m_3 + m_5 + m_6)')' \\
&= (m'_0\, m'_3\, m'_5\, m'_6)' \\
&= ((Z'Y'X')'\ (Z'YX)'\ (ZY'X)'\ (ZYX'))' \\
&= ((Z' \uparrow Y' \uparrow X')\ (Z' \uparrow Y \uparrow X)\ (Z \uparrow Y' \uparrow X)\ (Z \uparrow Y \uparrow X'))' \\
&= ((Z' \uparrow Y' \uparrow X') \uparrow (Z' \uparrow Y \uparrow X) \uparrow (Z \uparrow Y' \uparrow X) \uparrow (Z \uparrow Y \uparrow X'))
\end{aligned}$$



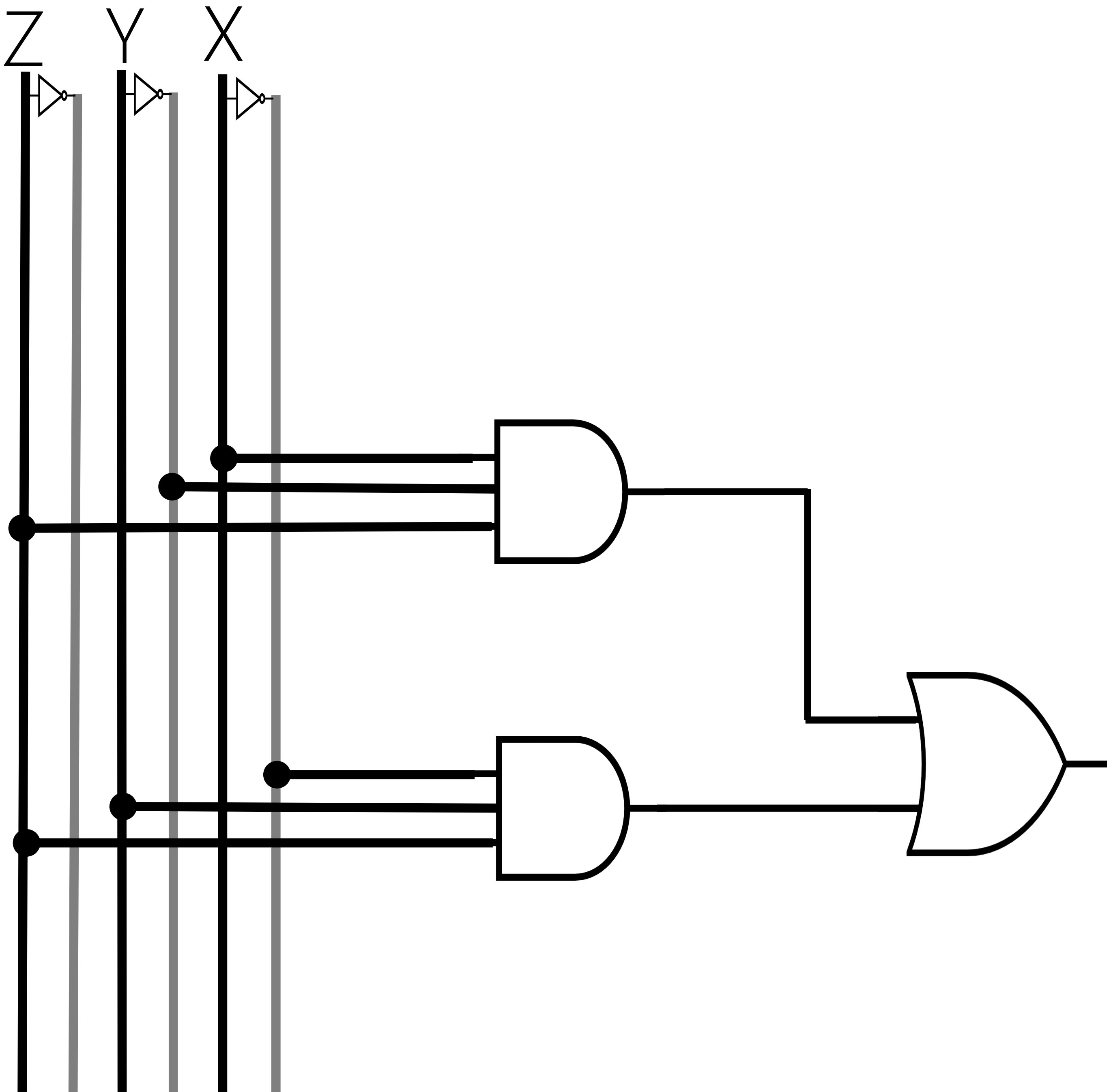
$$
\begin{aligned}
F_1 &= ((F_1)')' \\
&= ((m_0+m_3+m_5+m_6)')' \\
&= (m'_0\, m'_3\, m'_5\, m'_6)' \\
&= ((Z'Y'X')'\ (Z'YX)'\ (ZY'X)'\ (ZYX'))' \\
&= ((Z'\uparrow Y'\uparrow X')\ (Z'\uparrow Y\uparrow X)\ (Z\uparrow Y'\uparrow X)\ (Z\uparrow Y\uparrow X'))' \\
&= ((Z'\uparrow Y'\uparrow X')\uparrow(Z'\uparrow Y\uparrow X)\uparrow(Z\uparrow Y'\uparrow X)\uparrow(Z\uparrow Y\uparrow X'))
\end{aligned}
$$



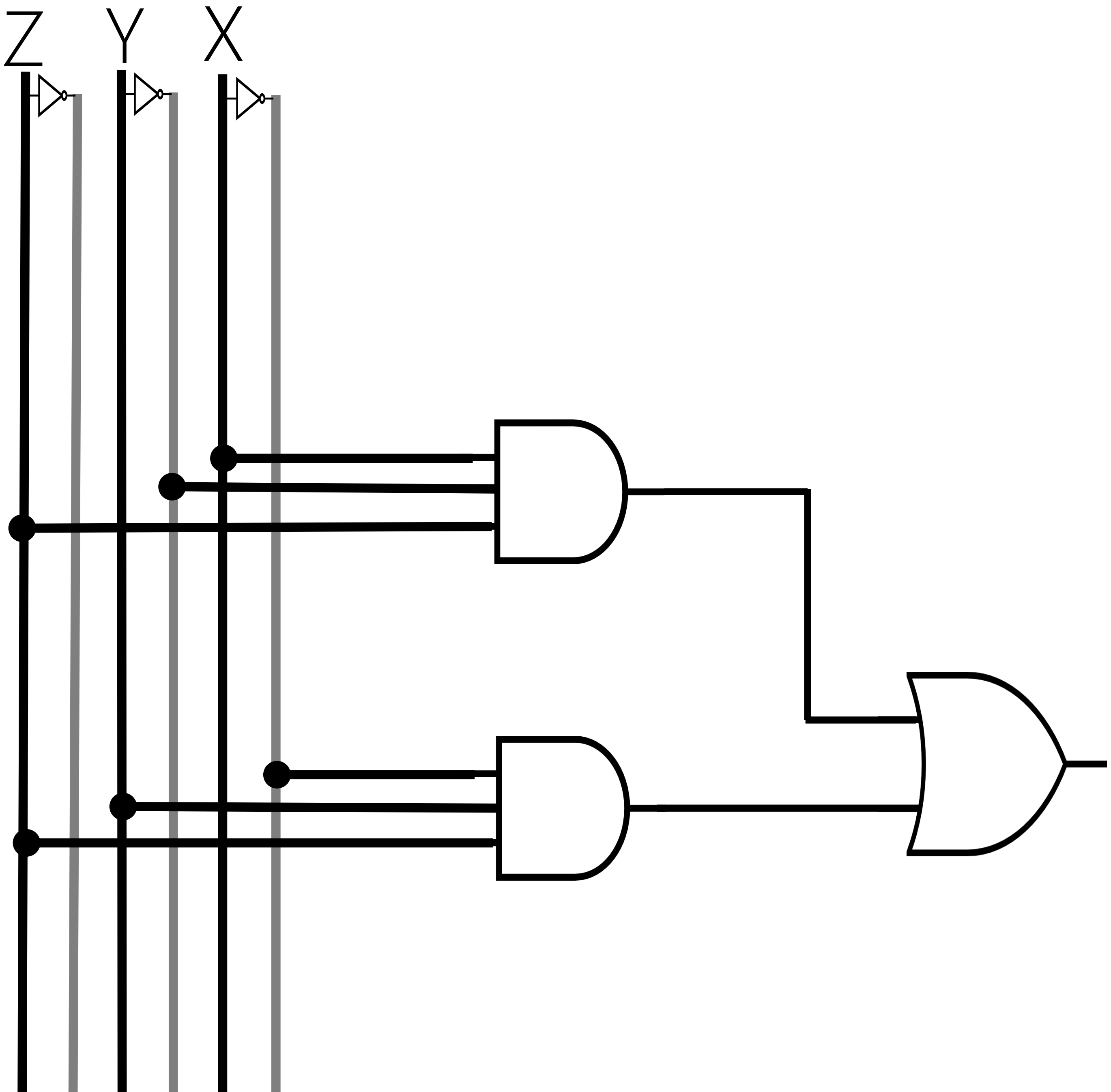
$F_2 = ZY'X' + ZY'X$



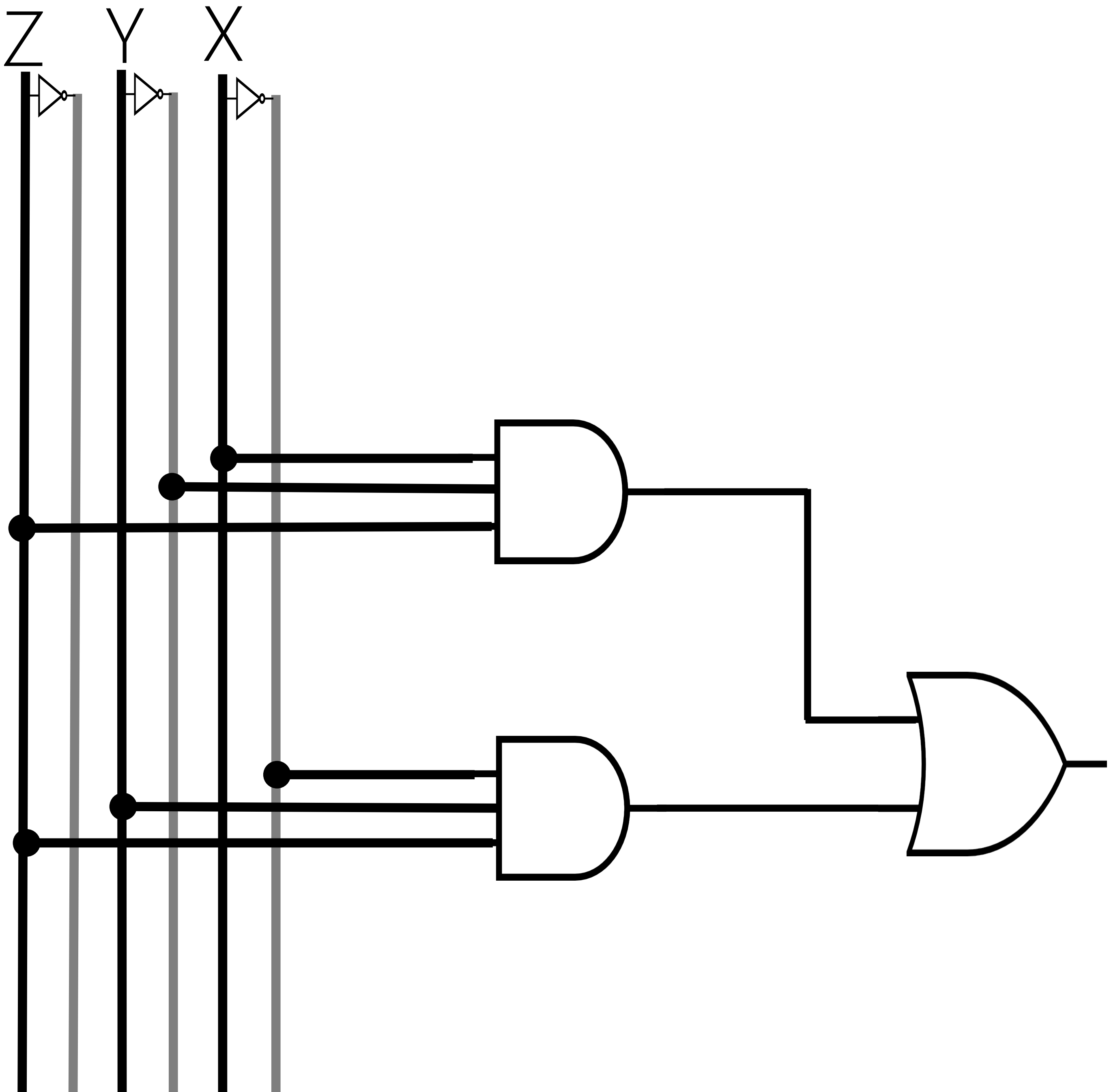
$$F_2 = m_4 + m_5$$
$$= ((F_2)')'$$



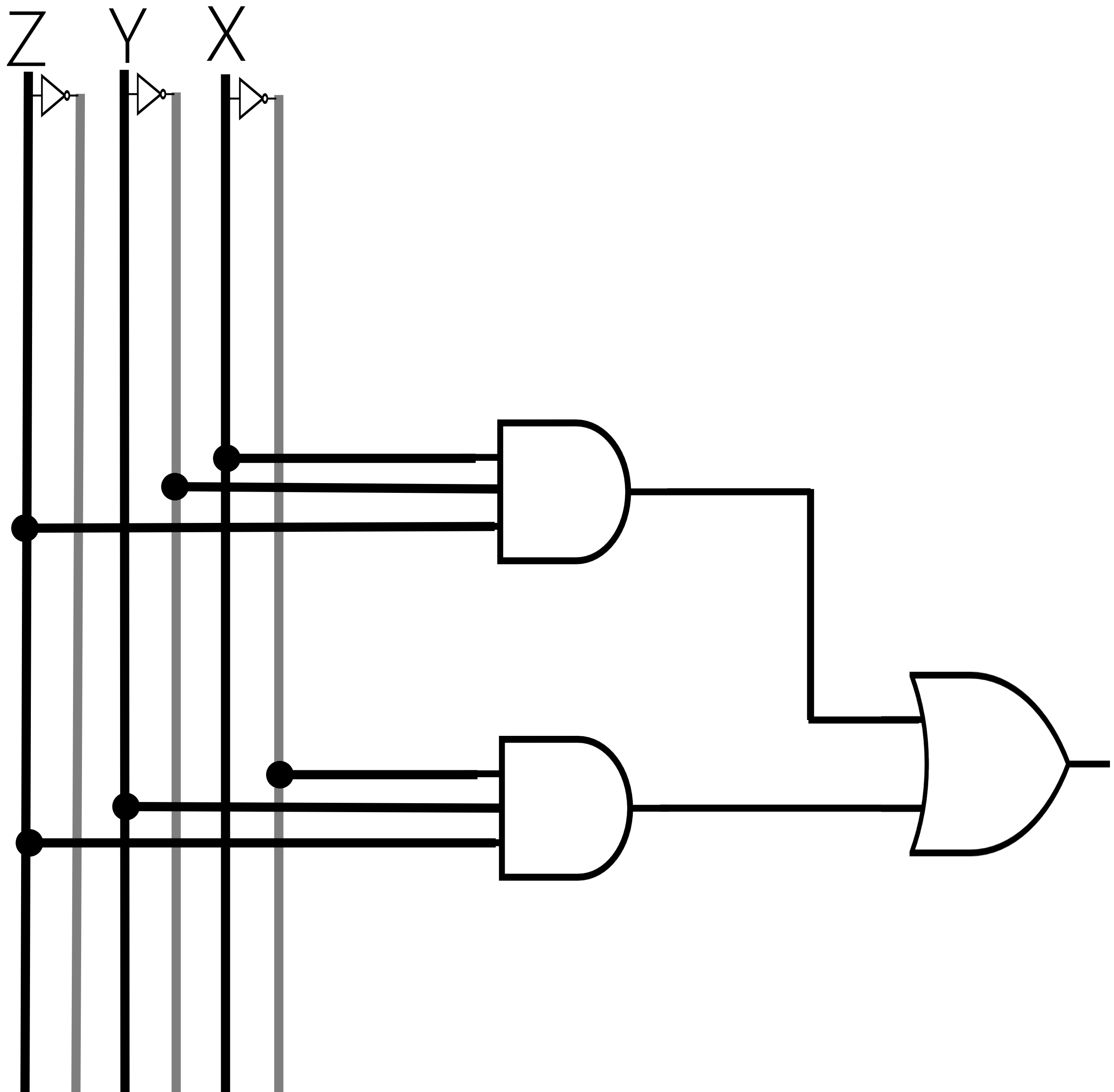
$$F_2 = m_4 + m_5$$
$$= ((F_2)')'$$
$$= ((m_4 + m_5)')'$$



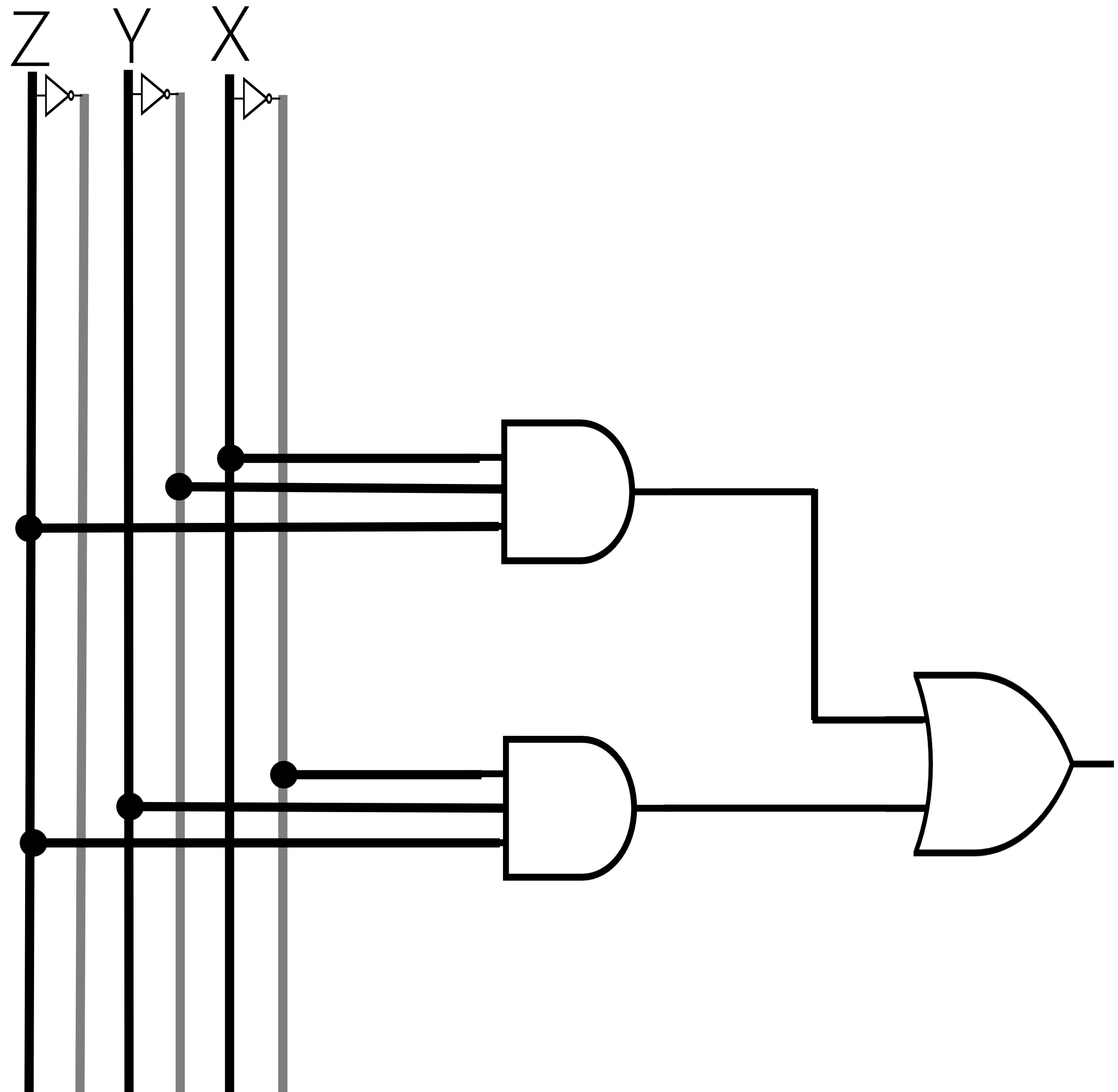
$$
\begin{aligned}
F_2 &= m_4 + m_5 \\
&= ((F_2)')' \\
&= ((m_4 + m_5)')' \\
&= (m'_4 m'_5)'
\end{aligned}
$$



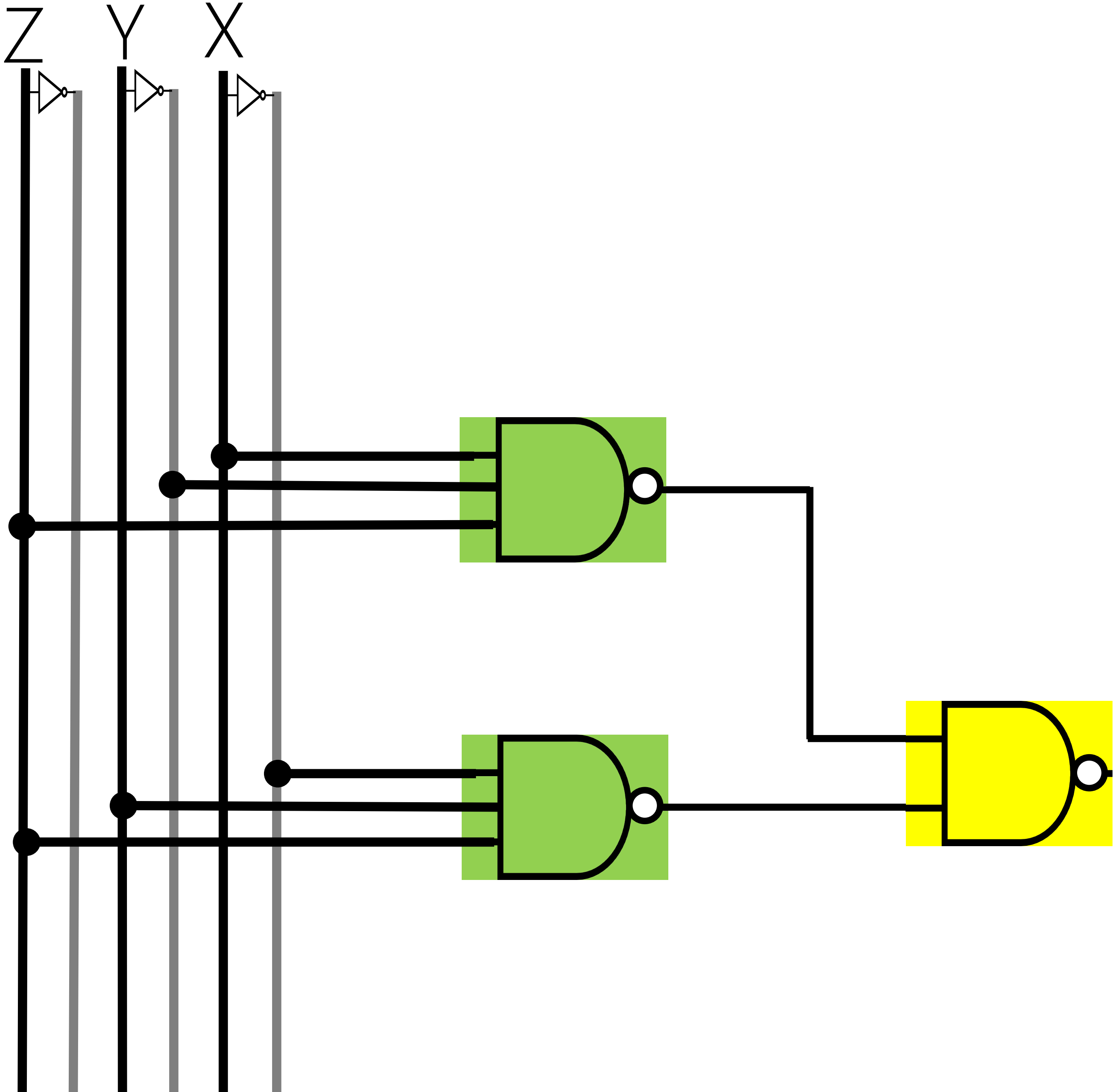
$$
\begin{aligned}
F_2 &= m_4 + m_5 \\
&= ((F_2)')' \\
&= ((m_4+m_5)')' \\
&= (m'_4 m'_5)' \\
&= ((ZY'X')' (ZY'X)')'
\end{aligned}
$$



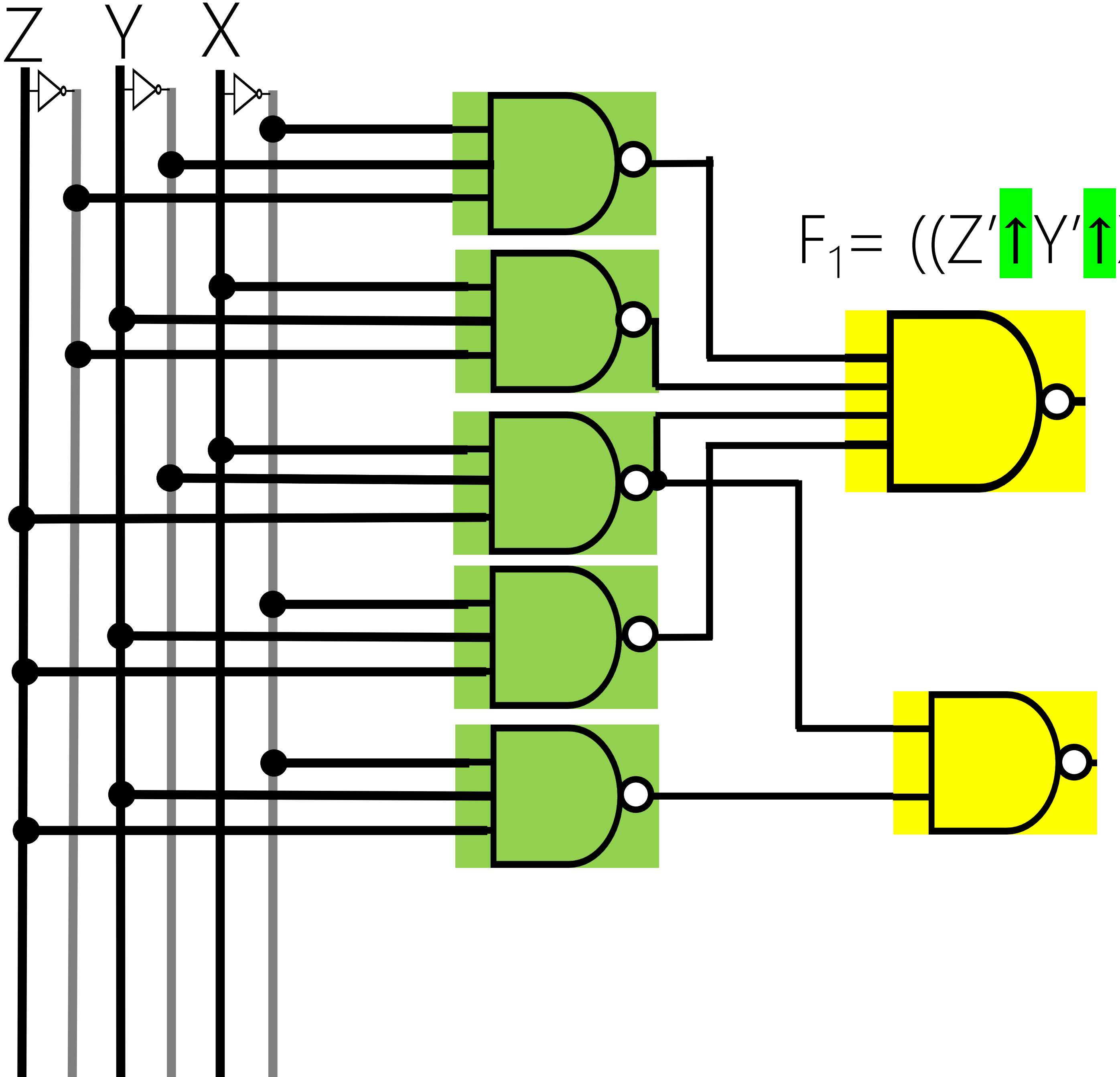
$$
\begin{aligned}
F_2 &= m_4 + m_5 \\
&= ((F_2)')' \\
&= ((m_4+m_5)')' \\
&= (m'_4 m'_5)' \\
&= ((ZY'X')' (ZY'X)')' \\
&= ((Z \uparrow Y' \uparrow X')\ (Z \uparrow Y' \uparrow X))'
\end{aligned}
$$



$$
\begin{aligned}
F_2 &= m_4 + m_5 \\
&= ((F_2)')' \\
&= ((m_4+m_5)')' \\
&= (m'_4 m'_5)' \\
&= ((ZY'X')'\,(ZY'X)')' \\
&= ((Z\uparrow Y'\uparrow X')\,(Z\uparrow Y'\uparrow X))' \\
&= ((Z\uparrow Y'\uparrow X')\uparrow(Z\uparrow Y'\uparrow X))
\end{aligned}
$$

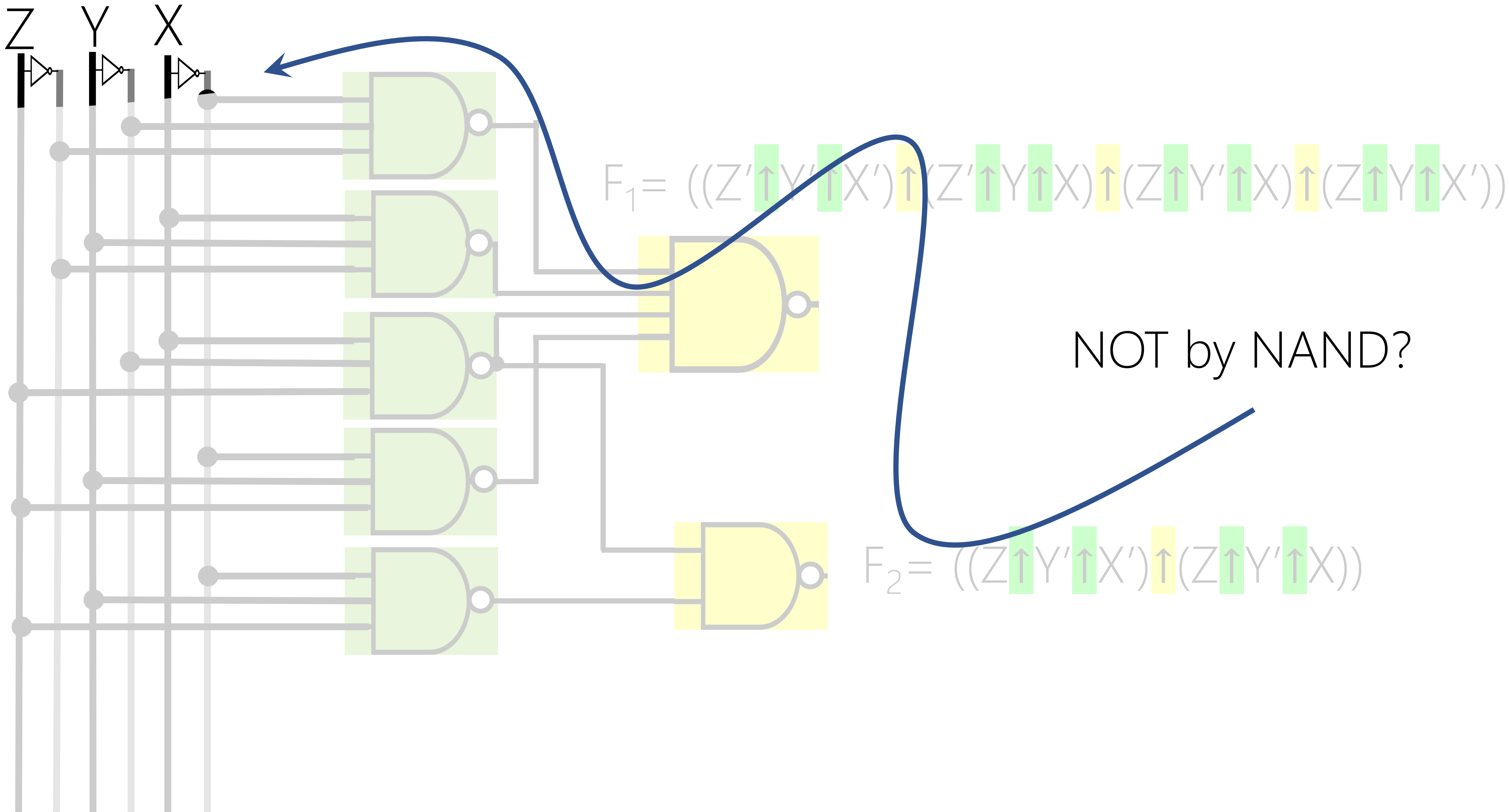


$$F_2 = ((Z \uparrow Y' \uparrow X') \uparrow (Z \uparrow Y' \uparrow X))$$

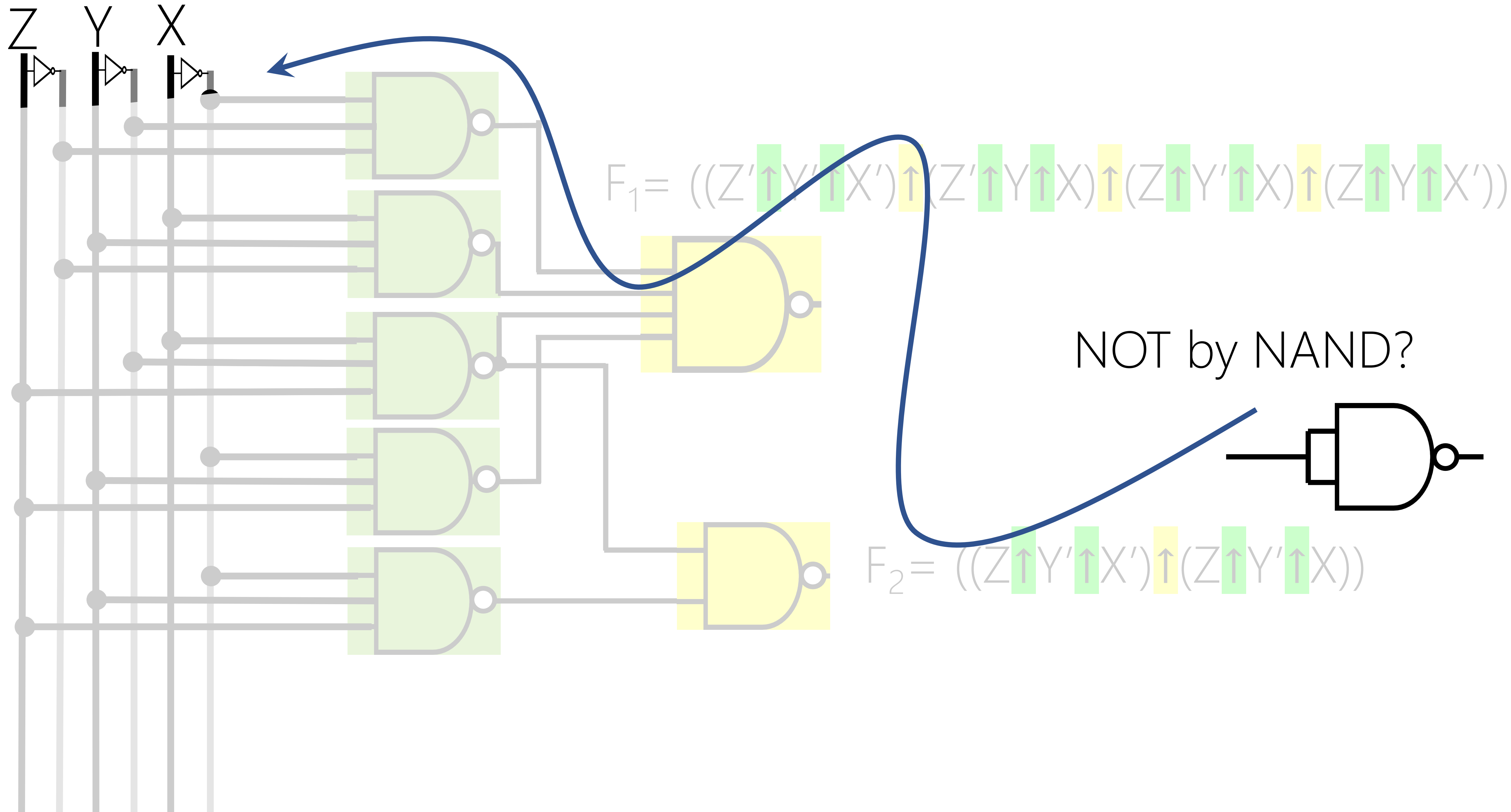


$$F_1 = ((Z' \uparrow Y' \uparrow X') \uparrow (Z' \uparrow Y \uparrow X) \uparrow (Z \uparrow Y' \uparrow X) \uparrow (Z \uparrow Y \uparrow X'))$$

$$F_2 = ((Z \uparrow Y' \uparrow X') \uparrow (Z \uparrow Y' \uparrow X))$$



Z
Y
X
F1= ((Z'↑Y'↑X')↑(Z'↑Y↑X)↑(Z↑Y'↑X)↑(Z↑Y↑X'))
F2= ((Z↑Y'↑X')↑(Z↑Y'↑X))
NOT by NAND?



Z Y X
$F_1 = ((Z' \uparrow Y' \uparrow X') \uparrow (Z' \uparrow Y \uparrow X) \uparrow (Z \uparrow Y' \uparrow X) \uparrow (Z \uparrow Y \uparrow X'))$
NOT by NAND?
$F_2 = ((Z \uparrow Y' \uparrow X') \uparrow (Z \uparrow Y' \uparrow X))$

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$F=(F')'=(SoP')'$

DESIGN II

a new algorithm for designing any logic circuits, given truth table

MAXTERM

aka. Standard Sum

X’ vs. X

1 binary variable appear either:

- in its normal form X, or
- in its complement form X’

$M_0 = m'_0$	$(X') = (X')' = X$
$M_1 = m'_1$	X'

$Y+X$ vs. $Y+X'$ vs. $Y'+X$ vs. $Y'+X'$

2 binary variables appear either in one of these forms:

$M_0 = m'_0$	$(Y'X')' = Y + X$
$M_1 = m'_1$	$(Y'X)' = Y + X'$
$M_2 = m'_2$	$(YX')' = Y' + X$
$M_3 = m'_3$	$(YX)' = Y' + X'$

$Z+Y+X$ vs. $Z+Y+X'$ vs. …

3 binary variables appear either in one of these forms: how many?

$Z+Y+X$ vs. $Z+Y+X'$ vs. …

3 binary variables appear either in one of these forms: how many?

Each variable can take 2 forms (normal and complement)

We have 3 variables, $2 \times 2 \times 2 = 2^3 = 8$

$M_0 = m'_0$	$(Z'Y'X')' = Z + Y + X$
$M_1 = m'_1$	$(Z'Y'X)' = Z + Y + X'$
$M_2 = m'_2$	$(Z'YX')' = Z + Y' + X$
$M_3 = m'_3$	$(Z'YX)' = Z + Y' + X'$
$M_4 = m'_4$	$(ZY'X')' = Z' + Y + X$
$M_5 = m'_5$	$(ZY'X)' = Z' + Y + X'$
$M_6 = m'_6$	$(ZYX')' = Z' + Y' + X$
$M_7 = m'_7$	$(ZYX)' = Z' + Y' + X'$

$$A_n + \cdots A_2 + A_1 + A_0 \text{ vs. } A_n + \cdots A_2 + A_1 + A'_0 \ldots$$

n binary variables appear either in one of these forms: how many?

Each variable can take 2 forms (normal and complement)

We have n variables, $2 \times 2 \times 2 \times \cdots \times 2 = 2^n$

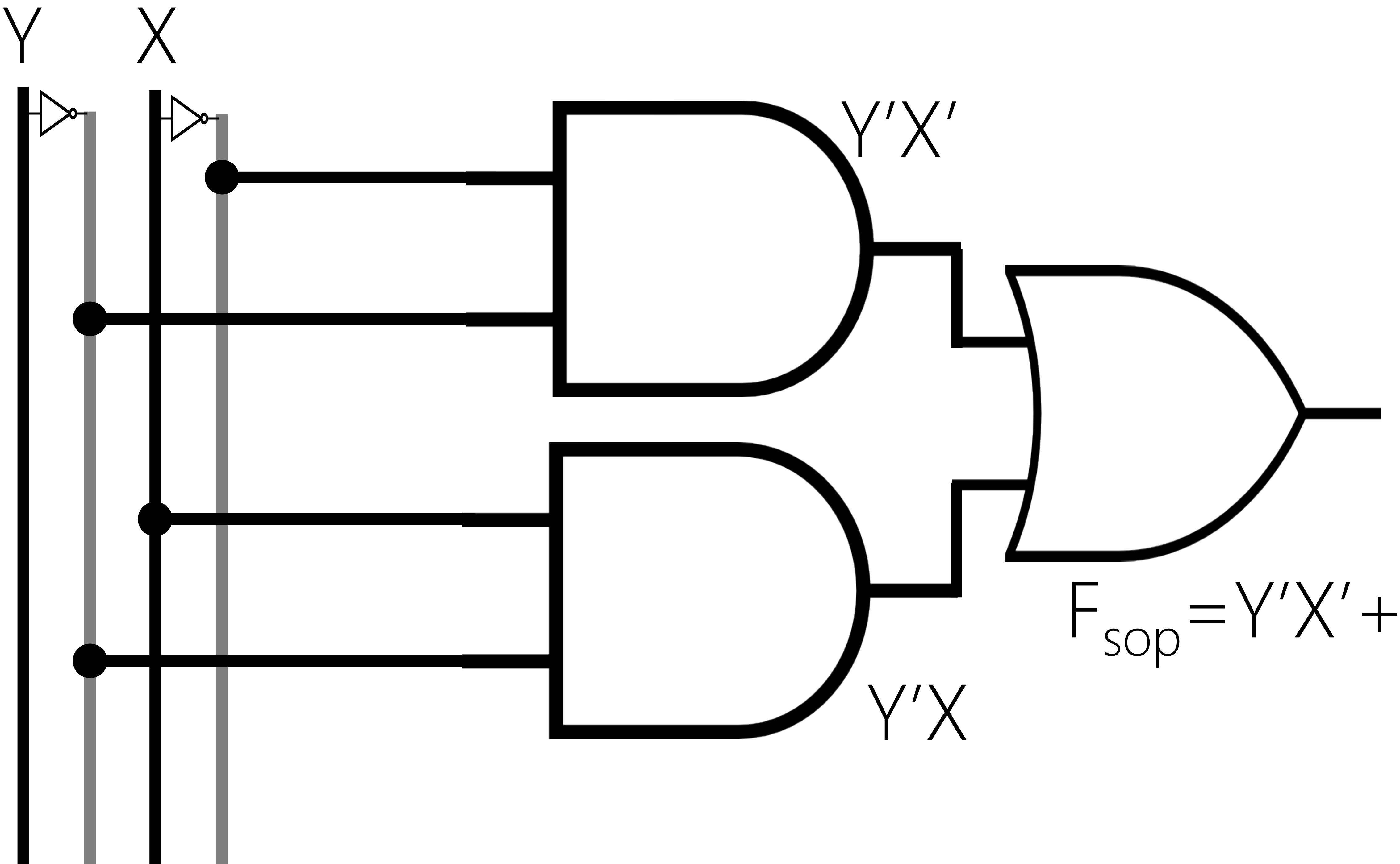
$M_0 = m'_0$	$A_n + A_2 + A_1 + A_0$
$M_1 = m'_1$	$A_n + \cdots A_2 + A_1 + A'_0$
$M_2 = m'_2$	$A_n + \cdots A_2 + A'_1 + A_0$
$\vdots$	$\vdots$
$\vdots$	$\vdots$
$M_{2^n-3} = m'_{2^n-3}$	$A'_n + \cdots A'_2 + A_1 + A'_0$
$M_{2^n-2} = m'_{2^n-2}$	$A'_n + \cdots A'_2 + A'_1 + A_0$
$M_{2^n-1} = m'_{2^n-1}$	$A'_n + \cdots A'_2 + A'_1 + A'_0$

TRUTH TABLE

en.wikipedia.org/wiki/Truth_table

TRUTH TABLE ↔ MAXTERM

Y	X	$F=m_0+m_1$
0	0	1
0	1	1
1	0	0
1	1	0



$$F_{sop} = Y'X' + Y'X$$

Y	X	$F=m_0+m_1$	$F'=m_2+m_3$
0	0	1	0
0	1	1	0
1	0	0	1
1	1	0	1

Y	X	$F=m_0+m_1$	$F'=m_2+m_3$	$(F')'=(m_2+m_3)'$
0	0	1	0	1
0	1	1	0	1
1	0	0	1	0
1	1	0	1	0

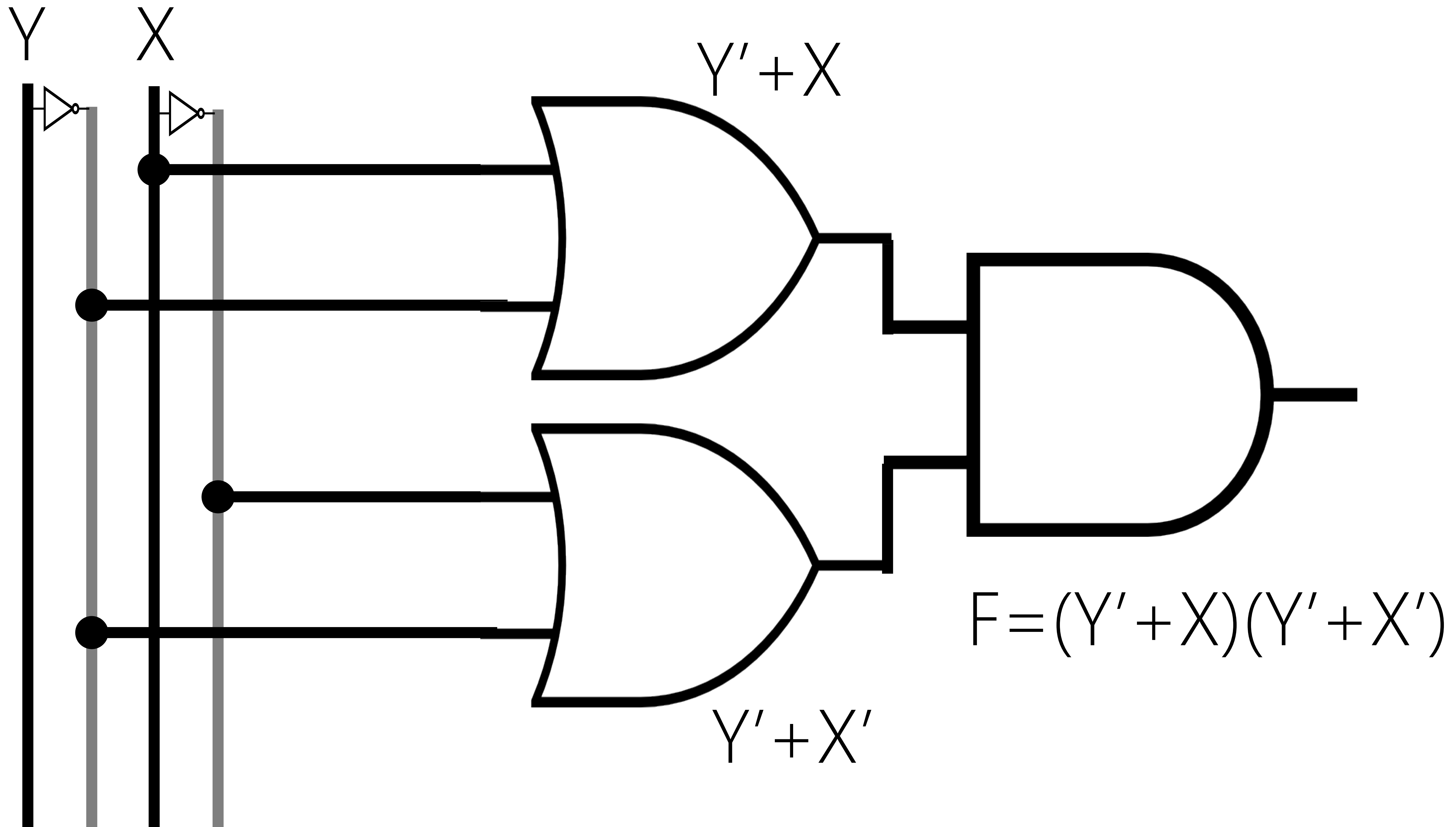
Y	X	$F=(F')'=(m_2+m_3)'=m'_2m'_3$
0	0	1
0	1	1
1	0	0
1	1	0

Y	X	$F=(F')'=m'_2m'_3=M_2M_3$
0	0	1
0	1	1
1	0	0
1	1	0

Y	X	$F=(F')'=m'_2m'_3=M_2M_3$
0	0	1
0	1	1
1	0	0
1	1	0

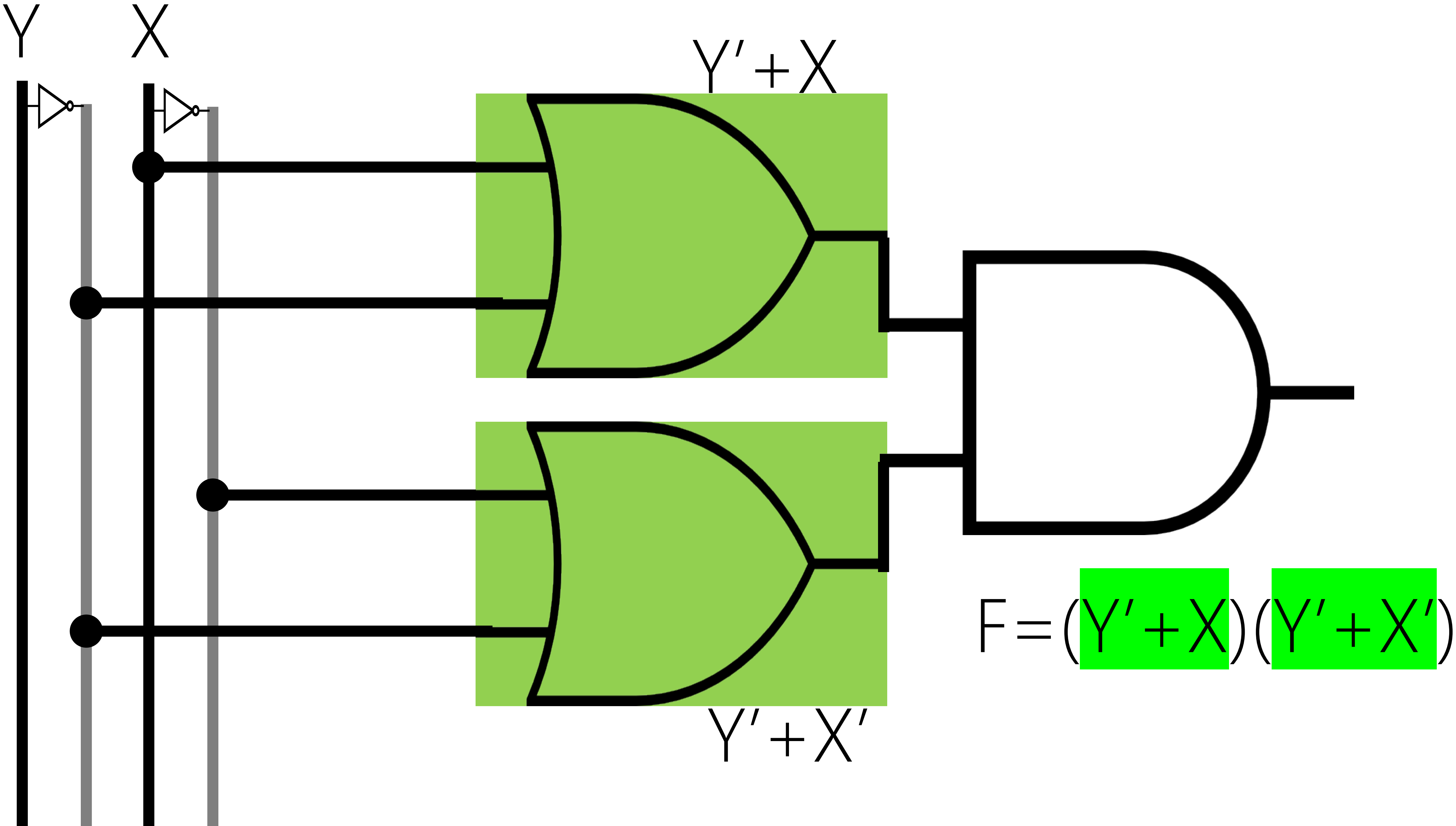
Y	X	$F = (F')' = M_2 M_3 = \prod \mathbf{M}(2,3)$
0	0	1
0	1	1
1	0	0
1	1	0

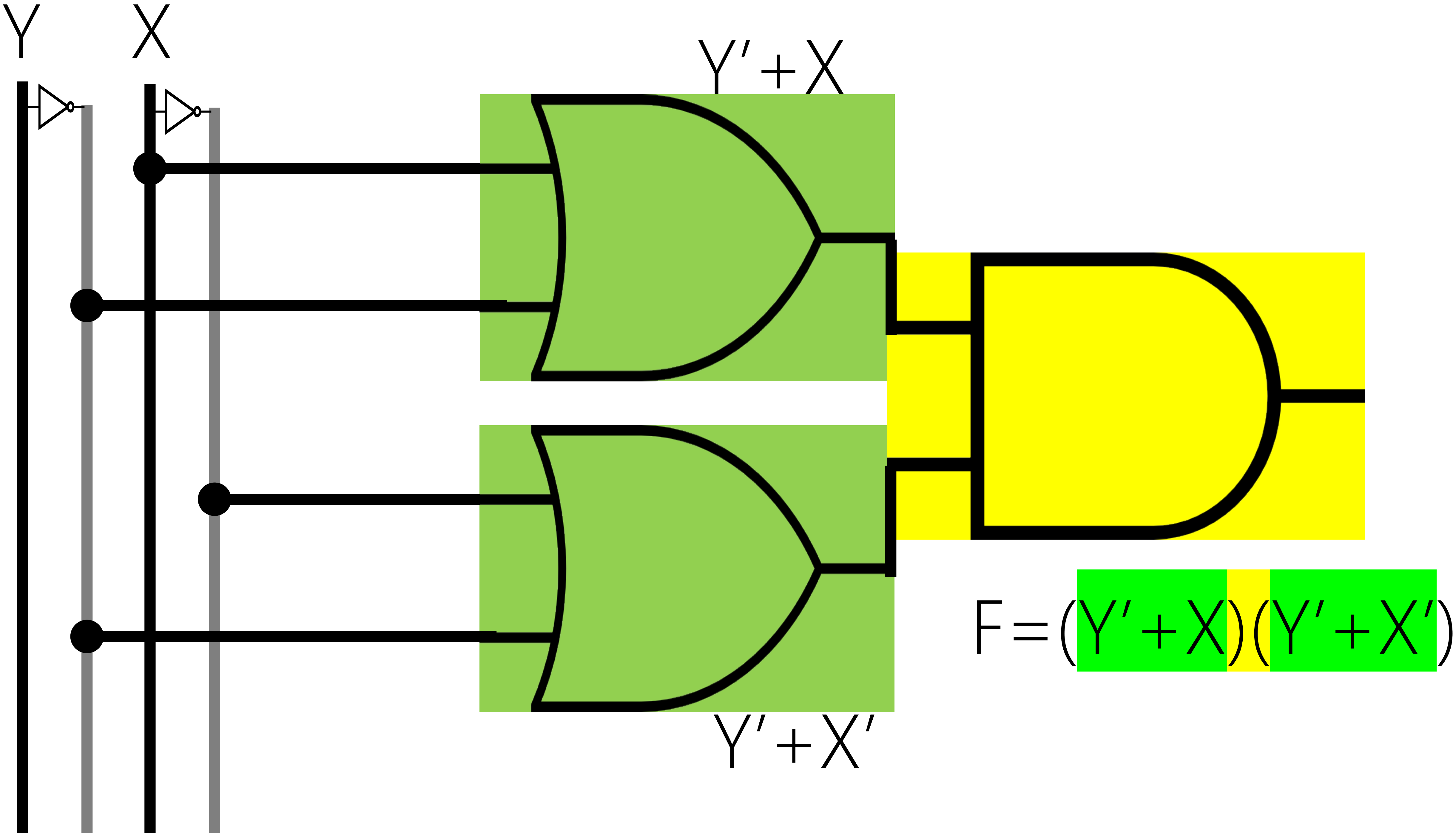
Y	X	$F=\prod M(2,3)=(Y'+X)(Y'+X')$
0	0	1
0	1	1
1	0	0
1	1	0



Y
X
Y'+X
Y'+X'
F=(Y'+X)(Y'+X')

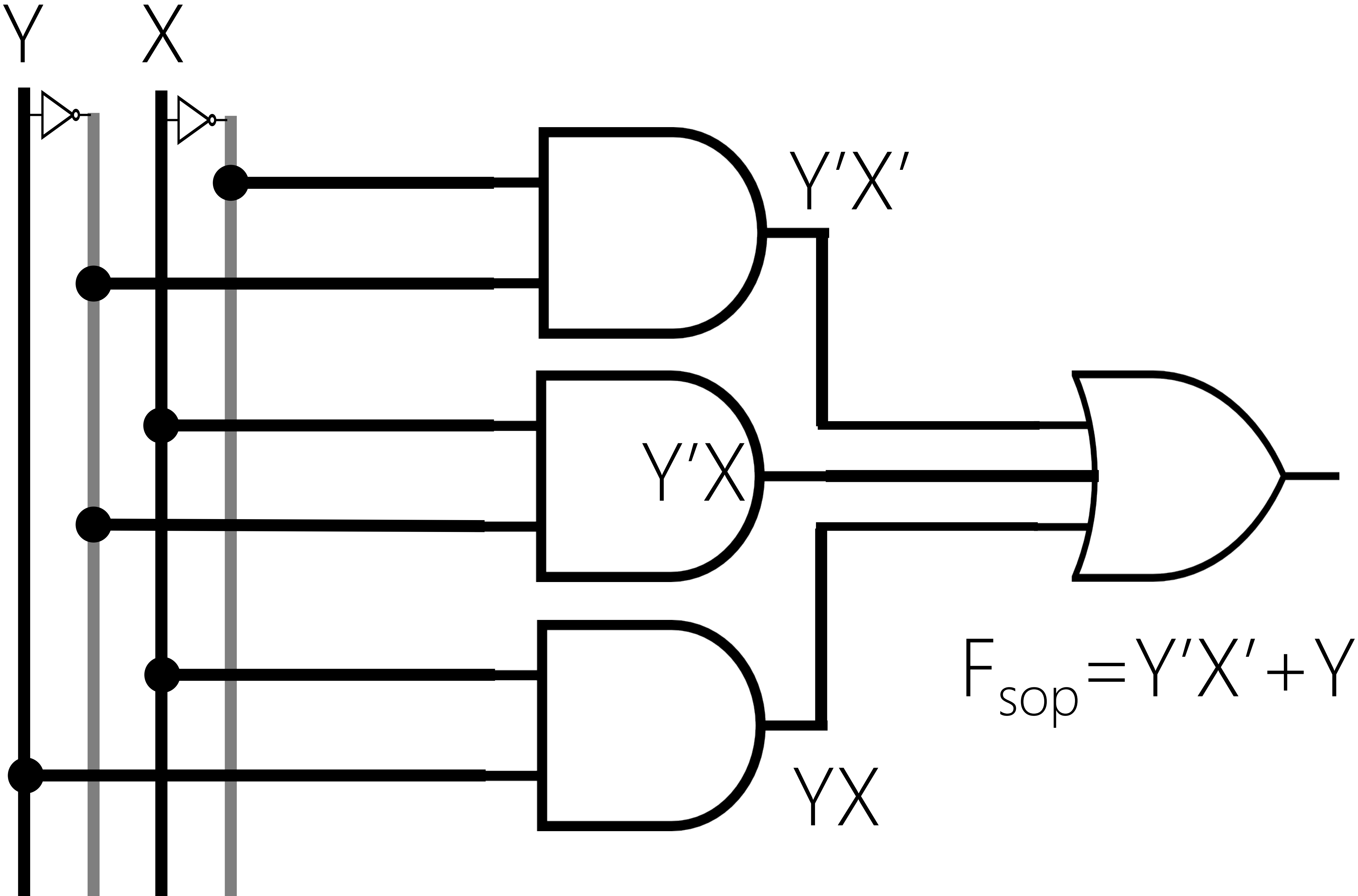
PRODUCT OF SUMS
(POS)


Y
X
Y′+X
Y′+X′
F=(Y′+X)(Y′+X′)


Y
X
Y′+X
Y′+X′
F=(Y′+X)(Y′+X′)

2 LEVELS
OR → AND

Y	X	$F=F(Y,X)=m_0+m_1+m_3=\sum \boldsymbol{m}(0,1,3)$
0	0	1
0	1	1
1	0	0
1	1	1

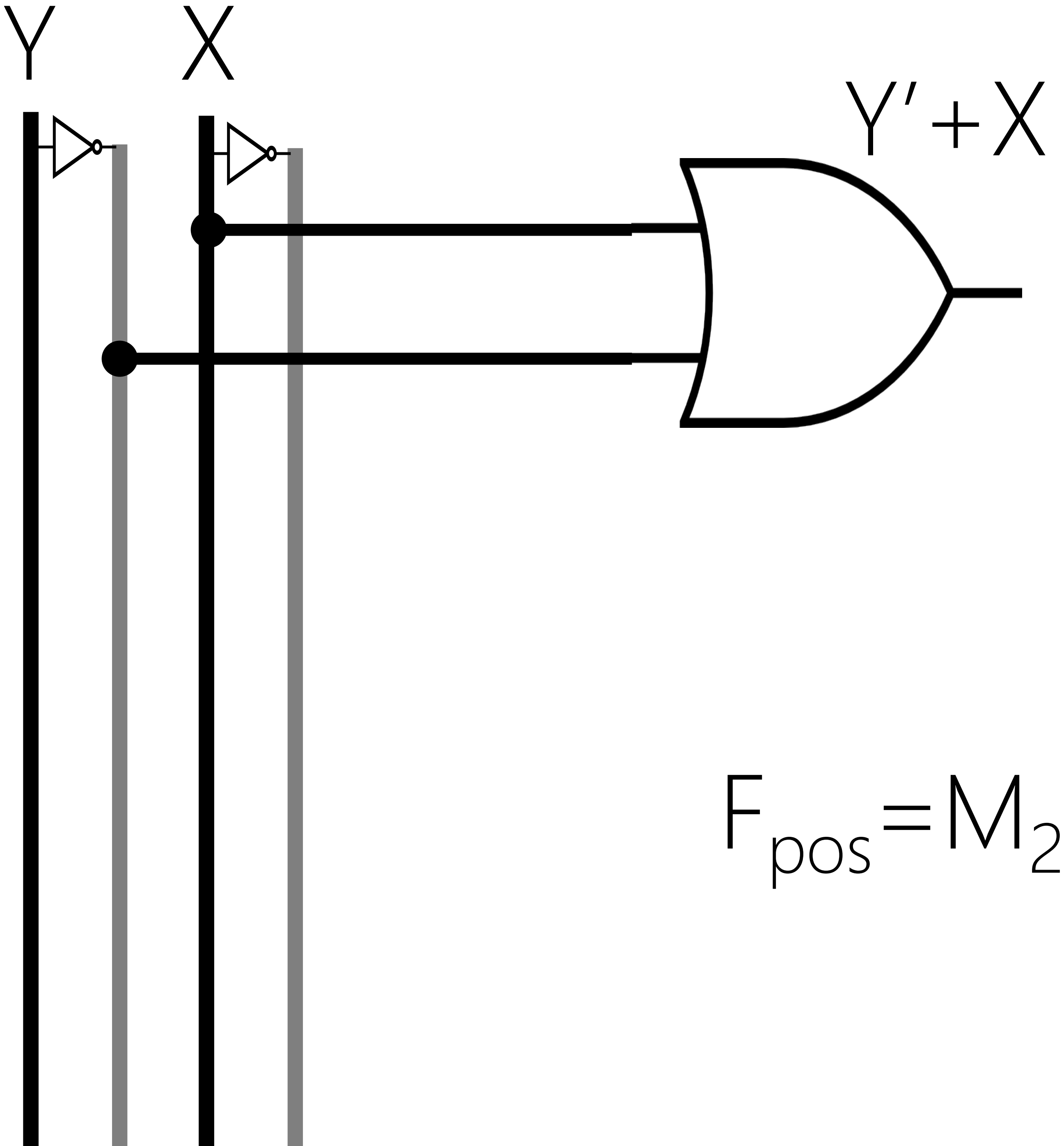


$$F_{sop} = Y'X' + Y'X + YX$$

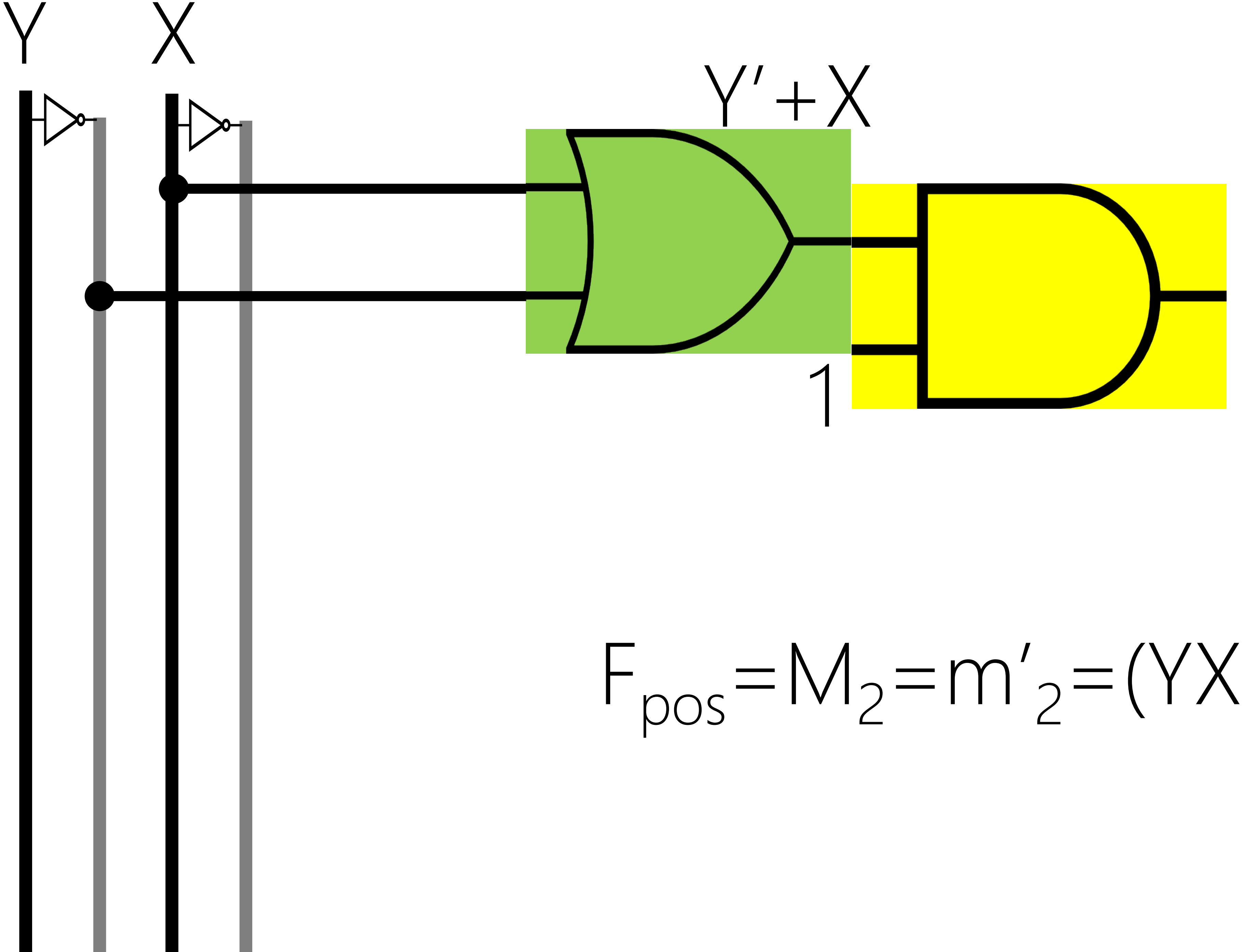
Y	X	$F=\sum \boldsymbol{m}(0,1,3)$	$F'=m_2$
0	0	1	0
0	1	1	0
1	0	0	1
1	1	1	0

Y	X	$F=\sum \boldsymbol{m}(0,1,3)$	$F'=m_2$	$(F')'=m'_2$
0	0	1	0	1
0	1	1	0	1
1	0	0	1	0
1	1	1	0	1

Y	X	$F=\sum \boldsymbol{m}(0,1,3)$	$F'=m_2$	$(F')'=M_2$
0	0	1	0	1
0	1	1	0	1
1	0	0	1	0
1	1	1	0	1



$$F_{pos}=M_2=m'_2=(YX')'=(Y'+X)$$

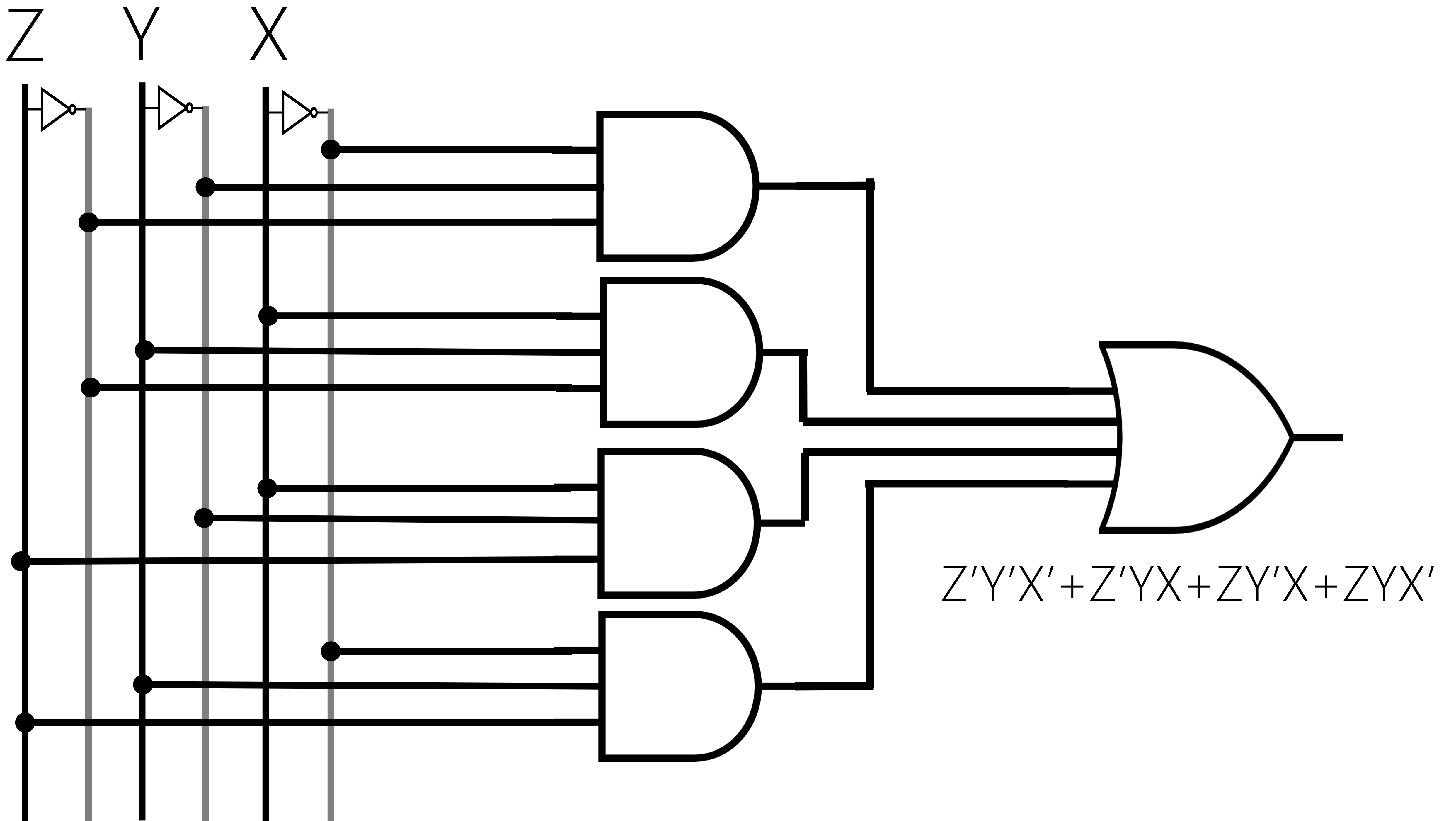


$$F_{pos}=M_2=m'_2=(YX')'=(Y'+X)(1)$$

DESIGN I vs. II
SoP vs. PoS

Lecture Assignment

Given 3 inputs, design a circuit to determine if there is even number of 1


Z
Y
X
Z'Y'X'+Z'YX+ZY'X+ZYX'

Z	Y	X	F(Z,Y,X)=?
0	0	0	?
0	0	1	?
0	1	0	?
0	1	1	?
1	0	0	?
1	0	1	?
1	1	0	?
1	1	1	?

Z	Y	X	F(Z,Y,X)=?
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

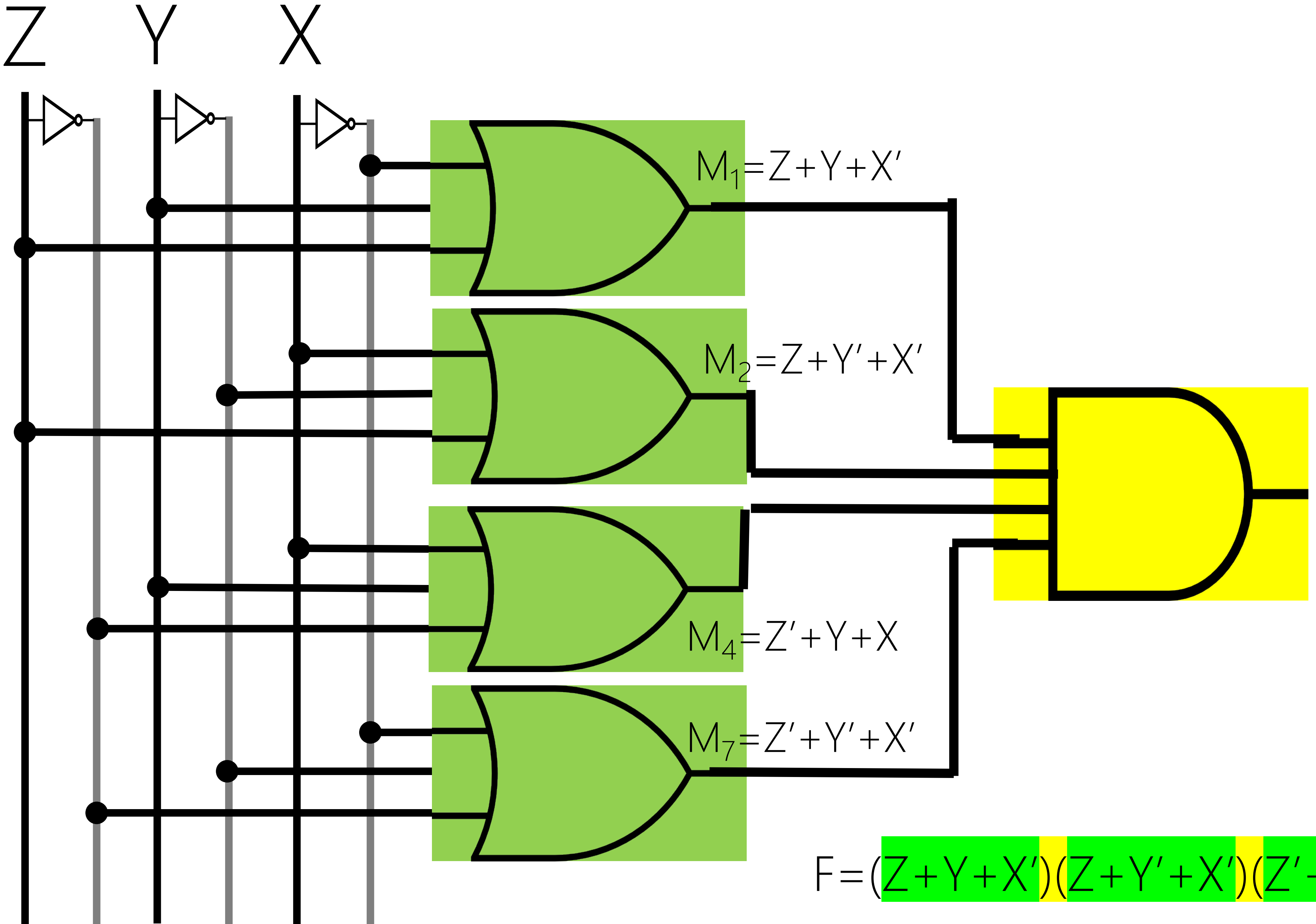
Z	Y	X	$F(Z,Y,X)=M_1$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Z	Y	X	$F(Z,Y,X)=M_1M_2$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Z	Y	X	$F(Z,Y,X)=M_1M_2M_4$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Z	Y	X	$F(Z,Y,X)=M_1M_2M_4M_7$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Z	Y	X	$F(Z,Y,X) = M_1 M_2 M_4 M_7 = \prod \mathbf{M}(1,2,4,7)$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0



$$F=(Z+Y+X')(Z+Y'+X')(Z'+Y+X)(Z'+Y'+X')$$

UNIVERSAL GATE {NOR}

Lecture Assignment

RECAP

Any Boolean Function F:

- Sum (OR) of Products (ANDs)
- Sum of minterms for Entries with 1
  - ANDs-OR
  - NAND via (F′)′

- Product (AND) of Sums (ORs)
- Product of MAXTERMS (NOT minterms) for Entries with 0
  - ORs-AND
  - NOR via (F′)′