UNIT 1 SYSTEMS OF LINEAR EQUATIONS

Structure

1.1 INTRODUCTION

In the last unit we introduced you to polynomial equations in one variable. In this unit we will start by considering linear equations in one or more variables. After that we shall consider ways of obtaining common solutions for several such equations. We call a set of linear equations a system of linear equations. Such systems of equations can arise while studying many practical problems. These include studying oscillations, the flow of currents, migration patterns, chemical contents of various solutions, input-output models of industrial production, and so on. Therefore, it is important that you spend some time studying them.

The first definite trace of systems of linear equations is found in Chui-chang Suan-shu, that is, Nine Chapters on the Mathematical Art. This is an ancient Chinese mathematical text which was probably written in 1100 B.C. Much later, in the third century B.C., the Greeks used some methods for solving certain systems of equations. Further notable developments in this area of mathematics took place in the 17th century. The Japanese mathematician Seki Kowa (around 1683) contributed greatly to the theory of systems of linear equations. About the same time the European mathematician Leibniz also discovered a method for solving systems of linear equations. In the next century the mathematicians Gauss and Cramer published methods that use the concepts of matrices and determinants for solving simultaneous equations.

In this unit we will discuss two methods for solving systems of linear equations. We will explain the method due to Cramer in the next unit.

Let us now list the objectives of this unit.

Objectives

After studying this unit you should be able to :

- obtain the solution set of a linear equations in one or more variables;
- define a system of m linear equations in n unknowns;
- apply the methods of substitution and elimination for solving simultaneous linear equations;
- choose the appropriate method, of the two methods discussed, for solving a given linear

Let us now start our discussion on linear equations.

1.2 LINEAR SYSTEMS

You know that the most general form of a **linear equation over R in one variable** x is $ax + b = 0$, $a, b \in R$, $a \neq 0$. You also know that this has a unique solution, namely, $x = -\frac{b}{a}$.

Now, can you think of a linear equation in two variables? What about $2x + 5y + 5 = 0$? According to the following definition, it is linear in two variables.

Definition: A linear equation in two variables x and y is an equation which can be written as

$$ax + by + c = 0,$$

where $a, b, c \in$ **R** and a and b are not both zero.

For example,

$-x + \frac{1}{2}y = 0$, $x = 25$ and $2s - 4t = 2$ are linear equations in two variables.

What about $xy = 1$, the equation of a hyperbola? Is it a linear equation in 2 variables? It is not, since x and y are both variables; and hence, it is not of the form

$ax + by + c = 0$, where $a, b, c \in$ **R**.

Try the following **exercises** now.

E 1) Which of the following equations are linear in 2 variables? Can you explain why ?

a) $2x + 3xy - 4y = 10$

b) $x + y^2 = 6$

c) $\sqrt{u} + v = 2$, where u and v are variables.

d) $2x = \frac{5x - 2y}{4} + 1$

E 2) "Every linear equation in one variable is also a linear equation in two variables." Is this statement true? Why or why not?

Now, what would any solution of the linear equation $2x + 3y + 1 = 0$ look like? It would consist of an ordered pair of real numbers say (a, b), such that $2a + 3b + 1 = 0$. For example, $(1, -1)$ is a solution, since $2(1) + 3(-1) + 1 = 0$.

You can check that $\left(\frac{1}{2}, -\frac{2}{3}\right)$ and $\left(-\frac{1}{2}, 0\right)$ are also solutions.

In fact, the given equation has infinitely many solutions given by $\left(x, \frac{-(2x+1)}{3}\right)$, as x varies in **R**. How do we get this general form of the solution? We can rewrite the equation as $y = \frac{-(2x+1)}{3}$. Then, for any value that we give to x, say $x = a$, we get a corresponding value $\frac{-(2a+1)}{3}$ for y. Thus, $\left(a, \frac{-(2a+1)}{3}\right)$ is a solution $\forall\, a \in$ **R**. Note that the solution set is a subset of $\mathbf{R}^2$.

Now, we could also have rewritten $2x + 3y + 1 = 0$ as $x = \frac{-(3y+1)}{2}$. Then the solution set would have been $\left\{\left(\frac{-(3y+1)}{2}, y\right) \mid y \in \mathbf{R}\right\}$. Are the two solution sets different? Not at all. If because from $-\left(\frac{3y+1}{2}\right) = x$, we get $-\left(\frac{2x+1}{3}\right) = y$ through suitable arithmetic operations.

This shows us that we can either obtain the solution set in terms of x or in terms of y.

Now, consider the equation $x - 2 = 0$ as a linear equation in two variables. What is its solution set? Whatever value y takes, x will always have to be 2. Thus, the solution set is $\{(2, y) \mid y \in \mathbf{R}\}$. It is the set of all points on the line $x - 2 = 0$ (Fig. 1). In fact,

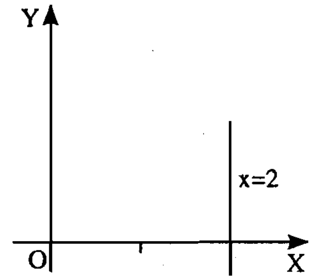


Fig. 1 : x = 2

> any linear equation in 2 variables can be geometrically represented by a straight line in the xy-plane.

Now let us define a linear equation in n variables, where $n \in \mathbf{N}$.

Definition: A linear equation over R in n variables $x_1, x_2, \ldots, x_n$ has the general form

$$a_1x_1 + a_2x_2 + \ldots + a_nx_n + b = 0.$$

where $a_1, a_2, \ldots\ldots\ldots, a_n, b \in \mathbf{R}$ and not all of $a_1, a_2, \ldots\ldots\ldots a_n$ equal zero.

Thus, $2x + 3y = 11z$ is a linear equation in 3 variables x, y and z. What does a solution of this look like? It will be ordered triple of real numbers that satisfies the equation. For example, (0, 0, 0) and (22, 0, 4) are solutions. But, (1, 1, 1) is not a solution.

Let us see what a solution of general linear equation looks like.

Definition: An n-tuple $(b_1, b_2, \ldots\ldots, b_n)$ in $\mathbf{R}^n$ is called a **solution** of the linear equation $a_1x_1 + a_2x_2 + \ldots + a_nx_n = b$, if

$$a_1b_1 + a_2b_2 + \ldots + a_nb_n = b.$$

In this case we also say that $x_1 = b_1, x_2 = b_2, \ldots\ldots, x_n = b_n$ satisfy the linear equation.

Note that the first element of the n-tuple is substituted for the first variable, the second for the second variable, and so on.

Remember that **a linear equation in two or more variables has infinitely many solutions. In general, m linear equations in n variables has infinitely many solutions, if m < n.**

Now why don't you see if you have absorbed what we have done so far.

E 3) Which of the following are solutions of $3x - 2y + 5z = 80$?

a) $(0, -40)$, (b) $(0, -40, 0)$, (c) $(2, 3, 15)$,

d) $(1, 1, \frac{79}{5})$.

E 4) Find the solution set of $x = y$. Also give its geometrical representation.

Studying only one linear equation at a time has been found inadequate for interpreting and solving real-world problems mathematically. The mathematical models of many problems consist of a set of several linear equations which need to be solved at the same time. For example, suppose the Indian Government has to suddenly send supplies of blood, medical kits, food and water to a quake-hit area. It knows the volume and weight of each unit of these items. It also knows that each aeroplane can take a maximum capacity of 600 cubic metres and a maximum weight of 20,000 Kg. These facts, put together, lead to the two equations

$$2x_1 + 3x_2 + 0.8x_3 + 0.6x_4 = 600$$
$$75x_1 + 50x_2 + 30x_3 + 35x_4 = 20{,}000.$$

where x_1, x_2, x_3, x_4 denote the number of containers of blood, medical kits, food and water respectively. We need to find common solutions to both these equations so as to get the amounts that can be sent. In other words, we need to solve these equations simultaneously. That is why we call such a set of equations **simultaneous linear equations.**

Definition: Any finite set of linear equations is called a **system of linear equations**, or a **linear system**, or **simultaneous linear equations.**

You have just seen one example involving emergency airlifting. For another example, consider the three equations.

$$\left.\begin{aligned} 2x + \frac{7}{2}y + 3z &= 1200 \\ 3x + \frac{5}{2}y + 2z &= 1150 \\ 4x + 3y + 2z &= 1400 \end{aligned}\right] \qquad \text{....(1)}$$

They form a linear system. This system is the mathematical formulation of the following problem:

A company produces 3 products, each of which must be processed through 3 divisions, A, B, and C. The number of hours taken by each unit of the product in each division, and the total number of hours available for production each week is given in Table 1

Table 1

Division	Product			Total number of hours per week
	1	2	3	
A	2	3.5	3	1200
B	3	2.5	2	1150
C	4	3	2	1400

What is the number of units of each product that should be produced so as to exhaust the weekly capacities of the 3 divisions?

How is the system (1) obtained from this problem? Well, if x, y and z denote the number of units of each product, we get the system (1).

In the following **exercises** you can see some more examples of linear systems arising from practical problems.

E 5) A dietitian is planning a noon meal for school children. It consists of 3 food types. He wants to ensure that the minimum daily requirements (MDR) for 4 vitamins are satisfied.

In Table 2 we summarise the vitamin content per unit of each food type in milligrams, and we give the MDR.

Table 2

Food Type	Vitamin content/unit (in mg.)			
	V_1	V_2	V_3	V_4
1	3	1	0	1
2	5	7	2	6
3	2	3	0	2
MDR	55	45	10	45

What is the mathematical formulation of this problem?

E 6) Thirty litres of a 50% alcohol solution are to be made by mixing 70% solution and 20% solution. We want to know how many litres of each solution should be used. Translate the problem into a linear system.

Let us now discuss what the set of solutions of a system of linear equations looks like.

Consider the following linear system in one variable:

$$cx + d = 0,$$

where $a, b, c, d \in \mathbf{R}$, $a \neq 0$, $c \neq 0$.

This will have a solution if and only if the two equations have a common solution, that is,

iff $-\dfrac{b}{a} = -\dfrac{d}{c}$. And then, $x = -\dfrac{b}{a}$ (or $-\dfrac{d}{c}$) is the unique solution.

For example, the system

$$\begin{aligned} x + 1 &= 0 \\ 3x + 3 &= 0 \end{aligned}$$

has the unique solution $x = -1$, while the system

$$\begin{aligned} 3x &= 0 \\ 2x + 5 &= 0 \end{aligned}$$

has no solution.

Now consider the system

$$\left.\begin{aligned} x + 2y &= 5 \\ x + y &= 3 \end{aligned}\right] \quad \text{.....(2)}$$

From the second equation we get $y = 3 - x$. Substituting this value in the first equation we get

$x + 2(3 - x) = 5$, that is, $x = 1$.

Then $y = 3 - 1 = 2$

So, (2) has a solution, namely, $x = 1$ and $y = 2$, that is, the ordered pair $(1, 2)$.

Now, recall that the solutions of a linear equation in two variables correspond to the points on the line representing the equation. Thus, the solutions of (2) would correspond to the points of intersection of the two lines representing the two equations. From Fig. 2(a) you can see that they intersect in only one point, namely, $(1, 2)$. Thus, (2) has a unique solution.

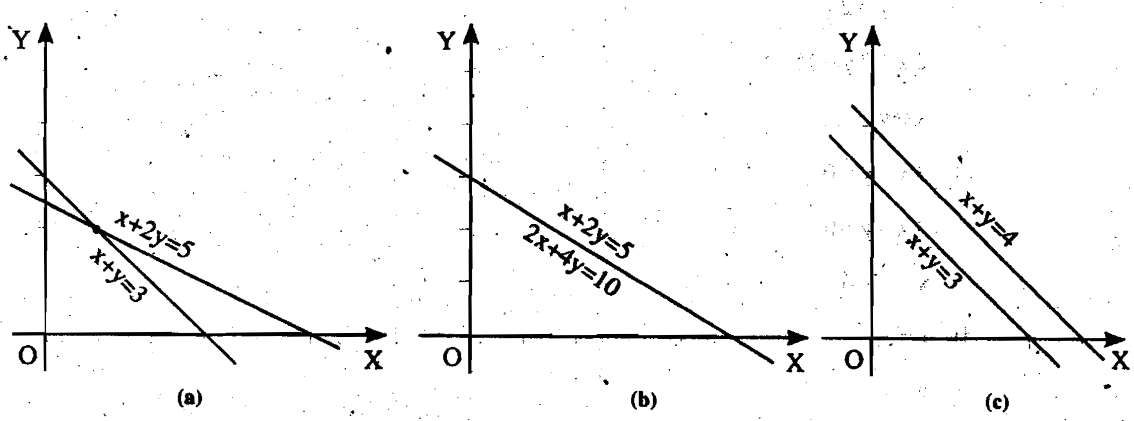


Fig. 2 : A linear system with (a) a unique solution; (b) infinitely many solutions; (c) no solution.

Now consider the system

$$\left.\begin{aligned} x + 2y &= 5 \\ 2x + 4y &= 10 \end{aligned}\right] \quad \text{....(3)}$$

You can check that for any $y \in \mathbf{R}$, the ordered pair $(5 - 2y, y)$ is a solution of (3). Thus, this system has infinitely many solutions.

Geometrically, since both the equations of (3) are multiples of each other, they represent the same line in the plane (see Fig. 2(b)). Thus, every point on the line is a common point. Hence the system (3) has infinitely many common points.

Finally, consider the system

$$\left.\begin{aligned} x + y &= 3 \\ x + y &= 4 \end{aligned}\right] \quad \text{....(4)}$$

You can see that this system of equations has no solution, since any solution would lead to the false statement $3 = 4$.
Geometrically, the two equations of (4) represent distinct parallel lines (see Fig. 2(c)). Thus, they have no point of intersection.

So you have seen three situations, namely,

i) a linear system can have a unique solution, or

ii) a linear system can have infinitely many solutions, or

iii) a linear system can have no solution.

In fact, these are the only situations possible for any system of linear equations. We shall not prove this statement here.

Now let us go back to a general linear system. We give the following definition.

Definition: If a system of linear equations has a solution, we call it **consistent**; otherwise we call it **inconsistent.**

Thus, (2) and (3) are consistent systems, while (4) is not.

Why don't you try the **following exercise** now?

E 7) Give the geometrical view of the following system of equations. Hence find out which of them are consistent.

a) $x+y=3$

$x=0$

$y=0$

b) $x+y=2$

$2x+2y=10$

$x=y$

c) $3x+y=0$

$3x-y=0$

$x-y=0$

d) $x=3$

$y=4$

Now let us discuss a method of solving a system of linear equations.

1.3 SOLVING BY SUBSTITUTION

Let us consider the following system of linear equations in one variable:

$$\left.\begin{array}{l} 3x+5=0 \\ 6x+10=0 \end{array}\right] \qquad \text{..............(5)}$$

From the first equation, we get $x = -\frac{5}{3}$. Substituting this value of x in the second equation, we get

$-5+5=0$, a true statement.

Thus, the equations in (5) are consistent, and the unique solution is $x=-\frac{5}{3}$. The method we have just used for solving (5) is called the **substitution method.**

Let us see how this method can be used for solving linear systems in two variables. Consider the system

$$\left.\begin{aligned} 2x+y&=7 \\ 5x+3y&=18 \end{aligned}\right] \quad \text{.....(6)}$$

We want to solve the equations in (6) simultaneously, that is, at the same time, **by substitution.** For this we first write one variable in terms of the other by using either of the equations. We will use the first one to write y in terms of x, as $y=7-2x$.

Then we substitute this value of y in the second equation, to get $5x+3(7-2x)=18$, that is, $21-x=18$.

This gives $x=3$.

Substituting this value of x in $y=7-2x$, we get $y=1$.

But, is (3, 1) a solution? We must double check by substituting these values in (6). We get $2\times 3+1\times 1=7$, which is true, and $(5\times 3)+(3\times 1)=18$, which is true. Thus, the system (6) has the unique solution (3, 1).

We can also solve (6) by using the second equation to write $x=\frac{18-3y}{5}$. Then substituting in the first equation, we get $2\left(\frac{18-3y}{5}\right)+y=7$, giving $y=1$.

And then $x=\frac{18-3y}{5}=\frac{18-(3\times 1)}{5}=3$.

To get some practice in solving by substitution, **try the following problems.**

E 8) Find solutions (if any) of the following sets of simultaneous equations by the substitution method.

a) $x+y=-2$

   $y=3$

b) $3a+7b=33$

   $a+3b=13$

c) $2s+t=20$

   $2s-5t=30$

d) $x+y=2$

   $2x+2y=4$

e) $3x=y+5$

   $9+y=3x$

The substitution method that we have employed for two equations in two unknowns can also be extended for solving several equations in several unknowns. But it becomes more and more difficult to apply as the number of equations and variables increase. In the next section we will discuss a better method of dealing with any number of equations in any number of variables.

Fig. 3 : Gauss in 1803

Two systems of equations are equivalent if they have the same solution set.

1.4 SOLVING BY ELIMINATION

This method of solving simultaneous linear equations is due to the great German mathematician Carl Friedrich Gauss (1777 – 1855). Because of his immense contribution to the development of mathematics, he is known as the 'prince of mathematics'. The method of solution is called **the Gaussian elimination** (or **successive elimination**) method. In this method we use multiplication and addition to **eliminate the variables**, one by one, from the equations. At each stage we transform the system of equations into an equivalent one.

Any of the following transformations are allowed:

1) changing the order of the equations of the system;
2) multiplying both sides of any equation of the system by a non-zero real number;
3) replacing an equation by the sum of that equation and a non-zero multiple of another equation in the system.

Let's work out a simple example, using this method. Consider the system

$$x+2y+x=4 \qquad \ldots(7)$$

$$3x-y-4z=-9 \qquad \ldots(8)$$

$$x+y+z=2 \qquad \ldots(9)$$

Let us begin by eliminating y from (8) and (9), by adding them. We get

$$4x-3z=-7 \qquad \ldots(10)$$

Now let us eliminate y from (7) and (8). For this we add (7) to 2 times (8). We get

$(x+2y+z)+2(3x-y-4z)=4+2(-9)$, that is,

$7x-7z=-14$

Dividing throughout by 7, we get

$$x-z=-2 \qquad \ldots(11)$$

Now, we can eliminate x from (10) and (11) by adding (–4) times (11) to (10). We get

$(4x-3z)-4(x-z)=-7-4(-2)$, that is,

$z=1.$

Substituting this value of z in (11) we get

$x=-2+1=-1.$

Substituting $x=-1$, $z=1$ in (9), we get

$y=2.$

We must verify if the ordered triple $(-1, 2, 1)$ satisfies all three equations.

On substituting this triple in each of the equations, we find that it is indeed the solution.

Whenever we use this method, or any method for solving a linear system we must keep the following remarks in mind.

Remark 1: Whenever we solve an equation or a system of equations, we must **always verify our solution.**

Remark 2: While solving a linear system, if we reach a false statement, it means that the system has no solution.

Now why don't you **try to solve** some linear systems.

E9) Solve the following systems by the Gaussian elimination method.

a) $2x+y+z=9$

$$-x-y+z=1$$
$$3x-y+z=9$$

b) $3x+4y+5z=6$
$$6x-7y=8$$
$$2x-3y-z=1$$

E 10) Solve the system that you got in E5, by elimination.

E 11) Determine, by elimination, the solution set of the system

$$-2x-y+3z=12$$
$$x-2y+5z=10$$
$$6x-3y+9z=24$$
$$5x-5y+22=0$$

In E10 and E11 you came across systems in which the number of equations was more than the number of variables. In such a situation also the system can have a unique solution, infinitely many solutions or no solution.

There can also be systems of equations with more variables than equations. Such a system will not have a unique solution. Thus, it will either be inconsistent, or it will have infinitely many solutions.

Let us consider the following example.

$$4x-y+z=0 \qquad \ldots(12)$$
$$x+y+z=5 \qquad \ldots(13)$$

We first eliminate x.

$(12)-4\times(13) \Rightarrow -5y-3z=-20$

$$\Rightarrow 5y+3z=20 \qquad \ldots(14)$$

We can't eliminate any more variables because playing around with (14) and the original system will only end in reintroducing x. Instead, we use (14) to write y in terms of z. We get

$$y=\frac{20-3z}{5}.$$

Then $(13) \Rightarrow x = 5-\left(\frac{20-3z}{5}\right)-z=\frac{5-2z}{5}.$

Substitute the triple $\left(\frac{5-2z}{5}, \frac{20-3z}{5}, z\right)$ where $z \in \mathbf{R}$, in (12) and (13), to verify that it is a solution. What do you find? for any $z \in \mathbf{R}$, the triple is a solution of the given system.

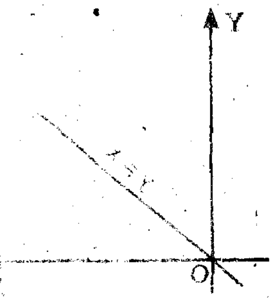
For example, when $z=0$ we get a solution $(1, 4, 0)$, and when $z=1$ we get a solution $\left(\frac{3}{5}, \frac{17}{5}, 1\right)$ and so on. Thus, the given linear system has infinitely many solutions.

We say that the solutions are $\left(\frac{5-2z}{5}, \frac{20-3z}{5}, z\right)$, where z is an arbitrary real number, or a parameter.

Now consider the system

$$x+2y+z=1 \qquad \ldots(15)$$
$$x+2y+z=-1 \qquad \ldots(16)$$

$(15)-(16) \Rightarrow 0=2$, a false statement.



Now why **don't you practice** the elimination method some more?

E 12) Solve the system you got in E6, if it is consistent.

E 13) Solve the system (1) that we gave at the beginning of Sec. 4.2.

E 14) Solve the system

$$\begin{aligned} x+y+z &= 20 \\ 10x+y-2z &= 5 \end{aligned}$$

E 15) Solve the systems

a) $$\begin{aligned} x+y+z &= 0 \\ y+2z &= 3 \end{aligned}$$

b) $$\begin{aligned} x+y+z &= 0 \\ y+2z &= 3 \\ z &= 4 \end{aligned}$$

So far we have discussed two methods of solving linear systems. In the next unit we will consider yet another method, which is specifically meant for a system of linear equations in which the number of equations is the same as the number of unknowns.

Let us now summarise what we have covered in this unit.

1.5 SUMMARY

In this unit we have discussed systems of linear equations. In particular you studied

1) what a linear system is and how it can arise from practical problems.

2) that a linear system can have a unique solution, infinitely many solutions or no solution.

3) the substitution method for solving "small" linear systems simultaneously.

4) the Gaussian elimination method, which is the method that is the most widely used.

We hope that you have tried all the exercises in the unit. You may like to see what our solutions to them are.

1.6 SOLUTIONS/ANSWERS

E 1) (a) is not, since the quadratic term xy occurs in it.

(b) is not, since the quadratic term y^2 occurs in it.

(c) is not; in fact, it is not even a polynomial equation.

(d) is linear, since it is equivalent to the linear equation $3x + 2y - 4 = 0$.

E 2) It is true because any linear equation in one variable is $ax + b = 0$, $a \neq 0$. This is equivalent to $ax + 0\,y + b = 0$, $a \neq 0$, which is linear in two variables.

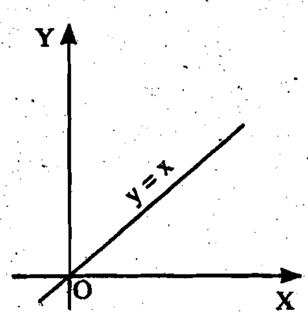


Fig. 4 : y = x.

E 3) (b) and (d) are, (a) $\in \mathbf{R}^2$, and hence can't be a solution. (c) is not, since $3(2) - 2(3) + 5(1$
80.

E 4) $\{(x, x) \mid x \in \mathbf{R}\}$

E 5) Let, x, y, z denote the units of each food type. Then

$$\begin{aligned} 3x+5y+2z &= 55 \\ x+7y+3z &= 45 \\ 2y &= 10 \\ x+6y+2z &= 45 \end{aligned}$$

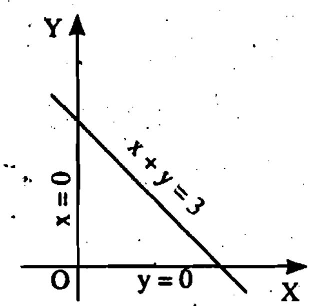


Fig. 5 : An inconsistent system

E 6) Say, we take x litres of the 70% solution and y litres of the 20% solution to make 30 litres of the 50% solution.

Then $\frac{70}{100}x + \frac{20}{100}y = \frac{50}{100} \times 30 \Rightarrow 7x + 2y = 150.$

Also $x + y = 30$.

Thus, the problem reduces to solving the linear system

$$\begin{aligned} 7x+2y &= 150 \\ x+y &= 30 \end{aligned}$$

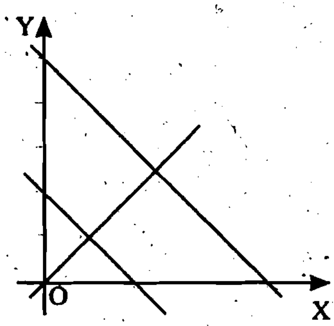


Fig. 6 : An inconsistent system

E 7) a) From Fig. 5 you can see that there is no point common to all three lines. Hence the system is inconsistent.

b) We have given the geometrical representation of this system in Fig. 6. Again, you can see that the system is inconsistent.

c) In Fig. 7 you can see that the three lines have a unique point of intersection, namely, (0, 0). Thus, the system has the unique solution (0, 0).

d) From Fig. 8 you can see that this system has the unique solution (3, 4).

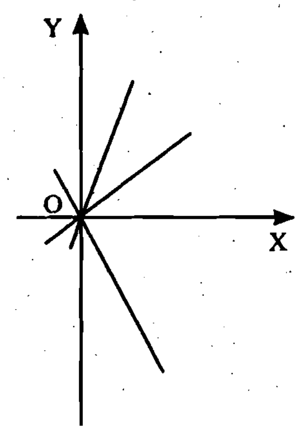


Fig. 7 :

E 8) a) The second equation says y = 3. Substituting this value in the first equation, we get $x + 3 = -2 \Rightarrow x = -5$.

$\therefore$ $(-5, 3)$ is the solution.

b) $a + 3b = 13 \Rightarrow a = 13 - 3b.$

$\therefore$ $3a + 7b = 33 \Rightarrow 3(13-3b) + 7b = 33 \Rightarrow 2b = 6 \Rightarrow b = 3.$

$\therefore$ $a = 13 - 3(3) = 4.$

$\therefore$ (4, 3) is the solution.

c) $2s + t = 20 \Rightarrow t = 20 - 2s$

$2s - 5t = 30 \Rightarrow 2s - 5(20 - 2s) = 30 \Rightarrow s = \frac{65}{6}$

$\therefore$ $t = 20\ \frac{65}{6} = -\frac{5}{3}$

$\therefore$ $\left(\frac{65}{6}, -\frac{5}{3}\right)$ is the solution.

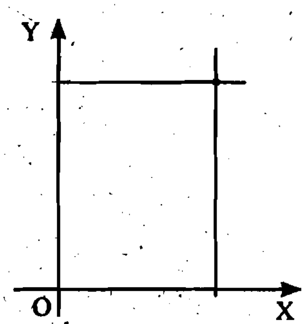


Fig. 8 :

d) $x + y = 2 \Rightarrow y = 2 - x$

$\therefore 2x + 2y = 4 \Rightarrow 2x + 2(2-x) = 4 \Rightarrow 0 = 0$

Note that the second equation is equivalent to the first one. Thus, any solution of the system is a solution of $x + y = 2$.

Thus, for any value of $x \in \mathbf{R}$, $(x, 2-x)$ is a solution. For example; (0, 2) is a solution.

This system has infinitely many solutions.

e) $3x = y + 5 \Rightarrow y = 3x - 5$

$\therefore 9 + y = 3x \Rightarrow 9 + 3x - 5 = 3x \Rightarrow 4 = 0$, a false statement.

This the system is inconsistent.

E 9) a)

$$\begin{aligned} 2x+y+z &= 9 \quad &\ldots(17) \\ -x-y+z &= 1 \quad &\ldots(18) \\ 3x-y+z &= 9 \quad &\ldots(19) \end{aligned}$$

To eliminate y we add (17) and (18). We get

To eliminate z we subtract (18) from (19). We get
$4x = 8$, that is, $x = 2$.
Substituting this value of x in (20), we get

$2z = 10 - 2 = 8 \Rightarrow z = 4$.
Then (17) gives us
$2(2) + y + 4 = 9 \Rightarrow y = 1$.
Thus, (2, 1, 4) is the solution. (Verify this!)

b) $\left(\frac{29}{41}, \frac{22}{41}, \frac{49}{41}\right)$

E 10)
$$3x + 5y + 2z = 55 \quad \ldots(21)$$
$$x + 7y + 3z = 45 \quad \ldots(22)$$
$$2y = 10 \quad \ldots(23)$$
$$x + 6y + 2z = 45 \quad \ldots(24)$$

$(23) \Rightarrow y = 5$.

Then $(21) \Rightarrow 3x + 2z = 30 \quad \ldots(25)$

and $(22) \Rightarrow x + 3z = 10 \quad \ldots(26)$

Eliminating x from (25) and (26), we get

$z = 0$.

Then $(24) \Rightarrow x + 6(5) + 2(0) = 45 \Rightarrow x = 15$.

Now we need to check if (15, 5, 0) satisfies all the equations in the system. It doesn't satisfy (21). But our calculations have been right.

Conclusion: the system is inconsistent!

The dietitian will have to alter his constraints!

E 11) $\left(-\frac{32}{15}, \frac{34}{15}, \frac{10}{3}\right)$

E 12) The system is

$$7x + 2y = 150$$
$$x + y = 30.$$

It has a unique solution, namely, (18, 12).

E 13) (200, 100, 150).

E14) The solution set is $\{(x, 15 - 4x, 3x + 5) \mid x \in \mathbf{R}\}$.

E 15) a) $\{(z - 3, 3 - 2z, z) \mid z \in \mathbf{R}\}$ is the solution set.

b) $(1, -5, 4)$ is the unique solution.