

**Definition 1.** Given categories  $\mathcal{C}$  and  $\mathcal{D}$ , a natural transformation  $\alpha$  from a functor  $F : \mathcal{C} \rightarrow \mathcal{D}$  to a functor  $G : \mathcal{C} \rightarrow \mathcal{D}$  is a family of functions  $\alpha_a : Fa \rightarrow Ga$  for all  $a \in \mathcal{C}$ , such that the naturality square condition holds: for  $a, b \in \mathcal{C}$ ,  $f : a \rightarrow b$ ,  $\alpha_b \circ Ff = Gf \circ \alpha_a$ . (Commutative diagram)

**Definition 2.** Functors from  $\mathcal{C}$  to  $\mathcal{D}$  forms a category called functor category, denoted as  $[\mathcal{C}, \mathcal{D}]$  or  $[\mathcal{C}; \mathcal{D}]$ , with functors  $\mathcal{C} \rightarrow \mathcal{D}$  as objects, and natural transformations between them as arrows.

**Example 1.** `safeHead :: List a -> Maybe a`

**Example 2.** `maybeSingleton :: Maybe a -> List a`

**Example 3.** `alpha :: (Unit -> a) -> Maybe a`, where `alpha _ = Nothing`

**Example 4.** `beta :: (Unit -> a) -> Maybe a`, where `alpha f = Just (f ())`

**Definition 3.** A terminal in  $\mathcal{C}$  is ... A product in  $\mathcal{C}$  is ...

**Definition 4.** A diagonal functor/constant functor  $\Delta_d : \mathcal{C} \rightarrow \mathcal{D}$  for some object  $d \in \mathcal{D}$  maps anything in  $\mathcal{C}$  to  $d$ , and maps every morphisms in  $\mathcal{C}$  to  $1_d$ .

**Definition 5.** A diagram  $F$  in  $\mathcal{C}$  with shape  $J$  is a functor  $F : J \rightarrow \mathcal{C}$ . A cone  $(n, \psi)$  is to  $F$  is a object  $n : \mathcal{C}$  along with a natural transformation  $\psi_x : \Delta_n x \rightarrow Fx$ .

**Definition 6.** Cones for  $F$  forms a category, with morphisms  $f : (m, \psi) \rightarrow (n, \phi)$  to be the same morphism  $f : m \rightarrow n$ . (**Cone**)<sup>1</sup>

**Definition 7.** A limit for  $F$  is the terminal in **Cone** category. In other words, a limit is a cone  $(l, \phi)$  that for every other cone  $(n, \psi)$ , there is a unique morphism  $u_n : n \rightarrow l$ .

**Example 5.** Take  $J$  to be empty category, a cone is an arbitrary object, and the limit is the terminal object.

**Example 6.** Take  $J$  to be discrete category 2, the limit is the product objects.

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<sup>1</sup>inaccurate,  $f$  needs satisfy an extra condition to be a morphism in **Cone**.