# SOME TESTS FOR HOMOSCEDASTICITY

STEPHEN M. GOLDFELD AND RICHARD E. QUANDT*
*Princeton University*

Two exact tests are presented for testing the hypothesis that the residuals from a least squares regression are homoscedastic. The results can be used to test the hypothesis that a linear [ratio] model explains the relationship between variables as opposed to the alternative that the ratio [linear] specification is correct. The first test is parametric and uses the $F$-statistic. The second test is nonparametric and uses the number of peaks in the ordered sequence of unsigned residuals. In conclusion, the results of some experimental calculations of the powers of the tests are discussed.

## 1. INTRODUCTION

CONSIDER the regression model

$$y_i = a_0 + a_1 x_{1i} + \cdots + a_m x_{mi} + u_i \tag{1}$$

where

(a) the independent variables are nonstochastic and identical in repeated samples, or if not, are distributed independently of the error term;

(b) the error term $u$ is normally distributed with $E(u_i)=0$, $E(u_i u_j)=0$ for $i \neq j$ and $E(u_i^2)=\phi_i^2\sigma^2$ where $\sigma^2$ is unknown but $\phi_i^2$ is known.

It is assumed that a sample of $n$ independent observations is available. It is known[1] that best estimates of $a_0, a_1, \cdots, a_m$ are the least-squares estimates of the coefficients in the regression of $y_i/\phi_i$ on $1/\phi i, x_{1i}/\phi_i, \cdots, x_{mi}/\phi_i$. In particular, when $\phi_i = 1$ for all $i$, the solution is given by the least squares estimates in the regression of $y_i$ on $x_{1i}, \cdots, x_{mi}$. When $\phi_i = x_{mi}$, the solution is given by the least-squares estimates in the regression

$$\frac{y_i}{x_{mi}} = a_0 \frac{1}{x_{mi}} + a_1 \frac{x_{1i}}{x_{mi}} + \cdots + a_m + v_i \tag{2}$$

The present paper gives two methods of discriminating between the two particular cases given by (1) and (2).[2]

Two illustrations from an economic context will suffice to show the importance of the choice between (1) or (2).

(a) Much attention has been devoted to the problem of predicting aggregate consumption on the basis of aggregate income on a time series basis. Investigators may face the choice between a model relating consumption in money terms to money income and the price level and a model relating real consumption (money consumption divided by the price level) to real income.

(b) Recently a great deal of interest has centered on analyzing the portfolio behavior of various economic units. In particular, portfolios have been decomposed into broad assets categories and the changes in asset composition

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[1] See [1].

[2] A somewhat less general alternative to (1) is discussed in [6, pp. 152–3] where (2) is written with $a_0 = 0$.

over time have been examined. In these models it is frequently assumed that the equilibrium holdings of all assets are homogeneous of degree one in dollar magnitudes. If $a_0=0$, then both (1) and (2) are consistent with this assumption and we must choose between them on statistical grounds.

The paper is organized as follows. Section 2 is devoted to a discussion of the tests and Section 3 presents and discusses some experimental results concerning the power of the tests.

## 2. SOME EXACT TESTS

We assume that there is given a sample of $n$ observations on $m+1$ independent variables $(x_0, \cdots, x_m)$ and a sample of $n$ observations on the dependent variable $y$. We further assume that $Y$ and $X$ are respectively $n\times 1$ and $n\times(m+1)$ matrices of the observations, $\beta$ is the $(m+1)\times 1$ vector of coefficients to be estimated, $U$ and $V$ are $n\times 1$ vectors of (unobservable) error terms and $[Y/x_m]$ and $[X/x_m]$ are matrices differing from $Y$ and $X$ only in that they contain the elements of $Y$ and $X$ each divided by the corresponding element of the last column of $X$.[3] The values of the dependent variable are thought to have been generated by one or the other of the following statistical models:

$$Y = X\beta + U \tag{3}$$

$$[Y/x_m] = [X/x_m]\beta + V. \tag{4}$$

The objective is to estimate $\beta$ and to accept one or the other (or possibly neither) model.

The ratio hypothesis is obtained directly from the linear hypothesis by division by $x_m$; hence if the linear model (3) is true and the ratio model (4) is fitted, the assumption of homoscedasticity of the residuals from the correct model implies that the residuals from the incorrectly specified model cannot be homoscedastic. To distinguish between (3) and (4) it appears natural to test both models for homoscedasticity. The following possibilities arise: (a) we cannot reject the hypothesis of homoscedasticity in either case. We shall then suspend judgment as to which model is preferable. (b) We reject the hypothesis of homoscedasticity for one but not the other case. We shall then accept the formulation leading to homoscedastic residuals as the true one. (c) We reject homoscedasticity in both cases. We are then again (as in (a)) unable to choose between the ratio and linear models but are forced to consider an enlargement of the set of possible alternatives.[4]

To test for the homoscedasticity of (3) we may proceed in one of two ways:

*A Parametric Test.* On the assumption that the error terms are normally distributed a test can be constructed in the following manner.

(a) Order the observations by the values of the variable $x_m$ which is the potential deflator; i.e., the new ordering is given in terms of the second subscript of $x_m$ indexed so that $x_{mi} \leqq x_{mj}$ if and only if $i<j$ and we then index the remaining variables so that the index values correspond with those of the $x_{mi}$.

[3] Of course the ratio hypothesis is meaningless if any $x_m$-value is zero; typically in economic applications the $x_m$-variable used as a deflator is strictly positive.

[4] There is no implication that, if we fail to reject the hypothesis of homoscedasticity, the corresponding model is "right."

(b) Given some choice of the number of central observations, $k$, to be omitted, we fit separate regressions (by least squares) to the first $(n-k)/2$ and last $(n-k)/2$ observations, provided also that $(n-k)/2>m+1$, the number of parameters to be estimated, and that the $(n-k)/2$ observations be distributed over at least $(m+1)$ distinct points in the $x$-space.

(c) Denoting by $S_1$ and $S_2$ the sum of the squares of the residuals from the regressions based on the relatively small and relatively large values of $x_m$ respectively, we form

$$R = \frac{S_2}{S_1} \cdot$$

The quantity $R$ clearly has the $F$-distribution with

$$\left(\frac{n-k-2m-2}{2}, \frac{n-k-2m-2}{2}\right)$$

degrees of freedom under the null hypothesis. Under the alternative hypothesis values of $R$ will tend to be large since, if the ratio hypothesis is true, $u=vx_m$ and $\text{Var}(u)=x_m^2 \cdot \text{Var}(v)=x_m^2 \cdot \text{constant}$. Since the values of $x_m^2$ are larger for the second set of residuals than for the first, the corresponding sum of squares of residuals will tend to be larger as well.[5]

The following observations are relevant:

(a) Since the sum of the squares of the residuals can be expressed as a quadratic form in the true errors,[6] the ratio $R$ is homogeneous of degree zero in the true error terms; hence the ratio is independent of $\sigma_u$ under the null hypothesis and $\sigma_v$ if the alternative hypothesis is true.

(b) The ratio $R$ is independent of the regression coefficients $\beta$.

(c) We normally cannot use the standard test for the homogeneity of several variances since we generally do not have repeated observations for given $x$'s.[7] In addition, the number of observations is frequently too small for an analogous multiple division of a given sample.

(d) The power of this test will clearly depend upon the value of $k$, the number of omitted observations; for very large values of $k$ the power will be small but it is not obvious that the power increases monotonically as $k$ tends to 0.[8]

(e) The power of the test will clearly depend on the nature of the sample of values for the variable which is the deflator. Thus, if the variance of $x_m$ is small relative to the mean of $x_m$ the power can be expected to be small and conversely.

*A Nonparametric Test.* A nonparametric test can be constructed in the following manner.

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[5] An analogous procedure holds when we test for the homoscedasticity of (4). The ordering procedure, of course, is carried out with $1/x_{mj}$ rather than $x_{mj}$.

If the linear model is true, then Var $(v)$ = Var $(u)/x_m^2$. However, since we order by $1/x_m$ for the ratio model, $R$ would also tend to be large if the linear hypothesis is correct and we estimate a ratio model.

It is to be noted that this test is an adaptation of the standard equality of variances test. See [3, pp. 259–60].

[6] $\Sigma\hat{u}^2 = u'(I - X(X'X)^{-1}X')u$.

[7] [2, p. 243].

[8] The reason for this conjecture is as follows. Although the two regressions will be based on relatively more observations when $k$ equals or is near zero, thus leading to high power, the inclusion of the centrally located observations will cause the residual variances to differ from each other by less than they would if $k$ were relatively large. This latter effect tends to diminish the power. It does not seem to be clear *a priori* which influence will predominate.

(a) Fit both the linear and the ratio models to the entire series of data. If the data were generated by the linear [ratio] model, the variances of residuals from the ratio [linear] model will be monotonically declining [increasing] as the value of the deflator variable increases.

(b) Confining our attention to the residuals from the linear model,[9] let $\hat{u}_i$ be the $i$th residual corresponding to the $i$th value of the deflator variable $x_i$. The set of residuals, $\{\hat{u}_j\}$, is assumed to be ordered in the following manner. As in the case of the parametric test, we order the values of the deflator variable $x_m$ so that $x_{mi} \leqq x_{mj}$ if and only if $i<j$. We then index the residuals $\hat{u}_i$ so that the index values correspond with those of $x_{mi}$.[10] Thus if $i<j$ then $\hat{u}_i$ appears to the left of $\hat{u}_j$ in the ordered list of residuals.

(c) Define a *peak* in the ordered residuals at observation $i$ to be an instance where $|\hat{u}_j| \geqq |\hat{u}_i|$ for all $i<j$. The first residual, $u_1$, does not constitute a peak.

(d) If the residuals are heteroscedastic such that the variance increases with $x_m$, the number of observed peaks will tend to be large, provided the variance of $x_m$ is large relative to $\mu_{xm}$.

(e) Otherwise, under the null hypothesis of homoscedasticity we can calculate the probability of $0, 1, \cdots, n-1$ peaks in a sequence of $n$ residuals as follows.

Define $N(n, k)$ as the number of permutations of $n$ absolute values of residuals yielding $k$ peaks. For convenience we define $N(1, 0)=1$. We make the observation that adjoining an $n$th residual to $n-1$ others can create a total of $k$ peaks in two ways: (i) if the preceding $n-1$ residuals yielded $k$ peaks and the last one creates no new one, and (*ii*) if the preceding $n-1$ residuals yielded $k-1$ peaks and the $i$th one does create an additional peak. This yields the following recursions:

$$
\begin{aligned}
N(n, n-1) &= 1 \\
N(n, n-2) &= (n-1)N(n-1, n-2) + N(n-1, n-3) \\
&\vdots \\
N(n, k) &= (n-1)N(n-1, k) + N(n-1, k-1) \\
&\vdots \\
N(n, 1) &= (n-1)N(n-1, 1) + N(n-1, 0) \\
N(n, 0) &= (n-1)N(n-1, 0).
\end{aligned}
$$

Since $n$ absolute values of residuals can appear in a total of $n!$ permutations the probability $P(n, k)$ of $n$ absolute values of residuals yielding exactly $k$ peaks is

$$P(n, k) = \frac{1}{n!} N(n, k).$$

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[9] Analogous remarks are valid if we confine our attention to the residuals from the ratio model. For a full test both procedures are undertaken.

[10] In the subsequent development it is assumed that all $x_{mi}$ are distinct. In the event, which may occur in practice, that $r$ of the $x_{mi}$ are tied, the reasonable practice seems to be to consider only the largest of the $(\hat{u}_i)$ corresponding to the tied $x_{mi}$ and to diminish the effective value of $n$ by $r-1$.

The values of $P(n, k)$ and of the cumulative probabilities

$$\sum_{i=0}^{k} P(n, i)$$

are displayed for selected values of $n$ and $k$ in Table 1.[11]

The concrete process for applying this test is then as follows: (a) we fit least squares regression lines to the data according to both the linear and ratio models; (b) we order the absolute values of the residuals from each regression such that the number of peaks will tend to be large if the hypothesis employed in fitting the particular relation (linear or ratio) is false;[12] (c) we count the number of peaks in both series of residuals and compare them with Table 1; depending on the outcome of the comparisons we accept one or the other or neither hypothesis analogously with the procedure discussed under the $F$-test.

TABLE 1. CUMULATIVE PROBABILITIES FOR THE DISTRIBUTION OF PEAKS

| | P (number of peaks $\leqq x$) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | $x=0$ | $x=1$ | $x=2$ | $x=3$ | $x=4$ | $x=5$ | $x=6$ | $x=7$ | $x=8$ | $x=9$ | $x=10$ |
| 5 | .2000 | .6167 | .9083 | .9917 | 1.0000 | | | | | | |
| 10 | .1000 | .3829 | .7061 | .9055 | .9797 | .9971 | .9997 | 1.0000 | | | |
| 15 | .0667 | .2834 | .5833 | .8211 | .9433 | .9866 | .9976 | .9997 | 1.0000 | | |
| 20 | .0500 | .2274 | .5022 | .7530 | .9056 | .9720 | .9935 | .9988 | .9998 | 1.0000 | |
| 25 | .0400 | .1910 | .4441 | .6979 | .8705 | .9559 | .9879 | .9973 | .9995 | .9999 | 1.0000 |
| 30 | .0333 | .1654 | .4001 | .6525 | .8386 | .9395 | .9815 | .9953 | .9990 | .9998 | 1.0000 |
| 35 | .0286 | .1462 | .3654 | .6144 | .8098 | .9234 | .9745 | .9929 | .9984 | .9997 | .9999 |
| 40 | .0250 | .1313 | .3373 | .5818 | .7837 | .9078 | .9674 | .9903 | .9975 | .9995 | .9999 |
| 45 | .0222 | .1194 | .3138 | .5536 | .7600 | .8930 | .9601 | .9874 | .9966 | .9992 | .9998 |
| 50 | .0200 | .1096 | .2940 | .5288 | .7383 | .8788 | .9530 | .9844 | .9956 | .9989 | .9998 |
| 55 | .0182 | .1014 | .2769 | .5068 | .7184 | .8653 | .9456 | .9813 | .9944 | .9986 | .9997 |
| 60 | .0167 | .0944 | .2620 | .4871 | .7001 | .8524 | .9384 | .9780 | .9932 | .9982 | .9996 |

### 3. AN EXPERIMENTAL CALCULATION OF POWER

Sampling experiments were performed on a simple model with one independent variable in order to obtain (experimental) estimates of the powers of the two tests. Four parameters were varied: (a) the total number of observation was either 30 or 60; (b) the number of (central) observations omitted was 0, 4, 8, 12, or 16 for the parametric test; the omission of central observations is not relevant for the nonparametric test; (c) the independent variable was identical in repeated samples and each particular sample of $x$'s was chosen from the uniform distribution with mean $\mu_x = 10, 20, 30, 40, 50$, and (d) standard deviation $\sigma_x = 5, 10, 15, 20, 25, 30$. Only those cases were used in which the actual sample of $x$'s generated satisfied $x \geqq 1$. For each $\mu_x$, $\sigma_x$ combination one sample of $x$'s was generated and for each such sample 100 samples of 30 (or 60)

[11] It is noteworthy that the recursions above also yield the number of permutations of $n$ objects with $k+1$ cycles. The numbers generated are therefore the unsigned Stirling numbers of the first kind. See [4, pp. 66–72].

[12] Thus for the linear hypothesis we order the residuals with increasing values of the deflator variable and for the ratio model with decreasing values.

TABLE 2. RELATIVE FREQUENCY OF CASES IN WHICH FALSE HYPOTHESIS IS REJECTED FOR $n=30$

| $\mu_x$ | $\sigma_x$ | $k=0$ | $k=4$ | $k=8$ | $k=12$ | $k=16$ |
|---|---|---|---|---|---|---|
| 20 | 5 | .410 | .450 | .450 | .400 | .390 |
| 30 | 5 | .240 | .190 | .240 | .210 | .210 |
| 30 | 10 | .820 | .840 | .810 | .820 | .730 |
| 30 | 15 | .990 | .990 | .990 | .990 | .980 |
| 30 | 20 | .980 | .990 | .990 | .990 | .990 |
| 40 | 5 | .140 | .140 | .100 | .130 | .080 |
| 40 | 10 | .470 | .490 | .520 | .530 | .430 |
| 40 | 15 | .860 | .920 | .920 | .900 | .800 |
| 40 | 20 | .980 | .990 | 1.000 | 1.000 | .990 |
| 50 | 5 | .150 | .150 | .150 | .150 | .060 |
| 50 | 10 | .370 | .360 | .330 | .240 | .190 |
| 50 | 15 | .450 | .490 | .510 | .510 | .500 |
| 50 | 20 | .920 | .900 | .900 | .870 | .810 |
| 50 | 25 | .940 | .950 | .950 | .960 | .930 |
| 50 | 30 | .990 | .990 | 1.000 | 1.000 | .980 |
| Average | | .647 | .656 | .657 | .647 | .605 |

$u$-values were generated from the normal distribution with zero mean and unit variance. Corresponding samples of $y$-values were calculated from[13]

$$y_i = a_0 + a_1 x_i + u_i$$

and the resulting samples of $x$ and $y$ values were used to calculate the $F$-ratio for both

$$\hat{y}_i = \hat{a}_0 + \hat{a}_1 x_i \tag{3-1}$$

and

$$\frac{\hat{y}_i}{x_i} = \hat{a}_0 \frac{1}{x_i} + \hat{a}_1. \tag{3-2}$$

Two statistics are of particular relevance: (a) the relative frequency (in 100 trials) of cases in which the false hypothesis (3-2) is rejected, this being an estimate of the power of the test and (b) the relative frequency of cases in which the correct statistical decision is reached, i.e., (3-1) is not rejected and (3-2) is, this being also an estimate of the power of the test. Since in the present experiments there is virtually no difference between the two statistics, we shall concentrate our attention on (a).

Tables 2 and 3 contain experimental estimates of the power of the parametric test for $n=30$ and $n=60$ respectively. Each case represents 100 replications of the experiment for a particular choice of a sample of $x$'s. For each case we rank the powers for the five values of $k$, the number of omitted central observations. If Kendall's coefficient of concordance $W$ is calculated for each table, duly taking into account the number of ties, we find $W=.379$ for Table 2

[13] The test, it should be recalled, is independent of the true values of the regression coefficients.

which is significantly different from zero on the .05 level and $W=.170$ for Table 3 which is not. Most of the significance of $W$ for Table 2 is due to the last column. The average powers over all cases are displayed in the last line of each table; if we pretended that we could use the ordinary test of the hypothesis that two percentages calculated from two *independent* samples are the same, we would find that the largest actual difference is significantly different from zero in Table 2 but not in Table 3. On the whole we cannot reject the null hypothesis that—within the ranges examined—the number of omitted central observations leaves power unaffected with the possible exception of the case where the number of omitted observations becomes very large compared to the total number of observations. It is nevertheless interesting to note that the highest mean power figures occur for $k=8$ when $n=30$ and $k=16$ for $n=60$.

TABLE 3. RELATIVE FREQUENCY OF CASES IN WHICH FALSE HYPOTHESIS IS REJECTED FOR $n=60$

| $\mu_x$ | $\sigma_x$ | $k=0$ | $k=4$ | $k=8$ | $k=12$ | $k=16$ |
|---|---|---|---|---|---|---|
| 10 | 5 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 20 | 5 | .700 | .720 | .690 | .710 | .780 |
| 20 | 10 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 30 | 5 | .500 | .480 | .510 | .480 | .500 |
| 30 | 10 | .960 | .970 | .970 | .960 | .960 |
| 30 | 15 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 40 | 5 | .290 | .290 | .300 | .320 | .340 |
| 40 | 10 | .740 | .750 | .740 | .770 | .770 |
| 40 | 15 | .970 | .970 | .980 | .970 | .970 |
| 40 | 20 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 50 | 5 | .310 | .290 | .290 | .310 | .310 |
| 50 | 10 | .550 | .560 | .550 | .610 | .580 |
| 50 | 15 | .920 | .920 | .920 | .940 | .960 |
| 50 | 20 | .960 | .960 | .960 | .970 | .970 |
| 50 | 25 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| Average | | .793 | .794 | .794 | .803 | .809 |

We thus concentrate now on $k=8$ for $n=30$ and $k=16$ for $n=60$. Table 4 displays the experimentally calculated power of the parametric test tabulated by the parameters of the distribution from which a sample of $x$'s was chosen for each set of 100 replications.[14] The tables confirm the conjecture that an increase in $\sigma_x$ relative to $\mu_x$ improves the power of the test. Table 4 also provides an estimate of the increase in power due to an increase in $n$.

Analogous results hold for the nonparametric peak test which was performed on precisely the same samples. Estimates of the power are displayed in Table 5. As expected, power generally increases with $n$ and $\sigma_x/\mu_x$. Perhaps more unexpectedly, the powers compare quite favorably with the parametric test, particularly when $\sigma_x/\mu_x$ and $n$ are both large. In fact, over a comparable subset

[14] The table does not contain entries in all corresponding positions since no calculations were performed when a sample of $x$'s chosen happened to contain an $x$ value less than one. Coincidentally, however, the table contains 15 cases for both $n=30$ and $n=60$.

TABLE 4. POWER OF THE PARAMETRIC TEST FOR $n=30$ $(k=8)$ and $n=60$ $(k=16)$

| $\sigma_x$ | $\mu_x$ | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|---|
| 5 | $n=30$ | — | .450 | .240 | .100 | .150 |
| | $n=60$ | 1.000 | .780 | .500 | .340 | .310 |
| 10 | $n=30$ | | — | .810 | .520 | .330 |
| | $n=60$ | | 1.000 | .960 | .770 | .580 |
| 15 | $n=30$ | | | .990 | .920 | .510 |
| | $n=60$ | | | 1.000 | .970 | .960 |
| 20 | $n=30$ | | | .990 | 1.000 | .900 |
| | $n=60$ | | | — | 1.000 | .970 |
| 25 | $n=30$ | | | | | .950 |
| | $n=60$ | | | | | 1.000 |
| 30 | $n=30$ | | | | | 1.000 |
| | $n=60$ | | | | | — |

TABLE 5. POWER OF THE NONPARAMETRIC TEST FOR $n=30$ AND $n=60$

| $\sigma_x$ | $\mu_x$ | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|---|
| 5 | $n=30$ | — | .27 | .16 | .16 | .10 |
| | $n=60$ | .75 | .37 | .24 | .18 | .21 |
| 10 | $n=30$ | | — | .33 | .30 | .18 |
| | $n=60$ | | .86 | .38 | .29 | .22 |
| 15 | $n=30$ | | | .56 | .45 | .24 |
| | $n=60$ | | | .84 | .54 | .54 |
| 20 | $n=30$ | | | .53 | .54 | .44 |
| | $n=60$ | | | — | .78 | .69 |
| 25 | $n=30$ | | | | | .52 |
| | $n=60$ | | | | | .78 |
| 30 | $n=30$ | | | | | .64 |
| | $n=60$ | | | | | — |

of cases computed for $n=60$ the power of the peak test is 63 per cent of the power of the $F$-test. It appears thus as a sensible alternative to the $F$-test when the distribution of residuals is not known.[15]

## 4. CONCLUSION

Two tests have been proposed for testing the hypothesis that a linear rather than a ratio model explains the relation between variables. A parametric $F$-test and the nonparametric peak test both rely upon testing residuals for homoscedasticity as against the alternative that residual variances are monotonic in the independent variable. It should be noted that the peak test can also be used as a two-tailed test when one examines whether the residual variances are monotonically increasing or decreasing in the independent variable. Finally, it should be mentioned that although we have not explicitly dealt with the case treated by Theil where the variance of the residuals is proportional to the square of the mean of the dependent variable,[16] the peak test appears applicable to this case as well if the residuals are considered ordered by the predicted value of the dependent variable. The power of both tests is estimated from sampling experiments and found satisfactory.

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[15] Since the peak test is relatively powerful, it would be reasonable and easy to incorporate it in standard computer programs for calculating regression coefficients.

[16] See [5].