A NEW TEST FOR HETEROSKEDASTICITY*

H. Glejser
University of Brussels

The quite general test for heteroskedasticity presented here regresses the absolute values of the residuals obtained by ordinary least-squares on some variable(s). Denoting the O.L.S. residuals by $|\hat{u}|$, one obtains, for instance, a regression like $|\hat{u}| = a + bz + \hat{\epsilon}$ where z is a variable, a and b regression coefficients and $\hat{\epsilon}$ the residuals of the new regression. We call the acceptance of a non-zero value for both a and b a case of "mixed heteroskedasticity", which we deem frequent in practice though neglected in handbooks.

The paper also summarizes another test due to S. M. Goldfeld and R. E. Quandt and examines the powers of the two by using Monte-Carlo simulations: the new test seems to compare favourably, except perhaps in the case of large samples.

1. A TEST USING THE ABSOLUTE VALUES OF THE RESIDUALS

Let

$$y = X'\beta + u \tag{1}$$

be the specification where y represents the value of the endogenous variable and the vector X' the values of the k predetermined variables, β the vector of the parameters to be estimated and u the disturbance. The u's are independently distributed with zero expectations and variances σ_u^2. We suppose that u can be written:

$$u = vP_g(z) = v\{m_0 + m_1 f(z) + m_2[f(z)]^2 + \cdots + m_g[f(z)]^g\} \tag{2}$$

where v is a random variable with zero expectation and constant variance, σ^2, and $P_g(z)$ is a polynomial of order g in any function of a mathematical variable z—usually z will be one of the variables appearing in X', say x_j, which means that x_j must be a mathematical variable. The function f and the order g are supposed to be known but at least some of the parameters, m, are unknown. Besides, the terms in the sum of (2) are all nonnegative.

In practice, $f(x_j)$ might be $x_j^{\frac{1}{2}}$ or $x_j^{-\frac{1}{2}}$ or $\log x_j$ and g would never be greater than 2. Equation (2) implies:

$$\sigma_u^2 = \sigma^2 [P_g(z)]^2 \tag{3}$$

Special cases of (2) are:

(1) $m_o \neq 0$, $m_1 = m_2 = \cdots = m_i = \cdots = m_g = 0$, which means homoskedasticity;

(2) $m_i \neq 0 (i \neq 0)$ and the rest of the m's equal zero, which is the kind of heteroskedasticity most usually assumed in textbooks [1]. In this case, homoskedastic disturbances can be obtained by using $y/[f(z)]_i$ as a regressor.

Taking absolute values and expectations in (2):

* I wish to express my gratitude to P. Hennart and A. Van Peeterssen who programmed all the Monte-Carlo experiments mentioned in this paper. It benefited also from stimulating comments of S. M. Goldfeld, R. E. Quandt, D. Smyth, J. Waelbroeck, the Editor and Associate Editor of this Journal and two unknown referees. Remaining errors are mine.

$$E(|u|) = E(|v|)\cdot P_g(z) = E(|v|)m_0 + E(|v|)m_1 f(z) + \cdots + E(|v|)m_i[f(z)]^i + \cdots + E(|v|)m_g[f(z)]^g \cdot \tag{4}$$

Since u_t, the value of the disturbance for the tth observation, is unknown, we shall try to estimate $E(|v|)m_i$ of (4) by regressing $|\hat{u}_t|$ on the $[f(z)]^i$ values. $|\hat{u}_t|$ is the absolute value of the residual obtained by estimating β of (1) by means of ordinary least-squares. Unfortunately, in

$$|\hat{u}_t| = E(|v|)\cdot P_g(z) + |\hat{u}_t| - E(|u_t|) \tag{5}$$

the disturbance $\epsilon_t = |\hat{u}_t| - E(|u_t|)$ does not generally have zero expectation. For example, suppose that u and $\hat{u}$ are normally distributed.

$$E(\epsilon) = E(|\hat{u}|) - E(|u|) = (\sigma_{\hat{u}} - \sigma_u)\left(\frac{2}{\pi}\right)^{1/2} < 0 \tag{6}$$

since $\sigma_{\hat{u}} < \sigma_u$ in the null hypothesis of homoskedasticity.

We shall neglect this bias effect, hoping that it will generally be unimportant in comparison with the first term in (5). Some small-sample cases will be studied by Monte-Carlo experiments in section III. Let us assume that, in practice, the $P_g(x)$ tested are of the form:

$$m_0 + m_1 x_j^{\frac{1}{2}} \quad \text{or } m_0 + m_1 x_j \quad \text{or } m_0 + m_1 x_j^{-\frac{1}{2}} \quad \text{or } m_0 + m_1 x_j^{-1} \tag{7}$$

Two relevant possibilities may then arise:

(1) The estimated intercept in (4), $E(|v|)m_o$, does not differ significantly from zero whereas $E(|v|)m_1$, the estimated slope, is positive and significant: accept the double hypothesis $m_0 = 0$ and $m_1 \neq 0$, i.e., the "classical" heteroskedasticity of textbooks and apply generalized least-squares to (1) by using the hypothesis:

$$E(uu') = \sigma^2 \Omega = \sigma^2 m_1^2 \begin{bmatrix} x_{1j}^{2h} & 0 & \cdots 0 \\ 0 & x_{2j}^{2h} & \cdots 0 \\ \vdots & \vdots & \\ 0 & 0 & x_{nj}^{2j} \end{bmatrix} \tag{8}$$

where $\sigma^2 m_1^2$ merely plays the role of a scale factor and where $h = \frac{1}{2}, 1, -\frac{1}{2}, -1$ according to the specification of (7);

(2) Both estimators, $E(|v|)m_0$ and $E(|v|)m_1$, are significant and $\hat{m}_0$ and $\hat{m}_1 x_j^h$ are positive over the relevant range: accept the double hypothesis $m_0 \neq 0$ and $m_1 \neq 0$, i.e., "generalized" mixed heteroskedasticity.

This means that: $\sigma_u^2 = \sigma^2(m_0 + m_1 x_j^h)^2$. The covariance matrix of u can thus be estimated as:

$$\sigma^2 \Omega^* = \sigma^2 \begin{bmatrix} (\hat{m}_0 + \hat{m}_1 x_{1j}^h)^2 & 0 & \cdots & 0 \\ 0 & (\hat{m}_0 + \hat{m}_1 x_{2j}^h)^2 & \cdots & 0 \\ & \vdots & \ddots & \\ & 0 & & (\hat{m}_0 + \hat{m}_1 x_{nj}^h)^2 \end{bmatrix} \tag{9}$$

where σ^2 merely plays the role of a scale factor. This leads to the weighted least-squares estimators

$$\beta^* = (\overline{X}'\Omega^{*-1}\overline{X})^{-1}\overline{X}'\Omega^{*-1}Y$$

where $\overline{X}$ represents the matrix of the observations of the regressors.[1] In all the other cases, the hypothesis of homoskedasticity is accepted.[2]

2. THE PARAMETRIC TEST OF GOLDFELD AND QUANDT

The approach of Goldfeld and Quandt [2] is as follows:

(a) Order the observations (x_t, y_t) by increasing values of $x_t (t=1, 2, \cdots, n)$.
(b) Given some choice of number of central observations, k, to be omitted, fit separate regressions (by least-squares) to the first $n-k/2$ and last $n+k/2$ observations of (x_t, y_t).
(c) Denoting by S_1 and S_2 the sum of the squares of the residuals from the regressions based on the relatively small and relatively large values of x_t respectively, form $R=S_2/S_1$ if, in (2) $P_g(x)$, is an increasing function of x, or alternatively $T=S_1/S_2$ if $P_g(x)$ is a decreasing function of x. (The method breaks down if $P_g(x)$ is not monotonic over the range considered.)

The quantity R (or T) clearly has the F distribution with $(n-k/2-2; n-k/2-2)$ degrees of freedom under the null hypothesis $P_g(x)=m_0>0$. Under the alternative hypothesis of $P_g(x)$ monotonically increasing (or decreasing) with x, the ratio R (or T) tends to be greater than 1.

3. AN EXPERIMENTAL COMPARISON OF POWERS

Sampling experiments were performed on a model of the form (1) in order to obtain estimates of the power of the two tests. Three parameters were first varied—the standard deviation of x, the size of the sample and the form of the heteroskedasticity:

(a) The values of the regressor were identical in repeated samples, being chosen from a normal distribution with mean 50 and a standard deviation taking the values 5, then 10 and finally 30 (3 cases). Generated values of x smaller than 1 were rejected.
(b) Each sample contained 20, then 30, then 60 observations (3 cases).
(c) The function (2) was taken successively as (8 cases):

$$u = v; \quad u = v\cdot x; \quad u = vx^{\frac{1}{2}}; \quad u = vx^{-1}; \quad u = vx^{-\frac{1}{2}};$$
$$u = v(x+10); \quad u = v(x^{\frac{1}{2}}+10); \quad u = v(x^2+50)^{\frac{1}{2}};$$

with v taken from a normal population with zero mean and unit standard deviation.

Thus, in total, 72 cases $(3\times 3\times 8)$ were studied. For every case, 100 samples

[1] An iterative procedure could be applied here, according to a suggestion made by the Editor, by regressing the absolute values of the new residuals, $|u_t^*|$, on the chosen regressor and so on.

[2] This paper was already completed when a note of R. E. Park [3] was published which shows some analogy with our approach. Starting from (2), one obtains $\sigma_u^2=\sigma^2[P_g(z)]^2$. Park assumes, $P_g(z)=z^\gamma$ and multiplies the right side of this expression by an additional disturbance term e^w where w is a well behaved disturbance. Taking logarithms

$$\ln \sigma_u^2 = \ln \sigma^2 + 2\gamma \ln z + w$$

Park proposes to use $\hat{u}_t^2$ as an estimate of $\sigma^2_{u_t}$ and to regress on $\ln z$ to estimate $\ln \sigma^2$ and γ. However, as in our case, the coefficients will be biased as $E(\ln u^2_t - \ln \sigma^2_{u_t})$ is not zero and this difference will usually be relatively high.

TABLE I. RELATIVE FREQUENCY OF CASES IN WHICH RIGHT OR APPROXIMATELY RIGHT HYPOTHESIS IS ACCEPTED FOR $\sigma_x = 5$*

Size of the sample	u is equal to (a) / Test	v (b)	vx (c)	$vx^{\frac{1}{2}}$ (c)	vx^{-1} (c)	$vx^{-\frac{1}{2}}$ (c)	$v(x+10)$ (c)	$v(x^{\frac{1}{2}}+10)$ (c)	$v(x^2+50)^{\frac{1}{2}}$ (c)
20	G − Q (*d*)	0.95	0.14	0.07	*0.15*	0.04	0.08	0.03	0.06
	G (*d*)	0.96	*0.21* and 0.00	*0.10* and 0.00	0.14 and 0.00	*0.08* and 0.00	*0.00* and *0.13*	*0.00* and *0.07*	*0.00* and *0.12*
30	G − Q (*d*)	0.98	0.16	0.07	0.15	0.06	0.10	*0.10*	0.21
	G (*d*)	0.98	*0.25* and 0.00	*0.15* and 0.00	*0.21* and 0.00	*0.10* and 0.00	*0.00* and *0.19*	0.00 and 0.09	*0.00* and *0.23*
60	G − Q (*d*)	0.97	0.19	0.11	0.15	0.17	0.19	0.05	0.18
	G (*d*)	0.97	*0.29* and 0.00	*0.16* and 0.00	*0.24* and 0.00	*0.20* and 0.00	*0.00* and *0.23*	*0.00* and *0.09*	*0.00* and *0.23*

* An italicized frequency is used to indicate which test is more powerful.
(a) The variable v has been defined in section II as normal with zero mean and unit variance.
(b) In this column, the expected value is 0.95.
(c) The hypothesis of mixed heteroskedasticity cannot be tested by the method of Goldfeld and Quandt: the figures in these columns denote the relative frequency in which the approximately right hypothesis of pure heteroskedasticity is accepted. As to our test, the first figure represents the relative frequency in which the right hypothesis is accepted and the second figure the relative frequency in which the approximately right hypothesis is accepted. (i.e., pure in lieu of mixed heteroskedasticity and vice versa).
(d) G—Q stands for the test of Goldfeld and Quandt, G for the test proposed in this article.

of vectors U and thus also of vectors Y were generated with the specification:

$$y = \beta_0 + \beta_1 x + u$$

The choice of the β's is irrelevant as the joint distribution of the $\hat{u}_t$'s does not involve β_0 and β_1.

In the test of Goldfeld and Quandt, the number k of central observations dropped is 4 for samples of size 20, 8 for samples of size 30 and 16 for samples of size 60.[3] The relevant statistic R or T was formed from the simple linear regression by the assumption that one knew whether heteroskedasticity was either increasing or decreasing.

In our test, it was found experimentally that in order to have a 5% probability for errors of type I (detection of heteroskedasticity when there is none) a significance level of approximately 11% should be taken for the two-tailed t distribution used to test the regression coefficients in (5). This is, of course, a crude result: it would have taken many more trials than were available to pin down the 5% fractile accurately. Besides, it can be expected on *a priori* grounds that the significance level is not invariant to sample size, although no indication of that kind is to be found in our simulation: with this 11% significance level, errors of type I (i.e., detection of heteroskedasticity when there is none) occur 17 times out of 300 for samples of size 20, 18 times for samples of size 30 and 10 times for samples of size 60 (see Column b of Tables I through III). Also the discrepancy from the expected number of type I errors in the test of Goldfeld

[3] The values 8 and 16 were recommended by these authors in their paper. The value 4 is our own choice.

TABLE II. RELATIVE FREQUENCY OF CASES IN WHICH RIGHT OR APPROXIMATELY RIGHT HYPOTHESIS IS ACCEPTED FOR $\sigma_x = 10$*

Size of the sample	u is equal to (a) / Test	v (b)	vx (c)	$vx^{\frac{1}{2}}$ (c)	vx^{-1} (c)	$vx^{-\frac{1}{2}}$ (c)	$v(x+10)$ (c)	$v(x^{\frac{1}{2}}+10)$ (c)	$v(x^2+50)^{\frac{1}{2}}$ (c)
20	G−Q	0.91	0.23	0.11	0.26	0.08	0.20	0.0	*0.18*
	G	0.94	*0.30* and 0.00	*0.15* and 0.00	*0.32* and 0.00	*0.10* and 0.00	0.00 and 0.20	*0.00* and *0.05*	0.00 and 0.16
30	G−Q	0.94	0.29	0.13	0.36	0.11	0.22	0.10	0.33
	G	0.92	*0.34* and 0.00	*0.17* and 0.00	*0.47* and 0.00	*0.14* and 0.00	*0.00* and *0.32*	*0.00* and *0.12*	0.00 and *0.42*
60	G−Q	0.96	*0.60*	0.19	0.59	0.24	0.42	0.10	*0.63*
	G	0.95	0.59 and 0.00	*0.31* and 0.00	*0.75* and 0.00	*0.36* and 0.00	*0.00* and *0.49*	*0.00* and *0.14*	0.00 and 0.60

* An italicized frequency is used to indicate which test is more powerful.
(a) The variable v has been defined in section II as normal with zero mean and unit variance.
(b) In this column the expected value is 0.95.
(c) Viz. (c) of Table I.

and Quandt is inconclusive: +2 for size 20, −4 for size 30 and +1 for size 60 (out of 300 samples).

Tables I, II and III present estimates of the power of the Goldfeld-Quandt test (row G–Q) and of our test (row G) in the case where the standard-deviation of x is respectively 5, 10 and 30. An *italicized* figure indicates which test is more powerful. Moreover, as our test can detect pure or mixed heteroskedasticity, two figures must be used, the first showing the relative frequency of acceptance of the right hypothesis and the second the relative frequency of acceptance of an approximately right hypothesis: in Table III, e.g., for a sample of 60 observations and for $u=v(x+10)$, i.e., mixed heteroskedasticity, the hypothesis of mixed heteroskedasticity is accepted 24 times out of 100 and that of pure heteroskedasticity (i.e., $u=vx$) 74 times. It should be pointed out that, in our tests, the absolute values of the residuals are regressed either on x and $x^{1/2}$ or on x^{-1} and $x^{1/2}$, according to whether x or $x^{1/2}$, on one hand, or x^{-1} or $x^{-1/2}$, on the other hand, appears in the distribution of u.[4] In practice, of course, we do not usually know which hypothesis to choose.

Our experiments show that there is hardly any drop in the estimated power of the test when the wrong regressor is chosen (x instead of $x^{1/2}$ or vice versa and similarly with x^{-1} and $x^{-\frac{1}{2}}$): very often, when one is significant, so is the other. One way of deciding between both, is a comparison of the correlation coefficients in (5): it appears that this criterion leads to the right conclusion about 6 times out of 10 in the Monte Carlo simulations which we made. Thus, in practice, a wrong function will often be accepted for (2). The results of the simulations show some of the implications of such errors. The relevant possibilities are then:

[4] In the case $u=v(x^2+50)^{\frac{1}{2}}$ neither x nor $x^{\frac{1}{2}}$ is, in fact, the right regressor: we chose x as approximately right.

TABLE III. RELATIVE FREQUENCY OF CASES IN WHICH RIGHT OR APPROXIMATELY RIGHT HYPOTHESIS IS ACCEPTED FOR $\sigma_x = 30$*

Size of the sample	u is equal to (a) / Test	v (b)	vx (c)	$vx^{\frac{1}{2}}$ (c)	vx^{-1} (c)	$vx^{-\frac{1}{2}}$ (c)	$v(x+10)$ (c)	$v(x^{\frac{1}{2}}+10)$ (c)	$v(x^2+50)^{\frac{1}{2}}$ (c)
20	G−Q	0.95	*0.69*	0.23	*0.75*	0.19	0.45	*0.14*	*0.85*
	G	0.93	0.68 and 0.01	*0.26* and 0.00	0.65 and 0.12	*0.35* and *0.02*	*0.01* and *0.45*	0.00 and 0.13	0.03 and 0.66
30	G−Q	0.94	*0.96*	0.41	*1.00*	*0.84*	0.68	0.08	*0.95*
	G	0.92	0.81 and 0.08	*0.48* and 0.00	0.76 and 0.22	0.76 and 0.04	*0.10* and *0.62*	*0.00* and *0.15*	0.08 and 0.85
60	G−Q	0.96	1.00	0.67	*1.00*	*0.93*	0.97	0.18	0.99
	G	0.98	0.86 and 0.14	*0.84* and 0.00	0.67 and 0.33	0.90 and 0.02	*0.24* and *0.74*	*0.00* and *0.24*	*0.21* and *0.76*

* An italicized frequency is used to indicate which test is more powerful.
(a) The variable v has been defined in section II as normal with zero mean and unit variance.
(b) In this column, the expected values is 0.95.
(c) Viz. (c) of Table I.

(1) That the wrong function of pure heteroskedasticity is accepted (i.e., $u = vmx$ instead of $u = vmx^{1/2}$ or conversely); as could be expected, the efficiency of the estimator is still higher than that of the ordinary least-squares estimator. For instance, in case I of table IV, the mean square error of the estimated slope using the BLUE estimator is 0.028: it rises to 0.033 when one accepts $u = vmx$ instead of $u = vmx^{1/2}$ and to 0.035 with ordinary least-squares.

(2) That mixed heteroskedasticity is accepted instead of pure heteroskedasticity: here again, the efficiency of the estimator is increased in comparison to ordinary least-squares.

(3) That pure heteroskedasticity is accepted in lieu of mixed heteroskedasticity: this can lead either to an increase or to a decrease in efficiency with respect to ordinary least-squares. The first possibility is exemplified by case III of table IV where $u = v(7+0.005x)$ and where the hypothesis $u = vmx^{1/2}$ leads to a standard error for the slope of 0.057; $u = vmx$ yields 0.060 and ordinary least-squares ($u = vm$) give 0.066 for that standard error. On the contrary, in case IV of table IV where $u = v(22+0.2x)$, hypothesis $u = vmx^{1/2}$ leads to a standard error of 0.237, $u = vmx$ to 0.223 and ordinary least-squares to 0.215.

This illustrates the importance of detecting mixed heteroskedasticity—which, in our opinion, must be frequent in practice.

Unfortunately, in this respect, our test only seems to help when:

—the size of sample is not too small (30 or more)
—the standard deviation of x is large (30 when the mean of x is 50)
—x appears with power no less than 1 in absolute value in the formula of u.[5]

[5] However, under those conditions, mixed (in lieu of pure) heteroskedasticity is quite frequently accepted. For instance, in table III, for sample size 60, $\sigma_x = 30$ and $u = vx$, the test leads to the acceptance of pure heteroskedasticity (which is correct) in 86% of cases and of mixed heteroskedasticity ($u = m_0 + m_1 x$) in 14% of cases. But as stated in the text, such an error means already an improvement in efficiency compared with ordinary least-squares.

TABLE IV. EFFECTS OF ERROR IN THE HETEROSKEDASTICITY FUNCTION ON THE EFFICIENCY OF THE ESTIMATORS

Model: $y=10+0.9\,x+u;\ x=100\ (10)\ 290$
Sample Size: 20; *Number of samples drawn:* 200
Significance level of t – test: 0.05
which corresponds to a probability of type I error close to 0.035

Procedure	Number of samples to which the procedure was applied	Mean square error of the estimated constant term	Mean square error of the estimated slope
Case I: $u=v(0.7x)^{1/2}$: procedure (B) should ideally be applied			
(A) Ordinary least-squares (O.L.S.)	200	6.74	0.035
(B) Weighted least-squares (W.L.S.) with $u=v(mx^{1/2})$	22	5.52	0.028
(C) Weighted least-squares (W.L.S.) with $u=v(mx)$	22	6.37	0.033
Case II: $u=v(0.6x)$: procedure (C) should ideally be applied			
(A) O.L.S.	200	87.2	0.495
(B) W.L.S. with $u=v(mx^{1/2})$	62	75.8	0.454
(C) W.L.S. with $u=v(mx)$	65	73.0	0.415
Case III: $u=v(7+0.005x)$ = W.L.S. with mixed heteroskedasticity never detected—should ideally be applied			
(A) O.L.S.	200	11.4	0.066
(B) W.L.S. with $u=v(mx^{1/2})$	34	9.1	0.057
(C) W.L.S. with $u=v(mx)$	35	9.9	0.060
Case IV: $u=v(22+0.2x)$: W.L.S. with mixed heteroskedasticity never detected—should ideally be applied			
(A) O.L.S.	200	36.8	0.215
(B) W.L.S. with $u=v(mx^{1/2})$	35	38.9	0.237
(C) W.L.S. with $u=v(mx)$	39	36.3	0.223

As can be seen from the *italicized* figures, the test of Goldfeld and Quandt usually seems less powerful than ours, except for pure heteroskedasticity when σ_x is large (see Table III). Out of 21 relevant cases, in each table, their F test performs best:

—in 2 cases for $\sigma_x=5$ (Table I)
—in 33 cases for $\sigma_x=10$ (Table II)
—in 11 cases for $\sigma_x=30$ (Table III).

This could be explained by the fact that the test of Goldfeld and Quandt takes no advantage of the variations of σ_u *inside* each of the two groups of observations considered.[6,7] The power of both tests is very low—practically always smaller than 20% for $\sigma_x = 5$ and sample size $= 20$ (see Table I).

4. GENERALIZATION OF THE TEST

When several variables $(z_1, z_2, \cdots, z_p)$ appear in the specification of u in equation (2), the generalization of our test is straightforward. Equation (2) is then replaced by:

$$u = V[m_0 + m_1 f_1(z) + m_2 f_2(z) + \cdots m_i f_i(z) + \cdots + m_k f_k(z)] \qquad (10)$$

where $f_i(z)$ represents a specified function of the vector $z = (z_1, z_2, \cdots, z_p)$.
Equation (5) then becomes:

$$|\hat{u}| = E(|v|)[m_0 + m_1 f_1(z) + m_2 f_2(z) + \cdots + m_k f_k(z)] + \epsilon \qquad (11)$$

the parameters of which are estimated by ordinary least-squares: the regressors with nonsignificant coefficients are dropped and the remaining parameters are reestimated. The estimated covariance matrix of u, Ω, takes thus a form similar to (9) which can be used in the estimation by weighted least-squares.

5. CONCLUSION

A new test for heteroskedasticity, using the absolute values of the residuals obtained from least-squares regression, has been proposed here. It can be utilized whether heteroskedasticity is due to one or several variables. Sampling experiments indicate that, in most cases, it compares favorably with the one due to S. M. Goldfeld and R. E. Quandt. They also show that the efficiency of the estimators is in general substantially increased in comparison with ordinary least-squares even when the wrong function of heteroskedasticity is used. An exception to this rule might be the case when pure heteroskedasticity—i.e., a variance of u equal a constant times, say, x—is assumed whereas mixed heteroskedasticity—i.e., the same type as before plus a constant term—is present in fact.

REFERENCES

[1] Goldberger, A. S. (1963). *Econometric Theory.* New York: John Wiley and Sons, pp. 235–236.
[2] Goldfeld, S. M., and R. E. Quandt (1965) "Some tests for homoskedasticity," *Journal of the American Statistical Association*, 60, 539–547.
[3] Park, R. E. (1966) "Estimation with heteroscedastic error terms," *Econometrica*, 34, 888.

---

[6] This explanation has been suggested to us by J. Waelbroeck.

On the other hand, the experiments show that the test of Goldfeld and Quandt often detects heteroskedasticity when ours does not: thus, out of 300 samples of 20 observations with $\sigma_u = 5$, and $u = vx$, heteroskedasticity was detected

—19 times by both tests
—30 times by our test only
—17 times by the test of Goldfeld and Quandt only.

One could thus think of utilizing the two at the same time. Unfortunately errors of type I, i.e., detection of heteroskedasticity when there is none, are seldom committed by both tests simultaneously: we found 20 of them in 900 samples ($2\frac{1}{3}$%). The probability of errors of type I for either of the two tests when both are applied thus reaches $7\frac{2}{3}$%.

[7] Similar Monte-Carlo experiments suggest that all the above conclusions hold true when x is a random instead of a mathematical variable.