FEEDBACK CONTROL SYSTEMS FINAL REPORT
CARAVAN CONTROL

TUCKER HAYDON AND CONNOR BRASHAR

ABSTRACT. This is the project abstract.

1. SCENARIO

You are an engineer in charge of designing and implementing an estimation and control system for a fleet of self-driving semi-trucks. Due to government regulation, the truck fleet can only operate in the self-driving mode when they are on long, straight stretches of highway between cities. In order to reduce costs and conserve fuel, the trucks drive very closely together at a pre-determined speed in 'caravan formation'. By driving very closely together, the trucks can draft off of one another, reduce drag, and save fuel by upwards of 21% [1].

For the system you are designing, only three trucks will be in the caravan. The caravan is equipped with the following sensor suite:

(1) The lead truck is equipped with a GPS receiver that measures its position at 1 Hz.
(2) The two following trucks are equipped with range sensors that measure the relative position between themselves and the truck in front of them at 10 Hz.
(3) All three trucks are equipped with an IMU that measures respective acceleration at 100 Hz.

Each truck may be independently controlled and may be instructed to either accelerate or decelerate.

2. NOMINAL SYSTEM DESCRIPTION

Define the system state.

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ v_1 \\ v_2 \\ v_3 \end{bmatrix} \tag{1}$$

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Define the system input.

$$\vec{u} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \tag{2}$$

Define the system dynamics.

$$\dot{\vec{x}} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{A} \vec{x} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{B} \vec{u} \tag{3}$$

Define the system observer equation.

$$\vec{y} = \begin{bmatrix} x_1 \\ \Delta x_{12} \\ \Delta x_{23} \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{C} \vec{x} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{D} \vec{u} \tag{4}$$

2.1. **Observability.** Using the nominal system, determine whether or not the system is observable. The nominal system is linear and time-invariant, so the observability Grammian is simplified:

$$W_o = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} \tag{5}$$

If the rank of $W_o = n$ then the system is observable. **The nominal system is observable**.

2.2. **Controllability.** Using the nominal system, determine whether or not the system is controllable. The nominal system is linear and time-invariant, so the controllability Grammian is simplified:

$$W_c = \begin{bmatrix} B & AB & A^2B & \dots & A^{n-1}N \end{bmatrix} \tag{6}$$

If the rank of $W_c = n$ then the system is controllable. **The nominal system is controllable**.

2.3. **Conclusion.** The fact that the nominal system is both observable and controllable indicates that it is well-designed and amenable to both estimation and control.

3. Estimator

TODO Connor

4. Control System

Controllers drive systems to the origin. The nominal system, however, should not be driven to the origin. Instead an error-state system should be specified where the error is the deviation of the nominal system from a reference signal.

First, define the reference signal. From the problem statement, the goal is to maintain a specified distance between the trucks and to keep that at a constant velocity.

$$\vec{x_r} = \begin{bmatrix} x_{1_r} \\ \Delta x_{12_r} \\ \Delta x_{23_r} \\ v_{1_r} \\ v_{2_r} \\ v_{3_r} \end{bmatrix} \tag{7}$$

The error signal is the difference between the nominal and the reference states. Note that not all states have a corresponding reference signal.

$$\vec{x_e} = \begin{bmatrix} x_1 - x_2 - \Delta x_{12_r} \\ x_2 - x_3 - \Delta x_{23_r} \\ v_1 - v_{1_r} \end{bmatrix} \tag{8}$$

The error signal is an affine transform using both the nominal state and the reference signal. The error signal dynamics don't conform to the standard $\dot{x} = Ax + Bu$ form — there is an affine transform between the error dynamics and the nominal and reference signals. However, the standard form can be composed via the following trick: append the reference signal to the nominal state and define zero dynamics and full observability for the reference substate.

Stack the nominal and reference states into the *stacked state*.

$$\vec{x_s} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ v_1 \\ v_2 \\ v_3 \\ \Delta x_{12_r} \\ \Delta x_{23_r} \\ v_{1_r} \end{bmatrix} \tag{9}$$

Define the stacked state dynamics.

$$\dot{\vec{x}}_s = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ a_1 \\ a_2 \\ a_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{A_s} \vec{x}_s + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{B_s} \vec{u} \tag{10}$$

Define the stacked state observer equation. Note that the reference states are specified and fully known so there is full state feedback for these states.

$$\vec{y} = \begin{bmatrix} x_1 \\ \Delta x_{12} \\ \Delta x_{23} \\ a_1 \\ a_2 \\ a_3 \\ \Delta x_{12_r} \\ \Delta x_{23_r} \\ v_{1_r} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{C_s} \vec{x}_s + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{D_s} \vec{u} \tag{11}$$

Now define the error state by linearly transforming the stacked state via a *similiarity transform*.

$$\vec{x}_e = \begin{bmatrix} x_1 \\ \Delta x_{12_e} \\ \Delta x_{23_e} \\ v_{1_e} \\ v_2 \\ v_3 \\ \Delta x_{12_r} \\ \Delta x_{23_r} \\ v_{1_r} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{T} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ v_1 \\ v_2 \\ v_3 \\ \Delta x_{12_r} \\ \Delta x_{23_r} \\ v_{1_r} \end{bmatrix} \tag{12}$$

Now, leverage the similarity transform to define the error state dynamics.

$$\dot{\vec{x}}_e = TA_sT^{-1}\vec{x}_e + TB_s\vec{u} \tag{13}$$

$$y = CT^{-1}\vec{x}_e + D\vec{u} \tag{14}$$

Finally, clean up the equation by redefining the system.

$$\begin{aligned} \vec{x} &\coloneqq \vec{x}_e \\ A &\coloneqq TA_sT^{-1} \\ B &\coloneqq TB_s \\ C &\coloneqq C_sT^{-1} \\ D &\coloneqq D \end{aligned}$$

Now the system is in the standard form.

$$\dot{\vec{x}} = A\vec{x} + B\vec{u} \tag{15}$$

$$y = C\vec{x} + D\vec{u} \tag{16}$$

5. Simulation

6. Results

References

[1] C. Bonnet and H. Fritz, "Fuel consumption reduction in a platoon: Experimental results with two electronically coupled trucks at close spacing," tech. rep., SAE Technical Paper, 2000.

The University of Texas at Austin

E-mail address: thaydon@utexas.edu, connor.brashar@utexas.edu