



Examenul final are 20 de întrebări de tipuri
cu un singur răspuns corect.
Plecarea întrebării are în medie 3
răspunsuri posibile.
Poziția examenului înaintește datele în secvenție

Part II: Concurrency Control

- 3 Concurrency Control: Notions of Correctness for the Page Model
- 4 Concurrency Control Algorithms
- 5 Multiversion Concurrency Control
- 6 Concurrency Control on Objects: Notions of Correctness
- 7 Concurrency Control Algorithms on Objects
- 8 Concurrency Control on Relational Databases
- 9 Concurrency Control on Search Structures
- 10 Implementation and Pragmatic Issues

3. Concurrency Control – Notions of Correctness for the Page Model

- 3.2 Canonical Synchronization Problems
- 3.3 Syntax of Histories and Schedules
- 3.4 Correctness of Histories and Schedules
- 3.5 Herbrand Semantics of Schedules
- 3.6 Final-State Serializability
- 3.7 View Serializability
- 3.8 Conflict Serializability
- 3.9 Commit Serializability
- 3.10 An Alternative Criterion: Interleaving Specifications
- 3.11 Lessons Learned

Lost Update Problem

| P1 | Time | P2 |
|----------------|-----------------|----------------|
| $r(x)$ | \downarrow | |
| $x := x + 100$ | $\frac{1}{2}$ | |
| $w(x)$ | \downarrow | $r(x)$ |
| | $\frac{4}{5}$ | $x := x + 200$ |
| | \downarrow | |
| | $\frac{6}{7}$ | $r(x)$ |
| | \downarrow | $w(x)$ |
| | $\frac{8}{9}$ | |
| | \downarrow | |
| | $\frac{10}{11}$ | |

↑
update "lost" \Rightarrow we lose the consistency from ACID

Observation: problem is the interleaving $r_1(x) \ r_2(x) \ w_1(x) \ w_2(x)$



index 1 \Rightarrow transaction 1
index 2 \Rightarrow transaction 2

$r(x) \Rightarrow$ read information x from DB
 $w(x) \Rightarrow$ write information x to DB
 $a(x) \Rightarrow$ abort operation

A transaction is $r(x) + w(x)$

Inconsistent Read Problem

| P1 | Time | P2 |
|------------------|-----------------|---------------|
| $sum := 0$ | \downarrow | |
| $r(x)$ | $\frac{1}{2}$ | $r(x)$ |
| $r(y)$ | $\frac{3}{4}$ | $x := x - 10$ |
| $sum := sum + x$ | $\frac{5}{6}$ | $w(x)$ |
| $sum := sum + y$ | $\frac{7}{8}$ | |
| | \downarrow | $r(y)$ |
| | $\frac{9}{10}$ | $y := y + 10$ |
| | \downarrow | $w(y)$ |
| | $\frac{11}{11}$ | |

↑
"sees" wrong sum

Observations:

problem is the interleaving $r_2(x) \ w_2(x) \ r_1(x) \ r_1(y) \ r_2(y) \ w_2(y)$
no problem with sequential execution

The scheduler we want to model
should not interleave operations.

Dirty Read Problem

| P1 | Time | P2 |
|--------------------------|------|--------------------------|
| $r(x)$ $x := x + 100$ | 1 | |
| $w(x)$ | 2 | |
| failure & rollback | 3 | |
| | 4 | $r(x)$ $x := x - 100$ |
| | 5 | |
| | 6 | $w(x)$ |
| | 7 | |

↑
cannot rely on validity
of previously read data

Observation: transaction rollbacks could affect concurrent transactions

Usually we will consider that all transactions commit.

In this slide we assume that operation $W_1(x)$ aborts and should be rolled back

$r_1(x) \quad W_1(x) \quad r_2(x) \quad a_1(x) \quad W_2(x)$
↑
 $a_1(x)$ should happen before $r_2(x)$

Chapter 3: Concurrency Control – Notions of Correctness for the Page Model

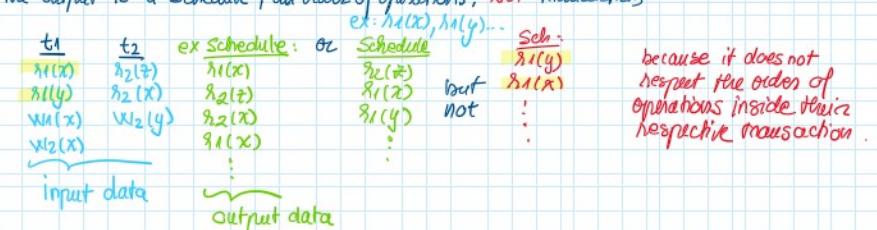
- 3.2 Canonical Synchronization Problems
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We want to order operations from the transactions.

The scheduler is an algorithm

We input transactions into the scheduler

The output is a Schedule, an order of operations, NOT transactions



We have 2 criteria for a schedule

- 1) **Correctness:** we pick only from schedules which order the operations in such a way so that the order of the operations reflect the ACID properties of the transaction they are a part of.
- (a) $\forall s \in \text{Schedules}$:
 (i) $\forall t_i \in s: \forall p_i \in op_t \forall c_i \in op_t: p_i \leq c_i$ (all operations in the history belong only to the transactions they are a part of)
 (ii) $\forall t_i \in s: \forall p_i \in op_t: \forall c_i \in op_t: p_i \leq c_i \Rightarrow \text{if } p_i \neq c_i \text{ then } p_i \leq c_i \Rightarrow \text{if a transaction commits, then it cannot abort, and vice versa}$
 (iii) $\forall t_i \in s: \forall p_i \in op_t: \forall c_i \in op_t: p_i \leq c_i \Rightarrow \text{at most one of the two operations of a transaction can be a write}$
 (iv) $\forall t_i \in s: \forall p_i \in op_t: \forall c_i \in op_t: p_i \leq c_i \Rightarrow \text{at least one of them is a write and both access the same data item: } p_i \leq c_i \text{ or } q_i \leq p_i$
 (v) $\forall t_i \in s: \forall p_i \in op_t: \forall c_i \in op_t: p_i \leq c_i \Rightarrow \text{operations are partially ordered}$

- 2) **Efficiency:** from the correct schedules we pick the schedule which is the most parallelized.

History = Complete Schedule

Partial Schedule = Schedule

Schedules and Histories

Definition 3.1 (Schedules and histories):

Let $T = \{t_1, \dots, t_n\}$ be a set of transactions, where each $t_i \in T$ has the form $t = (op(s), \leq)$ with op_i denoting the operations of t and \leq their ordering.

- (i) A **history** for T is a pair $s = (op(s), \leq)$ s.t. $\bigcup_{t \in T} op_t \subseteq op(s)$ and \leq is a partial ordering of $\bigcup_{t \in T} op_t$.
 (a) $op(s) \subseteq \bigcup_{t \in T} op_t \cup \bigcup_{t \in T} \{a_p, c_i\}$
 (b) for all $i, 1 \leq i \leq n$, $c_i \in op(s) \Leftrightarrow a_i \notin op(s) \Rightarrow$ if a transaction commits, then it cannot abort, and vice versa
 (c) $\leq_{op(s)} \subseteq \leq$
 (d) for all $i, 1 \leq i \leq n$, and all $p \in op_i: p \leq_{op(s)} c_i \text{ or } p \leq_{op(s)} a_i \Rightarrow$ at most one of the two operations of a transaction can be a write
 (e) for all $p, q \in op(s)$ s.t. at least one of them is a write and both access the same data item: $p \leq_{op(s)} q$ or $q \leq_{op(s)} p$

- (ii) A **schedule** is a prefix of a history.

Definition 3.2 (Serial history):

A history s is **serial** if for any two transactions t_i and t_j in s , where $i \neq j$, all operations from t_i are ordered in s before all operations from t_j or vice versa.

Schedules and Histories (2)

So, a history (for partially ordered transactions):

- has to contain all operations from all transactions;
- needs a distinct termination operation for every transaction;
- preserves all orders within the transactions;
- has the termination steps as the final steps in each transaction;
- orders conflicting operations.

Because of a) and b) a history is also called a complete schedule.

Exercises:

$S = r_1(x) \quad r_2(z) \quad r_3(x) \quad W_2(x) \quad W_1(x) \quad r_2(y) \quad r_1(y) \quad W_1(y) \quad W_2(y) \quad W_3(y)$

$t_trans(S) = \{t_1, t_2, t_3\}$

$commit(S) = \{t_1\}$

$abort(S) = \{t_3\}$

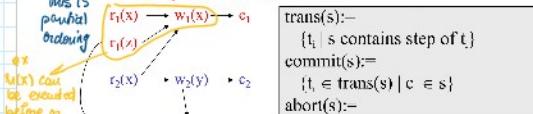
$active(S) = \{t_2\}$

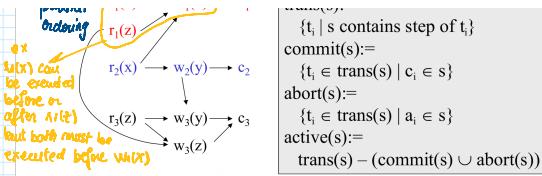
$t_1 = r_1(x), t_2 = r_2(z)$

$S_1 = r_1(x) \quad r_2(z) \quad x$

Example Schedules and Notation

Example 3.4: History (History is Committed or Aborted)





Example 3.6: Schedule (There are still running transactions)
This is totally ordered
 $r_1(x) \ r_2(x) \ r_3(x) \ w_1(y) \ w_2(y) \ w_3(z) \ c_1 \ c_2 \ c_3$

Time

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Correctness of Schedules

1. Define equivalence relation \approx on set S of all schedules.
 2. “Good” schedules are those in the equivalence classes of serial schedules.
- Equivalence must be efficiently decidable.
 - “Good” equivalence classes should be “sufficiently large”.

For the moment,
disregard aborts: assume that all transactions are committed.

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$active(s) = \{t_i \mid s \text{ contains step of } t_i\}$
 $commit(s) := \{t_i \in trans(s) \mid c_i \in s\}$
 $abort(s) := \{t_i \in trans(s) \mid a_i \in s\}$
 $active(s) := trans(s) - (commit(s) \cup abort(s))$

$t_1 = h_1(x), t_2 = h_2(x)$

$s_1 = \{t_1(x)\}$

$s_2 = \{t_2(x)\}$

$s_3 = \{t_1(x), t_2(x)\} \quad \text{executed in parallel}$

If person A and person B share a bank account with 10 lei and both people want to retrieve 10 lei, at the same time one of them is going to get an error

$A \rightarrow -10 \text{ lei} \rightarrow B - \text{error}$

$B \rightarrow -10 \text{ lei} \rightarrow A - \text{error}$

both of these situations is correct, even though it is different

So, there are cases where equal states at the end means equivalent, but there are always other cases!

$$\begin{array}{ll} x=0, y=1 & x=0, y=1 \\ \vdots & \vdots \\ x=100, y=101 & x=100, y=101 \end{array}$$

Herbrand Semantics of Schedules

Definition 3.3 (Herbrand Semantics of Steps):

- For schedule s the **Herbrand semantics** H_s of steps $r(x), w_i(x) \in op(s)$ is:
- $H_s[r(x)] := I_s(w_i(x))$ where $w_i(x)$ is the last write on x in s before $r(x)$.
 - $H_s[w_i(x)] := I_s(I_s[r(y_1)], \dots, I_s[r(y_m)])$ where the $r(y_j), 1 \leq j \leq m$, are all read operations of t , that occur in s before $w_i(x)$ and I_s is an uninterpreted m-ary function symbol.

Definition 3.4 (Herbrand Universe):

For data items $D = \{x, y, z, \dots\}$ and transactions $t_0, 1 \leq i \leq n$,

- the **Herbrand universe** HU is the smallest set of symbols $s.t.$
- $f_{t_0}(\cdot) \in HU$ for each $x \in D$ where f_{t_0} is a constant, and
 - if $w_i(x) \in op_s$ for some t_i , there are m read operations $r(y_1), \dots, r(y_m)$ that precede $w_i(x)$ in t_i and $y_1, \dots, y_m \in HU$, then $I_{t_0}(y_1, \dots, y_m) \in HU$.

Definition 3.5 (Schedule Semantics):

The **Herbrand semantics of a schedule** s is the mapping

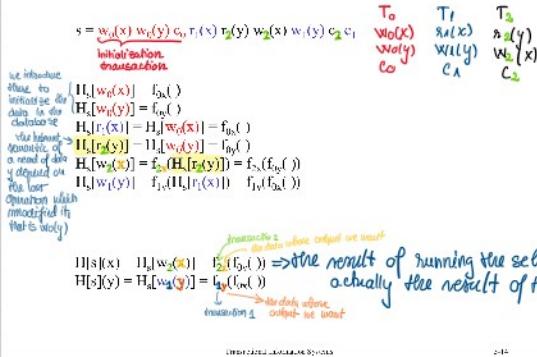
$$H[s] : D \rightarrow HU \text{ defined by } H[s](x) := H_s[w_i(x)].$$

where $w_i(x)$ is the last operation from s 's writing x , for each $x \in D$.

Transcript: exercise 1

2-15

Herbrand Semantics: Example



Example:

not the ref schedule

$$s = w_0(x) w_0(y) w_0(z) c_0 w_1(x) w_1(y) w_1(z) c_1$$

$$H[s](x) = H_s[w_1(x)] = f_{t_2}(H_s[w_0(x)], H_s[w_0(z)]) = f_{t_2}(H_s[w_0(x)], H_s[w_0(z)]) = f_{t_2}(f_{t_1}(f_{t_0}(x)), f_{t_1}(f_{t_0}(z)))$$

keep the order of the operation in the schedule

$$H[s](y) = H_s[w_1(y)] = f_{t_2}(H_s[w_0(y)], H_s[w_0(z)]) = f_{t_2}(f_{t_1}(f_{t_0}(y)), f_{t_1}(f_{t_0}(z))) = f_{t_2}(f_{t_1}(f_{t_0}(y)), f_{t_1}(f_{t_0}(z)))$$

$$H[s](z) = H_s[w_1(z)] = f_{t_2}(H_s[w_0(z)], H_s[w_0(y)]) = f_{t_2}(f_{t_1}(f_{t_0}(z)), f_{t_1}(f_{t_0}(y))) = f_{t_2}(f_{t_1}(f_{t_0}(z)), f_{t_2}(f_{t_1}(f_{t_0}(y))))$$

$$= f_{t_2}(f_{t_1}(f_{t_0}(z)), f_{t_2}(f_{t_1}(f_{t_0}(y))))$$

Curs 5 exerciti

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Transcript: exercise 2

2-17

Final-State Equivalence

Definition 3.6 (Final State Equivalence):

Schedules s and s' are called **final state equivalent**, denoted $s \approx_F s'$, if $op(s) = op(s')$ and $H(s) = H(s')$.

Example a:

$$\begin{aligned} s &= r_1(x) r_2(y) w_1(y) r_3(z) w_2(z) r_4(x) w_1(x) \\ s' &= r_2(z) w_3(z) r_1(y) r_3(x) r_4(x) w_1(y) w_1(x) \\ H[s](x) &= H_s[w_1(x)] = f_{t_0}(f_{t_0}(x)) = H_s[w_1(x)] = H[s'](x) \\ H[s](y) &= H_s[w_1(y)] = f_{t_0}(f_{t_0}(y)) = H_s[w_1(y)] = H[s'](y) \\ H[s](z) &= H_s[w_1(z)] = f_{t_0}(f_{t_0}(z)) = f_{t_0}(f_{t_0}(z)) = H_s[w_1(z)] = H[s'](z) \end{aligned}$$

$\Rightarrow s \approx_F s'$

Incorrect because
lost update

Serial

$$\begin{aligned} s &= r_1(x) r_2(y) w_1(y) r_3(z) \\ s' &= r_1(x) w_1(y) r_2(y) w_2(y) \\ H[s](y) &= H_s[w_1(y)] = f_{t_0}(f_{t_0}(y)) \\ H[s'](y) &= H_s[w_2(y)] = f_{t_0}(f_{t_0}(y)) \end{aligned}$$

introducing
since the Herbrand
semantics of s and s'
are different, s and s'
are not final state
equivalent

2-18

Reads-from Relation

Definition 3.7 (Reads-from Relation; Useful, Alive, and Dead Steps):

Given a schedule s , extended with an initial and a final transaction, t_0 and t_n ,

- $r(x)$ is **ready** in s from $w_i(x)$ if $w_i(x)$ is the last write on x s.t. $w_i(x) \prec r(x)$.
- The **reads-from relation** of s is

$$RT(s) := \{(t_0, x, t_1) \mid \text{an } r(x) \text{ reads } x \text{ from a } w_i(x)\}.$$

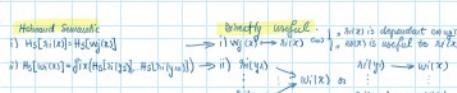
- Step p is **directly useful** for step q , denoted $p \rightarrow q$, if q reads from p .

or p is a read step and q is a subsequent write step of the same transaction.

\rightarrow , the '**useful relation**', denotes the reflexive and transitive closure of \rightarrow .

reflexivity
propagation
transitivity
 $\Rightarrow p \rightarrow p$

Herbrand Semantics
 $=$ output (x, y) of an operation $H[w_i(x)]$
 or a schedule $H[s](x)$
 $\# [s](y)$



Example 2:

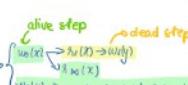
Let there be two schedules:

$$s = r_1(x) w_1(y) w_2(y)$$

$$s' = r_1(x) w_1(y) r_2(y) w_2(y)$$

We augment these 2 schedules with transactions to and t_n :

$$s = w_0(x) w_0(y) \rightarrow r_1(x) w_1(y) w_2(y) \rightarrow r_2(y) \rightarrow w_0(y) \rightarrow \lambda_{t_0}(y) \rightarrow \lambda_{t_n}(y)$$



- reflexive
p-p
transitive
p-p
symmetric
p-p
- (ii) The **reads-from relation** of s is $\text{RF}(s) = \{(t_p, x, t_q) \mid \text{an } r_p(x) \text{ reads } x \text{ from } w_q(x)\}$.
 - (iii) Step p is **directly useful** for step q , denoted $p \rightarrow q$, if q reads from p .
 - p is a read step and q is a subsequent write step of the same transaction.
 - * the "useful" relation denotes the reflexive and transitive closure of \rightarrow .
 - (iv) Step p is **alive** in s if it is useful for some step from t_s , and **dead** otherwise.
 - (v) The **live-reads-from relation** of s is $\text{LRF}(s) = \{(t_u, x, t_v) \mid \text{an alive } r_u(x) \text{ reads } x \text{ from } w_v(x)\}$

$\text{LRF}(s) \subseteq \text{RF}(s)$

Example 3.7: $s = r_1(x) r_2(y) w_1(y) w_2(y)$

$s' = r_1(x) w_1(y) r_2(y) w_2(y)$

$\text{RF}(s) = \{(r_1, x, t_1), (r_2, y, t_2), (w_1, x, t_1), (w_2, y, t_2)\}$

$\text{RF}(s') = \{(r_1, x, t_1), (r_2, y, t_2), (w_1, x, t_1), (w_2, y, t_2)\}$

$\text{LRF}(s) = \{(r_1, y, t_2), (r_2, x, t_1), (r_2, y, t_2)\}$

$\text{LRF}(s') = \{(r_1, x, t_1), (r_1, y, t_2), (r_2, x, t_1), (r_2, y, t_2)\}$

Trans. 1: $r_1 \rightarrow r_2 \rightarrow w_1 \rightarrow w_2$

1.17

Final-State Serializability

Theorem 3.1:

For schedules s and s' the following statements hold.

- (i) $s \approx s'$ iff $\text{op}(s) = \text{op}(s')$ and $\text{LRF}(s) = \text{LRF}(s')$.
- (ii) For s let the step graph $D(s) = (V, E)$ be a directed graph with vertices $V = \text{op}(s)$ and edges $E = \{(p, q) \mid p \rightarrow q\}$, and the reduced step graph $D_r(s)$ be derived from $D(s)$ by removing all vertices that correspond to dead steps. Then $\text{LRF}(s) = \text{LRF}(s')$ iff $D_r(s) = D_r(s')$.

Corollary 3.1:

Final-state equivalence of two schedules s and s' can be decided in time that is polynomial in the length of the two schedules.

Definition 3.8 (Final State Serializability):

A schedule s is **final state serializable** if there is a serial schedule s' s.t. $s \approx s'$. FSR denotes the class of all final-state serializable histories.

FSR: Example 3.9

Error

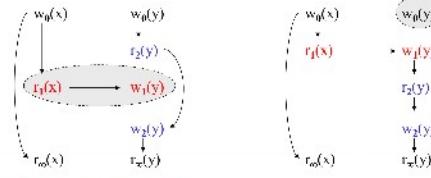
Last Update: this operation is overwritten

This is the correct
schedule since it is the serial one

$$s = r_1(x) r_2(y) w_1(y) w_2(y)$$

$$s' = r_1(x) w_1(y) r_2(y) w_2(y)$$

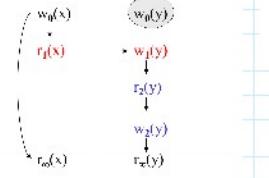
D(s):



Since the 2 graphs produced
here are different, the 2
schedules are not final
state equivalent

dead steps

D(s'):



The final State Equivalence presents some problems
because of the need to be checking the final
state serial form against all possible
final state serial forms, possible for
a specific schedule, thus it introduces
a factorial component.

This is why, even though it is specifically
good at detecting the Last update error,
we are going to drop it. Drawbacks
like Inconsistent Read anomaly occur, as the
final state of the bad schedule is the
same as of a serial permutation of the
Schedule. We discover that the bad
schedule is final state serializable!

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Exercise 3.1

Let there be two schedules:

$s = r_1(x) w_1(y) w_2(y)$

$s' = r_1(x) w_1(y) r_2(y) w_2(y)$

We argument these 2 schedules with transactions t_1 and t_2 :

$s = w(x) w(y) \rightarrow r_1(x) w(y) w_1(y) \rightarrow r_2(y) \rightarrow w_2(y) \rightarrow$

$s' = w(x) w(y) \rightarrow r_1(x) w(y) r_2(y) \rightarrow w_1(y) \rightarrow r_2(y) \rightarrow$

$\text{LRF}(s) = \{(t_1, x, t_1), (t_2, y, t_2), (t_2, y, t_1), (t_2, y, t_2), (t_2, y, t_1)\}$

$\text{LRF}(s') = \{(t_1, x, t_1), (t_2, y, t_2), (t_2, y, t_1), (t_2, y, t_2), (t_2, y, t_1)\}$

$\text{LRF}(s) = \{(t_1, x, t_1), (t_2, y, t_2), (t_2, y, t_1), (t_2, y, t_2), (t_2, y, t_1)\}$

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$\text{LRF}(s') = \{(t_1, x, t_1), (t_2, y, t_2), (t_2, y, t_1), (t_2, y, t_2), (t_2, y, t_1)\}$

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- 3.7 View Serializability**
- 3.8 Conflict Serializability
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$$RF(s_{00}) = \{(t_0, x, t_0), (t_2, x, t_1), (t_3, x, t_0)\}$$

Let's compute the "useful" relationship for this schedule:

$$w_0(x) \rightarrow r_2(x) \rightarrow w_1(x) \rightarrow r_1(x) \rightarrow w_2(x) \rightarrow r_3(x) \quad ; \text{ alive steps}$$

So, we have $LRF(s_{01}) \leftarrow RF(s_{00}) - \{(t_0, x, t_2), (t_2, x, t_1), (t_1, x, t_0)\}$.

So $LRF(s) \neq LRF(s_{01})$ and $LRF(s) \neq LRF(s_{10})$. So schedule s is not final-state serializable.

Canonical Anomalies Reconsidered

- Lost update anomaly:

$$L = r_1(x) r_2(x) w_1(x) w_2(x) c_1 c_2$$

$$\rightarrow \text{history is not FSR} \quad LRF(t_1, t_2) = \{(t_0, x, t_2), (t_2, x, t_0)\} \\ LRF(t_1, t_2) = \{(t_0, x, t_1), (t_1, x, t_2), (t_2, x, t_1)\} \\ LRF(t_2, t_1) = \{(t_0, x, t_2), (t_2, x, t_1), (t_1, x, t_0)\}$$

- Inconsistent read anomaly:

$$I = r_2(x) w_2(y) r_1(x) r_1(y) r_2(y) w_2(y) c_1 c_2$$

$$\rightarrow \text{history is FSR} \quad LRF(I) = \{(t_0, x, t_2), (t_0, y, t_2), (t_2, x, t_1), (t_2, y, t_0)\} \\ LRF(t_1, t_2) = \{(t_0, x, t_2), (t_0, y, t_2), (t_2, x, t_1), (t_2, y, t_0)\} \\ LRF(t_2, t_1) = \{(t_0, x, t_2), (t_0, y, t_2), (t_2, x, t_1), (t_2, y, t_0)\}$$

Observation: (Herbrand) semantics of all read steps matters!

New EQUIVALENCE CRITERIA : View Serializability

Definition 3.9 (View Equivalence):

Schedules s and s' are **view equivalent**, denoted $s \approx_s s'$, if the following hold:

- op(s) = op(s')
- $\Pi[s] = \Pi[s']$
- $H_p[p] = H_{s'}[p]$ for all (read or write) steps

Theorem 3.2:

For schedules s and s' the following statements hold.

- $s \approx_s s'$ iff $op(s) = op(s')$ and $RF(s) = RF(s')$ \rightarrow schedules are view equivalent if the set of operations is the same AND the Read From (RF) set contains the same tuples!
- $s \approx_s s'$ iff $D(s) = D(s')$ \rightarrow schedules are view equivalent if their STEP GRAPHS are the same!

\rightarrow the final data should be the same (final state serializability)

+ we add an additional request:

The Herbrand Semantic of each step should be the same
= they read and write the same values at each step

Inconsistent Read Reconsidered

- Inconsistent read anomaly:

$$I = r_2(x) w_2(x) r_1(x) r_1(y) r_2(y) w_2(y) c_1 c_2$$

\rightarrow history is not VSR ! ~~this is not in the first~~ \rightarrow this is not in the second

$$RF(I) = \{(t_0, x, t_1), (t_0, y, t_1), (t_1, y, t_2), (t_2, x, t_2), (t_2, y, t_0)\}$$

$$RF(t_1, t_2) = \{(t_0, x, t_1), (t_0, y, t_1), (t_0, x, t_2), (t_0, y, t_2), (t_2, x, t_2), (t_2, y, t_0)\}$$

$$RF(t_2, t_1) = \{(t_0, x, t_2), (t_0, y, t_2), (t_1, x, t_1), (t_1, y, t_1), (t_2, x, t_1), (t_2, y, t_1)\}$$

$$RF(I) \neq RF(t_1, t_2) \text{ and } RF(I) \neq RF(t_2, t_1)$$

\Downarrow thus

Observation: VSR properly captures our intuition

Relationship Between VSR and FSR

Theorem 3.3:
VSR \subset FSR.

Theorem 3.4:
Let s be a history without dead steps. Then $s \in \text{VSR}$ iff $s \in \text{FSR}$.

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On the Complexity of Testing VSR

Theorem 3.5:
The problem of deciding for a given schedule s whether $s \in \text{VSR}$ holds is NP-complete.

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Properties of VSR

Definition 3.11 (Monotone Classes of Histories)

Let s be a schedule and $T \subseteq \text{trans}(s)$. $\Pi_T(s)$ denotes the projection of s onto T . A class E of histories is called **monotone** if the following holds:
if s is in E , then $\Pi_T(s)$ is in E for each $T \subseteq \text{trans}(s)$.
VSR is not monotone.

Example:

$s = w_1(x) w_2(y) w_3(z) w_4(y) c_1 w_5(x) w_6(y) c_2$ $\rightarrow \in \text{VSR}$
 $\Pi_{\{t_1, t_2\}}(s) = w_1(x) w_2(x) w_3(y) c_2 w_4(y) c_1$ $\rightarrow \notin \text{VSR}$

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Conflict Serializability

Definition 3.12 (Conflicts and Conflict Relations):

Let s be a schedule, $t, t' \in \text{trans}(s)$, $t \neq t'$.

- (i) Two data operations $p \in t$ and $q \in t'$ are in **conflict** in s if they access the same data item and at least one of them is a write.
- (ii) $\{(p, q) \mid p, q \text{ are in conflict and } p <_s q\}$ is the **conflict relation** of s .

Definition 3.13 (Conflict Equivalence):

Schedules s and s^* are **conflict equivalent**, denoted $s \approx_c s^*$, if

$\text{op}(s) = \text{op}(s^*)$ and $\text{conf}(s) = \text{conf}(s^*)$.

Definition 3.14 (Conflict Serializability):

Schedule s is **conflict serializable** if there is a serial schedule s^* s.t. $s \approx_c s^*$.

CSR denotes the class of all conflict serializable schedules.

Example a: $r_1(x) r_2(x) r_1(z) w_1(x) w_2(y) r_3(z) w_3(y) c_1 c_2 w_3(z) c_3 \rightarrow \in \text{CSR}$

Example b: $r_2(x) w_2(x) r_1(x) r_1(y) r_2(y) w_2(y) c_1 c_2 \rightarrow \notin \text{CSR}$

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Properties of CSR

Theorem 3.8:

$\text{CSR} \subset \text{VSR}$

Example: $s = w_1(x) w_2(x) w_2(y) c_1 w_1(y) c_1 w_3(x) w_3(y) c_3$
 $s \in \text{VSR}$, but $s \notin \text{CSR}$.

Theorem 3.9:

- (i) CSR is monotone.
- (ii) $s \in \text{CSR} \Leftrightarrow \Pi_T(s) \in \text{VSR}$ for all $T \subseteq \text{trans}(s)$
(i.e., CSR is the largest monotone subset of VSR).

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Conflict Graph

Definition 3.15 (Conflict Graph):

Let s be a schedule. The **conflict graph** $G(s) = (V, E)$ is a directed graph

with vertices $V := \text{commit}(s)$ and

edges $E := \{(t, t') \mid t \neq t' \text{ and there are steps } p \in t, q \in t' \text{ with } (p, q) \in \text{conf}(s)\}$.

Theorem 3.10:

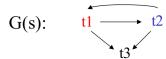
Let s be a schedule. Then $s \in \text{CSR}$ iff $G(s)$ is acyclic.

Corollary 3.4:

Testing if a schedule is in CSR can be done in time polynomial to the schedule's number of transactions.

Example 3.12:

$s = r_1(y) r_3(w) r_2(y) w_1(y) w_1(x) w_2(x) w_2(z) w_3(x) c_1 c_3 c_2$



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Proof of the Conflict-Graph Theorem

- (i) Let s be a schedule in CSR. So there is a serial schedule s^* with $\text{conf}(s) = \text{conf}(s^*)$. Now assume that $G(s)$ has a cycle $t_1 \rightarrow t_2 \rightarrow \dots \rightarrow t_k \rightarrow t_1$. This implies that there are pairs $(p_1, q_1), (p_2, q_2), \dots, (p_k, q_1)$ with $p_i \in t_i, q_i \in t_1, p_i <_s q_i$ and p_i in conflict with $q_{(i+1)}$. Because $s^* \approx_c s$, it also implies that $p_i <_s q_{(i+1)}$. Because s^* is serial, we obtain $t_i <_{s^*} t_{(i+1)}$ for $i=1, \dots, k-1$, and $t_k <_{s^*} t_1$. By transitivity we infer $t_1 <_{s^*} t_2$ and $t_2 <_{s^*} t_1$, which is impossible. This contradiction shows that the initial assumption is wrong. So $G(s)$ is acyclic.

- (ii) Let $G(s)$ be acyclic. So it must have at least one source node. The following topological sort produces a total order $<$ of transactions:
- start with a source node (i.e., a node without incoming edges),
 - remove this node and all its outgoing edges,
 - iterate a) and b) until all nodes have been added to the sorted list.
- The total transaction ordering order $<$ preserves the edges in $G(s)$; therefore it yields a serial schedule s^* for which $s^* \approx_c s$.

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! Commutativity and Ordering Rules

Commutativity rules:

- C1: $r_i(x) r_j(y) \sim r_j(y) r_i(x)$ if $i \neq j$
- C2: $r_i(x) w_j(y) \sim w_j(y) r_i(x)$ if $i \neq j$ and $x \neq y$
- C3: $w_i(x) w_j(y) \sim w_j(y) w_i(x)$ if $i \neq j$ and $x \neq y$

Ordering rule:

- C4: $o_i(x), p_j(y)$ unordered $\sim o_i(x) p_j(y)$
if $x \neq y$ or both o and p are reads

Example for transformations of schedules:

$$\begin{aligned} s &= w_1(x) \underbrace{r_2(x)}_{w_1(y)} \underbrace{w_1(y)}_{r_2(y)} \underbrace{w_1(z)}_{r_3(z)} \underbrace{w_2(y)}_{w_3(y)} \underbrace{w_3(y)}_{w_2(z)} w_3(z) \\ &\sim [C2] \quad w_1(x) \underbrace{w_1(y)}_{r_2(x)} \underbrace{r_2(y)}_{w_1(z)} \underbrace{w_2(y)}_{r_3(y)} \underbrace{r_3(z)}_{w_3(y)} w_3(z) \\ &\sim [C2] \quad w_1(x) w_1(y) w_2(z) \underbrace{r_2(x)}_{w_2(y)} \underbrace{w_3(y)}_{r_3(z)} r_3(z) w_3(y) w_3(z) \\ &= t_1 t_2 t_3 \end{aligned}$$

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Commutativity-based Reducibility

Definition 3.16 (Commutativity Based Equivalence):

Schedules s and s' s.t. $op(s)=op(s')$ are **commutativity based equivalent**, denoted $s \sim^* s'$, if s can be transformed into s' by applying rules C1, C2, C3, C4 finitely many times.

Theorem 3.11:

Let s and s' be schedules s.t. $op(s)=op(s')$. Then $s \approx_c s'$ iff $s \sim^* s'$.

Definition 3.17 (Commutativity Based Reducibility):

Schedule s is **commutativity-based reducible** if there is a serial schedule s' s.t. $s \sim^* s'$.

Corollary 3.5:

Schedule s is commutativity-based reducible iff $s \in CSR$.

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Order Preserving Conflict Serializability

Definition 3.18 (Order Preservation):

Schedule s is **order preserving conflict serializable** if it is conflict equivalent to a serial schedule s' and for all $t, t' \in trans(s)$: if t completely precedes t' in s , then the same holds in s' . OCSR denotes the class of all schedules with this property.

Theorem 3.12:

$OCSR \subset CSR$.

Example 3.13:

$$\begin{aligned} s &= w_1(x) \underbrace{r_2(x)}_{c_2} w_3(y) c_3 w_1(y) c_1 \quad \rightarrow \in CSR \\ &\quad \rightarrow \notin OCSR \end{aligned}$$

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Commit-order Preserving Conflict Serializability

Definition 3.19 (Commit Order Preservation):

Schedule s is **commit order preserving conflict serializable** if for all $t_i, t_j \in trans(s)$: if there are $p \in t_i, q \in t_j$ with $(p, q) \in conf(s)$ then $c_i \leq_s c_j$. COCSR denotes the class of all schedules with this property.

Theorem 3.13:

$COCSR \subset CSR$.

Theorem 3.14:

Schedule s is in COCSR iff there is a serial schedule s' s.t. $s \approx_c s'$ and for all $t_i, t_j \in trans(s)$: $t_i \leq_s t_j \Leftrightarrow c_i \leq_s c_j$.

Theorem 3.15:

$COCSR \subset OCSR$.

Example:

$$\begin{aligned} s &= w_3(y) c_3 w_1(x) \underbrace{r_2(x)}_{c_2} w_1(y) c_1 \quad \rightarrow \in OCSR \\ &\quad \rightarrow \notin COCSR \end{aligned}$$

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Commit Serializability

Definition 3.20 (Closure Properties of Schedule Classes):

Let E be a class of schedules.

For schedule s let $CP(s)$ denote the projection $\Pi_{\text{commit}(s)}(s)$.
 E is **prefix-closed** if the following holds: $s \in E \Leftrightarrow p \in E$ for each prefix of s .
 E is **commit-closed** if the following holds: $s \in E \Rightarrow CP(s) \in E$.

Theorem 3.16:

CSR is prefix-commit-closed, i.e., prefix-closed and commit-closed.

Definition 3.21 (Commit Serializability):

Schedule s is **commit- Θ -serializable** if $CP(p)$ is Θ -serializable for each prefix p of s , where Θ can be FSR, VSR, or CSR.

The resulting classes of commit- Θ -serializable schedules are denoted CMFSR, CMVSR, and CMCSR.

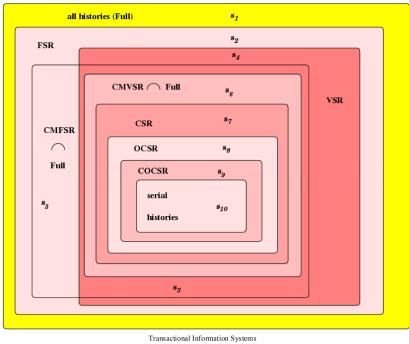
Theorem 3.17:

- (i) CMFSR, CMVSR, CMCSR are prefix-commit-closed.
- (ii) $CMCSR \subset CMVSR \subset CMFSR$

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Landscape of History Classes



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Interleaving Specifications: Motivation

Example: all transactions known in advance
transfer transactions or checking accounts a and b and savings account c:

$$\begin{aligned} t_1 &= r_1(a) w_1(a) r_1(c) w_1(c) \\ t_2 &= r_2(b) w_2(b) r_2(c) w_2(c) \end{aligned}$$

balance transaction:

$$t_3 = r_3(a) r_3(b) r_3(c)$$

audit transaction:

$$t_4 = r_4(a) r_4(b) r_4(c) w_4(z)$$

Possible schedules:

| | | |
|---|--------------------------|-------------------------------------|
| $r_1(a) w_1(a) r_2(b) w_2(b) r_3(c) w_3(c) r_4(c) w_4(c)$ | $\rightarrow \in CSR$ | application-tolerable interleavings |
| $r_1(a) w_1(a) r_3(a) r_2(b) r_3(c) r_1(c) w_1(c)$ | $\rightarrow \notin CSR$ | interleavings |
| $r_1(a) w_1(a) r_2(b) w_2(b) r_1(c) r_3(c) w_2(c) w_1(c)$ | $\rightarrow \notin CSR$ | non-admissible interleavings |
| $r_1(a) w_1(a) r_4(a) r_3(b) r_4(c) w_4(z) r_1(c) w_1(c)$ | $\rightarrow \notin CSR$ | interleavings |

Observations: application may tolerate non-CSR schedules

a priori knowledge of all transactions impractical

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Indivisible Units

Definition 3.22 (Indivisible Units):

Let $T = \{t_1, \dots, t_n\}$ be a set of transactions. For $t_i, t_j \in T, t_i \neq t_j$, an **indivisible unit of t_i relative to t_j** is a sequence of consecutive steps of t_i s.t. no operations of t_j are allowed to interleave with this sequence.

$IU(t_i, t_j)$ denotes the ordered sequence of indivisible units of t_i relative to t_j . $IU_k(t_i, t_j)$ denotes the k^{th} element of $IU(t_i, t_j)$.

Example 3.14:

| | |
|-------------------------------------|--|
| $t_1 = r_1(x) w_1(x) r_1(z) r_1(y)$ | $IU(t_1, t_2) = < [r_1(x) w_1(x)], [w_1(z) r_1(y)] >$ |
| $t_2 = r_2(y) w_2(y) r_2(x)$ | $IU(t_1, t_2) = < [r_1(x) w_1(x)], [w_1(z)], [r_1(y)] >$ |
| $t_3 = w_3(x) w_3(y) w_3(z)$ | $IU(t_2, t_3) = < [r_2(y)], [w_2(y) r_2(x)] >$ |
| | $IU(t_2, t_3) = < [r_2(y) w_2(y)], [r_2(x)] >$ |
| | $IU(t_3, t_1) = < [w_3(x) w_3(y)], [w_3(z)] >$ |
| | $IU(t_3, t_2) = < [w_3(x) w_3(y)], [w_3(z)] >$ |

Example 3.15:

$s_1 = r_1(y) r_1(x) w_1(x) w_2(y) r_2(x) w_1(z) w_3(x) w_3(y) r_1(y) w_3(z) \rightarrow$ respects all IUs

$s_2 = r_1(x) r_2(y) w_2(y) w_1(x) r_2(x) w_1(z) r_1(y) \rightarrow$ violates $IU_1(t_1, t_2)$ and $IU_2(t_2, t_1)$
but is conflict equivalent to an allowed schedule

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Relatively Serializable Schedules

Definition 3.23 (Dependence of Steps):

Step q directly **depends on** step p in schedule s, denoted $p \rightarrow q$, if $p \leq_s q$ and either p, q belong to the same transaction t and $p \leq_t q$ or p and q are in conflict. $\sim \rightarrow^*$ denotes the reflexive and transitive closure of $\sim \rightarrow$.

Definition 3.24 (Relatively Serial Schedule):

s is **relatively serial** if for all transactions t_i, t_j : if $q \in t_j$ is interleaved with some $IU_k(t_i, t_j)$, then there is no operation p in $IU_k(t_i, t_j)$ s.t. $p \rightarrow^* q$ or $q \rightarrow^* p$

Example 3.16:

$s_3 = r_1(x) r_2(y) w_1(x) w_2(y) w_3(x) w_1(z) w_3(y) r_1(y) w_3(z)$

Definition 3.25 (Relatively Serializable Schedule):

s is **relatively serializable** if it is conflict equivalent to a relatively serial schedule.

Example 3.17:

$s_4 = r_1(x) r_2(y) w_2(y) w_1(x) w_3(x) r_2(x) w_1(z) w_3(y) r_1(y) w_3(z)$

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Relative Serialization Graph

Definition 3.26 (Push Forward and Pull Backward):

Let $IU_k(t_i, t_j)$ be an IU of t_i relative to t_j . For an operation $p_i \in IU_k(t_i, t_j)$ let

- (i) **F(p, t)** be the last operation in $IU_k(t_i, t_j)$
- (ii) **B(p, t)** be the first operation in $IU_k(t_i, t_j)$.

Definition 3.27 (Relative Serialization Graph):

The **relative serialization graph RSG(s)** = (V, E) of schedule s is a graph with vertices $V := op(s)$ and edge set $E \subseteq V \times V$ containing four types of edges:

- (i) for consecutive operations p, q of the same transaction (p, q) $\in E$ (*I-edge*)
- (ii) if $p \rightarrow^* q$ for $p \in t_i, q \in t_j, t_i \neq t_j$, then $(p, q) \in E$ (*D-edge*)
- (iii) if (p, q) is a D-edge with $p \in t_i, q \in t_j$, then $(F(p, t_i), q) \in E$ (*F-edge*)
- (iv) if (p, q) is a D-edge with $p \in t_i, q \in t_j$, then $(p, B(q, t_j)) \in E$ (*B-edge*)

Theorem 3.18:

A schedule s is relatively serializable iff RSG(s) is acyclic.

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RSG Example

Example 3.19:

$$t_1 = w_1(x) \ r_1(z)$$

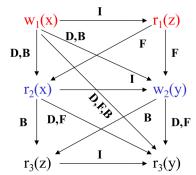
$$t_2 = r_2(x) \ w_2(y)$$

$$t_3 = r_3(z) \ r_3(y)$$

$IU(t_1, t_2) = < [w_1(x) \ r_1(z)] >$
 $IU(t_1, t_3) = < [w_1(x)], \ [r_1(z)] >$
 $IU(t_2, t_1) = < [r_2(x)], \ [w_2(y)] >$
 $IU(t_2, t_3) = < [r_2(x)], \ [w_3(y)] >$
 $IU(t_3, t_1) = < [r_3(z)], \ [r_3(y)] >$
 $IU(t_3, t_2) = < [r_3(z) \ r_3(y)] >$

$$s_5 = w_1(x) \ r_2(x) \ r_3(z) \ w_2(y) \ r_3(y) \ r_1(z)$$

RSG(s_5):



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Lessons Learned

- Equivalence to serial history is a natural correctness criterion
- CSR, albeit less general than VSR,
is most appropriate for
 - complexity reasons
 - its monotonicity property
 - its generalizability to semantically rich operations
- OCSR and COCSR have additional beneficial properties

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