

6.39) $\text{cmmdc}(21100, 12345)$ și coef. lui Bézant

$$21100 = 12345 \cdot 1 + 8755$$

$$12345 = 8755 \cdot 1 + 3590$$

$$8755 = 3590 \cdot 2 + 1575$$

$$3590 = 1575 \cdot 2 + 440$$

$$1575 = 440 \cdot 3 + 255$$

$$440 = 255 \cdot 1 + 185$$

$$225 = 185 \cdot 1 + 70$$

$$185 = 70 \cdot 2 + 45$$

$$70 = 45 \cdot 1 + 25$$

$$45 = 25 \cdot 1 + 20$$

$$25 = 20 \cdot 1 + 5$$

$$20 = 5 \cdot 4 + 0$$

$$\Rightarrow \text{cmmdc}(21100, 12345) = 5$$

$$x_{8755} = (1, 0) - 1(0, 1) = (1, -1)$$

$$x_{3590} = (0, 1) - 1(1, -1) = (-1, 2)$$

$$x_{1575} = (1, -1) - 2(-1, 2) = (3, -5)$$

$$x_{440} = (-1, 2) - 2(3, -5) = (-7, 12)$$

$$x_{255} = (3, -5) - 3(-7, 12) = (24, -41)$$

$$x_{185} = (-7, 12) - (24, -41) = (-31, 53)$$

$$x_{70} = (24, -41) - (-31, 53) = (55, -94)$$

$$x_{45} = (-31, 53) - 2(55, -94) = (-141, 241)$$

$$x_{25} = (55, -94) - (-141, 241) = (196, -335)$$

$$x_{20} = (-141, 241) - (196, -335) = (-337, 576)$$

$$x_5 = (196, -335) - (-337, 576) = (533, -911)$$

$$\Rightarrow 5 = 533 \cdot 21100 + (-911) \cdot 12345$$

7.39) $65^{-1} \pmod{127}$

$$\text{Calc. cmmdc}(127, 65)$$

$$127 = 65 \cdot 1 + 62$$

$$65 = 62 \cdot 1 + 3$$

$$62 = 3 \cdot 20 + 2$$

$$3 = 2 \cdot 1 + 1$$

$$2 = 1 \cdot 2 + 0$$

$$\Rightarrow \text{cmmdc}(127, 63) = 1 \Rightarrow \exists! 65^{-1} \pmod{127} \quad (1)$$

$$x_{62} = (1, 0) - 1(0, 1) = (1, -1)$$

$$x_3 = (0, 1) - 1(1, -1) = (-1, 2)$$

$$x_2 = (1, -1) - 20(-1, 2) = (21, -41)$$

$$x_1 = (-1, 2) - (21, -41) = (-22, 43)$$

$$\Rightarrow 1 = (-22) \cdot 127 + 43 \cdot 65 \Rightarrow 1 - 43 \cdot 65 = -22 \cdot 127 \Rightarrow 1 \equiv 43 \cdot 65 \pmod{127} \quad (2)$$

$$\text{Din (1) și (2)} \Rightarrow 65^{-1} = 43 \pmod{127}$$