Lab Assignment: **FOKC sine-test autotuner. Case Study: the VTOL platform**

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Course: Emerging Control Systems for Industry 5.0

Objectives

1. To understand and apply fractional order autotuners to a time-delay system, specifically a Vertical Take-off and Landing (VTOL) system.

2. To tune the controller based on a single sine test.

3. Implement the controller on the VTOL experimental unit.

1 Theory

The strategy resembles the previous method by using data related to the process: modulus, phase and derivative of the phase.

The novelty of this approach lies in defining a 'forbidden region' circle on the Nyquist diagram. This region is based upon the gain and phase margins which are related to the performance specifications that the closed loop should achieve. The controller parameters should be computed such that the open loop touches the forbidden region's border.

The main idea is described by Fig. 1, by moving a point B from the process's Nyquist plot to a new point A with the help of the controller. The loop frequency response, denoted by $L(j\omega) = P(j\omega)C(j\omega)$, should be tangent to the forbidden region circle, meaning that the slope of the tangent is equal to the slope of $L(j\omega)$. Point D is obtained with respect to an imposed gain margin (GM), while E is based on the phase margin (PM). Trigonometric equations inside the circle give the forbidden region center C and radius R as

$$C = \frac{GM^2 - 1}{2GM(GMcosPM - 1)} \qquad R = C - \frac{1}{GM}, \tag{1}$$

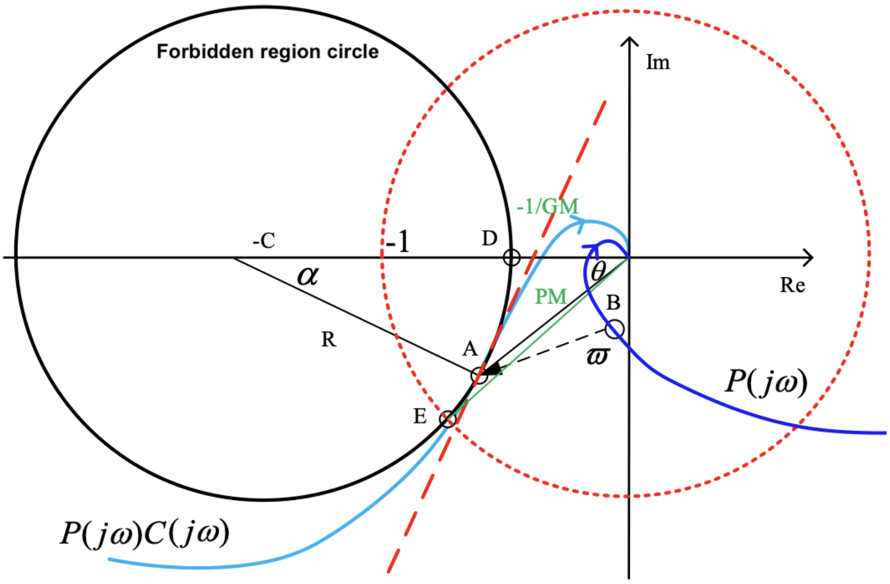

Figure 1: Tuning principle for the KC auto-tuner

which gives the slope of the forbidden region tangent in point A with respect to the angle α

$$\left.\frac{d\,Im}{d\,Re}\right|\alpha = \frac{-Re+C}{Im} = \frac{\cos\alpha}{\sin\alpha}. \tag{2}$$

The slope of $L(j\omega)$ can be computed with respect to the derivative

$$\begin{aligned}\frac{dL(j\omega)}{d\omega} &= P(j\omega)\frac{dC(j\omega)}{d\omega} + C(j\omega)\frac{P(j\omega)}{d\omega} \\ &= \frac{d\,Re_{PC}}{d\omega} + j\frac{d\,Im_{PC}}{d\omega}\end{aligned} \tag{3}$$

Equation (3) allows the computation of $\left.\frac{d\,Im_{PC}}{d\omega}\right|_{\omega=\bar{\omega}}$, where $\bar{\omega}$ is the chosen test frequency. The following equation holds for point A

$$M_A e^{j\varphi_A} = M_{PC}(j\bar{\omega})e^{j\varphi_{PC}(j\bar{\omega})} \tag{4}$$

which can be rewritten as

$$M_A = M_{PC}(j\bar{\omega}) = M_P(j\bar{\omega})M_C(j\bar{\omega}), \tag{5}$$
$$\varphi_a = \varphi_{PC}(j\bar{\omega}) = \varphi_P(j\bar{\omega}) + \varphi_C(j\bar{\omega}). \tag{6}$$

The modulus and phase can also be computed trigonometrically as

$$\begin{aligned} M_A &= \sqrt{C^2+R^2-2CR\cos\alpha} \\ \tan(\varphi_C+\varphi_P) &= \frac{R\sin\alpha}{C-R\cos\alpha} = \frac{tan\varphi_C + tan\varphi_P}{1-tan\varphi_C\, tan\varphi_P} \\ \implies tan\varphi_C &= \frac{Rsin\alpha - tan\varphi_P(C-Rcos\alpha)}{tan\varphi_P\, R\, sin\alpha + (C-Rcos\alpha)} \end{aligned} \tag{7}$$

$P(j\bar{\omega}), \varphi_P(j\bar{\omega})$ and $\left.\frac{dP(j\omega)}{d\omega}\right|$ are computed according to [?] resulting in $\left.\frac{d\,Im_{PC}}{d\,Re_{PC}}\right|_{\omega=\bar{\omega}}$. The design translates into a minimization problem

$$\min_{\alpha} \left\| \left.\frac{d\,Im}{d\,Re}\right|_{\alpha} - \left.\frac{d\,Im_{PC}}{d\,Re}\right|_{\omega=\bar{\omega}} \right\|, 0 \leq \alpha \leq 90^{\circ}. \quad (8)$$

2 Materials and equipment - the VTOL platform

The SOPDT process chosen to validate the previously presented fractional order PI tuning procedure is the Vertical Take-Off and Landing (VTOL) platform provided by National Instruments. The experimental stand exhibits motion dynamics with dead time, which are approximated to a second order model with dead time.

2.1 Case Study Description

The VTOL platform is a one degree of freedom helicopter providing an introduction to the concept of vertical take-off and landing concepts. Fig. 2 illustrates the experimental setup, as well as the add-on board NI ELVIS used for measurement and control purposes.

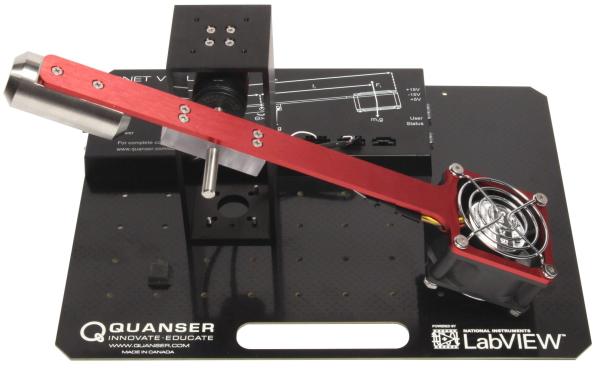

Figure 2: Experimental NI vertical take-off and landing platform (©*2017 National Instruments*)

The working principle lays in a one variable speed fan surrounded by safety guards, attached to the end of a cantilevered beam. On the opposite side of the beam, a balancing weight is present. The cantilevered beam is attached to an encoder shaft that measures the pitch between the beam and an imaginary axis horizontal on the pivot point. The input of the system is considered the voltage (V) applied to the motor rotating the fan, while the output is the pitch of the cantilevered beam (degrees). The process is highly nonlinear due to the circular shape created in the motion of the fan and the presence of the balancing weight at the end of the beam which also moves in a circular pattern. Another nonlinear aspect is introduced by the influence of the base platform on the air propelled by the fan.

The pitch measurements and control are realized using the LabVIEW graphical programming software with a sampling time $T_s = 0.005$ seconds.

2.2 System Identification

A manifold of process identification methods can be employed to determined second order plus dead time process models. The dynamics of the movement of the cantilevered arm are approximated to a SOPDT process model.

For system identification purposes, an input step signal of $6.3V$ has been applied to the rotating fan that moves the cantilevered arm from the initial pitch of -26 degrees to the 0 degrees position. For the pitch of 0 degrees, the beam is parallel to the base platform. The freedom of the arm's movement belongs to the interval of $[-26, 60]$ degrees exhibiting nonlinear behavior. An operating point around the 0 degrees position has been chosen to approximate the system's dynamics. The SOPDT model from (**??**) has been obtained with $k = 22.24$, $\zeta = 0.1514$, $\omega_n = 2.29$ and dead-time $\tau = 0.8$ seconds.

$$H_{SOPDT}(s) = \frac{22.24}{s^2 + 0.6934s + 5.244} e^{-0.8s} \tag{9}$$

3 Analysis and Calculations

3.1 Controller tuning

The following steps should be followed for the FO-KC auto-tuning procedure:

1. Select gain margin GM and phase margin PM values to ensure the stability of the closed loop system and desired performance.
2. Compute the forbidden region circle and α using (1).
3. Determine the slope of the forbidden region circle using (2).
4. Select the frequency $\bar{\omega}$ and perform an experimental sine tests on the process.
5. Compute the frequency response of $L(j\bar{\omega})$.
6. Taking points on the circle's border in small increments, compute the desired fractional order controller.
7. Search for the point where the loop frequency response is tangent to the forbidden region circle using (8).
8. Determine the controller that corresponds to the point found at Step 7.

The methodology has been validated for PM=45° and GM=2. It is a good idea to choose these values, especially for PM, in order to work with a right triangle which considerably reduces computational effort. The FO-KC method should work for any $\bar{\omega}$, however all available validations use the critical frequency for the sine tests. The minimization problem can be solved with a simple *for* loop, by taking alpha in 1° increments and saving the best solution.

The gain crossover frequency $\omega_{gc} = 0.185\ Hz = 1.1623\ rad/s$ (less than the critical frequency) and $\varphi_m = 80°$ (realistic phase margin expectation based on the phase introduced by the fractional order PI controller) have been chosen for the FO-KC auto-tuning.

The next step is to compute the forbidden region circle $C = 5.7587$, $R = 5.6713$ and $\alpha = 10°$. The tangent of the forbidden region circle in the working point is $\tan\alpha = 0.1763$.

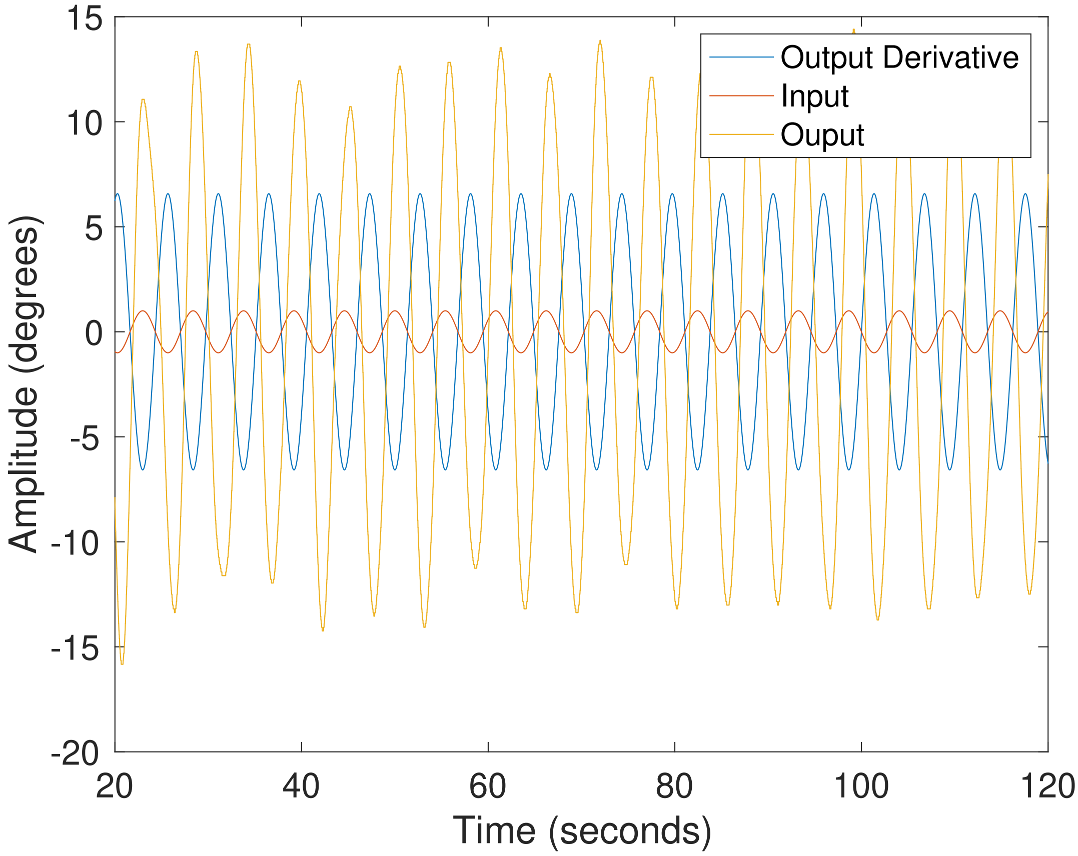

Figure 3: Normalized input, output and output derivative signals for the FO-KC auto-tuner - experimental data

The experimental platform is fed with a sinusoidal signal of amplitude 1V, frequency 0.185 Hz and offset 6.3V after the moving arm reaches 0°. The normalized input, output and phase derivative values are shown in Fig. 3. The phase derivative was computed using the same strategy as for the Sine Test auto-tuner.

The magnitude and phase are obtained as $M(j\omega_{gc} = 12.8413)$ and $\varphi(j\omega_{gc}) = -0.6376\ rad$. For the output derivative the following values are determined: $M(\bar{j}\omega_{gc}) = 6.577, \bar{\varphi} = -3.1698\ rad$, leading to $\left.\frac{d\angle P(j\omega)}{j\omega}\right|_{\omega=\omega_{gc}} = -0.42\ rad$.

The last step is to solve the minimization problem from (8). Several approaches can be used for this, such as for loops and graphical approaches or directly using Matlab built-in functions such as *fmincon* from the System Optimization Toolbox.

The obtained FO-KC controller with the FO-KC method is

$$C_{FO-KC}(s) = 0.031\left(1 + \frac{2.6048}{s^{0.9651}}\right). \tag{10}$$

3.2 Controller implementation

In order to implement a fractional order controller, it has to be discretized. There are two approaches

1. Indirect approach: approximate the fractional order term to an integer order transfer function and then discretize using a known method (i.e. zoh, tustin). There are a manifold of methods to aproximate the fractional order term, one of the most popular being the Recursive Oustaloup Approximation (ora foc.m function in Matlab)

2. Direct approach: approximate the fractional order transfer function directly to a discrete-time transfer function. Known methods are Muir, Continued Fraction Expansion or RDK.

Tasks

1. Excite the VTOL process with a sine wave amplitude 1V, frequency 0.185 Hz after the moving arm reaches 0°.

2. Compute the magnitude, phase and phase derivative.

3. Impose PM = 80^o and compute the FOPI controller.

4. Check that the frequency domain specifications are met using the Bode diagram.

5. Check the step response of the system.

6. Choose the desired discrete-time approximation and compute the discrete-time transfer function of the controller.

7. Compute the recurrence relation in order to obtain $c(k)$ with respect to previous error and control values.

8. Implement the discrete time controller on the VTOL platform.