Exam Project 1: Asset Liability Management in a pension fund

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1 Nomenclature

$t \in T = 0, \dots, \tau$ Time steps, measured relative to current year.

$i \in E$ ETF index

$m \in M$ Months subset index of T

$T_m \subset T$ Specified interval dates

$s \in \Omega$ Scenarios index

$st \in ST$ Index over weeks included in monthly scenarios

$P_{i,t}$ Price of ETF i at time t

$r_{i,t}$ Historical weekly return for asset i at week t.

$r^m_{i,t}$ Historical monthly return for asset i at week t.

$MRS_{i,s,m}$ Monthly return for asset i in scenario s in month m.

2 Data download and clustering analysis

Price data on the ETFs is downloaded from Google Finance using the data download functionality built in to the Pandas module for Python. We use the closing price of the ETF for each day to represent the day's price. Any ETF with less than 2400 data points is excluded, leaving 93 of the initial 100 ETFs to use for further processing.

To cluster the data, it is necessary to define a distance between assets. We compute the correlation coefficient C_{ij} between the log returns of assets i and j, and define the distance between them as

$$d(i, j) = 1 - C_{ij}^2 \in [0, 1]. \tag{1}$$

This distance will tend to group together assets that are more correlated and/or anti-correlated together. This is intuitively a reasonable measure, as (1) going long on an asset is the same as going short on an asset that is anti-correlated with it, and we wish to treat short and long assets the same in this analysis, and (2) the difference between a correlation of 1.00 and 0.95 is more significant than between 0.05 and 0.00.

To do hierarchical clustering, a method of combining clusters must be defined. We compare the results of using the various methods built in to the SciPy module for Python: complete, average, weighted and single. A dendrogram of the clusters found by these methods is shown on Figure 1. Both the complete, average and weighted methods lead to fairly balanced trees. We choose to use the complete method, as the final clusters found by this method are the most uniform in size. We choose to use only 15 clusters for the current work, since going to 25 clusters would lead to many single-asset cluster while not breaking up the largest cluster in Figure 1 — an indication that the assets in this cluster are very similar.

We can further examine the clusters by decomposing C into its principal components PC_i, and looking at the projection of the columns of C onto the first two. This will show

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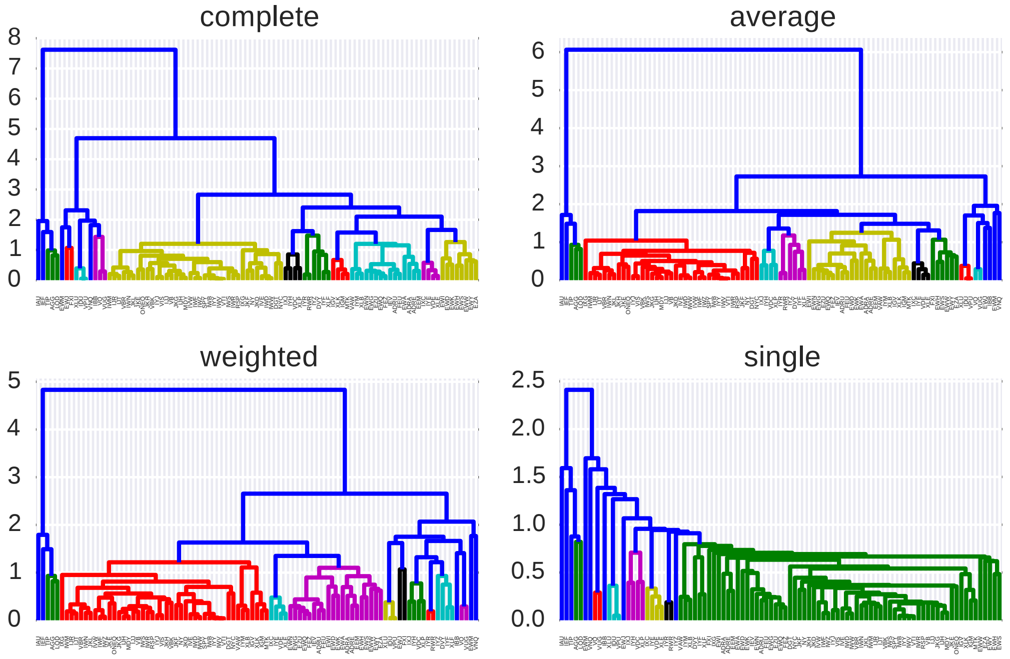

Figure 1: Dendrogram of clusters found by various methods. All non-blue leaves are trimmed by the max-return and the min-std criteria.

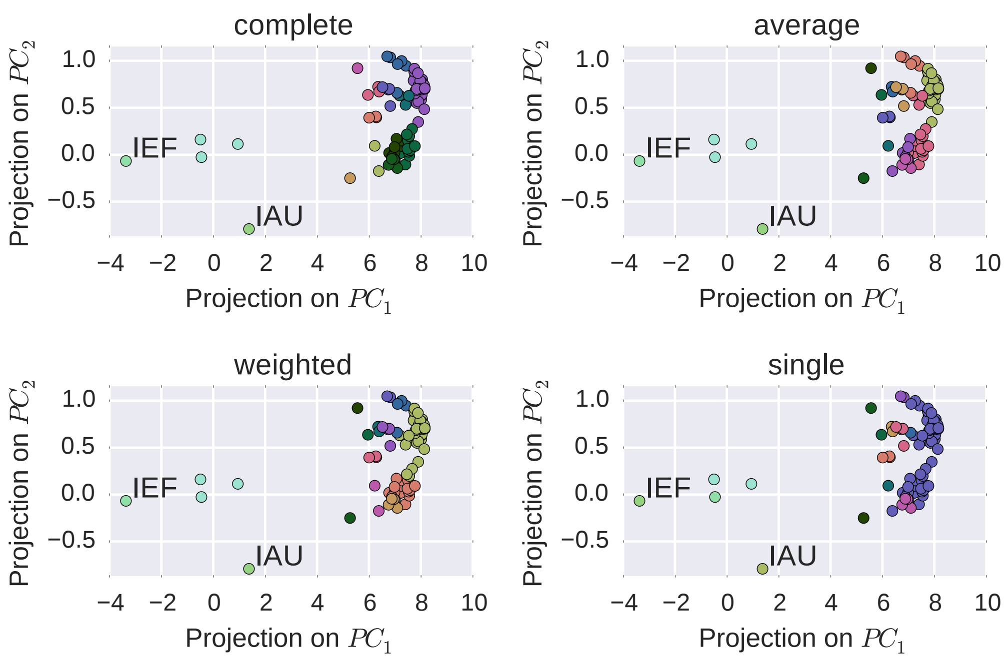

Figure 2: Projection of asset correlation onto the largest principal components. These directions account for 78.58% and 4.51% of the variance, respectively.

a slice of the space defined by $d(i,j)$, and reveal which assets tend to move in a similar manner across the time series. This slice is shown in fig 2, and clearly shows that 5 assets have radically different behaviour than the others. In particular, IAU, which tracks the gold price, and IEF, which tracks US treasury bonds, appear to move in a manner inconsistent with the overall market, and are hence clustered differently to these.

In order to prune the tree of assets, we pick from each cluster a single representative asset in two ways. The max-return criterion picks out the asset which has the highest return across the period, while the min-stdev criterion pick the asset with the smallest standard deviation, i.e. the most stable asset. The assets selected by these two criteria and their associated price histories for the entire period are shown in Figure 3. The price histories for the assets selected by the max-return criterion looks more diverse than for the min-stdev criterion, and we will restrict the following discussion to the assets found by the max-return criterion.

3 Scenario generation

The period of study considered in this work is between 02/02/2005 and 12/11/2014, with the aim to produce a back-test starting on 27/02/2008. In order to generate scenarios to be used for CVaR model, several assumptions are needed. The historical return for each ETF over the period t is given by

$$r_{i,t} = \frac{P_{i,t+1}}{P_{i,t}} - 1, \quad \forall i \in E, t \in T \tag{2}$$

The scenarios are randomly generated picking four dates in a specified interval and taking the historical returns for the ETF's as

$$WRS_{i,st,s,m} = r_{i,t} \text{ for } t \equiv RANDOM(T_m), \quad \forall i \in E, st \in ST, s \in \Omega, m \in M \tag{3}$$

where $WRS_{i,st,s,m}$ is the weekly return scenarios for each month m. The first specified interval T_1 where is picked the historical returns is between 28/01/2005 and 27/02/2008. For each m the specified interval shift itself one month until the end of the period of study (12/11/2014). The monthly return for each of scenario $MRS_{i,s,m}$ is based on accumulating the four weeks return scenario and is determined as

$$MRS_{i,s,m} = \prod_{st}(1 + WRS_{i,st,s,m}) - 1, \quad \forall i \in E, s \in \Omega, m \in M \tag{4}$$

4 Scenario generation via moment matching

For the moment matching scenarios, we are looking to match the mean, variance, skewness and kurtosis with historical data. Calculating these for each month m as

$$\mu_{m,i} = \frac{1}{|T_m|} \sum_{t \in T_m} r_{i,t}^m \tag{5}$$

$$\beta_{m,i} = \frac{1}{|T_m|} \sum_{t \in T_m} \left(r_{i,t}^m - \mu_{m,i}\right)^2 \tag{6}$$

$$\gamma_{m,i} = \frac{1}{|T_m|} \sum_{t \in T_m} \left(r_{i,t}^m - \mu_{m,i}\right)^3 \tag{7}$$

$$\eta_{m,i} = \frac{1}{|T_m|} \sum_{t \in T_m} \left(r_{i,t}^m - \mu_{m,i}\right)^4, \tag{8}$$

we solve the following optimization problem to find the scenario sets $\{\xi_{i,s}\}_m$:

$$\min \sum_i \left((\tilde{\mu}_i - \mu_{m,i})^2 + \left(\frac{\tilde{\beta}_i}{\beta_{m,i}} - 1\right)^2 + \left(\frac{\tilde{\gamma}_i - \gamma_{m,i}}{\beta_{m,i}^{3/2}}\right)^2 + \left(\frac{\tilde{\eta}_i - \eta_{m,i}}{\beta_{m,i}^2}\right)^2 \right) \tag{9}$$

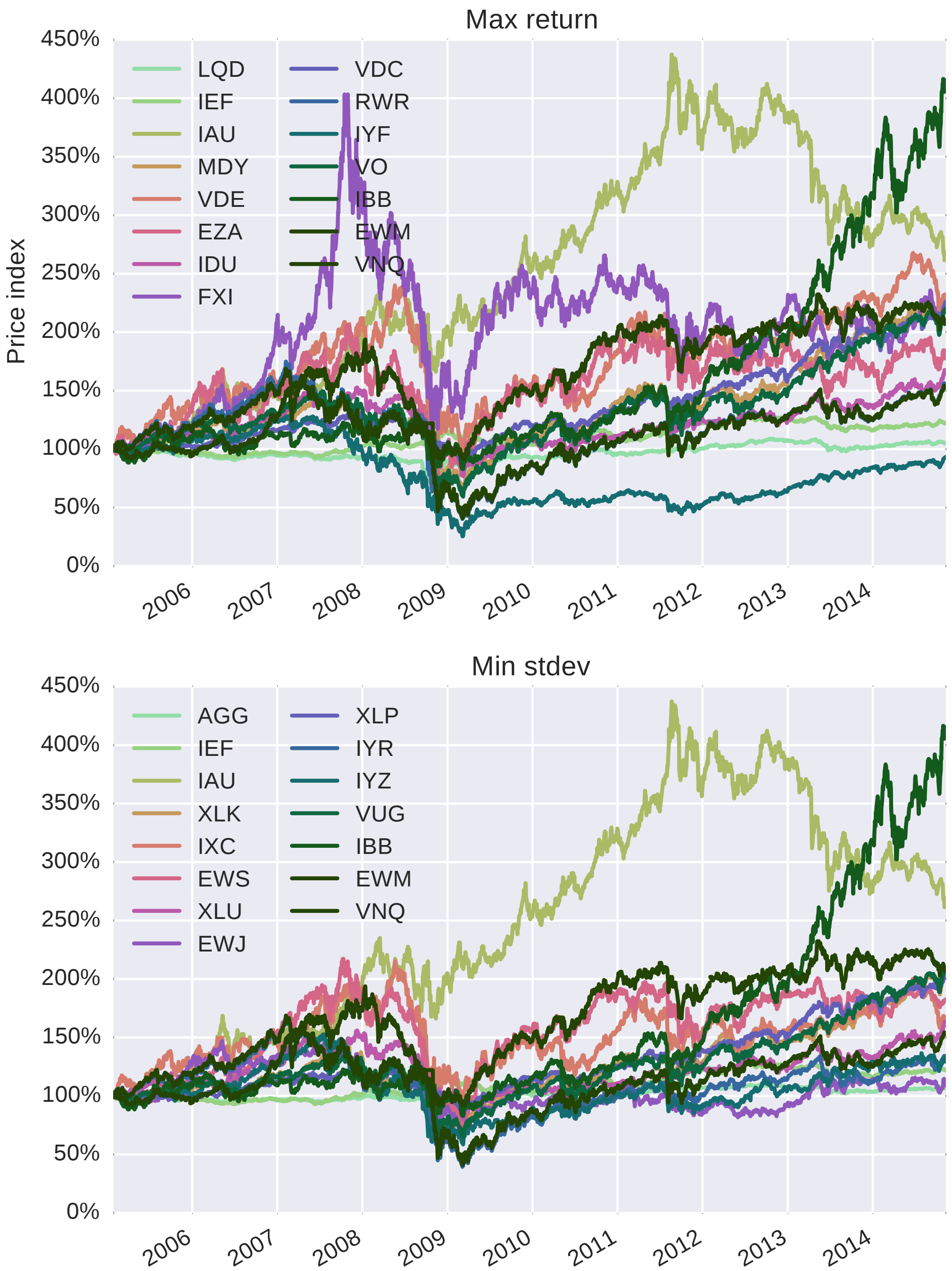

Figure 3: Price history for the assets selected by the max-return (top) and the min-stdev criteria (bottom).

S.t.

$$\tilde{\mu}_i = \frac{1}{|\Omega|}\sum_{s\in\Omega}\xi_{i,s} \quad \forall i \tag{10}$$

$$\tilde{\beta}_i = \frac{1}{|\Omega|}\sum_{s\in\Omega}\left(\xi_{i,s}-\tilde{\mu}_i\right)^2 \quad \forall i \tag{11}$$

$$\tilde{\gamma}_i = \frac{1}{|\Omega|}\sum_{s\in\Omega}\left(\xi_{i,s}-\tilde{\mu}_i\right)^3 \quad \forall i \tag{12}$$

$$\tilde{\eta}_i = \frac{1}{|\Omega|}\sum_{s\in\Omega}\left(\xi_{i,s}-\tilde{\mu}_i\right)^4 \quad \forall i \tag{13}$$

The normalization in (9) by powers of $\beta_{m,i}$ ensures that the objective is scale-invariant, and that each term has an equal weight, eliminating the need to weight the objective.

The following two chapters use the scenarios from the bootstrap method.

5 CVaR model

The CVaR model was built based on the first set of scenarios with start date 2008-02-27, obtained from scenario generaion chapter (section 3 and 4). Based on this set the current value and expected value $P_{i,s}$ is determined for each ETF i for each scenario s. In addition, an initial budget of 1 million kr is considered. The CVaR model is defined as

$$\sum_i x_i = \text{Budget} \tag{14}$$

$$\text{MeanReturn} \geq \mu_{\text{Target}}\text{Budget} \tag{15}$$

$$\text{VaRDev}_s \geq \text{Losses}_s - \text{VaR} \quad \forall s \tag{16}$$

$$\text{Losses}_s = \sum_i x_i - \sum_i \text{P}_{i,s}x_i \quad \forall s \tag{17}$$

$$\text{CVaR} = \text{VaR} + \frac{1}{1-\alpha}\sum_s \text{pr}_s\text{VaRDev}_s \tag{18}$$

$$\text{MeanReturn} = \sum_i \text{EP}_i x_i \tag{19}$$

where μ_{Target} is 0, $\text{P}_{i,s}$ is the value of each ETF i by scenario s, pr_s is the probability of each scenario s, EP_i is the expected value for each ETF i, and α is assumed to be 0.5. For this work, pr_s is assumed to be linearly distributed over the scenarios, i.e. $\text{pr}_s = \frac{1}{|\Omega|}\forall s$.

In order to obtain the efficient frontier based on 10 optimal solutions of CVaR model is neccesary to find the minimum CVaR solution, CVaR_{min}, as well as CVaR_{max} from the maximization of mean return. Between these extreme points, 10 values of $\text{CVaR}_{\text{Target}}$ are created by linear interpolation. A constraint for limit the space solution movement to the $\text{CVaR}_{\text{Target}}$ is included in the CVaR model, and is defined as

$$\text{CVaR} \leq \text{CVaR}_{\text{Target}} \tag{20}$$

For each of the 10 values, the CVaR model is optimized in order to maximize the MeanReturn variable. The efficient frontier of the 10 optimal solutions is presented in Figure 4. At low CVaR, a small increase in CVaR leads to a large increase in expected return, whereas the opposite holds true for the high values of CVaR.

The portfolio results for each of the 10 runs is illustrated in Figure 5. A CVaR bound of 0.0 indicates that CVaR is minimal, with 1.0 indicating that the optimization solely considers mean return. One can see that FXI index increases its weight in the portfolio according to the increase of CVaR target. This happens because the increase of CVaR target leads to the maximization of return. In the period before the trading date (2008-02-27), the scenario generation contains the historical information for the last three years, where FXI index has a higher return when compared to other assets (as one can see in Figure 3).

Figure 6 compares the scenario return at varying levels of the CVaR bound. One can verify that the possibility of gettting higher return increase according to the increase of CVaR target as expected. It is also clear that we accept a very large increase in risk (corresponding to scenarios for which we lose money), in exchange for a relatively modest increase in expected return.

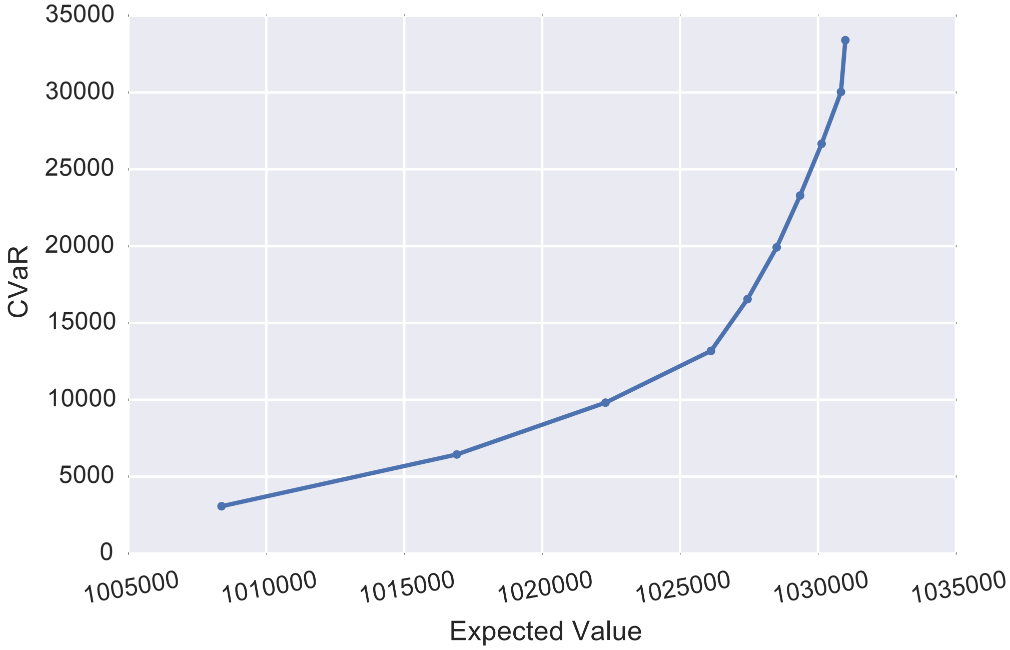

Figure 4: Optimal frontier for equidistant steps in CVaR. 'Expected value' is the mean value of the portfolios found based on the scenarios.

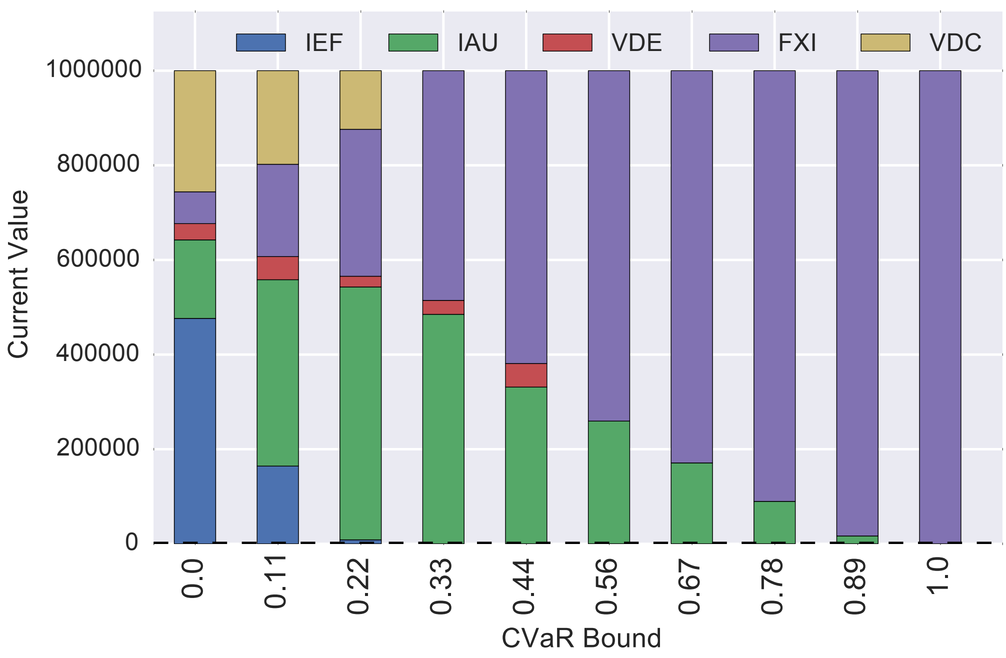

Figure 5: Portfolios at varying levels of the CVaR bound.

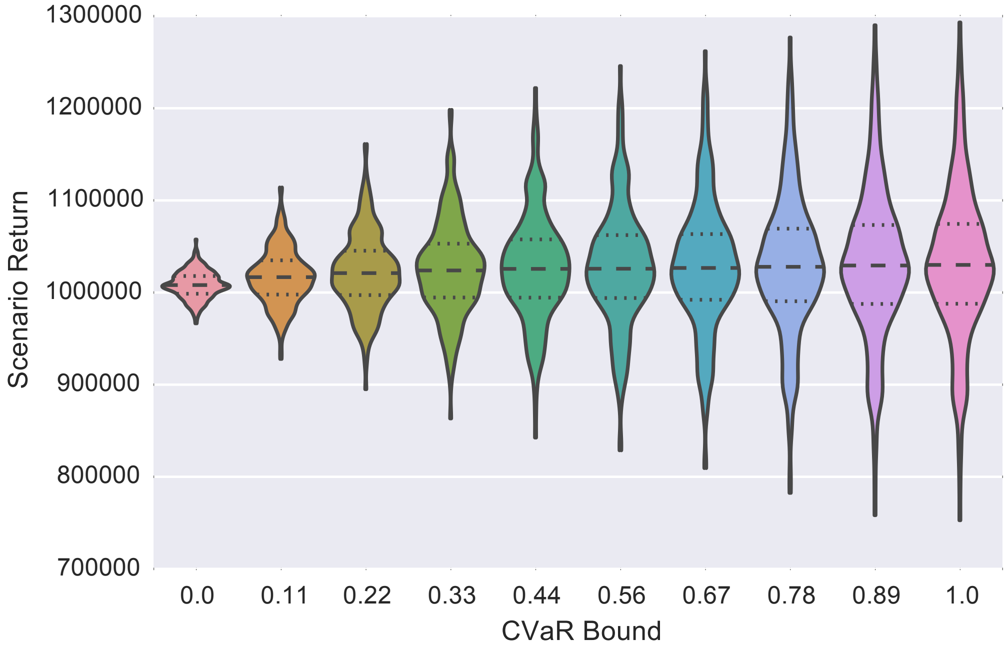

Figure 6: Scenario return for portfolios at varying levels of the CVaR bound. The distributions for each bound are mirrored on the vertical axis, with the mean (dashed) and standard deviation (dotted) shown.

6 Portfolio revision implementation

The portfolio revision model is an extension of the CVaR model to revise and evaluate portfolios after some time has past. In this work, is assumed that four weeks are gone and the previous portfolio on section 5 could no longer be credible and acceptable. So, a revision of the portfolio on the (2008-03-28) to maximize the return under two different strategies (risk averse and risk neutral) are considered. The portfolio revision entails some costs for the transactions deviations on the portfolio. Here, we assume a transaction cost of 0.1% of the traded amount in kr., and a minimum cost of at least 50 kr per trade. These costs are taken to be paid from our own pocket outside of the value of the assets.

The portfolio revision model is defined as

$$\text{TotalCost} = \lambda\text{CVaR} - (1-\lambda)\text{MeanReturn} + \text{TradeCost} \tag{21}$$

$$x_i^{\text{old}} + x_i^{\text{difference}} = x_i \ \ \forall i \tag{22}$$

$$\sum_i x_i^{\text{old}} = \sum_i x_i \tag{23}$$

$$x_i^{\text{difference}} \leq B_i\text{M} \ \ \forall i \tag{24}$$

$$-x_i^{\text{difference}} \leq B_i\text{M} \ \ \forall i \tag{25}$$

$$\text{TradeCost} = \sum_i \text{TC}_i \tag{26}$$

$$\text{TC}_i \geq B_i\text{penalty} \ \ \forall i \tag{27}$$

$$\text{TC}_i \geq 0.001 x_i^{\text{difference}} \ \ \forall i \tag{28}$$

$$\text{TC}_i \geq -0.001 x_i^{\text{difference}} \ \ \forall i \tag{29}$$

where λ is the level of risk of the strategy (1 - Risk Averse ; 0 - Risk Neutral), TradeCost is the total cost of the trades of all changes in portfolio, and x_i^{old} is the previous result of portfolio updated with the current historical monthly return. In this way, the first x_i^{old} is based on the results of section 5 using the risk averse result. The difference between the x_i^{old} and the new x_i is given by $x_i^{\text{difference}}$. B_i is a binary variable which characterizes if there is or is not a transacation in each ETF i, and M is a high constant value to allow the selection of trade or not trade, chosen here to be equal to the value of the portfolio. TC_i is the trade cost incurred due to trading in each ETF i, and penalty corresponds to the trade cost of 50 kr. In addition to the previous constraints, constraints (19) through (16) in the CVaR

Table 1: Stats for portfolios found by portfolio revision model.

	Expected Profit	CVaR	Trading Cost
Type			
Initial	7801.66	3105.22	0.00
Risk Averse	6617.95	2392.81	181.52
Risk Neutral	24172.14	36246.49	1807.48

model presented in section 5 is considered as constraints in the portfolio revision model.

The initial solution is obtained running the full portfolio revision mode where the value of x_i^{old} is based on the previous results of CVaR model updating with the current historical return (2008-03-26). x_i^{old} is given by

$$x_i^{\text{old}} = x_i(1 + r_{i,m}) \tag{30}$$

where $r_{i,m}$ is the histoical monthly return.

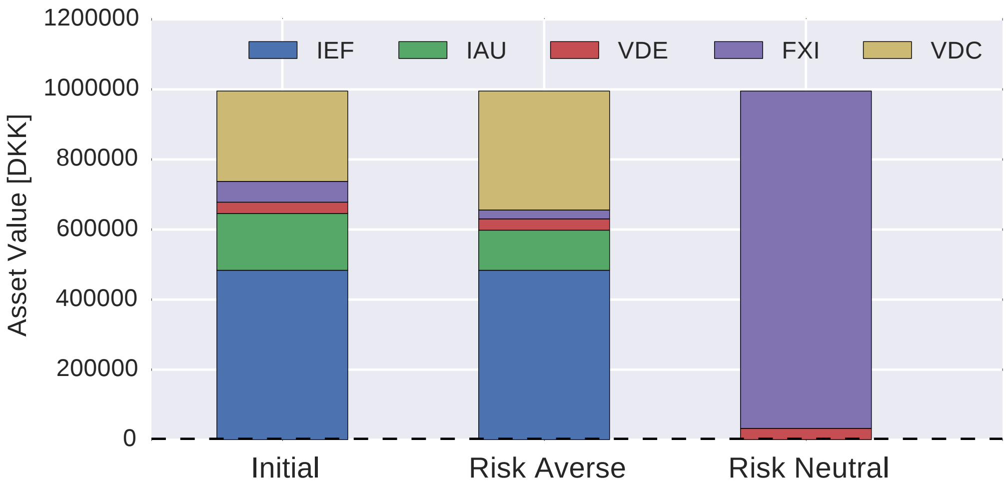

Figure 7: Initial portfolio versus those found by the two types of traders.

The aim of the present section is to minimize the TotalCost of the full portfolio revision model for each type of strategy (risk averse and risk neutral). The portfolios are shown on Figure 7, while the resulting expected value, CVaR and trading are presented in Table 1 considering a comparison between the different strategies and the initial solution.

The risk-averse strategy makes a small adjustment to the portfolio to decrease the CVaR, while the risk-neutral trader throws away pretty much the entire portfolio to nearly only include a single, high risk asset with high historic return. Trading a very large value in this way yields a very high trading cost, which is made up for by the large increase in expected profit.

7 Back-test results

The implementation of back-test results is based on running the portfolio revision model for each month until the last portfolio revision at 2014-06-18. The portfolio revision model is performed for each month, where x_i^{old} is updated every run, with the previous portfolio updated with the historical month return of the current month. I.e. $x_i^{\text{old}} \leftarrow (1 + HMR_{i,m}) * x_i$.

The optimal portfolio mix for each period considering all its revision for both risk averse and risk neutral strategies are illustrated in Figure 8. The risk averse strategy always obtain a mixed portfolio in order to minimize the risk of bad return. One can verify that the IAU assets is in general the base of the portfolio mix for risk averse strategy. Looking at the risk neutral strategy, on the other hand, it tries to bank on the single asset with the largest expected return. At early times, the FXI asset has high participation, since that ETF has an upward tendency until the crash. After the crash, IAU has a very good upward tendency. At this point IBB start slowly a upward tendency, which is more steeper in later periods. In

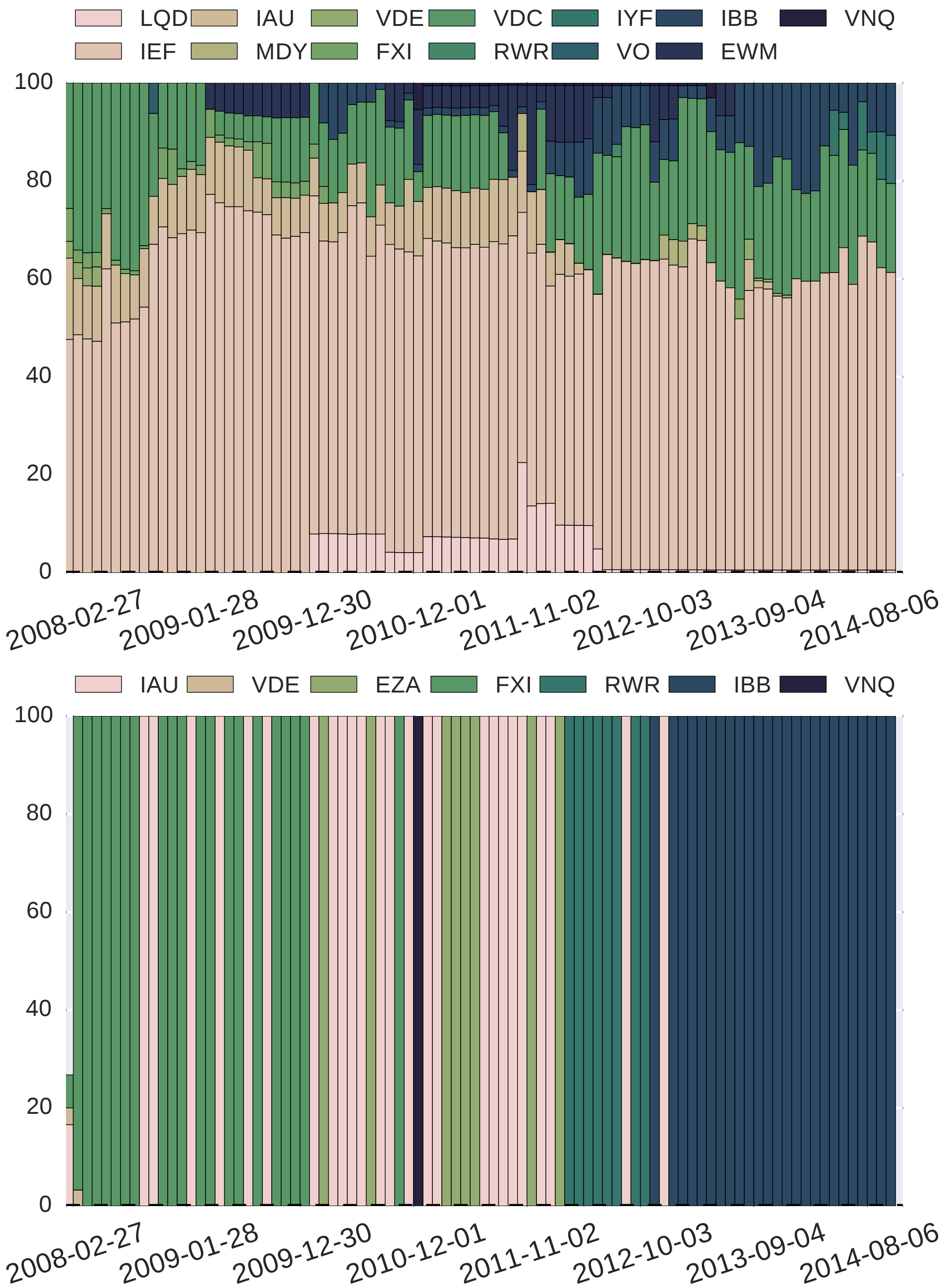

Figure 8: Portfolios found by the risk-averse (top) and risk-neutral (bottom) trading strategies.

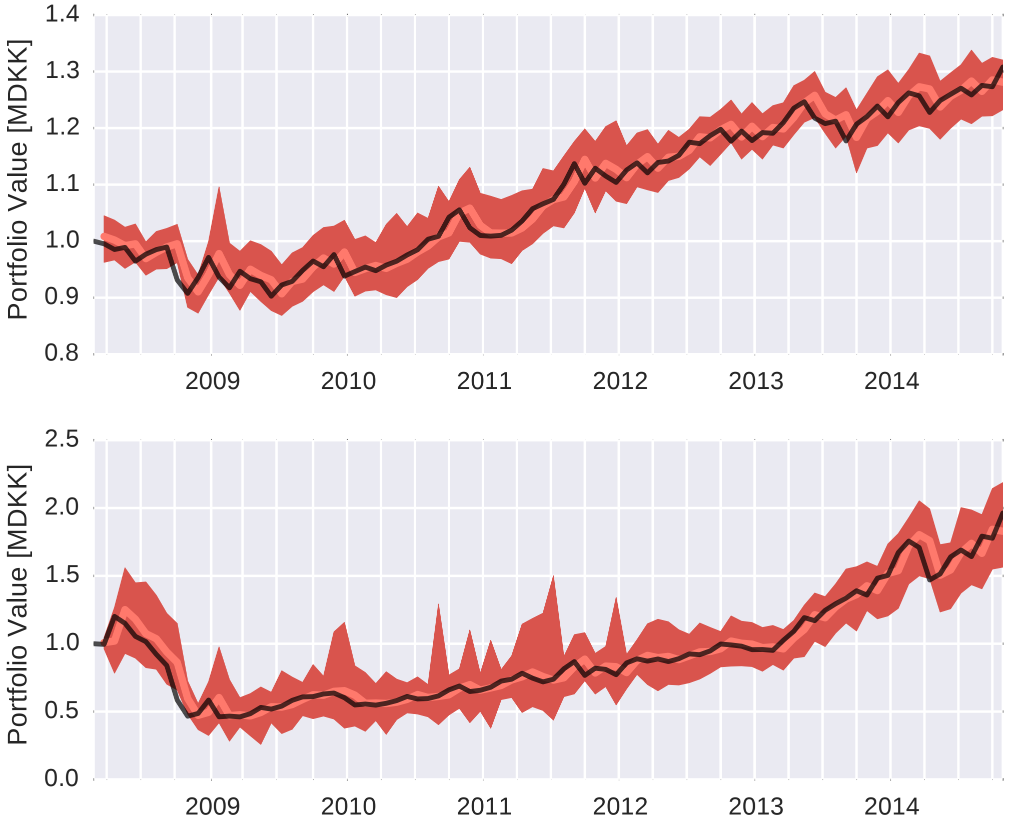

Figure 9: Nominal values of portfolios over time (Black line) versus forecasted mean value (light red). The shaded region indicates the maximum and minimum forecasted values of the ensembles. The risk avers (top) and risk neutral (bottom) strategies for portfolio valueare considered.

this way and based on the behaviour of the ETFs in Figure 3, the portfolio revision obtained for both perspectives is as expected.

The historical values of the portfolios found is compared to that forecasted by the scenarios of the previous trading period in Figure 9. The forecasted values always seem to trail the actual values by 1 month. This is explained as the result of the forecast essentially being a persistence forecast due to the small average return provided.

Looking at the spread the scenarios reveal that only in a few cases around mid-late-2008 do the returns fall outside the interval predicted by the forecast, indicating that the scenarios are able to capture the risk in the market, despite missing the movement of the portfolio mean.

Figure 10 present the portfolio net value for different strategies and methods. A comparison between risk averse and risk neutral strategy for bootstrap method and moment matching method is considered. These strategies are compared with a simple trading scheme ($1/N$ strategy). On the first trading day we invest an equal amount into each asset, and then leave the assets be for the entire trading period. Throughout the entire trading period, one can identify that $1/N$ strategy has a "middle-term" bahaviour when compared with the remaining strategies. In this way, $1/N$ strategy present a higher portfolio value than risk averse strategy for both bootstrap and moment matching strategies. On the other hand, risk neutral strategy for both bootstrap and moment matching achieve higher portfolio value than $1/N$ strategy.

As concerns the behaviour of the risk neutral strategy for both scenario generation methods, it can be seen that the moment matching method quickly recover from the crash in early 2009, while the bootstrap method take years to recover. If we were to end the backtest in early 2013, the bootstrap method would not ensure a good portfolio value. However, the bootstrap method recovers very quickly after 2013 which results in very close portfolio value to the moment matching method in the end of the trading period. Looking at the performance of the risk averse strategy for the two different methods of scenario generation, the bootstrap method ensures better portfolio value than moment matching strategy

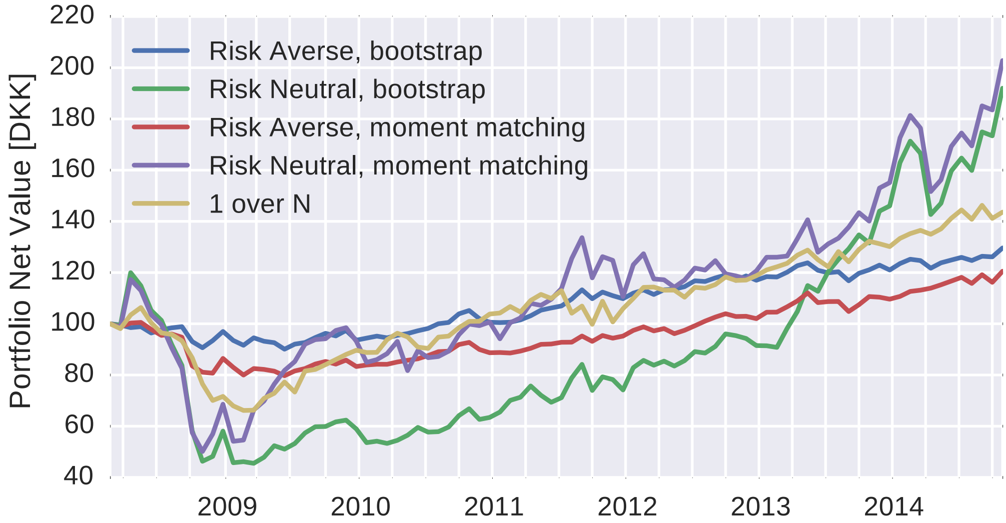

Figure 10: Net values of portfolios over time. (Net value = nominal value minus cumulative trading costs)

Table 2: Comparison of trading strategies

	Final Nominal Value [DKK]	Trading Costs [DKK]	Final Net Profit [DKK]	Annualized Return
Risk Averse, bootstrap	1259714	10810	248904	3.6 %
Risk Neutral, bootstrap	1639882	43283	596599	7.7 %
Risk Averse, moment matching	1172530	5797	166733	2.5 %
Risk Neutral, moment matching	1701345	8787	692558	8.7 %
1 over N	1462676	0	462676	6.2 %

in most of the trading period. The risk averse strategy is very sensitive to correlations in the market and is not accounted in the moment matching method. One can conclude that the performance of the strategy strongly depends on the chosen scenerio generation method and vice-versa.

A comparison between the final numbers of the trading strategies is shown in Table 2. Four things are noteworthy here:

Firstly, the highest return for the risk averse strategies is found by the boostrap scenarios rather than the moment matching ones, whereas they perform oppositely for the risk neutral strategy. This is reasonable, as the risk averse strategy is sensitive to correlations in the data, which are destroyed by the moment matching scenarios, while the risk-neutral strategy needs an accurate representation of the mean, which is captured more closely by the moment matching scenarios.

Secondly, the trading costs are much higher for the bootstrap scenarios than for the moment matching scenarios. This may be understood as the bootstrap scenarios can change quite a lot over time, while the moment matching scenarios will tend to jump around less in between time steps.

Thirdly, the trading costs are higher for the risk neutral strategy than for the risk averse strategy. This is due to the risk neutral strategy constantly throwing away the entire portfolio as shown on Figure 8.

Fourthly, the 1 over N strategy manages to outperform both risk averse strategies, and would have outperformed the risk neutral strategies as well if the market hadn't experienced a steady upswing in 2013 and 2014.

As a closing remark, taking the old advice to just bet on gold, and invest everything in IAU, cashing out in 2012 would have given an annualized return across the period of 9.4%. If this money had then been reinvested in IBB, the annualized return would have been $\sim 27\%$. In other words, clairvoyance outperforms any mortal strategy.

A Gams code

All code from this project is available at `www.github.com/TueVJ/OptFinFinalExam` .