Sparse Block-Encoding for Linear Combinations of Ladder Operators

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In this work, we detail the construction of quantum circuit oracles that create block-encodings for observables described as a linear combination of products of ladder operators acting on fermionic, antifermionic, and bosonic modes. We refer to this constuction as LOBE (Ladder Operator Block-Encoding) and show how it can be used to simulate Hamiltonians involving interactions between these different types of particles. Our work builds off of similar sparse-oracle constructions in the literature, but generalizes prior works to establish a clear connection with block-encoding methods that are commonly refered to as LCU (Linear Combination of Unitaries). In addition to extnding LCU to more general observables that are given as a Linear combination of products of ladder operators, we also demonstrate how these oracles can be extended to include bosonic ladder operators. To our knowledge, this is the first block-encoding construction that allows for interactions between fermions, antifermions, and bosons, paving the way for simulation of more complicated quantum systems such as those that arise in high-energy physics.

I. INTRODUCTION

Wop, wop, wop, wop, wop, Dot, fuck 'em up
Wop, wop, wop, wop, wop, I'ma do my stuff [1]

II. THEORY

A. Encoding

Here we'll discuss how we encode the physical states we are interested in terms of qubits/registers.

B. Ladder Operators

1. Feromons and Antiferomons

Fermions (and antifermions) obey the Pauli-exclusion principle citation and therefore the occupation of a (anti)fermionic mode can only be occupied ($|1\rangle$) or unoccupied ($|0\rangle$). Fermionic (and antifermionic) ladder operators only act non-trivially on the qubits encoding the mode that the ladder operator acts on and we define their action as follows.

The fermionic creation operator is given by:

$$b_i^\dagger |\psi_{b_i}\rangle = \begin{cases} (-1)^{\sum_{j<i} b_j} |1\rangle & when\, |\psi_{b_i}\rangle\, is\, |0\rangle \\ 0 & when\, |\psi_{b_i}\rangle\, is\, |1\rangle \end{cases} \tag{1}$$

where b denotes a fermionic ladder operator, i is the index of the mode that is being acted on, $|\psi\rangle$ is the whole system and $|\psi_{b_i}\rangle$ is the subsystem of the b^{th} fermionic mode. An antifermionic creation operator is defined as above with the symbol d to denote that the operator acts on antifermions.

For a fermionic creation operator, if the mode being acted upon is unoccupied, then the creation operator "creates" a fermion in that mode and applies a phase determined by the parity of the occupation of the previous modes. Therefore the ordering of the modes in the encoding has an implication on the action of the operator that must be accounted for. Since fermionic modes can only be either occupied or unoccupied, then if the mode is already occupied the operator zeroes the amplitude of the quantum state, thereby "destoying" that portion of the quantum state.

The fermionic annihilation operator is given by:

$$b_i |\psi_{b_i}\rangle = \begin{cases} (-1)^{\sum_{j<i} b_j} |0\rangle & when\, |\psi_{b_i}\rangle\, is\, |1\rangle \\ 0 & when\, |\psi_{b_i}\rangle\, is\, |0\rangle \end{cases} \tag{2}$$

and the antifermionic annihilation operator is likewise defined for d instead of b.

The action of the annihilation operators is similar (and opposite) to the creation operators. If the mode is already occupied, then the annihilation operator "annihilates" the fermion at that mode by setting the occupation to zero and applies a phase based on the parity of the occupation of the preceeding modes. If the mode is unoccupied before the operator is applied, then the annihilation operator zeroes the amplitude.

2. Bosos

C. Observables

1. Products of Ladder Operators (Terms)

2. Linear Combinations of Terms

III. LADDER OPERATOR BLOCK-ENCODING (LOBE)

In this section of the text, we'll do the following:

A. Defining "Block-Encoding"

B. Prior Works

1. Linear Combination of Unitaries

2. Sparse Block-Encoding of Pairing Hamiltonians

stuff from Liu et al

C. Circuit Construction

Show big-picture schematic of circuit, then breakdown each sub-oracle

D. Hamiltonian Rescaling

E. Analytical Cost Analysis

Detail analytic cost of different variations

F. Example

Step-by-step example (intention is to move this to an appendix)

IV. RESULTS / NUMERICAL BENCHMARKING

here we'll "benchmark" (aka numerically compute the cost - number of qubits and gates - of creating a lobe block-encoding) for some systems

ideas of systems to benchmark on:

- Fermi-Hubbard
- Something with just bosons
- Something with fermions, antifermions, and bosons

V. CONCLUSIONS

[1] K. Lamar, *Not Like Us*, 1st ed. (Interscope Records, 2024).