Project for "Numerical Methods in Finance"

Stochastic process related with the Asset Price modelling

Session 05-2023

AMSE, Aix-Marseille University

Goal Nr 1 : *Standard Brownian Motion - simulation*

- Definition and theoretical properties.
- Simulate the path of a Standard Brownian Motion (here after denoted by **BM**). Use $\mathbf{n} = 256$ discrete steps.
- Write a Matlab function "BM.m" doing that simulation.
- Visualize three paths in the same figure **Fig 1** under Matlab/Octave calling "BM.m" inside a script

Goal Nr 2 : *Standard Brownian Motion - Monte Carlo analysis*

- Do the Monte Carlo empirical analysis of a **BM** stochastic process. Let $\mathbf{N_{sim}} = \mathbf{300}$. Simulate $\mathbf{N_{sim}}$ paths of **BM** and visualize them all in the same figure **Fig 2**.
- Compute the empirical mean and the empirical variance of the $\mathbf{N_{sim}}$ simulated paths.
- Visualize in a figure **Fig 3** the evolution in time of these empirical moments for $\mathbf{0 < t < T}$, where $\mathbf{T = 1}$.

Goal Nr 3 : *Generalized Brownian Motion - simulation*

- Definition and theoretical properties. Let $\boldsymbol{\mu} = 0.5$, $\boldsymbol{\sigma} = 0.25$ and $\mathbf{X}(1) = 0.2$.
- Simulate the path of a Generalized Brownian Motion (Brownian Motion with Drift, here after denoted by **BMD**). Use $\mathbf{n} = 256$ discrete steps.
- Visualize three paths in the same figure **Fig 4** under Matlab/Octave calling "BM.m" inside a script
- Write a Matlab function "BMD.m" doing that simulation, and which call the function "BM.m" inside it.

Goal Nr 4 : *Generalized Brownian Motion - Monte Carlo analysis*

- Monte Carlo empirical analysis of a **BMD** stochastic process. Let $\mathbf{N_{sim}} = \mathbf{300}$.
- Simulate $\mathbf{N_{sim}}$ paths of **BMD** and visualize them all in the same figure **Fig 5**.
- Compute the empirical mean and the empirical variance of the $\mathbf{N_{sim}}$ simulated paths, for each step of the path.
- Visualize in a figure **Fig 6** the evolution in time of these empirical moments for $\mathbf{0 < t < T}$, where $\mathbf{T = 1}$ (year)

Goal Nr 5 : *Geometric Brownian Motion - simulation*

- Definition and theoretical properties of $\mathbf{S_t}$ following a Geometric Brownian Motion (here after denoted by **GBM**). Let $\mathbf{r}$ and $\boldsymbol{\sigma}$ be the parameters definig the diffusion of a **GBM** process.
- Prove using the Itô Lemma that the process $\mathbf{ln(S_t)}$ is a **BMD** process and give its new associated parameters of interest.
- Simulate the path of a **GBM** process using the method used in **Goal nr 3**.
- Write a Matlab function "GBM.m" doing that simulation, and which call the function "BM.m" inside it.
- Visualize 3 paths in the same figure **Fig 7** under Matlab/Octave calling "BM.m" inside a script. Let $\mathbf{S_1} = 100$, $\mathbf{r} = 0.05$ and $\boldsymbol{\sigma} = 0.25$.

Goal Nr 6 : *Geometric Brownian Motion - Monte Carlo analysis for the last observation* $\mathbf{S_T}$

- Simulate $\mathbf{N_{sim}} = \mathbf{1000}$ paths of **GBM** , each one with $\mathbf{n} = 256$ steps (days) and visualize them all in the same figure **Fig 5**.
- Compute the empirical mean and the empirical variance of $\mathbf{S_T}$ (the future last step of each path, unknown at the $\mathbf{1^{st}}$step) for all the $\mathbf{N_{sim}}$ simulated paths.
- Simulate the Call for an European Stock Option based on a strike $\mathbf{K}$ and parameters $\mathbf{r}$, $\boldsymbol{\sigma}$, $\mathbf{S}_0$, $\mathbf{N}$ steps using the Monte Carlo analysis based on $\mathbf{N_{sim}} = \mathbf{10000}$ simulated paths of **GBM** process. The theoretical formula used is : $\mathbf{Call\_MC} = \mathbf{e^{-rT}E}[\max(\mathbf{0}, \widetilde{\mathbf{S}}_\mathbf{T} - \mathbf{K})]$ where $\widetilde{\mathbf{S}}_\mathbf{T}$ is the random future value of the Stock Price at date $\mathbf{T = 1}$ (year) and $\mathbf{E}$ is the expectation operator.

Goal Nr 7 : *Geometric Brownian Motion - Link with the European Stock Options*

- Black and Scholes formula for computing a Call for an European Stock Option.
- Write a Matlab function "BS_Call.m" computing the Call as function of $\mathbf{S}_0$, $\mathbf{r}$, $\boldsymbol{\sigma}$, $\mathbf{T}$ and $\mathbf{n}$ steps.
- Compare the simulated value by Monte Carlo for the Call with the value given by "BS_Call.m" function. Comments. Remember the first course where we discussed the misspecification of the economic and financial models.

Note : Partially, the Matlab code was already sent for some parts of these Goals covered by Exercices 1 and 2. And other scripts will be found on AMETICE with some homeworks to complete by the students. Use all the other PDF and JPG documents I already sent you during the small course.