

ÖZYEĞİN UNIVERSITY FACULTY OF ENGINEERING DEPARTMENT OF COMPUTER SCIENCE

CS 454

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Homework1

Parametric Classification

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Part A

1) First of all, the data in the training.csv file in the project was separated according to their classes. There are 3 classes and each class has 50 data.

Result:

```
Number_Of_Class_1 = 50
Number_Of_Class_2 = 50
Number_Of_Class_3 = 50
```

2) The prior of each class was found.

Result:

Priors Formula:

$$\frac{Number\ of\ Class1}{Number\ of\ Instances} = P(\ C = 1)$$

$$\frac{Number\ of\ Class2}{Number\ of\ Instances} = P(\ C = 2)$$

$$\frac{Number\ of\ Class3}{Number\ of\ Instances} = P(\ C = 3)$$

3) Mean and Standard Deviation were found

Results of Mean:

```
Mean_of_Class_1 = 24.48
Mean_of_Class_2 = 34.12
Mean_of_Class_3 = 49.44
```

Mean Formula:

$$\frac{\sum_{i=1}^{n} x_i}{n} = \mu$$

$$\Sigma x_i \rightarrow x_1 + x_2 + x_{3+...}x_n$$

 $n \rightarrow Total \ number \ of \ elements \ in \ each \ class$

$$\mu \rightarrow \textit{Mean}$$

Results of Standard Deviation:

Standard_Deviation_Of_Class_1 = 1.992385504865963 Standard_Deviation_Of_Class_2 = 4.1358916813669095 Standard_Deviation_Of_Class_3 = 5.091797325110258

Standard Deviation Formula:

$$\sigma = \sqrt{\frac{\Sigma(x-\mu)^2}{n}}$$

4) Likelihoods values were found with Gaussian distribution. Previously found mean and standard deviation were used. As input, all numbers between the smallest value and the largest value of the ages in the training.csv file were tried.

Gaussian Distribution Formula:

$$y = \frac{1}{\sigma\sqrt{2\pi}} * e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

5) Posterior were found along with likelihoods and priors values.

For class 1:

$$P(c = 1 \mid x = n) = \frac{P(x = n \mid c = 1) * P(c = 1)}{P(x = n \mid c = 1) * P(c = 1) + P(x = n \mid c = 2) * P(c = 2) + P(x = n \mid c = 3) * P(c = 3)}$$

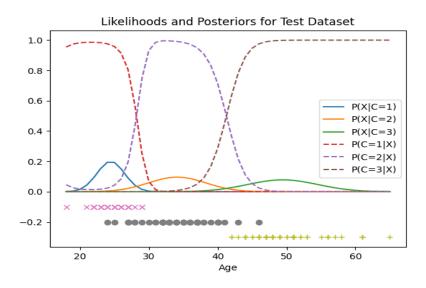
For class 2:

$$P(c = 2 \mid x = n) = \frac{P(x = n \mid c = 2) * P(c = 2)}{P(x = n \mid c = 1) * P(c = 1) + P(x = n \mid c = 2) * P(c = 2) + P(x = n \mid c = 3) * P(c = 3)}$$

For class 3:

$$P(c = 3 \mid x = n) = \frac{P(x = n \mid c = 3) * P(c = 3)}{P(x = n \mid c = 1) * P(c = 1) + P(x = n \mid c = 2) * P(c = 2) + P(x = n \mid c = 3) * P(c = 3)}$$

6) After finding all these values Likelihoods, Posteriors and, Instances are shown using matplotlib.



We assumed that the distributions of the samples were Gaussian. This distribution seems to fit when we draw the examples with the likelihood functions.

PART B

#0-1 Loss Training Part

0/1 loss training chart is drawn. Misclassifications have been identified. Shown in the 3 x 3 confusion matrix chart. Both the training set and the testing set were used.

For Training Set

```
C1_C1 = 49

C2_C1 = 1

C3_C1 = 0

------

C1_C2 = 5

C2_C2 = 42

C3_C2 = 3

-----

C1_C3 = 0

C2_C3 = 2

C3_C3 = 48

-------
```

Training Set		Actual		
		C1	C2	C3
Action	C1	49	5	0
	C2	1	42	2
	C3	0	3	48

For Test Set

```
T1_T1 = 48

T2_T1 = 2

T3_T1 = 0

------

T1_T2 = 7

T2_T2 = 41

T3_T2 = 2

-----

T1_T3 = 0

T2_T3 = 0

T3_T3 = 50
------
```

Test Set		Actual		
		T1	T2	T3
Action	T1	48	7	0
	T2	2	41	0
	T3	0	2	50

We see that the model makes misclassifications in both the test set and the training set. This is because of the outlier. A dataset, unlike other data, may contain outliers that are outside the expected range. These are called outliers.

Rejection and Minimum expected Risk Part

The cost (loss) of assigning an input to an incorrect class was determined of 4 and the cost of rejecting a sample was 1, the decision threshold for minimum expected risk was calculated. It was also shown in the 3 x 4 confusion matrix.

If there is a high probability of making a wrong decision, it can be said that I will refuse, I do not decide.

If rejections are less costly than misclassifications. Then rejection can be made.

Loss Table		Truth			
		C1	C2	C3	
Action	α1	0	4	4	
	α2	4	0	4	
	α3	4	4	0	
	REJECT	1	1	1	

To set the threshold value formula

$$R(\alpha_1|x) = loss_{c_1\alpha_1} * P(c_1|x) + loss_{c_2\alpha_1} * P(c_2|x) + loss_{c_3\alpha_1} * P(c_3|x)$$

$$R(\alpha_2|x) = loss_{c_1\alpha_2} * P(c_1|x) + loss_{c_2\alpha_2} * P(c_2|x) + loss_{c_3\alpha_2} * P(c_3|x)$$

$$R(\alpha_3|x) = loss_{c_1\alpha_3} * P(c_1|x) + loss_{c_2\alpha_3} * P(c_2|x) + loss_{c_3\alpha_3} * P(c_3|x)$$

Result:

$$R(\alpha_{1}|x) = 0 * P(c_{1}|x) + 4 * P(c_{2}|x) + 4 * P(c_{3}|x) = 4 \left(P(c_{2}|x) + P(c_{3}|x)\right) = 4 \left(1 - P(c_{1}|x)\right)$$

$$R(\alpha_{2}|x) = 4 * P(c_{1}|x) + 0 * P(c_{2}|x) + 4 * P(c_{3}|x) = 4 \left(P(c_{1}|x) + P(c_{3}|x)\right) = 4 \left(1 - P(c_{2}|x)\right)$$

$$R(\alpha_{3}|x) = 4 * P(c_{1}|x) + 4 * P(c_{2}|x) + 0 * P(c_{3}|x) = 4 \left(P(c_{1}|x) + P(c_{2}|x)\right) = 4 \left(1 - P(c_{3}|x)\right)$$
IF Choose Action $\alpha_{1}|c_{1}$

$$R(\alpha_1|x) < R(\alpha_2|x)$$
 & $R(\alpha_1|x) < R(\alpha_3|x)$ & $R(\alpha_1|x) < Reject$

1)
$$4(1 - P(c_1|x)) < 4(1 - P(c_2|x))$$

 $P(c_1|x) > P(c_2|x)$

2)
$$4(1 - P(c_1|x)) < 4(1 - P(c_2|x))$$

 $P(c_1|x) > P(c_3|x)$

3)
$$4(1 - P(c_1|x)) < 1$$

 $P(c_1|x) > 3/4$

Training Set for rejection

Training Set		Actual		
		P1	P2	Р3
Action	P1	47	5	0
	P2	0	41	2
	Р3	0	1	47
	REJECT	3	3	1

Testing Set for Rejection

Test Set		Actual		
		P1	P2	Р3
Action	P1	47	5	0
	P2	0	35	0
	Р3	0	2	48
	REJECT	3	8	2

As you can see in the tables, there are some rejections. This is because if the tendency to make mistakes is high, and the cost value of rejection is lower than misclassifications, rejection can be selected.

CODING PART

```
import pandas as pd
import math
Number_Of_1 = 0
Number_Of_2 = 0
Number_Of_3 = 0
print( )
print("Number_Of_Class_1 = " + str(Number_Of_1) )
print("Number_Of_Class_2 = " + str(Number_Of_1) )
print("Number_Of_Class_3 = " + str(Number_Of_1) )
Priors_Of_1 = Number_Of_1 / Total_Number_Of_Data
Priors_Of_2 = Number_Of_2 / Total_Number_Of_Data
Priors Of 3 = Number Of 3 / Total_Number Of_Data
print("Priors_Of_Class_1 = " + str(Priors_Of_1) )
print("Priors_Of_Class_2 = " + str(Priors_Of_2) )
print("Priors_Of_Class_3 = " + str(Priors_Of_3) )
Find_Total_Number_Of_Age_Of_Class(1)  # 1224
Find_Total_Number_Of_Age_Of_Class(2)  # 1706
Find_Total_Number_Of_Age_Of_Class(3)  # 2472
Mean_of_Class_1 = Find_Total_Number_Of_Age_Of_Class(1) / Number_Of_1
Mean_of_Class_2 = Find_Total_Number_Of_Age_Of_Class(2) / Number_Of_2
Mean_of_Class_3 = Find_Total_Number_Of_Age_Of_Class(3) / Number_Of_3
print("Mean_of_Class_1 = " + str(Mean_of_Class_1 ) )
print("Mean_of_Class_2 = " + str(Mean_of_Class_2 ) )
print("Mean_of_Class_3 = " + str(Mean_of_Class_3 ) )
```

```
def Standard Deviation Of Class(n):
                 number_temp = Number Of 1
         count = count / number_temp
print("Standard_Deviation_Of_Class_1 = " + str(Standard_Deviation_Of_Class(1) ) )
print("Standard_Deviation_Of_Class_2 = " + str(Standard_Deviation_Of_Class(2) ) )
print("Standard_Deviation_Of_Class_3 = " + str(Standard_Deviation_Of_Class(3) ) )
p1 = []
p2 = []
p3 = []
         likelihoods_for_class = 0
# For class 1
        p1.append(find_likelihoods_of_data(N,1))
p2.append(find_likelihoods_of_data(N,2))
p3.append(find_likelihoods_of_data(N,3))
```

```
def find_posteriors_of_data(n, class_type):
                     + (find_likelihoods_of_data(n, 2) * Priors_Of_2)
+ (findlikelihoods of data(n, 3) * Priors_Of_3)))
                       t1.append(find_posteriors_of_data(N, 1))
t2.append(find_posteriors_of_data(N, 2))
t3.append(find_posteriors_of_data(N, 3))
plt.plot( x1, p1 )
plt.plot( x1, p1 )
plt.plot( x1, p2 )
plt.plot( x1, p3 )
plt.plot( x2, t1 , '--')
plt.plot( x2, t2 , '--')
plt.plot( x2, t3 , '--')
class_y = [-0.1, -0.2, -0.3]
plt.plot( testing_file[testing_file['class']==1]['age'] , [-0.1 for N in range (50)] , 'x')
plt.plot( testing_file[testing_file['class']==2]['age'], [-0.2 for N in range (50)] , 'o')
plt.plot(testing_file[testing_file['class']==3]['age'] , [-0.3 for N in range (50)], '+')
plt.legend(['P(X|C=1)','P(X|C=2)','P(X|C=3)','P(C=1|X)','P(C=2|X)','P(C=3|X)'])
plt.title('Likelihoods and Posteriors for Test Dataset')
 plt.xlabel('Age')
```

```
print("------")
print("C1 C2 = " + str(C1 C2))
print("C2 C2 = " + str(C2 C2))
print("C3 C2 = " + str(C3 C2))
print("------")
print("C1_C3 = " + str(C1_C3))
print("C2_C3 = " + str(C2_C3))
print("C3_C3 = " + str(C3_C3))
T1_T1 = 0
T2_T1 = 0
T1_T2 = 0
T2_T2 = 0
T3_T2 = 0
T1_T3 = 0
T2_T3 = 0
T3_T3 = 0
```

```
T1_T2 = T1_T2 + 1

elif ((find_posteriors_of_data(row['age'], 2) > find_posteriors_of_data(row['age'],1))

and (find_posteriors_of_data(row['age'], 2) >

find_posteriors_of_data(row['age'], 3))):
print("-----")
print("T1 T1 = " + str(T1 T1))
print("T2 T1 = " + str(T2 T1))
print("T3 T1 = " + str(T3 T1))
print( 13_T1 = " + str(T3_T1))
print("-----")
print("T1_T2 = " + str(T1_T2))
print("T2_T2 = " + str(T2_T2))
print("T3_T2 = " + str(T3_T2))
print("-----")
print("-------")
print("T1_T3 = " + str(T1_T3))
print("T2_T3 = " + str(T2_T3))
print("T3_T3 = " + str(T3_T3))
print("------")
P1_P1 = 0
P2_P1 = 0
P1_P2 = 0
P2_{P2} = 0
```

```
and (find_posteriors_of_data(row['age'], 1) > 3 / 4)):
P1_P2 = P1_P2 + 1
               (find_posteriors_of_data(row['age'], 2) > 3 / 4)):
P2_P2 = P2_P2 + 1
             print("P1_P2 = " + str(P1_P2))
print("P2_P2 = " + str(P2_P2))
print("P3 P2 = " + str(P3 P2))
print( P3 P2 = " + Str(P3 P2))
print("REJECT_P2 = " + Str(REJECT_P2))
print("------")
print("P1 P3 = " + Str(P1 P3))
print("P2 P3 = " + Str(P2 P3))
print("P3 P3 = " + Str(P3 P3))
print("REJECT_P3 = " + str(REJECT_P3))
print("----")
```

```
\begin{array}{ccc}
Q1 \underline{\quad} Q1 &=& 0\\
Q2 \underline{\quad} Q1 &=& 0
\end{array}
Q1_Q2 = 0
Q2_Q2 = 0
                Q1_Q1 = Q1_Q1 + 1

elif ((find_posteriors_of_data(row['age'], 2) > find_posteriors_of_data(row['age'], 1))

and (find_posteriors_of_data(row['age'], 2) >
  find_posteriors_of_data(row['age'], 3)) and
  (find_posteriors_of_data(row['age'], 2) > 3/4) ):
                     and (find_posteriors_of_data(row['age'], 3) >
   find_posteriors_of_data(row['age'], 2)) and
        (find_posteriors_of_data(row['age'], 3) > 3/4) ):
```