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Report

“Dynamics of Non-Linear Robotic Systems”

Homework-01

1. Forward Kinematic

KUKA LBR iiwa 14 R820

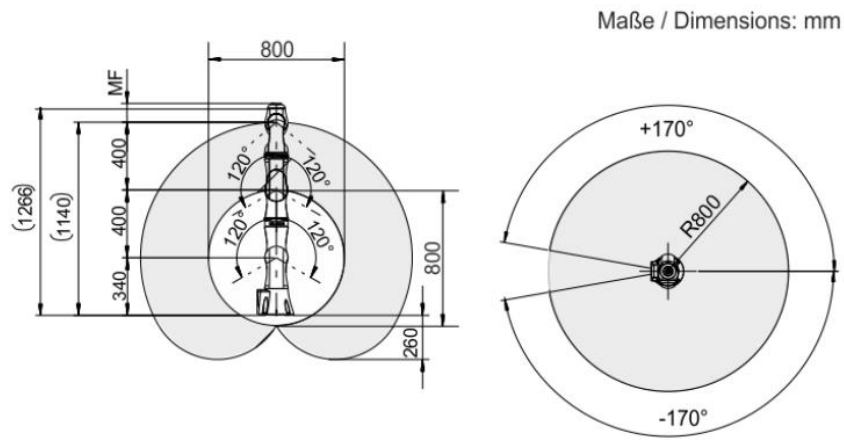

Figure 1 The manipulator structure

Denavit-Hartenberg convention

In Denavit-Hartenberg method, the homogeneous transformation matrix is represented by a product of four basic transformations:

$$A_i = Rot_{x,\theta_i} Trans_{z_i d_i} Trans_{x,a_i} Rot_{x,a_i} \tag{1.1}$$

$$= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\times \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In this case the robot KUKA LBR iiwa 14 R820 has 7 dof so the transformation from the base to the end-effector is given by

$$T_7^0 = T_1^0 T_2^1 T_3^2 T_4^3 T_5^4 T_6^5 T_7^6$$

Assigning the coordinate frames

By using Matlab with the robotics tool box

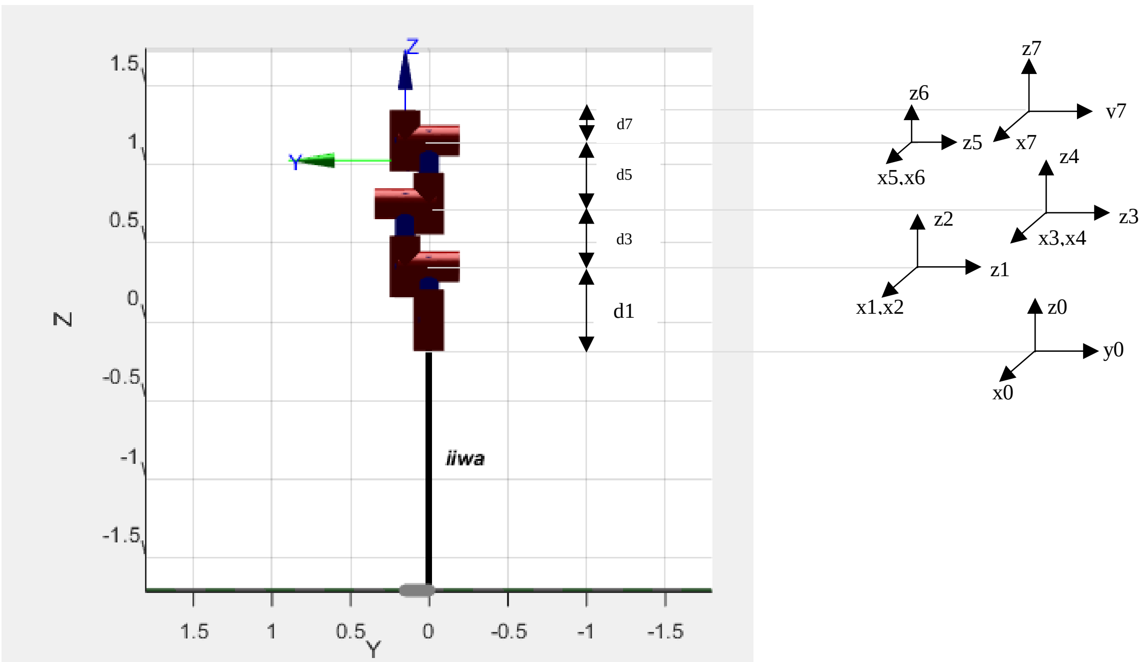

Figure 2 coordinate frame of robot for DH

Table 1 DH-parameter

J	Θ_i	d_i, mm	a_{i-1}	α_{i-1}	Joint limit
1	q1	340	0	$-\pi/2$	$\pm 170^\circ$
2	q2	0	0	$\pi/2$	$\pm 120^\circ$
3	q3	400	0	$-\pi/2$	$\pm 170^\circ$
4	q4	0	0	$\pi/2$	$\pm 120^\circ$
5	q5	400	0	$-\pi/2$	$\pm 170^\circ$
6	q6	0	0	$\pi/2$	$\pm 120^\circ$
7	q7	126	0	0	$\pm 170^\circ$

The homogeneous transformation matricesT70 are computed by substituting the above parameters into equation for each joint.

$$T_1^0 = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_i & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix},\ T_2^1 = \begin{bmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},T_3^2 = \begin{bmatrix} C_3 & 0 & -S_3 & 0 \\ S_3 & 0 & C_3 & 0 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^3 = \begin{bmatrix} C_4 & 0 & -S_4 & 0 \\ S_4 & 0 & -C_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},\ T_5^4 = \begin{bmatrix} C_5 & 0 & -S_5 & 0 \\ S_5 & 0 & C_5 & 0 \\ 0 & 1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} T_6^5 = \begin{bmatrix} C_6 & 0 & S_6 & 0 \\ S_6 & 0 & -C_6 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$T_7^6 = \begin{bmatrix} C_7 & -S_7 & 0 & 0 \\ S_7 & C_7 & 0 & 0 \\ 0 & 1 & 0 & d_7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_7^0 = T_1^0 T_2^1 T_3^2 T_4^3 T_5^4 T_6^5 T_7^6 = \begin{bmatrix} R_{11} & R_{12} & R_{13} & P_x \\ R_{21} & R_{22} & R_{23} & P_y \\ R_{31} & R_{32} & R_{33} & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} R11 = {} & C7S6(S4(S1S3 - C1C2C3) - \\ & C1C4S2)C6(C5(C4(S1S3 - C1C2C3) + C1S2S4) + S5(C3S1 + C1C2S3)) \\ & +S7(S5(C4(S1S3 - C1C2C3) + C1S2S4) - C5(C3S1 + C1C2S3)) \end{aligned}$$

$$\begin{aligned} R12 = {} & C7(S5(C4(S1S3 - C1C2C3) + C1S2S4) - C5(C3S1 + C1C2S3)) - \\ & S7(S6(S4(S1S3 - C1C2C3) - C1C4S2) \\ & -C6(C5(C4(S1S3 - C1C2C3) + C1S2S4) + S5(C3S1 + C1C2S3))) \end{aligned}$$

$$\begin{aligned} R13 = {} & -C6(S4(S1S3 - C1C2C3) - C1C4S2) - \\ & S6(C5(C4(S1S3 - C1C2C3) + C1S2S4) + S5(C3S1 + C1C2S3)) \end{aligned}$$

$$\begin{aligned} R21 = {} & -[C7(S6(S4(C1S3 + C2C3S1) + C4S1S2) - C6(C5(C4(C1S3 + \\ & C2C3S1) - S1S2S4) + S5(C1C3 - C2S1S3)))] \\ & -S7(S5(C4(C1S3 + C2C3S1) - S1S2S4) - C5(C1C3 - C2S1S3)) \end{aligned}$$

$$\begin{aligned} R22 = {} & -[S7(S6(S4(C1S3 + C2C3S1) + C4S1S2) - C6(C5(C4(C1S3 + \\ & C2C3S1) - S1S2S4) + S5(C1C3 - C2S1S3)))] \\ & -S7(S5(C4(C1S3 + C2C3S1) - S1S2S4) - C5(C1C3 - C2S1S3)) \end{aligned}$$

$$\begin{aligned} R23 = {} & C6(S4(C1S3 + C2C3S1) + C4S1S2) + \\ & S6(C5(C4(C1S3 + C2C3S1) - S1S2S4) + S5(C1C3 - C2S1S3)) \end{aligned}$$

$$R33 = C6(C2C4 - C3S2S4) - S6(C5(C2S4 + C3C4S2) - S2S3S5)$$

$$p_x = d_3C1S2 - d_7(S4(S1S3 - C1C2C3) - C1C4S2) -$$
$$[d7\ (C6(S4(S1S3 - C1C2C3) - C1C4S2) + S6(C5(C4(S1$$
$$S3 - C1C2C3) + C1S2S4) + S5(C3S1 + C1C2S3)))]$$

$$p_y = [d_7(C6(S4(C1S3 + C2C3S1 + C4S1S2) + S6(C5(C4$$
$$(C1S3 + C2C3S1) - S1S2S4) + S5(C1C3 - C2S1S3)))]$$
$$d_5(S4(C1S3 + C2C3S1) + C4S1S2) + d_3S1S2$$

$$p_z = d_1 + d_5(C2C4 - C3S2S4) -$$
$$[(\ d_7(S6(C5(C2S4 + C3C4S2) - S2S3S5) - C6(C2C4 - C3S2S4)) + d_3C2]$$

2. Inverse Kinematics

Kinematic decoupling (Pieper's Solution)

The manipulator model in this study has 7 joint angles while the given position and the orientations of the end-effector can be specified by six parameters meaning that one extra degree of freedom in the manipulator is available. The extra degree of freedom is representing the redundancy. In this case study the joint 3 is fixed so the model has 6 joint angles. The kinematic decoupling method (Pieper's Solution) is used for solving inverse kinematics while the manipulator is divided into part the first 3 link (1-4) is an arm link and the last 3d link is a wrist link. So we can solve inverse kinematics problem into two simpler problems and arm link will give inverse position kinematics, and wrist link will give inverse orientation kinematics.

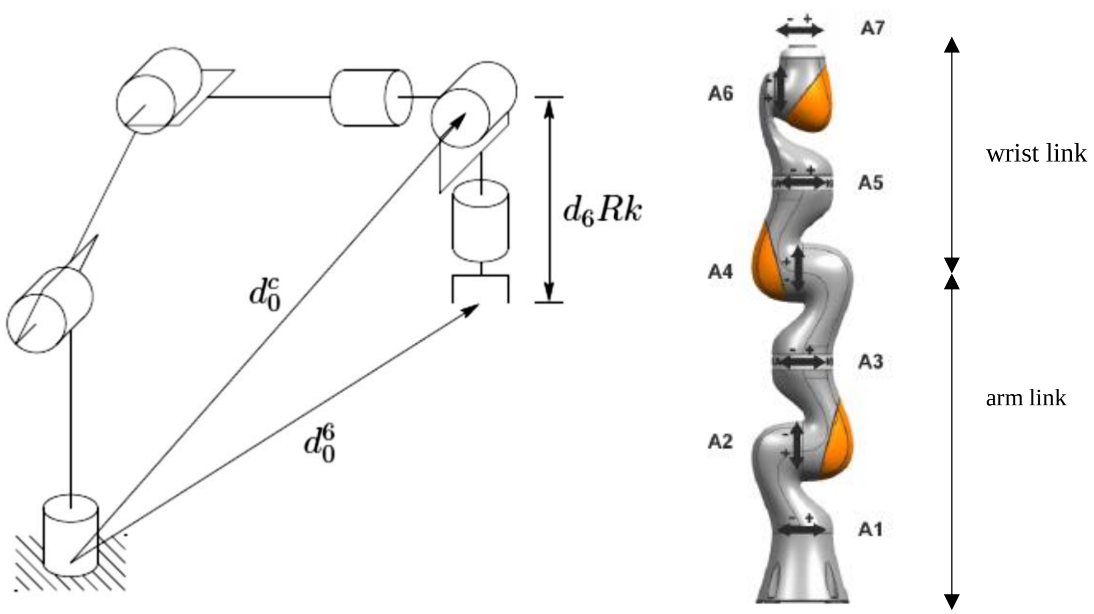

Figure 3 Kinematic decoupling

$$o = o_c^o - d_6 R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Arm link configuration

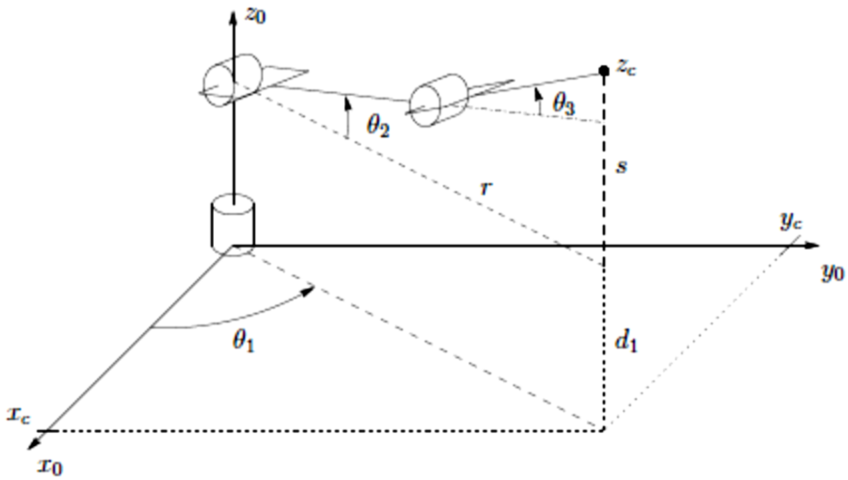

Figure 4 arm link configuration

$r_c = \sqrt{x_c^2 + y_c^2}$; $s = z_c - d_0$; $\theta_1 = atan2\left(\frac{x_c}{y_c}\right)$

$cos(\theta_3) = \frac{r^2+s^2-d_1^2-d_3^2}{2d_1d_3}$:= D

since $r_c = \sqrt{x_c^2 + y_c^2}$ $\quad s = z_c - d_0$.

Hence, $\theta_3 = atan2(D, \pm\sqrt{1 - D^2})$;

$\theta_2 = atan2(s, r) - atan2(d_3 s_3, d1 + d_3 c_3)$

singular configuration

$x_c = 0, y_c = 0$

$\theta_1 \in [0,2\pi]$

In this case θ_1 has infinite solution from 0 to 2π.

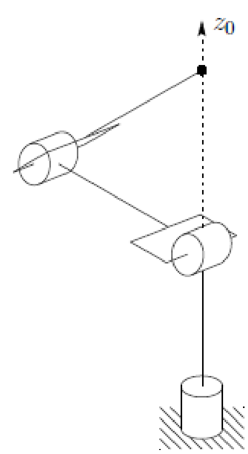

Figure 5 singular configuration

Wrist link configuration

$$R = R_3^0 R_6^3$$

$$R_6^3 = R_3^{0^T} R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$R_6^3 = Rot(x, \theta_4) Rot(z, \theta_5) Rot(x, \theta_6)$$

So, the Euler angle (XZX) solution is applied to this equation.

$$R_6^3 = \begin{bmatrix} c_4 c_5 c_6 - s_4 s_6 & -c_4 c_5 s_6 - s_4 c_6 & c_4 s_5 \\ s_4 c_5 c_6 + c_4 s_6 & -s_4 c_5 s_6 + c_4 c_6 & s_4 s_5 \\ -s_5 c_6 & s_5 s_6 & c_5 \end{bmatrix}$$

Hence the we can use the last column to solve

$$\theta_4 = atan2(r_{23}, r_{13})$$

$$\theta_5 = atan2(\sqrt{r_{13}^2 + r_{23}^2}, r_{33})$$

$$\theta_6 = atan2(r_{32}, r_{31})$$

Singular configuration

$$sin\theta_5 = 0 \;\Rightarrow r_{33} = \pm 1$$

$$\theta_5 = \mathrm{acos}(r11) = \{0, \pm\pi\}$$

$$\Rightarrow \theta_5\,, \theta_6 \;\; no\ unique\ solution$$

MATLAB code:

- `HW1.m` Run a script file is for solving solutions (forward and inverse kinematics)
- `Tr.m` a function for Transformation matrices
- `FKinematics.m` a function for direct kinematics
- `iiwa_show.m` a function for showing robot model
- `inv_kin.m` a function for inverse kinematics