State Estimation Of Permanent Magnet Stepper Motor Using Kalman Filter

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***Abstract*— Originally developed for use in spacecraft navigation, the Kalman filter turns out to be useful for many applications. It is mainly used to estimate system states. Filtering is desirable in many situations in engineering and embedded systems. In the present paper, applications of Kalman filter is extended to state estimation of permanent magnet stepper motor and results of state estimation are compared for simple discrete Kalman filter, Extended Kalman filter and Uncented Kalman filter. Comparison shows better estimation results in case of Uncented Kalman filter approach. The paper elaborates design and implementation of Kalman filter and its variants for state estimation of a practical problem. The proposed results provide a tool for obtaining the reliable estimates, in case of permanent magnet stepper motor of states.**

***Index Terms*—Kalman filter, Extended Kalman filter, Uncented Kalman filter, permanent magnet Stepper motor.**

I. INTRODUCTION

In the field of navigation, Kalman filtering technique provides a practical solution to number of problems that were previously considered difficult to analyze. Kalman filter is simply an optimal recursive data processing algorithm [1]. Due to its recursive nature it is more suitable to the problems where measurements are in a serial manner [2]. This well known filter made the transition from relatively abstract theory to application in many systems within a very short period during the early 1960's [3]. In order to use a Kalman filter to remove noise from a signal, the process under observation is required to be described by a linear system. In general Kalman filter is a minimum variance estimator where system is linear stochastic dynamical system [4]. Some of the practical applications of Kalman filter as estimator can be found in [5]. Many physical processes, such as a vehicle driving along a road, a satellite orbiting the earth, a motor shaft driven by winding currents, or a sinusoidal radio-frequency carrier signal, can be approximated as linear systems. A linear system is simply a process that can be described by the following two equations:

State equation:

$$x_k = Ax_{k-1} + Bu_{k-1} + w_k \tag{1}$$

Output equation:

$$z_k = Hx_k + v_k \tag{2}$$

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In the above equations *A*, *B*, and *C* are matrices; *k* is the time index; x is called the state vector of the system; u is a known input to the system; z is the measured output. The variable w is called the process noise, and v is called the measurement noise. All these quantities are vectors and therefore, contain more than one element. The vector x contains all of the information about the present state of the system, butcannot be measured directly. Instead, output z which is a function of state x and is corrupted by the noise v can be measured. So output z can be used to obtain an estimate of x. The Kalman filter removes noise by assuming a predefined model of a system therefore; the Kalman filter model must be meaningful. State process noise are required to be modeled initially and then refinement of filter is done based on observations.

II. KALMAN FILTER THEORY AND ALGORITHM

Filtering and estimation are two of the most pervasive tools of engineering. Whenever the state of a system is estimated from noisy sensor information, some kind of state estimator is employed to fuse the data from different sensors together to produce an accurate estimate of the true system state [6]. Suppose a linear system model as described in equations (1) and (2) is given. This dynamics describes the system behavior and also provides information about measurements. So for determining the best estimate of the state x estimator should satisfy the following criteria:

Firstly, it is desired that the average value of state estimate should be equal to the average value of the true state. That is, estimate should not be biased one way or the other. Mathematically, it simply means that in uncertain environment, expected value of the estimate should be equal to the expected value of the state. Secondly, it is expected that a state estimate should vary from the true state as little as possible. That is, not only one desires the average of the state estimate to be equal to the average of the true state, but also it is desired that estimator should result in the smallest possible variation of the state estimate. Mathematically, requirement is to find an estimator with the smallest possible error variance [2]. The Kalman filter can be used for a variety of end-purposes. Its basic function is to provide estimates of the current state of the system. But it also serves as the basis for predicting future values of prescribed variables or for improving estimates of variables at earlier times [7]. The Kalman filter estimates a process by using a form of feedback control: : the filter estimates the process state at some time

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and then obtains feedback in the form of (noisy) measurements. As such, the equations for the Kalman filter fall into two groups: time update equations and measurement update equations. The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the *a priori* estimates for the next time step. The measurement update equations are responsible for the feedback i.e. for incorporating a new measurement into the *a priori* estimate to obtain an improved *a posteriori* estimate. The time update equations can also be thought of as *predictor* equations, while the measurement update equations can be thought of as *corrector* equations [8]. The Kalman filter propagates the first two moment of the distribution of the state of the system and has a predictor corrector structure [9].

Let $\boldsymbol{w}$ is the process noise and $\boldsymbol{z}$ is the measurement noise. therefore,

Process noise covariance:

$$Q = E\left(w_k w_k^T\right)$$

Measurement noise covariance:

$$R = E\left(z_k z_k^T\right)$$

Where $\boldsymbol{w}_k^T$ and $\boldsymbol{z}_k^T$ indicate the transpose of the $\boldsymbol{w}$ and $\boldsymbol{z}$ random noise vectors, and E (·) means the expected value. The Kalman filter equations provide an extremely convenient procedure for digital computers implementation [10]. There are many alternative ways to express the Kalman filter equations. One of the formulations is given as follows:

Predict:

$$\begin{aligned} \hat{\boldsymbol{x}}_k^- &= A\hat{\boldsymbol{x}}_{k-1} + B\boldsymbol{u}_{k-1} \\ P_{\bar{k}} &= AP_{k-1}A^T + Q \end{aligned} \tag{3}$$

Update:

$$\begin{aligned} K_k &= P_k^- H^T \left(HP_k^- H^T + R\right)^{-1} \\ \hat{x}_k &= \hat{x}_k^- + K_k\left(z_k - H\hat{x}_k^-\right) \\ P_k &= \left(I - K_k H\right)P_k^- \end{aligned} \tag{4}$$

where,

A: State transition matrix (i.e. transition between states)

$\boldsymbol{u}$: Control variable

B: Control matrix (i.e. mapping control to state variables)

P : State variance matrix (i.e. error or estimation)

Q : Process variance matrix (i.e. error due to process)

H : Measurement matrix (i.e. mapping measurements onto state)

K : Kalman gain

R : Measurement variance matrix (i.e. error from measurements)

In the present paper, applications of Kalman filter is extended to state estimation of permanent magnet stepper motor and compassed is made with Extended Kalman filter based estimation and Uncented Kalman filter estimator.

III. STATE ESTIMATION OF PERMANENT MAGNET STEPPER MOTOR

A . Problem statement

Consider a simple problem of state estimation of permanent magnet stepper motor. Permanent magnet stepper motor is having nonlinear dynamics and goal here is to estimate the states of permanent magnet stepper motor with the help of Kalman filter. As the direct Kalman filter is not well suited for non linear systems, so idea here is to analyze the relative performance of different variants of Kalman filter for non linear systems. Originally, stepper motors were designed to provide precise position and velocity control within a fixed number of steps. For a two phase permanent magnet stepper motor (PMSM) following state space model is considered [11]:

$$\begin{aligned} \dot{I}_a &= -\frac{R_a}{L}I_a + \frac{w\lambda}{L}\sin\theta + \frac{u_a + \Delta u_a}{L} && \text{(current through winding a)} \\ \dot{I}_b &= -\frac{R_a}{L}I_b + \frac{w\lambda}{L}\cos\theta + \frac{u_b + \Delta u_b}{L} && \text{(current through winding b)} \\ \dot{w} &= -\frac{3\lambda}{2J}I_a\sin\theta + \frac{3\lambda}{2J}I_b\cos\theta - \frac{Fw}{J} + \Delta\alpha && \text{(load acceleration)} \\ \dot{\theta} &= w && \text{(average velocity)} \end{aligned} \tag{5}$$

where, u_a and u_b are the voltages that are applied across the two motor windings; I_a and I_b are the currents in two motor windings; Δu_a and Δu_b are noise terms due to errors in voltages u_a and u_b ; $\Delta\alpha$ is a noise term due to uncertainty in the load torque; J = moment of inertia of the motor shaft and its load; F = coefficient of viscous friction; $\boldsymbol{w}$ = rotational velocity of rotor; R = resistance of the coils; L = inductance of the coil λ = flux constant of motor; θ = actual rotor position. To implement the Kalman filter for the stepper motor, one has to simulate the motor model and then the states of permanent magnet stepper motor can easily be obtained using Kalman filter.

Here, model in (5) can be written state space form as:

$$\begin{aligned} \dot{x}_1 &= -\frac{R_a}{L}x_1 + \frac{x_3\lambda}{L}\sin(x_4) + \frac{u_a + \Delta u_a}{L} \\ \dot{x}_2 &= -\frac{R_a}{L}x_2 + \frac{x_3\lambda}{L}\cos(x_4) + \frac{u_b + \Delta u_b}{L} \\ \dot{x}_3 &= -\frac{3\lambda}{2J}x_1\sin(x_4) + \frac{3\lambda}{2J}x_2\cos(x_4) - \frac{Fx_3}{J} + \Delta\alpha \\ \dot{x}_4 &= x_3 \end{aligned}$$

Where $x_1 = I_a, x_2 = I_b, x_3 = w$ and $x_4 = \theta$

B. Motor Model for Discrete Time Kalman Filter

Here the discrete time Kalman filter has been applied to estimate the motor's position and speed using current measurement. The state and measurement equations for the Kalman filter for this motor involves following matrices

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$$A = \begin{bmatrix} -\frac{R_a}{L} & 0 & \frac{\lambda}{L}*\sin(x_4) & 0 \\ 0 & -\frac{R_a}{L} & \frac{\lambda}{L}*\cos(x_4) & 0 \\ -\frac{3}{2}*\frac{\lambda}{J}*\sin(x_4) & \frac{3}{2}*\frac{\lambda}{J}*\cos(x_4) & -\frac{F}{J} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Measurement equation:

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$P = I$ = Initial state estimation covariance

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$Q =$ Process noise covariance

$$\begin{bmatrix} 0.1111 & 0 & 0 & 0 \\ 0 & 0.1111 & 0 & 0 \\ 0 & 0 & 0.2500 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

R = Measurement noise covariance

$$\begin{bmatrix} 0.0100 & 0 \\ 0 & 0.0100 \end{bmatrix}$$

The Kalman filter equations are;
New optimal state equation:

$$\hat{x}_k = \hat{x}_k^- + K*(z - H\hat{x})$$

Kalman gain equation:

$$K_k = P_k^- H^T (HP_k^- H^T + R)^{-1}$$

Updated error covariance:

$$P_k = (I - K_k H) P_k^-$$

By using above mentioned model to find the time update and measurement update equation in simulation results are:

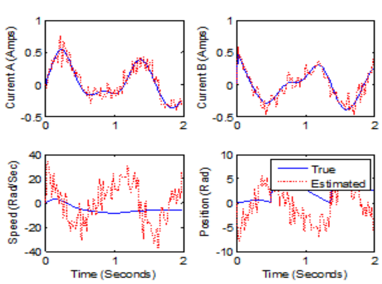

Figure 1.1 Motor's state estimation using Kalman filter

Standard deviation of estimation errors in different states comes out as = 0.096911, 0.098586, 20.183 and 4.7588, respectively. Here in figure 1.1 the dashed lines show the estimated results by Kalman filter and thick lines show the true results for the states of motor. Results shows that Kalman filter is not giving the optimum results for the non linear systems. Therefore Extended Kalman filter is suitable for such types of systems. In next part, performance of Extended Kalman filter for the same non linear system problem is analyzed. The Extended Kalman filter simply linearizes all nonlinear transformations and substitutes Jacobian matrices for the linear transformations in the Kalman filter equations. The extended Kalman filter algorithm is an optimal recursive estimation algorithm for nonlinear systems. It processes all available measurements regardless of their precision, to provide a quick and accurate estimate of the variables of interest, and also achieves a rapid convergence [12].

C. Motor Model for Discrete Time Extended Kalman Filter

The Extended Kalman filter equations are;
Extrapolation equation

$$x_k = \hat{x}_k + A_k(x_{k-1} - \hat{x}_{k-1}) + W_k w_{k-}$$

Error covariance equation

$$\bar{P}_k = A_k P_{k-1} A_k^T + W_k Q_{k-1} W_k^T$$

Kalman gain

$$K_k = \bar{P}_k H_k^T \left(H_k \bar{P}_k H_k^T + V_k R_k V_k^T\right)^{-1}$$

New optimal state

$$\hat{x}_k = \hat{x}_k^- + K_k\left[z_k - H_k \hat{x}_k^-\right]$$

Updated error covariance

$$P_k = (I - K_k H_k)\bar{P}_k$$

Where

A_k is the Jacobian matrix of partial derivatives of f with respect to x,

W_k is the Jacobian matrix of partial derivatives of f with respect to *w*,

H_k is the Jacobian matrix of partial derivatives of h with respect to *x*,

V_k is the Jacobian matrix of partial derivatives of h with respect to *v*,

The Jacobian of the stepper motor model is described as follows:

$$\varphi = \begin{bmatrix} -\frac{R}{L} & 0 & \frac{\lambda}{L}*\sin(x_4) & x_3*\frac{\lambda}{L}*\cos(x_4) \\ 0 & -\frac{R}{L} & -\frac{\lambda}{L}*\cos(x_4) & x_3*\frac{\lambda}{L}*\sin(x_4) \\ -\frac{3}{2}*\frac{\lambda}{J}*\sin(x_4) & \frac{3}{2}*\frac{\lambda}{J}*\cos(x_4) & -\frac{F}{J} & -\frac{3}{2}*\frac{\lambda}{J}*(x_1*\cos(x_4)+x_2*\sin(x_4)) \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

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Measurement matrix:

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

The above mentioned EKF model is used to find the time update and measurement update equation. The motor state estimation results using EKF has been shown in figure 1.2. Standard deviation of estimation errors in different states comes out as = 0.12948, 0.14867, 0.35538, 0.89364, respectively. In figure1.2, dashed line shows the estimated results by Extended Kalman filter and thick lines correspond to the true results for the states of motor. From the results it is clear that Extended Kalman filter works better for the non linear systems state estimation. The same estimation problem is also addressed using Unscented Kalman filter in next part of simulation.

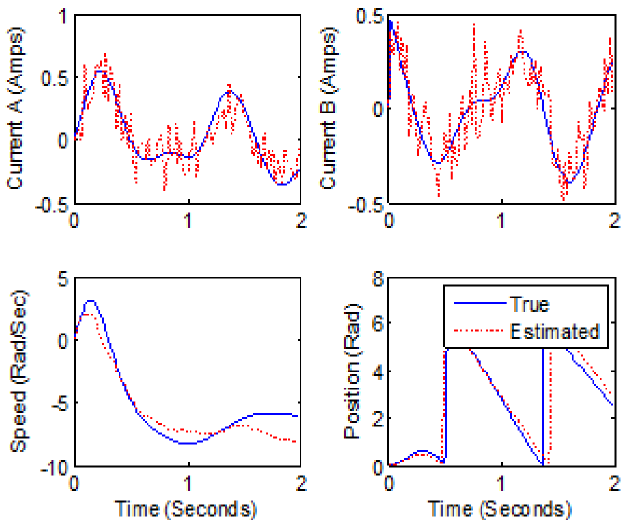

Figure 1.2 Motor's state estimation using Extended Kalman filter

D. Motor Model For Discrete Time Uncented Kalman Filter

Unscented Kalman filter uses the principle that a set of discrete sampled points can be used to parameterize mean and covariance. The estimator yields performance equivalent to the Kalman filter for linear systems and generalizes elegantly to nonlinear systems without the linearization steps as required by the Extended Kalman filter. The heart of Unscented Kalman filter is unscented transformation. The unscented transformation (UT) was developed to address the deficiencies of linearization by providing a more direct and explicit mechanism for transforming mean and covariance information [13]. First, we create number of sigma points which can be calculated by this equation,

npts = 2*n+1

where,

n= number of states

Defining some scaling parameters as,

alpha = scaling parameter 1

beta = extra weight on zero'th point

kappa = scaling parameter 2 (usually set to 0)

Calculating kappa according to scaling parameters by the following equation,

$kappa = \text{alpha}^\wedge 2 * (n + kappa) - n$

Matrix square root of weighted covariance matrix can be calculated by using equation as,

$Psqrtm = (n + kappa) * chol(P)'$

Array of sigma points is calculated by this equation,

xpts = [zeros(size(p,1),1)- Psqrtm Psqrtm]

Array of weights for each sigma point is created by the equation given below,

wPts = [kappa0.5*ones(1,nPts-1) 0] / (n+kappa)

There after calculated the zero'th covariance term weight,

wPts (nPts+1) = wPts(1)+(1- α^2)+ β

After that unscented transformation of process and measurement is used for transferring sigma points into process and measurements equations.

$$\boldsymbol{x}_{i,k} = F\left[\boldsymbol{x}_{i,k-1}, \mathbf{u}_{k-1}\right]; \mathbf{Z}_{i,k} = H\left[\boldsymbol{x}_{i,k-1}\right]$$

$$\hat{\boldsymbol{x}}_k^- = \sum_{i=0}^{2n} W_i \boldsymbol{x}_{i,k}$$

Transformed covariance for process is,

$$P_k^- = \sum_{i=0}^{2n} W_i \left\{\boldsymbol{x}_{i,k} - \hat{\boldsymbol{x}}_k^-\right\}\left\{\boldsymbol{x}_{i,k} - \hat{\boldsymbol{x}}_k^-\right\}^T$$

$$\hat{\mathbf{z}}_k^- = \sum_{i=0}^{2n} W_i \mathbf{Z}_{i,k}$$

Transformed covariance for measurement is calculated by the equation as;

$$P_{\mathbf{z}_k \mathbf{z}_k} = \sum_{i=0}^{2n} W_i \left\{\mathbf{Z}_{i,k} - \hat{\mathbf{z}}_k^-\right\}\left\{\mathbf{Z}_{i,k} - \hat{\mathbf{z}}_k^-\right\}^T$$

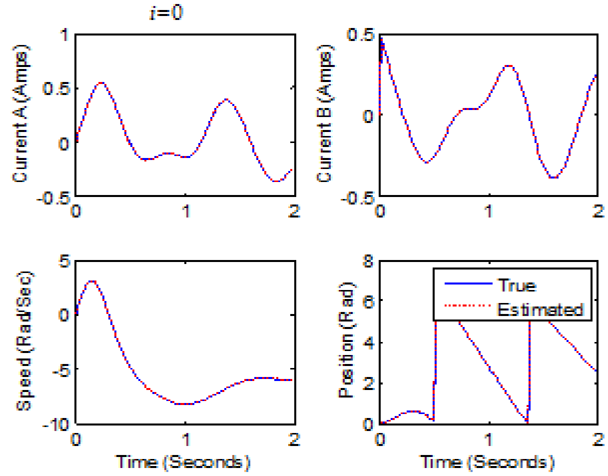

Figure 1.3 Motor's state estimation using Unscented Kalman filter

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TABLE I.

Nonlinear system: Permanent magnet stepper motor	
Filter	Standard deviation of estimation errors
Kalman filter	0.096911, 0.098586, 20.183, 4.7588
Extended Kalman filter	0.12948, 0.14867, 0.35538, 0.89364
Unscented Kalman filter	0.00060793, 0.00066092, 0.011073, 0.0017944

Transformed cross covariance is calculated as,

$$P_{x_k z_k} = \sum_{i=0}^{2n} W_i \left\{ \mathbf{x}_{i,k} - \hat{\mathbf{x}}_k^- \right\} \left\{ \mathbf{Z}_{i,k} - \hat{\mathbf{z}}_k^- \right\}^T$$

Kalman gain is calculated by the following equation,

$$K = P_{x_k z_k} P_{z_k z_k}^{-1}$$

Finally, the covariance update equation is given by,

$$P_k = P_k^- - K_k P_{\mathbf{z}_k \mathbf{z}_k} K_k^T$$

And State update equation is given by,

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + K_k \left(\mathbf{z}_k - \hat{\mathbf{z}}_k^- \right)$$

After applying all these equations into the motor model and coding in simulation, the results obtained by Unscented Kalman filter are shown in figure 1.3.

CONCLUSIONS

In this paper, design and implementation of various variants of Kalman filter is analyzed for state estimation of permanent magnet stepper motor. The analyses revealse that Kalman filter works well for linear systems and for non linear systems its performance is degraded. Extended Kalman filter is suitable for non linear systems but for highly non linear systems it has some limitations. Unscented Kalman filter performance is better than the Extended Kalman filter for highly non linear systems, and by solving a practical problem of a non linear system state estimation it has been observed that Unscented Kalman filter provides much better results for state estimation to Extended Kalman filter and Kalman filter

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REFERENCES

[1] Peter S. May beck., “Stochastic Models Estimation and Control,” vol. 1, pp 4, Academic Press, New York, 1999

[2] Zhengyou , “A Tutorial with Application to the Conic Fitting,” Vol.15, no.1,pp 59-76,Image and Vision Computing, 1997

[3] Stanly F.Schmidt, “The Kalman filter: its Recognition and Development for the Aerospace Applications,” vol. 4, no.1, Journal of Guidance, Control and Dynamics, 1981

[4] Jean Philippe Drecourt, “Kalman Filtering in Hydrological Modeling,” pp 10, Daihm Technical Report, 2003

[5] Barry Cipra, “Engineering Look to Kalman Filtering For Guidance,” vol. 26, no.5, SIAM news, 1993

[6] Simon J. Julior and Jeffery K.Uhlamann,”A new extension of the Kalman filter to non linear system,”pp1, International Symposium on Aerospace / Defence Sensing, Simulation and Controls, Orlando, 1997

[7] Harolwd Sorenson, “Guest Editorial on Application of Kalman Filtering,” vol. ac 28, no3, IEEE Transactions on Automatic Control,1983

[8] Welch and Bishop,. “An introduction to the Kalman filter ,” pp 4-5, SIGGRAPH, Los Angels, 2001

[9] Simon Julier,Jeffrey Uhlamann and Hugh F. Durrant –Whyte, “ A New Method for the Nonlinear Transformation of Means and Covariances in Filters and Estimators,” pp 477, vol. 45, IEEE Transactions on Automatic Control, 2000

[10] H.W. Sorenson, “Least Square Estimation: from Gauss to Kalman,” pp 63-68, vol. 7, IEEE Spectrum, 1970

[11] Dan Simon , “Optimal State Estimation, Kalman , H Infinity, and Nonlinear Approaches,” John Wiley & Sons, 2006

[12] R.Dhaouadi, L.Norum, N. Mohan, “Design and Implementation of an Extended Kalman Filter for the State Estimation of a Permanent Magnet Synchronous Motor ,” vol. 6, issue. 3, pp. 491- 497, IEEE Transactions on Power Electronics, 1991.

[13] S.Julier, J.Uhlmann, H.F. Durrant Whyte, “A New Method for the Nonlinear Transformation of Means and Covariances in Filters and Estimators,” vol. 45, issue. 3, pp. 477 - 482 IEEE Transactions on Automatic Control ,2000.

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