Let's look on the network graph where nodes have correlated values and distribution of values on all the nodes has the same mean μ and variance σ^2. Now let's assume that correlation between the nodes depends on the distance between them in the following way: nodes at the distance 1 have correlation ρ, at the distance 2 correlation ρ^2 and so on. If the distance between nodes i and j is k then $corr(X_i, X_j) = \rho^k$.

First, let's look at the line where nodes are correlated as described above. Then $corr(X_i, X_{i+h}) = \rho^h$. Now let's start to collect the values along the line starting from the first node, $X_1, X_2, ..., X_n$. Then we can count variation of the mean of $X_1, X_2, ..., X_n$.

$$var\left[\frac{X_1, X_2, ..., X_n}{n}\right] = var\left[\bar{X}\right] = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n} cov(X_i, X_j) =$$

$$= \frac{\sigma^2}{n^2}\left(n + 2(n-1)\rho + 2(n-2)\rho^2 + ... + 2 \cdot 2\rho^{n-2} + 2 \cdot 1\rho^{n-1}\right) =$$

$$= \frac{\sigma^2}{n^2}\left(n + 2\sum_{i=1}^{n-1}(n-i)\rho^i\right) = \frac{\sigma^2}{n^2}\left(n + 2n\sum_{i=1}^{n-1}\rho^i - 2\sum_{i=1}^{n-1} i\rho^i\right) =$$

$$= \frac{\sigma^2}{n}\left(n + 2n\frac{\rho - \rho^n}{1-\rho} - 2\rho\sum_{i=0}^{n-2}(\rho^{i+1})'\right) =$$

$$= \frac{\sigma^2}{n}\left(n + 2n\frac{\rho - \rho^n}{1-\rho} - 2\rho\left(\frac{\rho - \rho^n}{1 - rho}\right)'\right) =$$

$$= \frac{\sigma^2}{n}\left(n + 2n\frac{\rho - \rho^n}{1-\rho} - 2\rho\frac{(1 - n\rho^{n-1})(1-\rho) + \rho - \rho^n}{(1-\rho)^2}\right) =$$

$$= \frac{\sigma^2}{n^2}\frac{n - n\rho^2 - 1\rho + 2\rho^{n+1}}{(1-\rho)^2}$$

Let's simplify a bit expression for variance by approximated one.

$$var\left[\bar{X}\right] = \frac{\sigma^2}{n^2}\frac{n - n\rho^2 - 2\rho + 2\rho^{n+1}}{(1-\rho)^2} = \frac{\sigma^2}{n}\frac{n - n\rho^2 - 2\rho + 2\rho^{n+1}}{n(1-\rho)^2} =$$

$$= \frac{\sigma^2}{n}\frac{1 - \rho^2 - 2\rho/n + 2\rho^{n+1}/n}{(1-\rho)^2} \simeq \frac{\sigma^2}{n}\frac{1-\rho^2}{(1-\rho)^2} = \frac{\sigma^2}{n}\frac{1+\rho}{1-\rho}$$

Approximation is especially good with big n and ρ.

If random variables $X_1, X_2, ..., X_n$ were independent then the variance of $\bar{X}$ would be $var_{ind}[\bar{X}] = \frac{\sigma^2}{n}$.

But we consider random variables $X_1, X_2, ..., X_n$ that are dependent with known correlation and the variance in this case is bigger.

$$var\left[\bar{X}\right] = var_{ind}[\bar{X}]\frac{1+\rho}{1-\rho} = var_{ind}[\bar{X}]\left(1 + \frac{2\rho}{1-\rho}\right) > var_{ind}[\bar{X}]$$

Variance with skipping

Let's look at the variance of the next random variable:

$$\bar{X}^k = \frac{X_1 + X_{1+k} + X_{1+2k} + ... + X_{1+(n-1)k}}{n}$$

So $corr(X_{1+ik}, X_{1+(i+h)k}) = \rho^{kh}$. Now let's introduce new random variable $Y_1, Y_2, ..., Y_n$ such that $Y_1 = X_1, Y_2 = X_{1+k}, ..., Y_n = X_{1+(n-1)k}$ and $r = \rho^k$, $\bar{Y} = \bar{X}^k$. Then $corr(Y_i, Y_{i+h}) = corr(X_{1+(i-1)k}, X_{1+(i+h-1)k}) = \rho^{kh} = r^h$.

To sum up we have random variables $Y_1, Y_2, ..., Y_n$ where $corr(Y_i, Y_{i+h} = \rho^{kh} = r^h$. But we already know that

$$var\left[\bar{Y}\right] \simeq \frac{\sigma^2}{n}\frac{1+r}{1-r}$$

Then

$$var\left[\bar{X}^k\right] \simeq \frac{\sigma^2}{n}\frac{1+\rho^k}{1-\rho^k}.$$

In RDS context

B - budget

C_1 - cost of one step of walk (individuals just provide the correct number of their contacts)

C_2 - cost of participation (cost of interview with individuals)

n - number of steps

m - number of participants from n

The next equality should be true:

$$B = n \cdot C_1 + m \cdot C_2$$

If we want to skip k steps between taking the node as a participant then

$$B = nC_1 + \frac{n}{k+1}C_2$$

Here $m = \frac{n}{k+1}$ as we take each $k+1$ node as a participant. So having budget B and skipping each k node allows as to perform $n = \frac{(k+1)B}{(k+1)C_1+C_2}$ steps with $m = \frac{B}{(k+1)C_1+C_2}$ number of participants.

Then variance:

$$\frac{\sigma^2}{\frac{B}{(k+1)C_1+C_2}} \frac{1+\rho^{k+1}}{1-\rho^{k+1}}$$

The goal is to minimize variance. Let's look on the next function of k:

$$f(k) = \frac{\sigma^2}{\frac{B}{kC_1+C_2}} \frac{1+\rho^k}{1-\rho^k}$$

.

It has minimum when k is a solution for the following equation.

$$2C_1 \log(\rho)\rho^k k - C_1\rho^{2k} + 2C_2 \log(\rho)\rho^k + C_1 = 0$$

I don't know if there is explicit expression for the solution.