Gibbs sampling

We have graph with n nodes. Each node i has a value $x_i \in \{1, 2, 3, ..., k\} = P$.

Algorithm

1. Create random configuration of properties on all nodes.
2. Choose the node i
   - according to some distribution $q = q_1, ..., q_n$ or
   - visiting each node consequently (periodic Gibbs sampler)
3. For each value $x \in P$ count the local energy on chosen node i as

$$E_i(x) = \sum_{j|i\sim j} (x - x_j)^2$$

4. Choose a new value x_i according to probability

$$\frac{e^{-\frac{E_i(x)}{T}}}{\sum\limits_{x' \in P} e^{-\frac{E_i(x')}{T}}}$$

   where T is temperature.
5. Continue 2-3 needed number of iterations.

Simulations

First, random geometric graph with 200 nodes and radius 0.13 was created, $RGG(200, 0.13)$. The set of values is $P = \{1, 2, ..., 10\}$. According to the first step of algorithm for each node was generated random property. The properties are depicted on the pictures as colors. Following pictures describe the properties of the graph after 2000 iterations of 2-3 steps for different temperature.

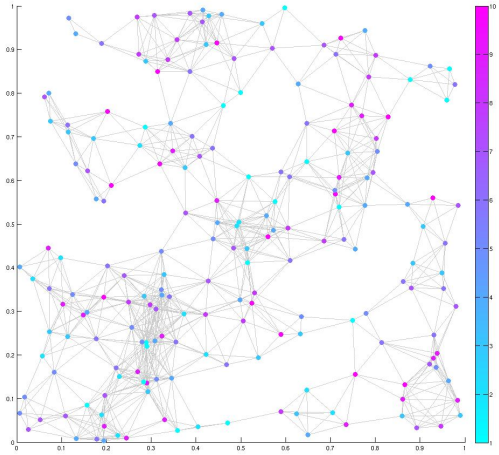

Figure 1: Random field

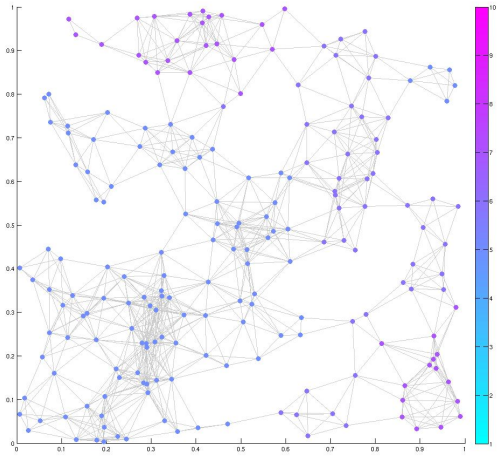

Figure 2: T = 0.1

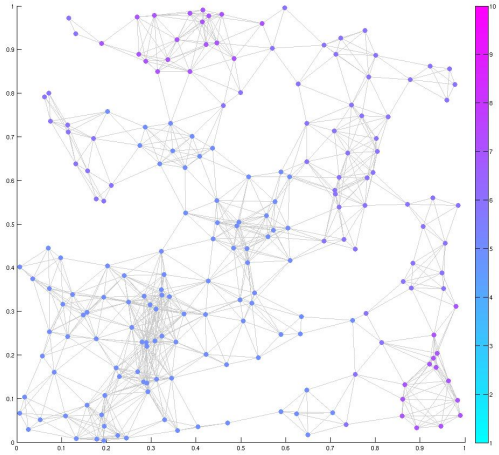

Figure 3: T = 1

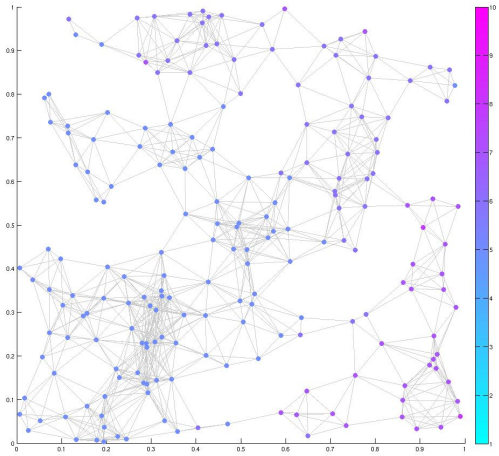

Figure 4: T = 2

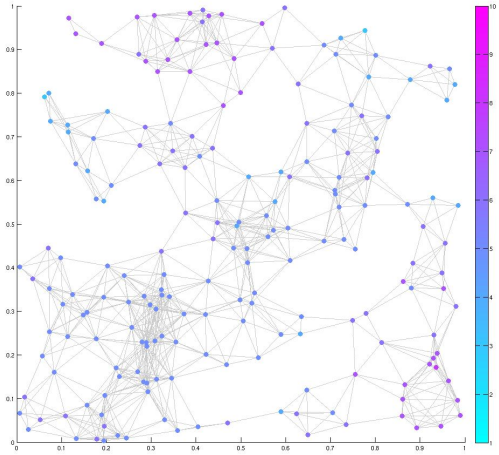

Figure 5: T = 4

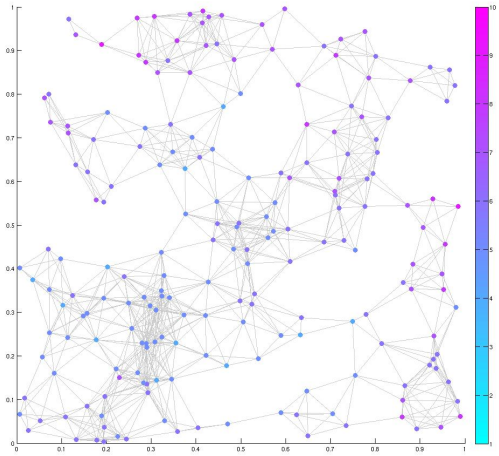

Figure 6: T = 8

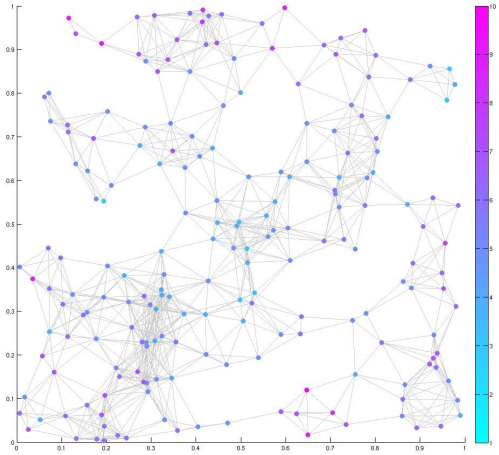

Figure 7: T = 16

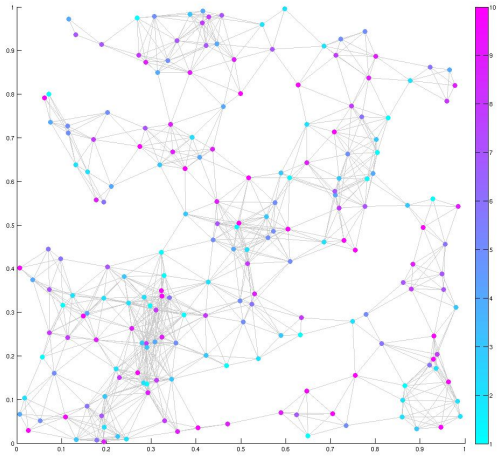

Figure 8: T = 2048