Machine Learning HW2 handwriting

1)

$$y = \delta(x) = \frac{1}{1+e^{-x}} \Rightarrow \delta(x) = (1+e^{-x})^{-1}$$

a) prove $\frac{\partial \delta(x)}{x} \Rightarrow \delta(x)(1-\delta(x))$

"Chain Rule" $$\frac{d\delta(x)}{d(x)} \Rightarrow -(1+e^{-x})^{-2} * (-e^{-x}) \Rightarrow \frac{e^{-x}}{(1+e^{-x})^2} \Rightarrow$$

Can be written as $\delta(x) = \frac{1}{1+e^{-x}} \Rightarrow e^{-x} = \frac{1-\delta(x)}{\delta(x)}$ Subsitude

$$\frac{d\delta(x)}{d(x)} \Rightarrow \frac{\frac{1-\delta(x)}{\delta(x)}}{(1+e^{-x})^2} \Rightarrow \frac{1-\delta(x)}{\cancel{\delta(x)}} * \delta(x)^{\cancel{2}} \Rightarrow (1-\delta(x))\,\delta(x)$$ ▲

b) prove $\delta(-x) \Rightarrow 1-\delta(x)$

$$\delta(-x) \Rightarrow \frac{1}{1+e^{x}} \Rightarrow \frac{e^{-x}}{e^{-x}(1+e^{x})} \Rightarrow \frac{e^{-x}}{1+e^{-x}} \Rightarrow 1 - \underbrace{\frac{1}{1+e^{-x}}}_{\delta(x)} \Rightarrow$$

"$1+e^{-x} - 1 \Rightarrow e^{-x}$"

$$\Rightarrow 1-\delta(x)$$ ▲

c) prove $x = \delta^{-1}(y) = \ln\left(\frac{y}{1-y}\right)$

$$y = \frac{1}{1+e^{-x}} \Rightarrow \frac{1}{y} \Rightarrow 1+e^{-x} \Rightarrow$$

$$e^{-x} \Rightarrow \frac{1}{y} - 1 \Rightarrow \frac{1-y}{y} \Rightarrow$$

$$e^{x} \Rightarrow \frac{y}{1-y} \qquad \ln(e^{x}) \Rightarrow x$$

$$\ln(e^{x}) \Rightarrow \ln\left(\frac{y}{1-y}\right) \Rightarrow x = \ln\left(\frac{y}{1-y}\right)$$ ▲

2)
$$\begin{cases} \hat{y}_n = \sigma(W^T \phi_n) \\ E(w) = -\ln p(t|w) \Rightarrow -\sum_{n=1}^{N} \{t_n \ln y_n + (1-t_n)\ln(1-y_n)\} \\ \nabla E(w) = \sum_{n=1}^{N} (y_n - t_n)\phi_n \end{cases}$$

Derivative =>

$$\frac{d}{dw}\Big(\underbrace{t_n \ln y_n}_{\mathrm{I}} + \underbrace{(1-t_n)\ln(1-y_n)}_{\mathrm{II}}\Big)$$

I => $\frac{d}{dw}(t_n \ln y_n) \Rightarrow t_n \frac{d}{dw}(\ln y_n) \Rightarrow$

$$t_n \frac{1}{y_n} \frac{dy_n}{dw}$$

Using result from "q1" $\frac{d\sigma(x)}{dx} \Rightarrow \sigma(x)(1-\sigma(x))$ and substitude $y_n = \sigma(w^T\phi_n)$

$$\frac{dy_n}{dw} \Rightarrow y_n(1-y_n)\phi_n \Rightarrow$$

$$\frac{d}{dw}(t_n \ln y_n) \Rightarrow t_n \frac{1}{\cancel{y_n}} * \cancel{y_n}(1-y_n)\phi_n \Rightarrow t_n(1-y_n)\phi_n$$

II $\frac{d}{dw}\big((1-t_n)\ln(1-y_n)\big) \Rightarrow (1-t_n)\frac{d}{dw}\big(\ln(1-y_n)\big) \Rightarrow$

$$-(1-t_n)\frac{1}{1-y_n} * \frac{d(1-y_n)}{dw}$$

$$\frac{d(1-y_n)}{dw} \Rightarrow -y_n(1-y_n)\phi_n$$

$$\frac{d}{dw}\big((1-t_n)\ln(1+y_n)\big) \Rightarrow -(1-t_n)\frac{1}{\cancel{(1-y_n)}} * -y_n\cancel{(1-y_n)}\phi_n \Rightarrow (1-t_n)y_n\phi_n$$

$$-\sum_{n=1}^{N} t_n(1-y_n)\phi_n - (1-t_n)y_n\phi_n \Rightarrow$$

$$-\sum_{n=1}^{N}(t_n - \cancel{t_n y_n} - y_n + \cancel{t_n y_n})\phi_n \Rightarrow -\sum_{n=1}^{N}(t_n - y_n)\phi_n$$

$$\sum_{n=1}^{N}(y_n - t_n)\phi_n$$

3)

$\hat{y}_n = \sigma(W^T \phi_n)$

$L(w) \Rightarrow -\sum_{n=1}^{N} \{ y_n \ln(\hat{y}_n) + (1-y_n) * \ln(1-\hat{y}_n) \}$ "BCEL" function

$\nabla L(w) \Rightarrow \sum_{n=1}^{N} (\hat{y}_n - y_n) \phi_n$

$\phi \Rightarrow$ input
$W \Rightarrow$ weight
$\sigma \Rightarrow$ sigmoid

3.1 $\frac{d}{dW} \ln(\hat{y}) \Rightarrow \frac{1}{\hat{y}_n} \frac{d\hat{y}_n}{dW} \Rightarrow \frac{1}{\cancel{\hat{y}_n}} \times + \cancel{\hat{y}}_n (1-\hat{y}_n) \phi \Rightarrow (1-\hat{y}_n) \phi_n$

3.2 $\frac{1}{dW} \ln(1-\hat{y}_n) \Rightarrow \frac{1}{1-\hat{y}_n} \frac{d(1-\hat{y}_n)}{dW} \Rightarrow \frac{1}{\cancel{1-\hat{y}_n}} - y_n \cancel{(1-\hat{y}_n)} \phi_n \Rightarrow -\hat{y}_n \phi_n$

3.1 $\frac{d\hat{y}_n}{dW} \Rightarrow \frac{d\,\sigma(W^T \phi_n)}{dW} \Rightarrow \hat{y}_n (1-\hat{y}_n) \phi_n$

3.2 $\frac{d(1-\hat{y}_n)}{dW} \Rightarrow -\hat{y}_n (1-\hat{y}_n) \phi$

$$-\sum_{n=1}^{N} \left[y_n (1-\hat{y}_n) \phi_n - (1-y_n) \hat{y}_n \phi_n \right]$$

$$\left[y_n \phi_n - \cancel{y_n \hat{y}_n \phi_n} - \hat{y}_n \phi_n + \cancel{y_n \hat{y}_n \phi_n} \right] \Rightarrow$$

$$-\sum_{n=1}^{N} (y_n - \hat{y}_n) \phi_n \Rightarrow \underbrace{\sum_{n=1}^{N} (\hat{y}_n - y_n) \phi_n}_{\nabla L(w)}$$