January 2024 CSE 220
Online on the Convolution of Linear Time-Invariant Systems
Subsections: B1, B2

October 27, 2024

Suppose you have the stock market prices of XYZ company for the last few days. For trend analysis, it is often convenient to work with the moving averages of the stock prices. There are many ways to do this. One way is to simply take the *average* of the last n days of stock prices. Here n is called the **window size**. This average is called the Simple or Unweighted Moving Average. However, this method gives the same weight to each of the last n days. In stock price analysis, the later days usually have more influence on the future prices.

One way to solve this problem is to put more weight on the last day, a bit less weight on the day before that and so on. This method is called the **Weighted Moving Average** and it puts different weights on each of the n days. For this assignment, please set the weight of the last day to n, the weight of the day before that to $n-1$, and so on. Please remember that you need to normalize these weights so that their sum equals to 1. Take a look at the sample I/O for a better understanding.

Your task in this assignment is to determine the Simple or **Unweighted Moving Average** and the **Weighted Moving Average** of stock prices for the window size n. You must make use of convolution to solve both problems. Using Python library functions to determine any of the two averages is strictly prohibited. You should use the classes and functions implemented during your Offline 1 assignment.

I/O Format

You shall need to take stock market prices and the window size as inputs and print the two moving averages mentioned above as outputs.

In the first line, take the stock prices as input in the form of space separated integers. Then in the next line, you should take the moving window size n as the input.

As for the output, you should print out the moving average as the outputs. Please refer to the sample I/O for a better understanding.

A template Python file already containing these codes has been be given. You need not implement I/O.

Sample I/O

Case 1

Input

```
Stock Prices: 1, 2, 3, 4, 5, 6, 7, 8
```

```
Window size: 4
```

Output

```
Unweighted Moving Average: 2.50, 3.50, 4.50, 5.50, 6.50
Weighted Moving Average:   3.00, 4.00, 5.00, 6.00, 7.00
```

Explanation

Stock Prices	1	2	3	4	5	6	7	8	Unweighted Moving Averages	
Unweighted Average Window	0.25	0.25	0.25	0.25					= (1+2+3+4)*0.25	**= 2.5**
		0.25	0.25	0.25	0.25				= (2+3+4+5)*0.25	**= 3.5**
			0.25	0.25	0.25	0.25			= (3+4+5+6)*0.25	**= 4.5**
				0.25	0.25	0.25	0.25		= (4+5+6+7)*0.25	**= 5.5**
					0.25	0.25	0.25	0.25	= (5+6+7+8)*0.25	**= 6.5**

Stock Prices	1	2	3	4	5	6	7	8	Weighted Moving Averages	
Weighted Average Window	0.1	0.2	0.3	0.4					= 1*0.1 + 2*0.2 + 3*0.3 + 4*0.4	**= 3.0**
		0.1	0.2	0.3	0.4				= 2*0.1 + 3*0.2 + 4*0.3 + 5*0.4	**= 4.0**
			0.1	0.2	0.3	0.4			= 3*0.1 + 4*0.2 + 5*0.3 + 6*0.4	**= 5.0**
				0.1	0.2	0.3	0.4		= 4*0.1 + 5*0.2 + 6*0.3 + 7*0.4	**= 6.0**
					0.1	0.2	0.3	0.4	= 5*0.1 + 6*0.2 + 7*0.3 + 8*0.4	**= 7.0**

Case 2

Input

```
Stock Prices: 5, -2, 3, 1, 0, -6, 4, -2, 1
Window Size:  3
```

Output

```
Unweighted Moving Average: 2.00, 0.67, 1.33, -1.67, -0.67, -1.33, 1.00
Weighted Moving Average:   1.67, 1.17, 0.83, -2.83, 0.00, -0.67, 0.50
```

Hints

- Try to relate the following formula of the Convolution Sum to the formula of the moving average. What should be $x[k]$ and how is it related to the stock prices? Note that $x[k]$ is the same for both cases of the moving averages. What should you choose as the impulse response signal $h[n-k]$? How is it related to the moving average windows? Take a look at the two tables above.

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] \times h[n-k]$$

Marks Distribution

- Taking the input in the given format: **1 Mark**
- Determining an **input signal** for determining the **both Moving Averages using convolution**: **1 Mark**
- Determining an **impulse response** for determining the **Unweighted Moving Average using convolution**: **1 Mark**
- Determining the **Unweighted Moving Average** correctly: **2 Marks**
- Determining an **impulse response** for determining the **Weighted Moving Average** using convolution: **2 Marks**
- Determining the **Weighted Moving Average** correctly: **2 Marks**
- Printing the output in the given format to the console: **1 Mark**