January 2024 CSE 220
Online on the Convolution of Linear Time-Invariant Systems
Subsections: C1, C2

October 26, 2024

Polynomials are algebraic expressions that consist of variables and coefficients. An example of a polynomial with one variable is $x^2 - 3x - 1$. The degree of a polynomial is defined as the highest exponent of a variable within a polynomial. For example: $5x^4 + x^3 - 2x^2 + 1$ has degree 4. We can multiply two polynomials with each other. For example: $(3x^2 - 2x + 1) \times (2x^3 - 3x + 1) = 6x^5 - 4x^4 - 7x^3 + 9x^2 - 5x + 1$.

Your task in this assignment is to implement the polynomial multiplication using discrete-time convolution operation. You should make use of the Python classes and functions implemented in your Offline assignment.

I/O Format

You shall need to take two polynomials as inputs and print the result of their multiplication as the output. You can assume that the polynomials have only one variable x.

In the first line, take the degree d_1 of the first polynomial as input. Then in the next line, take $(d_1 + 1)$ number of space separated integers as input. They are the coefficients of the decreasing exponents of x i.e. the coefficients of x^{d_1}, x^{d_1-1}, x^{d_1-2}, ... , x, 1, in this order. Similarly, take the second polynomial as input.

As for the output, you should multiply these two polynomials and print out the degree, followed by the coefficients in the next line. Please refer to the sample I/O for a better understanding.

Sample I/O

Case 1

Input

```
Degree of the first Polynomial: 2
Coefficients: 3 -2 1
Degree of the second Polynomial: 3
Coefficients: 2 0 -3 1
```

Output

```
Degree of the Polynomial: 5
Coefficients: 6 -4 -7 9 -5 1
```

Explanation

The first input denotes the polynomial $3x^2 - 2x + 1$ (degree=2) where the coefficients of x^2, x and x^0 are consecutively 3, -2 and 1. Similarly the second polynomial is $2x^3 - 3x + 1$ (degree=3). Their multiplication results in the polynomial $6x^5 - 4x^4 - 7x^3 + 9x^2 - 5x + 1$ (degree=5).

Case 2

Input

```
Degree of the first Polynomial: 2
Coefficients: 1 2 -1
Degree of the second Polynomial: 4
Coefficients: 2 -1 0 3 2
```

Output

```
Degree of the Polynomial: 6
Coefficients: 2 3 -4 4 8 1 -2
```

Case 3

Input

```
Degree of the first Polynomial: 4
Coefficients: 3 -2 0 1 -5
Degree of the second Polynomial: 3
Coefficients: 1 2 -3 -1
```

Output

```
Degree of the Polynomial: 7
Coefficients: 3 4 -13 4 -1 -13 14 5
```

Hints

- Try to relate the following formula of the Convolution Sum to the polynomial multiplication. What should be $x[k]$? What should you choose as the impulse response signal $h[n-k]$?

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] \times h[n-k]$$

- The following table shows the process of multiplying Polynomial A = $(2x^3 - 3x + 1)$ with Polynomial B = $(3x^2 - 2x + 1)$. The result is the Polynomial $6x^5 - 4x^4 - 7x^3 + 9x^2 - 5x + 1$. Pay attention to how the coefficients of x^5, x^4, ... , x, 1 are determined.

Polynomial A			2	0	-3	1			Coefficients
Coefficients of Polynomial B	1	-2	3						1*0+(-2)*0+3*2 = 6
		1	-2	3					1*0+(-2)*2+3*0 = -4
			1	-2	3				1*2+(-2)*0+3*(-3) = -7
				1	-2	3			1*0+(-2)*(-3)+3*1 = 9
					1	-2	3		1*(-3)+(-2)*1+3*0 = -5
						1	-2	3	1*1+(-2)*0+3*0 = 1

Marks Distribution

- Taking the input in the given format: **1 Mark**
- Defining an appropriate impulse response signal of the LTI system: **2 Marks**
- Defining an appropriate input signal to the LTI system: **2 Marks**
- Generating the multiplication of the two polynomials accurately using DT convolution: **4 Marks**
- Printing the output in the given format to the console: **1 Mark**