Bachelor thesis draft - comparison of GARCH-family models with realized volatility models in estimating volatility of time series

František Vlach, 66851139

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1 Value at risk

More theoretical discussion about VaR

Since 1998, banks with substantial trading activity have been required to set aside capital in order to insure for the case of extreme portfolio losses. The set-aside capital, also called the market risk capital requirement, is linked to a measure of portfolio risk. Currently, portfolio risk is measured with the use of its Value-at-risk (VaR), which is defined to be the loss which is expected to be exceeded only with $\alpha\%$ probability, i. e. only $\alpha\%$ of the time over a fixed time interval. Current regulatory framework requires that financial institutions use their own internal risk models to calculate and report a 1% value-at-risk, the VaR(0.01) over a 10-day horizon.

I am using 66-days forecast - check and describe

Check what VaR is actually computed - we have a 66 days forecast but the VaR is related to what length?

The VaR is defined as

$$VaR_t\left(\alpha\right) = -F^{-1}\left(\alpha|\Omega_t\right)$$

where $F^{-1}\left(\cdot|\Omega_t\right)$ represents the quantile function of the profit and loss distribution which varies over time as market conditions and the portfolio's composition, represented by Ω_t change. Accurate means of examining whether the reported VaR represents an accurate measure of actual risk level are necessary since financial institutions use their own internal risk models to determine the specific level of VaR, based on which they adhere to risk-based capital requirements. [Campbell, 2005]

1.1 Statistical framework of VaR backtests

A variety of tests have been proposed since 1990's in order to measure the accuracy of a VaR model. Many of these focus on a particular transformation of the reported VaR and realized profit or loss. We maY consider the event that the loss on a portfolio exceeds its reported VaR, that is, $VaR_t\left(\alpha\right)$. Denoting the daily profit or loss on the portfolio over a fixed time interval, we can defined the hit function as follows:

$$\begin{cases} 1, \ if \ x_{t,t+1} \leq -VaR_t(\alpha) \\ 0, \ if \ x_{t,t+1} > -VaR_t(\alpha) \end{cases}$$

which results in a sequence of 1 and 0 which carries the history of whether or not a loss in excess of the reported VaR has been realized. [Campbell, 2005]

[Bera et al., 1970] reduces the problem of determining the accuracy of a VaR model to determining whether the hit sequence

$$[I_{t+1}(\alpha)]_{t=1}^{t=T}$$

satisfies two propertis:

1. The unconditional coverage property states that the probability of realizing a loss in excess of the reported VaR must be precisely $\alpha\%$. If the losses in excess of the reported VaR occur more frequently, then it is a suggestion that the VaR measure systematically understates the portfolio's actual level of risk and vice versa, finding too few VaR violations may signal a systematic overstating of the risk level.

2. The idependence property places a restriction on the ways in which these violations may occur. Specificially, any two elements of the hit sequence must be independent from each other. This condition requires that the previous history of VaR violations must not convey any information about whether an additional VaR violation will occur. If previous VaR violations presage a future VaR violation, it suggests a general inadequacy in the reported VaR measure. An example of such inadequacy may be bunching, that is, the occurence of violations of VaR is cumulated together. This represents a violation of the independence property that signals a lack of responsiveness in the reported VaR measure as changing market risks fail to be fully incorporated into the reported VaR measure which makes successive runs of VaR violations more likely.

Therefore I should probably test bunching too - maybe Christoffersen test

These two properties are separate and distinct and must be both satisfied by an accurate VaR model. Only hit sequences that satisfy both properties can be described as evidence of an accurate VaR model. The two properties of the "hit" sequence, $[I_{t+1}(\alpha)]_{t=1}^{t=T}$, are often combined into a single statement:

$$I_t(\alpha) \overset{i.i.d}{\sim} B(\alpha)$$

i. e. the hit sequence is identically and independently distributed as a Bernoulli random variable with probability α.

1.2 Tests of VaR accuracy

1.2.1 Unconditional coverage tests

The earliest proposed VaR backtests, such as the commonly used Kupiec proportion of failure (POF) test [Kupiec, 1995], focus only on the first property, that is, unconditional coverage. These tests test only whether the reported VaR level is violated more or less than $\alpha\%$ of the time. The Kupiec tests statistic has the form

$$POF = 2 \cdot \left(\left(\frac{1-\hat{\alpha}}{1-\alpha} \right)^{T-I(\alpha)} \cdot \left(\frac{\hat{\alpha}}{\alpha} \right)^{I(\alpha)} \right)$$

$$\hat{\alpha} = \frac{1}{T} \cdot I(\alpha)$$

$$I(\alpha) = \sum_{t=1}^{T} I_t(\alpha)$$

where T is the sample size. We can see that if the proportion of VaR vilations is exactl equal to $\alpha\%$, then the POF test statistic is equal to zero. As the proportion differs from $\alpha\%$, the POF test statistic grws, indicating an evidence that the portfolio's underlying level of risk is either systematically underestimated or overestimated by the proposed VaR measure.

Other tests also exist to assess the unconditional coverage property of a given VaR model. One alternative is to simply base a test directly on the sample average of the number of VaR violations over a given time period, $\hat{\alpha}$. Under the assumption that the VaR under consideration is accurate, then a scaled version of $\hat{\alpha}$,

$$z = \frac{\sqrt{T} \cdot (\hat{\alpha} - \alpha)}{\sqrt{\alpha} \cdot (1-\alpha)}$$

has an approximate $N(0,1)$ distribution and since the exact finite distribution of z is known and so hypothesis tests can be conducted in exactly the same way that hypothesis tests are conducted in the case of Kupiec's POF statistic.

The tests of unconditional coverage, while useful in providing a benchmark for assessing the accuracy of a given VaR model, have two disadvantages: First, they are known to have difficulty to detect VaR measures which systematically under report risk. [Kupiec, 1995]. Second, they focus exclusively on the unconditional coverage property of an adequate VaR measure and do not examine the extent to which the independence property is satisfied. Therefore, they may naturally fail to detect VaR measures that exhibit correct unconditional coverage but dependent VaR violations, which may result in losses that exceed the reported VaR in clusters or streaks, which may result in even more stress on a financial institution than large unexpected losses that occur somewhat more frequently than expected but are spread out over time.

It is safe to conclude that as dependent VaR violations signal a lack of responsiveness to changing market conditions and inadequate risk reporting, relying solely on unconditional coverage tests appears problematic. [Campbell, 2005]

Independence tests from the same paper

2 Methods used for estimating volatility

Joint tests

? Check Joint tests based on multiple α

Add theory on how VaR can be estimated using volatility

The generalized autoregressive conditional heteroskedasticity (GARCH) model was introduced in [Bollerslev, 1987] Bollerslev in 1987 as a generalization of the earlier ARCH model defined by [Engle, 1982] and since then, it has been widely used for studying the volatility of time series. The original general specification of the GARCH model was

$$r_t = \phi_0 + \phi_1 \cdot r_{t-1} + a$$

$$a_t = \sigma_t \cdot \epsilon_t$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i \cdot a_{i-1}^2 + \sum_{j=1}^{s} \beta_j \cdot \sigma_{t-j}^2$$

or for the commonly used GARCH(1,1)

$$\sigma^2 = \alpha_0 + \alpha_i \cdot a_{t-1}^2 + \beta_1 \cdot \sigma_{t-1}^2 d$$

Several extensions were proposed by other authors later.

The GARCH in mean (GARCH-M) model which connects the return of an asset to its volatility was introduced by [Engle et al., 1987]

$$r_t = \mu + c \cdot \sigma_t^2 + a_t$$

$$a_t = \sigma_t \cdot \epsilon_t$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i \cdot a_{i-1}^2 + \sum_{j=1}^{s} \beta_j \cdot \sigma_{t-j}^2$$

The integrated GARCH (IGARCH), defined in [Engle and Bollerslev, 1986] model which is a unit-root integrated GARCH model in which the past squared shock is persistent:

Does it make much sense to speak about these methods and not use them? Or shold I use them to also have a comparison?

$$r_t = \phi_0 + \phi_1 \cdot r_{t-1} + a$$

$$a_t = \sigma_t \cdot \epsilon_t$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} (1 - \beta_i) \cdot a_{i-1}^2 + \sum_{j=1}^{s} \beta_j \cdot \sigma_{t-j}^2$$

[Zakoian, 1994] introduces the threshold autoregressive GARCH (TAR-GARCH), which is able to take into account the assymetric response in the volatility equation to the sign of a shock, which is supported empirically:

$$r_t = \phi_0 + \phi_1 \cdot r_{t-1} + a$$

$$a_t = \sigma_t \cdot \epsilon_t$$

$$\sigma_t^2 = \begin{cases} \alpha_0 + \alpha_1 \cdot a_{t-1}^2 + \beta_1 \cdot \sigma_{t-1}^2, & a_{t-1} \leq 0 \\ \alpha_2 + \alpha_3 \cdot a_{t-1}^2 + \beta_2 \cdot \sigma_{t-1}^2, & a_{t-1} > 0 \end{cases}$$

The Glosten-Jagannathan-Runkle GARCH (GJR-GARCH), which is a simple version of a threshold GARCH, as defined by [GLOSTEN et al., 1993]:

$$\sigma_t^2 + \alpha_0 + \beta_1 \cdot \sigma_{t-1}^2 + \alpha_1 \cdot a_{t-1}^2 + \alpha_1 \cdot a_{t-1}^2 + \gamma \cdot I\left(a_{t-1} \leq 0\right) \cdot a_{t-1}^2$$

$$I\left(a_{t-1} \leq 0\right) = 0 \text{ if } a_{t-1} \leq 0 \text{ and } I\left(a_{t-1} \leq 0\right) = 1 \text{ if } a_{t-1} > 0,$$

The Exponential GARCH (EGARCH) model as defined by [Nelson, 1991]:

$$a_t = \sigma_t \epsilon_t$$

$$a_t = \sigma_t \cdot \epsilon_t$$
$$log\left(\sigma_t^2\right) = \omega + \sum_{k=1}^{q} \beta_k \cdot g\left(Z_t\right) + \sum_{k=1}^{p} \alpha_k \cdot log\left(\sigma_{t-k}^2\right)$$
$$g\left(Z_t\right) = \Theta \cdot Z_t + \lambda \cdot \left(|Z_t| - E\left(Z_t\right)\right)$$

where Z_t is a standard normal variable, so $g\left(Z_t\right)$ allows the sign and the magnitude of Z_t to have separate effects on the volatility, which is especially useful in asset pricing context. Another class of models used especially for high-frequency data is those which utilize realized variance, as introduced in [Barndorff-Nielsen and Shephard, 2002]. Realized variance can be computed as

$$RV_{i,t} = \sum_{j=1}^{m} r_{i,t-1+j \cdot n}^2$$

Using the realized variance, we can estimate an AR(p) process as

$$RV_t = \beta_0 + \sum_{i=1}^{p} \beta_i \cdot RV_{t-i} + \epsilon_t$$

The basic Realized GARCH model was introduced by [HANSEN et al., 2012] and can be constructed as

$$h_t = \omega + \sum_{i=1}^{p} \beta_i h_{t-i} + \sum_{j=1}^{q} \alpha \cdot r_{t-1}^2$$

Another method which we can use for studying conditional heteroskedasticity with the use of realized volatility is the Heterogeneous Autoregression (HAR), introduced by [Corsi, 2009]:

$$RV_t = \alpha_0 + \eta_1 \cdot RV_{t-1} + \beta_2 \cdot RV_{t-1}^{(5)} + \beta_3 \cdot RV_{t-1}^{(22)} + u_t$$

where the $RV_{T-1}^{(H)}$ is h-period realized variance, so $RV_{t-1}^{(5)}$ corresponds to 1 week and $RV_{t-1}^{(22)}$ corresponds to one month, and u_t is a normally distributed error term.
We can further decompose the realized volatility into positive semivariances, developed by [Barndorff-Nielsen et al., 2010] as $RV_t = RS_t^- + RS_t^+$ where

$$RS_{i,t}^- = \sum_{j=1}^{m} r_{i,t-1+j \cdot n}^2, \text{ if } r_{i,t-1+j \cdot n} < 0$$

$$RS_{i,t}^{+} = \sum_{j=1}^{m} r_{i,t-1+j\cdot n}^{2}, \text{ if } r_{i,t-1+j\cdot n} > 0$$

and use them these in the model, as proposed by [Patton and Sheppard, 2015]:

$$RV_t = \alpha_0 + \beta_1^{+} \cdot RS_{t-1}^{+} + \beta_1^{-} \cdot RS_{t-1}^{-} + \beta_2 \cdot RV_{t-1}^{(5)} + \beta_3 \cdot RV_{t-1}^{(22)} + u_t$$

We can also compute realized skewness and and kurtosis as

$$RSkew_{i,t} = \frac{\sqrt{m}\sum_{i=1}^{m} r_{i,t-1+j\cdot n}^{3}}{RV_t^{\frac{3}{2}}}$$

and

$$RKurt_{i,t} = \frac{\sqrt{m}\sum_{i=1}^{m} r_{i,t-1+j\cdot n}^{4}}{RV_t^{2}}$$

and add them into the model, as proposed by [Amaya et al., 2015]:

$$RV_t = \alpha_0 + \beta_1^{\cdot} RS_{t-1} + \beta_2 \cdot RV_{t-1}^{(5)} + \beta_3 \cdot RV_{t-1}^{(22)} + \beta_s \cdot RSkew_{t-1} + \beta_k \cdot RKurt_{t-1} + u_t$$

or

$$RV_t = \alpha_0 + \beta_1^{+} \cdot RS_{t-1}^{+} + \beta_1^{-} \cdot RS_{t-1}^{-} + \beta_2 \cdot RV_{t-1}^{(5)} + \beta_3 \cdot RV_{t-1}^{(22)} + \beta_s \cdot RSkew_{t-1} + \beta_k \cdot RKurt_{t-1} + u_t$$

Add theory regarding different forecasting schemes

3 Model evaluation

The dataset we use includes 35 time series with daily granularity in the period starting on 2010-01-05 and ending on 2016-01-22 on stock returns with precalculated Realized Volatility, positive realized semi-volatility, negative realized semi-volatility, realized skewness and realized kurtosis. The specific stocks which we use are listed in the table in appendix.

The process of comparing the models will be by following steps for each model:

First, a full-sample size model will be used for fitting the in-sample model. Then there will be a discussion of the fits and comparison of qualitative differences of the estimates. Since the true conditional variance is latent, it needs to be substituted by some ex-post estimator based on observed quantities as they become available. Possible candidates to serve as unbiased proxies for volatility are, for example, the daily squared returns or RV. One way this can be done is visually by plotting the in-sample fits compared to the selected proxy. The loss function measures the difference between realization and the forecast. Some of the commonly used loss functions are Mean absolute error (MAE):

$$\mathcal{L}\left(RV_{t+1}, \hat{RV}_{t+1|t}\right) = \left|RV_{t+h} - \hat{RV}_{t+h|t}\right|$$

Mean squared error (MSE):

$$\mathcal{L}\left(RV_{t+1}, \hat{RV}_{t+1|t}\right) = \left(RV_{t+h} - \hat{RV}_{t+h|t}\right)^2$$

Quasi-likelihood (QLIKE):

$$\mathcal{L}\left(RV_{t+1}, \hat{RV}_{t+1|t}\right) = \left(log\left(\hat{RV_{t+h|t}}\right) + \frac{RV_{t+h}}{\hat{RV}_{t+h|t}}\right)$$

Furthermore, we need to check whether the ARMA-GARCH and RGARCH models have normally distributed residuals by graphical representation, the Q-Q plot and formally by the Jarque-Bera test with the null hypothesis of normal distribution. The test statistics of the Jarque-Bera test, as defined in [Jarque and Bera, 1980] is:

$$JB = \frac{n}{6}\left(S^2 + \frac{1}{4}(K-3)^3\right)$$

where n is the number of observations, or degrees of freedom, S is the sample skewness:

$$S = \frac{\hat{\mu}_3}{\hat{\sigma}^3} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^3}{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2\right)^{\frac{3}{2}}}$$

and K is the sample kurtosis:

$$K = \frac{\hat{\mu}_4}{\hat{\sigma}^4} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^4}{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2\right)^2}$$

where

$$JB \sim \chi^2(2)$$

in case that the data is normally distributed.

Second, we compare the out-of-sample forecasting performance of the selected models using different forecasting schemes: Expanding window, in which the sample used to estimate the parameters of the model grows as the forecaster makes predictions for successive observations, and Rolling window , in which the sequence of forecasts is based on parameters estimated using a rolling sample of fixed size. Then we can plot and compare the forecast errors from all models, compute the loss functions (MSE and MAE), compare model performance by the Diebold-Mariano test, introduced in [Diebold and Mariano, 2002]:

$$DM - T = \frac{\sqrt{T}\cdot\overline{d}}{\sqrt{\omega}} \overset{a}{\sim} N(0,1)$$

where $d_t = \mathcal{L}_{1,t} - \mathcal{L}_{2,t}$ is the loss differential, which is assumed to be stationary, and the null hypothesis of the DM test is $E[d_t] = 0$, and $\overline{d} = T^{-1}\sum_t d_t$ and

ω is its asymptotic variance, estimated as a sample variance of d_t. We can also utilize the Mincer-Zarnowitz regression, which regresses realized values on forecasts, as defined by [Mincer and Zarnowitz, 1969]:

$$\sigma_{t+1} = \beta_0 + \beta_1 \cdot \hat{\sigma}_{t+1}$$

and test the joint hypothesis, where not rejecting it means that the forecast is unbiased.

$$\beta_0 = 0, \;\; \beta_1 = 1$$

Third, the results from the above described methods shall be summarised and we will observe whether the comparison of results of performance of respective models is consistent across time series, or whether they differ. If they differ, we will discuss what may be determining the performance of respective models for each respective time series.

4 Data

To be able to observe a trend in performance in each model, we are using a set of daily data for 80 stocks. From the 100 most traded US stocks as of 7th May 2024, we eliminated these which had a long break of missing value in the middle, did not have a sufficient number of observations prior to outbreak of covid (at least 1000 observations before 29th November 2019), and several stocks in which some of the models failed to converge, even with careful parameter tuning. The used stocks and their basic properties are shown in Tables 5 and 6. For each stock we have the following values for every date:

Source for 1000 obs

source

- Close price
- Realized variance
- Realized positive semivariance
- Realized negative semivariance
- Realized skewness
- Realized kurtosis

We use close price to compute returns as simply

$$r_{i,t} = \frac{P^{close}_{i,t} - P^{close}_{i,t-1}}{P^{close}_{i,t}}$$

For each stock, we run the Augmented Dickey-Fuller (ADF) test for a presence of unit root on the returns. We also perform the Ljung-Box (LB) test for autocorrelation and Jarque-Bera (JB) test for normality. The p-values of these tests are summarized in Tables 1 and 2. From the p-values of the Jarque-Bera test, we can see that returns are not normally distributed, therefore in the VaR

Add Ljung-Box test to methodology

estimation section, we shall prefer to assume t-distributed returns rather than normally-distributed. Furthermore, we can see from the Augmented Dickey-Fuller test that unit root is not found in any of the return time series. Lastly, the p-values of the Ljung-Box suggest that there is a small amount of return time series which do exhibit serial correlation, however the most of them are serially independent.

Does it actually make any sense?

How is serial correlation/independence relevant for the methods? What if the results differ as here?

5 Model estimation and evaluation

We estimate the following models, described in section 2:

- GARCH
- AR(1)-RV
- R-GARCH
- HAR
- HAR-AS
- HAR-RS
- HAR-RSRK

Check consistency of names

Tables 3 and 4 show the p-values of the Jarque-Bera test for normality on the residuals from each model rounded to two decimals. We can clearly see that the residuals from each model for every stock is not normally distributed in any case.

Check GARCH orders

What does it mean, what should we do with that? t-distributed GARCH?

6 Forecasting

Tables 9 and 10 show the mean absolute errors for forecasts using expanding scheme. We can see that in general the GARCH models perform the best while the AR-RV model performs the worst, while the performance of the HAR models is in the middle and there are small differences between individual types of HAR models.

Does this even make any sense?

Tables 11 and 12 show the mean absolute errors of forecasts using rolling scheme. The conclusion is similar to the case of expanding forecast as there are only marginal differences between the type of forecasting window.

Update ALL tables with correct RGARCH values

Tables 13 and 14 show the mean square errors of forecasts using expanding scheme. The results are similar to that of mean absolute error, GARCH models having the smallest loss function and AR-RV the highest with HAR models in between.

Check properly, change the table - add order into brackets

Tables 15 and 16 show the mean square error of forecasts using rolling scheme. Again, the results are similar to that of models with expanding window forecast since there are only small differences depending on the type of forecasting window.

Check properly, change the table - add order

Tables 17 and 18 show which forecasting scheme performs better according to mean absolute error. Tables 19 and 20 show which forecasting scheme performs better according to mean square error.

Table 7 shows an overview of how many forecasts of respective stocks perform better with expanding or rolling window for each model according to mean absolute error. Table 8 shows an overview of how many forecasts of respective stocks perform better with expanding or rolling window for each model according to mean square error. Altogether we can say that there are only small nuances and we cannot observe a clear pattern stating that one type of forecasting window performs better for specific type of model, the only exception being the HAR-RSRK, in which rolling forecasting scheme outperforms expanding forecasting scheme for all the stocks we study. Therefore, for the VaR estimation, we will use results obtained using both schemes.

Doublecheck whether this really makes sense and try to reason why

Tables 21 and 22 show the mean value and standard deviation of the p-values of Diebold-Mariano test used to compare the forecasting performance of different models. Table 23 shows the percentage of tests for each stock in which the p-value was lower than 0.05, i. e. that the models differ in forecasting performance for the respective stock. In a Diebold-Mariano test, the null hypothesis states that both models are equivalently good, so a high p-value means that we do not have evidence that the models are different, whereas a small (¡0.05) p-value suggests that the models perform differently. Since table 23 shows the proportion of cases in which the p-value is lower than 0.05, we can assume that for the combinations in which this proportion is relatively high, the model performance is indeed different, whereas for combinations with a low ratio, there is no such evidence. We can definitely see that the performance of RGARCH differs from the performance of the AR(1)-RV, HAR and GARCH models, GARCH furthermore differs from the AR(1)-RV model, and HAR differs frm the AR(1)-RV model. However, the results of the DM test do not state anything about which model performs better than the other.

7 Value at risk estimation

Tables

24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, show the p-values of the Kupiec test for each stock and model. Tables 36, 37, 38 show the summary values of Kupiec test p-value across all stocks.

The null hypothesis of the Kupiec test states that a probability of an exception is equal to the significance level of VaR. Therefore, with a p-value lower than 0.05 and rejecting the null, the computed VaR is assumed to not correspond to the significance level. For a high number of p-values lower than 0.05 which can be seen in the summary tables, we can conclude that the model does not compute the VaR properly. For The Kupiec test, we can see mixed results for individual significancel levels. The results for 99% VaR are misleading because for this significance level, the computation is very biased due to a small (66) sample size. . A surprising result is that in some cases, there are very big differences between expanding window forecast and rolling window forecast, even though in simple forecast evaluation, these always performed relatively similarily. This may be attributed to the fact that only small nuances in forecasts may result in relatively big differences when assigning the t-distribution quantile in the VaR estimation equation.

Add Christoffersen test

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	Jarque-bera test	Augmented Dickey-Fuller test	LJung-Box test
AAL	0.00	0.01	0.00
AAPL	0.00	0.01	0.00
ABBV	0.00	0.01	0.80
ACN	0.00	0.01	0.00
ADBE	0.00	0.01	0.00
AMAT	0.00	0.01	0.00
AMD	0.00	0.01	0.74
AMGN	0.00	0.01	0.00
AMZN	0.00	0.01	0.00
ANET	0.00	0.01	0.56
AVGO	0.00	0.01	0.00
BA	0.00	0.01	0.00
BAC	0.00	0.01	0.00
BKNG	0.00	0.01	0.17
C	0.00	0.01	0.00
CAT	0.00	0.01	0.61
CMCSA	0.00	0.01	0.00
CMG	0.00	0.01	0.04
COP	0.00	0.01	0.00
COST	0.00	0.01	0.00
CRM	0.00	0.01	0.10
CSCO	0.00	0.01	0.00
CVS	0.00	0.01	0.00
CVX	0.00	0.01	0.00
DIS	0.00	0.01	0.00
EMR	0.00	0.01	0.00
FCX	0.00	0.01	0.45
FTNT	0.00	0.01	0.07
GE	0.00	0.01	0.20
GME	0.00	0.01	0.00
GOOG	0.00	0.01	0.06
GS	0.00	0.01	0.00
HD	0.00	0.01	0.01
HES	0.00	0.01	0.00
IBM	0.00	0.01	0.03
INTC	0.00	0.01	0.00
JNJ	0.00	0.01	0.00
JPM	0.00	0.01	0.00
KO	0.00	0.01	0.00
LRCX	0.00	0.01	0.06

Table 1: This table shows the p-values of the Jarque-Bera test, Augmented Dickey-Fuller test and Ljung-Box test on returns for the first half of stocks. For the ADF test, a value of 0.01 means in fact that the p-value is < 0.01

	Jarque-bera test	Augmented Dickey-Fuller test	LJung-Box test
MA	0.00	0.01	0.00
MCD	0.00	0.01	0.00
MCHP	0.00	0.01	0.00
MELI	0.00	0.01	0.00
META	0.00	0.01	0.83
MRK	0.00	0.01	0.26
MSFT	0.00	0.01	0.00
MSTR	0.00	0.01	0.49
MU	0.00	0.01	0.04
NEE	0.00	0.01	0.00
NFLX	0.00	0.01	0.00
NKE	0.00	0.01	0.01
NOW	0.00	0.01	0.01
NVDA	0.00	0.01	0.35
NXPI	0.00	0.01	0.00
ORCL	0.00	0.01	0.00
PANW	0.00	0.01	0.62
PEP	0.00	0.01	0.00
PFE	0.00	0.01	0.00
PG	0.00	0.01	0.00
PYPL	0.00	0.01	0.56
QCOM	0.00	0.01	0.00
SBUX	0.00	0.01	0.00
SHOP	0.00	0.01	0.17
SMCI	0.00	0.01	0.90
SO	0.00	0.01	0.00
SPGI	0.00	0.01	0.01
SYK	0.00	0.01	0.00
TJX	0.00	0.01	0.00
TMO	0.00	0.01	0.02
TMUS	0.00	0.01	0.00
TSN	0.00	0.01	0.52
TXN	0.00	0.01	0.00
UNH	0.00	0.01	0.51
V	0.00	0.01	0.00
VRTX	0.00	0.01	0.59
WDAY	0.00	0.01	0.16
WFC	0.00	0.01	0.00
WMT	0.00	0.01	0.00
XOM	0.00	0.01	0.00

Table 2: This table shows the p-values of the Jarque-Bera test, Augmented Dickey-Fuller test and Ljung-Box test on returns for the second half of stocks. For the ADF test, a value of 0.01 means in fact that the p-value is < 0.01

	AR1-RV	HAR	HAR-AS	HAR-RS	HAR-RSRK	RGARCH	GARCH
AAL	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AAPL	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ABBV	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ACN	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ADBE	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AMAT	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AMD	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AMGN	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AMZN	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ANET	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AVGO	0.00	0.00	0.00	0.00	0.00	0.00	0.00
BA	0.00	0.00	0.00	0.00	0.00	0.00	0.00
BAC	0.00	0.00	0.00	0.00	0.00	0.00	0.00
BKNG	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CAT	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CMCSA	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CMG	0.00	0.00	0.00	0.00	0.00	0.00	0.00
COP	0.00	0.00	0.00	0.00	0.00	0.00	0.00
COST	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CRM	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CSCO	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CVS	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CVX	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DIS	0.00	0.00	0.00	0.00	0.00	0.00	0.00
EMR	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FCX	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FTNT	0.00	0.00	0.00	0.00	0.00	0.00	0.00
GE	0.00	0.00	0.00	0.00	0.00	0.00	0.00
GME	0.00	0.00	0.00	0.00	0.00	0.00	0.00
GOOG	0.00	0.00	0.00	0.00	0.00	0.00	0.00
GS	0.00	0.00	0.00	0.00	0.00	0.00	0.00
HD	0.00	0.00	0.00	0.00	0.00	0.00	0.00
HES	0.00	0.00	0.00	0.00	0.00	0.00	0.00
IBM	0.00	0.00	0.00	0.00	0.00	0.00	0.00
INTC	0.00	0.00	0.00	0.00	0.00	0.00	0.00
JNJ	0.00	0.00	0.00	0.00	0.00	0.00	0.00
JPM	0.00	0.00	0.00	0.00	0.00	0.00	0.00
KO	0.00	0.00	0.00	0.00	0.00	0.00	0.00
LRCX	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 3: This table shows the p-values of the Jarque-Bera test of normality for residuals of all 7 models for the first half of stocks.

	AR1-RV	HAR	HAR-AS	HAR-RS	HAR-RSRK	RGARCH	GARCH
MA	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MCD	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MCHP	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MELI	0.00	0.00	0.00	0.00	0.00	0.00	0.00
META	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MRK	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MSFT	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MSTR	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MU	0.00	0.00	0.00	0.00	0.00	0.00	0.00
NEE	0.00	0.00	0.00	0.00	0.00	0.00	0.00
NFLX	0.00	0.00	0.00	0.00	0.00	0.00	0.00
NKE	0.00	0.00	0.00	0.00	0.00	0.00	0.00
NOW	0.00	0.00	0.00	0.00	0.00	0.00	0.00
NVDA	0.00	0.00	0.00	0.00	0.00	0.00	0.00
NXPI	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ORCL	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PANW	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PEP	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PFE	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PG	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PYPL	0.00	0.00	0.00	0.00	0.00	0.00	0.00
QCOM	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SBUX	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SHOP	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SMCI	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SO	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SPGI	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SYK	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TJX	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TMO	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TMUS	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TSN	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TXN	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UNH	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V	0.00	0.00	0.00	0.00	0.00	0.00	0.00
VRTX	0.00	0.00	0.00	0.00	0.00	0.00	0.00
WDAY	0.00	0.00	0.00	0.00	0.00	0.00	0.00
WFC	0.00	0.00	0.00	0.00	0.00	0.00	0.00
WMT	0.00	0.00	0.00	0.00	0.00	0.00	0.00
XOM	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 4: This table shows the p-values of the Jarque-Bera test of normality for residuals of all 7 models for the second half of stocks.

	Start date	End date	Observations	Observations before 2019-11-29
AAL	2013-12-10	2022-12-09	2267	1503
AAPL	1998-01-05	2022-12-09	6276	5512
ABBV	2013-01-03	2022-12-09	2503	1739
ACN	2001-07-20	2022-12-09	5383	4619
ADBE	1998-01-05	2022-12-09	6276	5512
AMAT	1998-01-05	2022-12-09	6276	5512
AMD	1998-01-05	2022-12-09	6276	5512
AMGN	1998-01-05	2022-12-09	6276	5512
AMZN	1998-01-05	2022-12-09	6276	5512
ANET	2014-06-09	2022-12-09	2144	1380
AVGO	2009-08-07	2022-12-09	3360	2596
BA	1998-01-05	2022-12-09	6276	5512
BAC	1998-01-05	2022-12-09	6276	5512
BKNG	2007-05-01	2022-12-09	3932	3168
C	1998-01-05	2022-12-09	6276	5512
CAT	1998-01-05	2022-12-09	6276	5512
CMCSA	1998-01-05	2022-12-09	6276	5512
CMG	2006-01-27	2022-12-09	4248	3484
COP	2002-09-04	2022-12-09	5104	4340
COST	1998-01-05	2022-12-09	6276	5512
CRM	2004-06-24	2022-12-09	4650	3886
CSCO	1998-01-05	2022-12-09	6276	5512
CVS	1998-01-05	2022-12-09	6276	5512
CVX	2001-10-11	2022-12-09	5329	4565
DIS	1998-01-05	2022-12-09	6276	5512
EMR	1998-01-05	2022-12-09	6276	5512
FCX	1998-01-05	2022-12-09	6276	5512
FTNT	2009-11-19	2022-12-09	3287	2523
GE	1998-01-05	2022-12-09	6276	5512
GME	2002-05-13	2022-12-09	5183	4419
GOOG	2014-03-28	2022-12-09	2193	1429
GS	1999-05-05	2022-12-09	5941	5177
HD	1998-01-05	2022-12-09	6276	5512
HES	2007-03-14	2022-12-09	3966	3202
IBM	1998-01-05	2022-12-09	6276	5512
INTC	1998-01-05	2022-12-09	6276	5512
JNJ	1998-01-05	2022-12-09	6276	5512
JPM	1998-01-05	2022-12-09	6276	5512
KO	1998-01-05	2022-12-09	6276	5512
LRCX	1998-01-05	2022-12-12	6277	5512

Table 5: This table shows the overview for the first half of stocks of basic end date, start date, number of observations and number of observations for the before covid training set, i. e. observations prior to 2019-11-29

	Start date	End date	Observations	Observations before 2019-11-29
MA	2006-05-26	2022-12-12	4166	3401
MCD	1998-01-05	2022-12-12	6277	5512
MCHP	1998-01-05	2022-12-12	6277	5512
MELI	2007-08-13	2022-12-12	3862	3097
META	2012-05-21	2022-12-12	2659	1894
MRK	1998-01-05	2022-12-12	6277	5512
MSFT	1998-01-05	2022-12-12	6277	5512
MSTR	2002-08-29	2022-12-12	5108	4343
MU	1998-01-05	2022-12-12	6277	5512
NEE	1998-01-05	2022-12-12	6277	5512
NFLX	2002-05-24	2022-12-12	5175	4410
NKE	1998-01-05	2022-12-12	6277	5512
NOW	2012-07-02	2022-12-12	2630	1865
NVDA	1999-01-25	2022-12-12	6012	5247
NXPI	2010-08-09	2022-12-12	3109	2344
ORCL	1998-01-05	2022-12-12	6277	5512
PANW	2012-07-23	2022-12-12	2616	1851
PEP	1998-01-05	2022-12-12	6277	5512
PFE	1998-01-05	2022-12-12	6277	5512
PG	1998-01-05	2022-12-09	6276	5512
PYPL	2015-07-07	2022-12-12	1874	1109
QCOM	1998-01-05	2022-12-12	6277	5512
SBUX	1998-01-05	2022-12-12	6277	5512
SHOP	2015-05-22	2022-12-12	1904	1139
SMCI	2007-03-30	2022-12-12	3955	3190
SO	1998-01-05	2022-12-12	6277	5512
SPGI	2007-04-30	2022-12-12	3935	3170
SYK	1998-01-05	2022-12-12	6277	5512
TJX	1998-01-05	2022-12-12	6277	5512
TMO	1998-01-05	2022-12-12	6277	5512
TMUS	2007-04-20	2022-12-12	3941	3176
TSN	1998-01-05	2022-12-12	6277	5512
TXN	1998-01-05	2022-12-12	6277	5512
UNH	1998-01-05	2022-12-12	6277	5512
V	2008-03-20	2022-12-12	3710	2945
VRTX	1998-01-05	2022-12-12	6277	5512
WDAY	2012-10-15	2022-12-12	2557	1792
WFC	1998-01-05	2022-12-12	6277	5512
WMT	1998-01-05	2022-12-12	6277	5512
XOM	1999-12-02	2022-12-12	5795	5030

Table 6: This table shows the overview for the second half of stocks of basic end date, start date, number of observations and number of observations for the before covid training set, i. e. observations prior to 2019-11-29

	AR(1)-RV	HAR	HAR-AS	HAR-RS	HAR-RSRK	RGARCH	GARCH
Rolling	52.00	24.00	39.00	37.00	80.00	41.00	42.00
Expanding	28.00	56.00	41.00	43.00	0.00	39.00	38.00

Table 7: This table shows a summary of how many stocks for each model perform better with expanding or rolling forecasting scheme according to mean absolute error.

	AR(1)-RV	HAR	HAR-AS	HAR-RS	HAR-RSRK	RGARCH	GARCH
Rolling	53.00	34.00	59.00	56.00	80.00	36.00	42.00
Expanding	27.00	46.00	21.00	24.00	0.00	44.00	38.00

Table 8: This table shows a summary of how many stocks for each model perform better with expanding or rolling forecasting scheme according to mean square error.

	AR(1)-RV	HAR	HAR-AS	HAR-RS	HAR-RSRK	RGARCH	GARCH
AAL	7.466	7.478	7.470	7.861	7.858	7.159	9.645
AAPL	11.870	6.042	6.043	6.187	6.144	5.379	6.566
ABBV	5.506	4.972	4.963	5.161	5.154	4.723	5.580
ACN	9.095	5.979	5.661	6.101	5.829	6.915	4.575
ADBE	11.410	5.939	5.887	6.113	6.116	4.410	5.674
AMAT	9.750	5.988	5.942	6.029	5.792	5.150	7.355
AMD	17.045	9.811	9.804	9.887	9.882	7.518	12.119
AMGN	8.399	4.963	4.991	5.056	5.062	3.959	4.728
AMZN	25.439	13.357	12.487	14.034	14.669	5.478	8.607
ANET	10.548	9.472	9.249	9.551	9.836	5.764	13.213
AVGO	7.533	6.228	6.229	6.352	6.447	5.138	7.578
BA	6.725	6.059	6.039	6.064	6.000	5.644	6.844
BAC	9.906	5.261	5.249	5.238	5.356	3.806	5.262
BKNG	10.204	6.930	6.888	6.826	6.868	5.694	5.758
C	15.975	9.076	9.075	9.239	9.268	4.345	5.129
CAT	8.393	5.209	5.203	5.272	5.310	4.315	6.131
CMCSA	8.225	5.130	5.062	5.241	5.457	4.063	5.371
CMG	9.686	5.655	5.658	5.846	5.946	4.360	6.697
COP	6.163	6.315	6.369	6.390	6.330	5.427	5.475
COST	11.678	7.651	7.653	7.663	7.650	3.794	4.177
CRM	10.004	5.980	5.895	6.055	6.129	5.104	7.207
CSCO	8.512	5.148	5.132	5.369	5.408	3.957	5.709
CVS	6.823	4.920	4.981	4.948	4.774	4.157	4.695
CVX	5.941	4.674	4.645	4.638	4.947	4.207	5.070
DIS	8.049	5.824	5.845	6.096	6.080	4.276	6.985
EMR	7.686	5.079	5.074	5.097	5.112	4.119	5.671
FCX	8.304	7.075	6.970	7.159	7.189	6.921	9.984
FTNT	14.862	9.702	9.651	9.769	9.930	5.696	7.226
GE	10.756	9.894	10.840	10.183	10.165	4.870	8.245
GME	24.747	20.707	20.595	20.755	20.866	19.488	23.984
GOOG	6.840	6.218	6.248	6.351	6.488	4.680	5.218
GS	6.325	4.579	4.507	4.737	4.887	3.972	5.096
HD	8.840	4.438	4.300	4.513	4.537	5.323	5.361
HES	8.818	7.709	7.875	7.888	7.652	6.966	8.473
IBM	9.481	8.625	8.593	8.568	8.698	4.447	5.621
INTC	10.739	7.343	7.402	7.411	7.329	5.160	7.671
JNJ	4.922	4.221	4.289	4.407	4.442	3.627	3.721
JPM	9.881	9.049	9.058	9.518	9.231	5.715	6.618
KO	5.426	3.703	3.655	3.929	3.874	3.405	3.732
LRCX	11.583	7.308	7.236	7.654	8.183	6.480	8.317

Table 9: This table shows the mean absolute errors of expanding forecast scheme for each type of model for the first half of stocks. The values are multiplied by 1000

	AR(1)-RV	HAR	HAR-AS	HAR-RS	HAR-RSRK	RGARCH	GARCH
MA	11.037	7.362	7.366	7.389	7.376	4.480	5.076
MCD	7.635	4.138	4.140	4.171	4.136	2.991	4.410
MCHP	11.800	10.027	10.026	10.088	10.114	7.992	9.515
MELI	11.572	7.427	7.387	7.682	7.822	6.618	8.562
META	9.919	9.219	9.289	9.329	9.493	5.386	6.210
MRK	6.537	4.733	4.725	4.803	4.796	3.952	4.696
MSFT	10.024	6.189	6.144	6.241	6.294	4.239	5.652
MSTR	10.005	6.451	6.447	6.617	6.808	5.286	5.489
MU	14.038	7.905	7.902	7.992	7.964	6.550	9.925
NEE	5.910	3.735	3.735	3.688	3.709	3.301	3.852
NFLX	14.441	7.277	7.235	7.340	7.365	6.152	12.171
NKE	8.399	5.665	5.602	5.706	5.661	4.016	4.873
NOW	9.970	9.173	9.231	9.386	9.700	7.018	10.134
NVDA	14.586	7.333	7.412	7.442	7.484	5.797	8.042
NXPI	8.137	6.138	6.045	6.280	6.251	5.406	5.677
ORCL	11.029	6.073	6.072	6.221	6.180	4.159	5.372
PANW	12.732	10.104	10.008	10.261	10.319	6.148	10.930
PEP	5.506	3.550	3.501	3.545	3.553	3.085	3.310
PFE	8.543	6.240	5.484	6.248	6.325	3.656	4.110
PG	4.753	3.621	3.488	3.727	3.763	3.293	3.199
PYPL	7.489	7.344	7.384	7.522	7.653	5.400	7.435
QCOM	9.582	6.191	6.190	6.267	6.217	5.197	6.608
SBUX	10.081	5.216	5.206	5.347	5.366	3.790	5.077
SHOP	8.499	7.703	7.717	7.837	7.725	6.912	8.366
SMCI	14.031	13.779	13.774	13.885	14.068	10.110	14.621
SO	4.700	3.905	3.891	3.788	3.816	3.527	4.255
SPGI	6.518	4.475	4.547	4.895	4.941	3.813	4.294
SYK	7.411	5.539	5.393	5.629	5.627	3.998	5.622
TJX	12.234	8.908	8.867	8.867	8.908	3.452	4.303
TMO	10.675	8.522	8.522	8.545	8.543	4.078	4.422
TMUS	13.397	8.396	8.246	8.475	8.263	3.776	6.114
TSN	7.800	4.547	4.524	4.697	4.719	4.430	6.262
TXN	9.650	5.693	5.696	5.826	5.805	4.667	6.505
UNH	6.441	4.844	4.752	5.008	4.989	4.134	5.305
V	7.198	5.180	5.190	5.338	5.323	4.140	4.636
VRTX	19.081	9.301	9.303	9.334	9.346	5.776	8.584
WDAY	7.398	8.046	8.052	8.381	8.647	5.634	9.516
WFC	8.266	4.635	4.633	4.775	4.885	3.595	5.924
WMT	6.921	3.712	3.718	3.826	3.859	2.758	3.429
XOM	5.263	4.947	4.878	5.071	4.991	4.523	5.224

Table 10: This table shows the mean absolute errors of expanding forecast scheme for each type of model for the second half of stocks. The values are multiplied by 1000

	AR(1)-RV	HAR	HAR-AS	HAR-RS	HAR-RSRK	RGARCH	GARCH
AAL	7.465	7.483	7.375	7.796	7.858	7.217	9.582
AAPL	11.369	6.029	6.029	6.175	6.144	5.364	6.563
ABBV	5.499	4.967	4.946	5.163	5.154	4.734	5.563
ACN	9.128	5.572	5.166	5.783	5.829	4.917	4.540
ADBE	11.410	5.945	5.891	6.117	6.116	4.417	5.667
AMAT	9.739	5.992	5.946	6.031	5.792	5.149	7.333
AMD	17.041	9.809	9.800	9.884	9.882	7.524	12.153
AMGN	8.406	4.963	4.989	5.054	5.062	3.944	4.732
AMZN	22.025	13.695	13.024	14.634	14.669	5.424	8.660
ANET	10.399	9.610	9.370	9.656	9.836	5.869	13.068
AVGO	7.476	6.223	6.221	6.357	6.447	5.134	7.620
BA	6.725	6.060	6.039	6.064	6.000	5.644	6.831
BAC	9.915	5.270	5.257	5.246	5.356	3.876	5.096
BKNG	10.220	6.937	6.895	6.830	6.868	5.683	5.721
C	16.008	9.099	9.097	9.260	9.268	4.405	5.118
CAT	8.392	5.211	5.204	5.272	5.310	4.311	6.140
CMCSA	8.211	5.136	5.068	5.250	5.457	4.066	5.386
CMG	9.672	5.641	5.641	5.813	5.946	4.387	6.797
COP	6.164	6.315	6.365	6.386	6.330	5.419	5.465
COST	11.654	7.648	7.648	7.657	7.650	3.788	4.180
CRM	9.915	5.972	5.878	6.020	6.129	5.239	7.178
CSCO	8.508	5.153	5.135	5.372	5.408	3.969	5.705
CVS	6.828	4.921	4.981	4.950	4.774	4.163	4.712
CVX	5.935	4.676	4.646	4.640	4.947	4.191	5.073
DIS	8.060	5.829	5.848	6.098	6.080	4.279	6.961
EMR	7.688	5.080	5.074	5.097	5.112	4.106	5.664
FCX	8.301	7.075	6.969	7.158	7.189	6.826	9.974
FTNT	14.673	9.776	9.721	9.843	9.930	5.662	7.131
GE	10.773	9.916	10.831	10.201	10.165	4.883	8.448
GME	24.751	20.711	20.560	20.720	20.866	19.563	23.972
GOOG	6.799	6.226	6.244	6.350	6.488	4.695	5.159
GS	6.313	4.581	4.508	4.735	4.887	3.977	5.090
HD	8.854	4.445	4.305	4.518	4.537	4.726	5.357
HES	8.836	7.713	7.878	7.889	7.652	6.956	8.459
IBM	9.484	8.634	8.596	8.572	8.698	4.452	5.635
INTC	10.756	7.347	7.401	7.413	7.329	5.156	7.698
JNJ	4.921	4.222	4.289	4.405	4.442	3.627	3.715
JPM	9.886	9.053	9.056	9.521	9.231	5.719	6.458
KO	5.426	3.704	3.657	3.930	3.874	3.407	3.732
LRCX	11.557	7.309	7.238	7.650	8.183	6.479	8.317

Table 11: This table shows the mean absolute errors of rolling forecast scheme for each type of model for the first half of stocks. The values are multiplied by 1000

	AR(1)-RV	HAR	HAR-AS	HAR-RS	HAR-RSRK	RGARCH	GARCH
MA	11.037	7.362	7.366	7.389	7.376	4.480	5.076
MCD	7.635	4.138	4.140	4.171	4.136	2.991	4.410
MCHP	11.800	10.027	10.026	10.088	10.114	7.992	9.515
MELI	11.572	7.427	7.387	7.682	7.822	6.618	8.562
META	9.919	9.219	9.289	9.329	9.493	5.386	6.210
MRK	6.537	4.733	4.725	4.803	4.796	3.952	4.696
MSFT	10.024	6.189	6.144	6.241	6.294	4.239	5.652
MSTR	10.005	6.451	6.447	6.617	6.808	5.286	5.489
MU	14.038	7.905	7.902	7.992	7.964	6.550	9.925
NEE	5.910	3.735	3.735	3.688	3.709	3.301	3.852
NFLX	14.441	7.277	7.235	7.340	7.365	6.152	12.171
NKE	8.399	5.665	5.602	5.706	5.661	4.016	4.873
NOW	9.970	9.173	9.231	9.386	9.700	7.018	10.134
NVDA	14.586	7.333	7.412	7.442	7.484	5.797	8.042
NXPI	8.137	6.138	6.045	6.280	6.251	5.406	5.677
ORCL	11.029	6.073	6.072	6.221	6.180	4.159	5.372
PANW	12.732	10.104	10.008	10.261	10.319	6.148	10.930
PEP	5.506	3.550	3.501	3.545	3.553	3.085	3.310
PFE	8.543	6.240	5.484	6.248	6.325	3.656	4.110
PG	4.753	3.621	3.488	3.727	3.763	3.293	3.199
PYPL	7.489	7.344	7.384	7.522	7.653	5.400	7.435
QCOM	9.582	6.191	6.190	6.267	6.217	5.197	6.608
SBUX	10.081	5.216	5.206	5.347	5.366	3.790	5.077
SHOP	8.499	7.703	7.717	7.837	7.725	6.912	8.366
SMCI	14.031	13.779	13.774	13.885	14.068	10.110	14.621
SO	4.700	3.905	3.891	3.788	3.816	3.527	4.255
SPGI	6.518	4.475	4.547	4.895	4.941	3.813	4.294
SYK	7.411	5.539	5.393	5.629	5.627	3.998	5.622
TJX	12.234	8.908	8.867	8.867	8.908	3.452	4.303
TMO	10.675	8.522	8.522	8.545	8.543	4.078	4.422
TMUS	13.397	8.396	8.246	8.475	8.263	3.776	6.114
TSN	7.800	4.547	4.524	4.697	4.719	4.430	6.262
TXN	9.650	5.693	5.696	5.826	5.805	4.667	6.505
UNH	6.441	4.844	4.752	5.008	4.989	4.134	5.305
V	7.198	5.180	5.190	5.338	5.323	4.140	4.636
VRTX	19.081	9.301	9.303	9.334	9.346	5.776	8.584
WDAY	7.398	8.046	8.052	8.381	8.647	5.634	9.516
WFC	8.266	4.635	4.633	4.775	4.885	3.595	5.924
WMT	6.921	3.712	3.718	3.826	3.859	2.758	3.429
XOM	5.263	4.947	4.878	5.071	4.991	4.523	5.224

Table 12: This table shows the mean absolute errors of rolling forecast scheme for each type of model for the second half of stocks. The values are multiplied by 1000

	AR(1)-RV	HAR	HAR-AS	HAR-RS	HAR-RSRK	RGARCH	GARCH
AAL	0.193	0.180	0.185	0.193	0.192	0.181	0.188
AAPL	0.166	0.077	0.077	0.081	0.081	0.062	0.076
ABBV	0.065	0.062	0.061	0.068	0.068	0.045	0.065
ACN	0.106	0.067	0.062	0.070	0.065	0.194	0.046
ADBE	0.146	0.064	0.064	0.070	0.070	0.046	0.067
AMAT	0.118	0.064	0.063	0.064	0.060	0.051	0.082
AMD	0.338	0.147	0.147	0.149	0.149	0.105	0.196
AMGN	0.088	0.051	0.052	0.054	0.054	0.041	0.045
AMZN	0.743	0.283	0.249	0.306	0.309	0.137	0.171
ANET	0.140	0.117	0.113	0.119	0.126	0.068	0.211
AVGO	0.078	0.067	0.067	0.068	0.070	0.052	0.078
BA	0.066	0.067	0.067	0.068	0.067	0.060	0.067
BAC	0.117	0.060	0.059	0.062	0.062	0.038	0.048
BKNG	0.138	0.101	0.100	0.100	0.102	0.081	0.083
C	0.283	0.114	0.114	0.118	0.115	0.048	0.051
CAT	0.092	0.055	0.055	0.057	0.057	0.044	0.062
CMCSA	0.079	0.046	0.045	0.050	0.052	0.039	0.052
CMG	0.110	0.065	0.065	0.069	0.071	0.044	0.073
COP	0.096	0.101	0.102	0.103	0.101	0.089	0.079
COST	0.155	0.085	0.085	0.086	0.085	0.045	0.049
CRM	0.124	0.070	0.068	0.071	0.069	0.058	0.109
CSCO	0.088	0.055	0.055	0.061	0.062	0.041	0.056
CVS	0.066	0.046	0.047	0.048	0.045	0.033	0.038
CVX	0.074	0.070	0.070	0.070	0.071	0.066	0.072
DIS	0.085	0.065	0.066	0.070	0.070	0.052	0.075
EMR	0.084	0.061	0.060	0.061	0.062	0.042	0.067
FCX	0.118	0.100	0.101	0.103	0.100	0.087	0.141
FTNT	0.245	0.116	0.114	0.116	0.120	0.069	0.078
GE	0.170	0.155	0.210	0.164	0.164	0.068	0.121
GME	1.415	0.983	0.973	0.970	0.978	0.978	1.363
GOOG	0.081	0.073	0.074	0.076	0.079	0.053	0.054
GS	0.053	0.039	0.038	0.045	0.049	0.030	0.047
HD	0.090	0.043	0.041	0.045	0.045	0.050	0.060
HES	0.234	0.239	0.242	0.235	0.229	0.215	0.234
IBM	0.118	0.103	0.103	0.104	0.104	0.058	0.068
INTC	0.147	0.118	0.120	0.121	0.119	0.085	0.112
JNJ	0.044	0.042	0.044	0.049	0.049	0.040	0.047
JPM	0.225	0.208	0.208	0.215	0.211	0.143	0.144
KO	0.051	0.037	0.036	0.038	0.038	0.038	0.043
LRCX	0.170	0.104	0.102	0.110	0.122	0.092	0.125

Table 13: This table shows the mean square errors of expanding forecast scheme for each type of model for the first half of stocks. The values are multiplied by 1000

	AR(1)-RV	HAR	HAR-AS	HAR-RS	HAR-RSRK	RGARCH	GARCH
MA	0.155	0.094	0.094	0.096	0.095	0.046	0.044
MCD	0.070	0.036	0.036	0.037	0.036	0.026	0.042
MCHP	0.204	0.230	0.230	0.228	0.229	0.178	0.206
MELI	0.159	0.094	0.092	0.102	0.109	0.072	0.099
META	0.140	0.130	0.131	0.135	0.138	0.084	0.094
MRK	0.060	0.044	0.045	0.046	0.046	0.036	0.047
MSFT	0.128	0.073	0.073	0.075	0.075	0.046	0.054
MSTR	0.119	0.058	0.058	0.061	0.063	0.045	0.053
MU	0.226	0.095	0.095	0.099	0.098	0.070	0.131
NEE	0.053	0.036	0.037	0.036	0.036	0.038	0.039
NFLX	0.237	0.071	0.071	0.072	0.072	0.051	0.173
NKE	0.090	0.064	0.063	0.065	0.065	0.042	0.058
NOW	0.228	0.242	0.247	0.241	0.238	0.190	0.227
NVDA	0.242	0.101	0.102	0.105	0.106	0.078	0.106
NXPI	0.081	0.058	0.057	0.061	0.059	0.047	0.058
ORCL	0.138	0.066	0.066	0.070	0.069	0.048	0.068
PANW	0.472	0.432	0.429	0.432	0.434	0.378	0.446
PEP	0.044	0.035	0.034	0.037	0.037	0.033	0.040
PFE	0.089	0.058	0.043	0.060	0.061	0.034	0.035
PG	0.040	0.036	0.035	0.039	0.039	0.031	0.027
PYPL	0.127	0.128	0.130	0.136	0.140	0.102	0.121
QCOM	0.114	0.070	0.070	0.070	0.068	0.056	0.075
SBUX	0.118	0.048	0.048	0.051	0.052	0.036	0.050
SHOP	0.147	0.150	0.150	0.156	0.155	0.132	0.152
SMCI	0.280	0.275	0.275	0.284	0.290	0.219	0.295
SO	0.039	0.039	0.039	0.039	0.038	0.042	0.037
SPGI	0.058	0.041	0.042	0.046	0.046	0.032	0.031
SYK	0.072	0.057	0.055	0.058	0.059	0.040	0.060
TJX	0.160	0.093	0.092	0.092	0.092	0.029	0.036
TMO	0.133	0.094	0.094	0.094	0.094	0.038	0.038
TMUS	0.207	0.081	0.079	0.083	0.080	0.025	0.047
TSN	0.080	0.050	0.050	0.057	0.057	0.049	0.069
TXN	0.111	0.053	0.054	0.056	0.056	0.041	0.064
UNH	0.061	0.048	0.046	0.051	0.050	0.031	0.046
V	0.074	0.062	0.062	0.065	0.066	0.043	0.039
VRTX	0.402	0.107	0.107	0.108	0.108	0.060	0.100
WDAY	0.101	0.157	0.158	0.159	0.167	0.095	0.147
WFC	0.084	0.048	0.048	0.051	0.054	0.034	0.055
WMT	0.058	0.033	0.033	0.035	0.035	0.023	0.038
XOM	0.072	0.080	0.078	0.081	0.080	0.077	0.065

Table 14: This table shows the mean square errors of expanding forecast scheme for each type of model for the second half of stocks. The values are multiplied by 1000

	AR(1)-RV	HAR	HAR-AS	HAR-RS	HAR-RSRK	RGARCH	GARCH
AAL	0.193	0.181	0.182	0.192	0.192	0.178	0.184
AAPL	0.154	0.077	0.077	0.081	0.081	0.061	0.075
ABBV	0.065	0.062	0.060	0.068	0.068	0.045	0.064
ACN	0.102	0.059	0.053	0.063	0.065	0.050	0.045
ADBE	0.146	0.064	0.064	0.070	0.070	0.046	0.067
AMAT	0.118	0.064	0.063	0.064	0.060	0.051	0.082
AMD	0.338	0.147	0.147	0.149	0.149	0.105	0.198
AMGN	0.088	0.051	0.051	0.053	0.054	0.041	0.045
AMZN	0.593	0.283	0.248	0.301	0.309	0.138	0.172
ANET	0.137	0.120	0.115	0.121	0.126	0.071	0.209
AVGO	0.078	0.066	0.066	0.068	0.070	0.052	0.078
BA	0.066	0.067	0.067	0.068	0.067	0.060	0.067
BAC	0.117	0.060	0.059	0.062	0.062	0.038	0.049
BKNG	0.138	0.101	0.100	0.100	0.102	0.082	0.081
C	0.284	0.114	0.115	0.118	0.115	0.050	0.051
CAT	0.092	0.055	0.055	0.057	0.057	0.044	0.062
CMCSA	0.079	0.046	0.046	0.050	0.052	0.039	0.052
CMG	0.110	0.065	0.065	0.068	0.071	0.044	0.074
COP	0.096	0.101	0.102	0.102	0.101	0.089	0.079
COST	0.154	0.085	0.085	0.086	0.085	0.045	0.049
CRM	0.122	0.069	0.068	0.071	0.069	0.060	0.109
CSCO	0.088	0.055	0.055	0.061	0.062	0.042	0.056
CVS	0.066	0.046	0.047	0.048	0.045	0.033	0.038
CVX	0.074	0.070	0.070	0.070	0.071	0.066	0.072
DIS	0.085	0.065	0.066	0.070	0.070	0.052	0.074
EMR	0.084	0.061	0.060	0.061	0.062	0.042	0.067
FCX	0.118	0.100	0.101	0.103	0.100	0.089	0.140
FTNT	0.240	0.117	0.115	0.117	0.120	0.069	0.078
GE	0.170	0.155	0.208	0.165	0.164	0.068	0.123
GME	1.416	0.984	0.971	0.969	0.978	0.980	1.363
GOOG	0.080	0.073	0.074	0.076	0.079	0.053	0.053
GS	0.053	0.039	0.038	0.045	0.049	0.030	0.047
HD	0.090	0.043	0.041	0.045	0.045	0.046	0.059
HES	0.234	0.239	0.242	0.235	0.229	0.215	0.236
IBM	0.118	0.104	0.103	0.104	0.104	0.058	0.068
INTC	0.147	0.118	0.120	0.121	0.119	0.085	0.113
JNJ	0.044	0.042	0.044	0.049	0.049	0.040	0.047
JPM	0.225	0.208	0.208	0.215	0.211	0.144	0.148
KO	0.051	0.037	0.036	0.038	0.038	0.038	0.043
LRCX	0.169	0.104	0.102	0.110	0.122	0.092	0.125

Table 15: This table shows the mean square errors of rolling forecast scheme for each type of model for the first half of stocks. The values are multiplied by 1000

	AR(1)-RV	HAR	HAR-AS	HAR-RS	HAR-RSRK	RGARCH	GARCH
MA	0.155	0.094	0.094	0.096	0.095	0.046	0.044
MCD	0.070	0.036	0.036	0.037	0.036	0.026	0.042
MCHP	0.204	0.230	0.230	0.228	0.229	0.178	0.206
MELI	0.159	0.094	0.092	0.102	0.109	0.072	0.099
META	0.140	0.130	0.131	0.135	0.138	0.084	0.094
MRK	0.060	0.044	0.045	0.046	0.046	0.036	0.047
MSFT	0.128	0.073	0.073	0.075	0.075	0.046	0.054
MSTR	0.119	0.058	0.058	0.061	0.063	0.045	0.053
MU	0.226	0.095	0.095	0.099	0.098	0.070	0.131
NEE	0.053	0.036	0.037	0.036	0.036	0.038	0.039
NFLX	0.237	0.071	0.071	0.072	0.072	0.051	0.173
NKE	0.090	0.064	0.063	0.065	0.065	0.042	0.058
NOW	0.228	0.242	0.247	0.241	0.238	0.190	0.227
NVDA	0.242	0.101	0.102	0.105	0.106	0.078	0.106
NXPI	0.081	0.058	0.057	0.061	0.059	0.047	0.058
ORCL	0.138	0.066	0.066	0.070	0.069	0.048	0.068
PANW	0.472	0.432	0.429	0.432	0.434	0.378	0.446
PEP	0.044	0.035	0.034	0.037	0.037	0.033	0.040
PFE	0.089	0.058	0.043	0.060	0.061	0.034	0.035
PG	0.040	0.036	0.035	0.039	0.039	0.031	0.027
PYPL	0.127	0.128	0.130	0.136	0.140	0.102	0.121
QCOM	0.114	0.070	0.070	0.070	0.068	0.056	0.075
SBUX	0.118	0.048	0.048	0.051	0.052	0.036	0.050
SHOP	0.147	0.150	0.150	0.156	0.155	0.132	0.152
SMCI	0.280	0.275	0.275	0.284	0.290	0.219	0.295
SO	0.039	0.039	0.039	0.039	0.038	0.042	0.037
SPGI	0.058	0.041	0.042	0.046	0.046	0.032	0.031
SYK	0.072	0.057	0.055	0.058	0.059	0.040	0.060
TJX	0.160	0.093	0.092	0.092	0.092	0.029	0.036
TMO	0.133	0.094	0.094	0.094	0.094	0.038	0.038
TMUS	0.207	0.081	0.079	0.083	0.080	0.025	0.047
TSN	0.080	0.050	0.050	0.057	0.057	0.049	0.069
TXN	0.111	0.053	0.054	0.056	0.056	0.041	0.064
UNH	0.061	0.048	0.046	0.051	0.050	0.031	0.046
V	0.074	0.062	0.062	0.065	0.066	0.043	0.039
VRTX	0.402	0.107	0.107	0.108	0.108	0.060	0.100
WDAY	0.101	0.157	0.158	0.159	0.167	0.095	0.147
WFC	0.084	0.048	0.048	0.051	0.054	0.034	0.055
WMT	0.058	0.033	0.033	0.035	0.035	0.023	0.038
XOM	0.072	0.080	0.078	0.081	0.080	0.077	0.065

Table 16: This table shows the mean square errors of rolling forecast scheme for each type of model for the second half of stocks. The values are multiplied by 1000

	AR(1)-RV	HAR	HAR-AS	HAR-RS	HAR-RSRK	RGARCH	GARCH
AAL	Rolling	Expanding	Rolling	Rolling	Rolling	Expanding	Rolling
AAPL	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling
ABBV	Rolling	Rolling	Rolling	Expanding	Rolling	Expanding	Rolling
ACN	Expanding	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling
ADBE	Rolling	Expanding	Expanding	Expanding	Rolling	Expanding	Rolling
AMAT	Rolling	Expanding	Expanding	Expanding	Rolling	Rolling	Rolling
AMD	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding	Expanding
AMGN	Expanding	Expanding	Rolling	Rolling	Rolling	Rolling	Expanding
AMZN	Rolling	Expanding	Expanding	Expanding	Rolling	Rolling	Expanding
ANET	Rolling	Expanding	Expanding	Expanding	Rolling	Expanding	Rolling
AVGO	Rolling	Rolling	Rolling	Expanding	Rolling	Rolling	Expanding
BA	Expanding	Expanding	Expanding	Rolling	Rolling	Rolling	Rolling
BAC	Expanding	Expanding	Expanding	Expanding	Rolling	Expanding	Rolling
BKNG	Expanding	Expanding	Expanding	Expanding	Rolling	Rolling	Rolling
C	Expanding	Expanding	Expanding	Expanding	Rolling	Expanding	Rolling
CAT	Rolling	Expanding	Expanding	Expanding	Rolling	Rolling	Expanding
CMCSA	Rolling	Expanding	Expanding	Expanding	Rolling	Expanding	Expanding
CMG	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding	Expanding
COP	Expanding	Expanding	Rolling	Rolling	Rolling	Rolling	Rolling
COST	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding
CRM	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding	Rolling
CSCO	Rolling	Expanding	Expanding	Expanding	Rolling	Expanding	Rolling
CVS	Expanding	Expanding	Expanding	Expanding	Rolling	Expanding	Expanding
CVX	Rolling	Expanding	Expanding	Expanding	Rolling	Rolling	Expanding
DIS	Expanding	Expanding	Expanding	Expanding	Rolling	Expanding	Rolling
EMR	Expanding	Expanding	Rolling	Expanding	Rolling	Rolling	Rolling
FCX	Rolling	Expanding	Rolling	Rolling	Rolling	Rolling	Rolling
FTNT	Rolling	Expanding	Expanding	Expanding	Rolling	Rolling	Rolling
GE	Expanding	Expanding	Rolling	Expanding	Rolling	Expanding	Expanding
GME	Expanding	Expanding	Rolling	Rolling	Rolling	Expanding	Rolling
GOOG	Rolling	Expanding	Rolling	Rolling	Rolling	Expanding	Rolling
GS	Rolling	Expanding	Expanding	Rolling	Rolling	Expanding	Rolling
HD	Expanding	Expanding	Expanding	Expanding	Rolling	Rolling	Rolling
HES	Expanding	Expanding	Expanding	Expanding	Rolling	Rolling	Rolling
IBM	Expanding	Expanding	Expanding	Expanding	Rolling	Expanding	Expanding
INTC	Expanding	Expanding	Rolling	Expanding	Rolling	Rolling	Expanding
JNJ	Rolling	Expanding	Expanding	Rolling	Rolling	Rolling	Rolling
JPM	Expanding	Expanding	Rolling	Expanding	Rolling	Expanding	Rolling
KO	Expanding	Expanding	Expanding	Expanding	Rolling	Expanding	Expanding
LRCX	Rolling	Expanding	Expanding	Rolling	Rolling	Rolling	Rolling

Table 17: This table shows which forecasting scheme performs better on the training set, according to mean absolute error for each model and the first half of stocks.

	AR(1)-RV	HAR	HAR-AS	HAR-RS	HAR-RSRK	RGARCH	GARCH
MA	Rolling	Expanding	Expanding	Expanding	Rolling	Expanding	Expanding
MCD	Expanding	Expanding	Rolling	Rolling	Rolling	Expanding	Expanding
MCHP	Rolling	Expanding	Rolling	Rolling	Rolling	Rolling	Rolling
MELI	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding
META	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding
MRK	Expanding	Expanding	Expanding	Expanding	Rolling	Expanding	Expanding
MSFT	Expanding	Expanding	Expanding	Expanding	Rolling	Rolling	Expanding
MSTR	Rolling	Expanding	Expanding	Expanding	Rolling	Expanding	Expanding
MU	Rolling	Expanding	Expanding	Expanding	Rolling	Expanding	Expanding
NEE	Expanding	Expanding	Expanding	Expanding	Rolling	Rolling	Expanding
NFLX	Rolling	Expanding	Expanding	Expanding	Rolling	Rolling	Expanding
NKE	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding	Rolling
NOW	Rolling	Rolling	Rolling	Expanding	Rolling	Expanding	Rolling
NVDA	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding	Expanding
NXPI	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling
ORCL	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding
PANW	Rolling	Expanding	Rolling	Rolling	Rolling	Expanding	Expanding
PEP	Rolling	Expanding	Expanding	Expanding	Rolling	Rolling	Rolling
PFE	Expanding	Expanding	Expanding	Expanding	Rolling	Rolling	Rolling
PG	Rolling	Expanding	Rolling	Expanding	Rolling	Expanding	Rolling
PYPL	Rolling	Expanding	Expanding	Rolling	Rolling	Expanding	Rolling
QCOM	Rolling	Expanding	Expanding	Expanding	Rolling	Expanding	Rolling
SBUX	Rolling	Expanding	Expanding	Expanding	Rolling	Rolling	Expanding
SHOP	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding
SMCI	Rolling	Expanding	Expanding	Expanding	Rolling	Expanding	Expanding
SO	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding
SPGI	Expanding	Expanding	Expanding	Expanding	Rolling	Expanding	Rolling
SYK	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling
TJX	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding	Expanding
TMO	Expanding	Expanding	Expanding	Rolling	Rolling	Rolling	Expanding
TMUS	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling
TSN	Rolling	Expanding	Rolling	Rolling	Rolling	Rolling	Expanding
TXN	Rolling	Expanding	Expanding	Expanding	Rolling	Expanding	Rolling
UNH	Expanding	Rolling	Rolling	Rolling	Rolling	Expanding	Rolling
V	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding	Rolling
VRTX	Rolling	Expanding	Expanding	Expanding	Rolling	Rolling	Expanding
WDAY	Expanding	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding
WFC	Expanding	Expanding	Expanding	Expanding	Rolling	Rolling	Rolling
WMT	Rolling	Expanding	Expanding	Expanding	Rolling	Expanding	Expanding
XOM	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding

Table 18: This table shows which forecasting scheme performs better on the training set, according to mean absolute error for each model and the second half of stocks.

	AR(1)-RV	HAR	HAR-AS	HAR-RS	HAR-RSRK	RGARCH	GARCH
AAL	Expanding	Expanding	Rolling	Rolling	Rolling	Rolling	Rolling
AAPL	Rolling	Expanding	Rolling	Rolling	Rolling	Rolling	Rolling
ABBV	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling
ACN	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling
ADBE	Rolling	Expanding	Rolling	Rolling	Rolling	Expanding	Rolling
AMAT	Rolling	Expanding	Rolling	Rolling	Rolling	Rolling	Rolling
AMD	Rolling	Rolling	Rolling	Expanding	Rolling	Expanding	Expanding
AMGN	Expanding	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding
AMZN	Rolling	Expanding	Rolling	Rolling	Rolling	Expanding	Expanding
ANET	Rolling	Expanding	Expanding	Expanding	Rolling	Expanding	Rolling
AVGO	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding
BA	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding	Rolling
BAC	Expanding	Expanding	Expanding	Rolling	Rolling	Expanding	Expanding
BKNG	Expanding	Expanding	Expanding	Expanding	Rolling	Expanding	Rolling
C	Expanding	Expanding	Expanding	Expanding	Rolling	Expanding	Expanding
CAT	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding
CMCSA	Rolling	Expanding	Expanding	Expanding	Rolling	Expanding	Expanding
CMG	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding	Expanding
COP	Expanding	Rolling	Rolling	Rolling	Rolling	Expanding	Rolling
COST	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding
CRM	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding	Rolling
CSCO	Rolling	Expanding	Rolling	Rolling	Rolling	Expanding	Rolling
CVS	Expanding	Rolling	Rolling	Rolling	Rolling	Expanding	Expanding
CVX	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling
DIS	Expanding	Expanding	Expanding	Rolling	Rolling	Expanding	Rolling
EMR	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling
FCX	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding	Rolling
FTNT	Rolling	Expanding	Expanding	Expanding	Rolling	Rolling	Expanding
GE	Expanding	Expanding	Rolling	Expanding	Rolling	Expanding	Expanding
GME	Expanding	Expanding	Rolling	Rolling	Rolling	Expanding	Rolling
GOOG	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding	Rolling
GS	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding	Rolling
HD	Expanding	Expanding	Rolling	Rolling	Rolling	Rolling	Rolling
HES	Expanding	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding
IBM	Rolling	Expanding	Expanding	Expanding	Rolling	Expanding	Expanding
INTC	Expanding	Rolling	Rolling	Rolling	Rolling	Expanding	Expanding
JNJ	Rolling	Expanding	Rolling	Rolling	Rolling	Rolling	Rolling
JPM	Expanding	Rolling	Rolling	Rolling	Rolling	Expanding	Expanding
KO	Rolling	Expanding	Expanding	Expanding	Rolling	Expanding	Expanding
LRCX	Rolling	Expanding	Expanding	Rolling	Rolling	Expanding	Rolling

Table 19: This table shows which forecasting scheme performs better on the training set, according to mean square error for each model and the first half of stocks.

	AR(1)-RV	HAR	HAR-AS	HAR-RS	HAR-RSRK	RGARCH	GARCH
MA	Rolling	Expanding	Expanding	Expanding	Rolling	Rolling	Rolling
MCD	Expanding	Rolling	Rolling	Rolling	Rolling	Expanding	Expanding
MCHP	Rolling	Expanding	Rolling	Rolling	Rolling	Rolling	Rolling
MELI	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding
META	Rolling	Expanding	Rolling	Rolling	Rolling	Rolling	Expanding
MRK	Expanding	Expanding	Rolling	Rolling	Rolling	Expanding	Expanding
MSFT	Expanding	Expanding	Rolling	Rolling	Rolling	Rolling	Rolling
MSTR	Rolling	Expanding	Expanding	Expanding	Rolling	Rolling	Expanding
MU	Rolling	Expanding	Expanding	Rolling	Rolling	Expanding	Rolling
NEE	Expanding	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding
NFLX	Rolling	Expanding	Expanding	Expanding	Rolling	Rolling	Expanding
NKE	Rolling	Rolling	Rolling	Expanding	Rolling	Expanding	Expanding
NOW	Rolling	Expanding	Rolling	Expanding	Rolling	Expanding	Rolling
NVDA	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding	Expanding
NXPI	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding	Rolling
ORCL	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding	Expanding
PANW	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling
PEP	Rolling	Expanding	Expanding	Expanding	Rolling	Rolling	Rolling
PFE	Expanding	Expanding	Expanding	Expanding	Rolling	Rolling	Rolling
PG	Rolling	Expanding	Rolling	Expanding	Rolling	Expanding	Rolling
PYPL	Expanding	Expanding	Expanding	Expanding	Rolling	Rolling	Expanding
QCOM	Rolling	Expanding	Rolling	Rolling	Rolling	Expanding	Rolling
SBUX	Rolling	Expanding	Rolling	Rolling	Rolling	Rolling	Rolling
SHOP	Rolling	Expanding	Rolling	Rolling	Rolling	Expanding	Expanding
SMCI	Rolling	Expanding	Expanding	Expanding	Rolling	Expanding	Expanding
SO	Rolling	Expanding	Rolling	Rolling	Rolling	Rolling	Expanding
SPGI	Expanding	Expanding	Expanding	Expanding	Rolling	Expanding	Rolling
SYK	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling
TJX	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling
TMO	Expanding	Expanding	Rolling	Rolling	Rolling	Rolling	Rolling
TMUS	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding	Rolling
TSN	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding
TXN	Rolling	Expanding	Rolling	Rolling	Rolling	Expanding	Rolling
UNH	Expanding	Rolling	Rolling	Rolling	Rolling	Expanding	Rolling
V	Expanding	Expanding	Expanding	Expanding	Rolling	Expanding	Rolling
VRTX	Rolling	Expanding	Expanding	Expanding	Rolling	Rolling	Expanding
WDAY	Expanding	Rolling	Rolling	Rolling	Rolling	Rolling	Expanding
WFC	Expanding	Expanding	Rolling	Rolling	Rolling	Expanding	Expanding
WMT	Rolling	Expanding	Rolling	Expanding	Rolling	Rolling	Expanding
XOM	Expanding	Expanding	Rolling	Expanding	Rolling	Expanding	Expanding

Table 20: This table shows which forecasting scheme performs better on the training set, according to mean square error for each model and the second half of stocks.

	AR(1)-RV	HAR	HAR-AS	HAR-RSV	HAR-RSRK	RGARCH	GARCH
AR(1)-RV	-	0.19	0.37	0.4	0.4	0.07	0.19
HAR	0.19	-	0.79	0.72	0.71	0.13	0.3
HAR-AS	0.37	0.79	-	0.39	0.42	0.4	0.59
HAR-RSV	0.4	0.72	0.37	-	0.36	0.35	0.57
HAR-RSRK	0.4	0.71	0.4	0.38	-	0.35	0.56
RGARCH	0.07	0.13	0.41	0.35	0.35	-	0.19
GARCH	0.18	0.3	0.58	0.56	0.56	0.18	-

Table 21: This table shows the means of p-values of the Diebold-Mariano test for respective combinations of models. The values below the diagonal are for rolling window forecast, the values above the diagonal are for the expanding window forecast.

	AR(1)-RV	HAR	HAR-AS	HAR-RSV	HAR-RSRK	RGARCH	GARCH
AR(1)-RV	-	0.31	0.35	0.36	0.36	0.17	0.32
HAR	0.31	-	0.17	0.18	0.18	0.23	0.3
HAR-AS	0.35	0.17	-	0.28	0.31	0.26	0.32
HAR-RSV	0.36	0.18	0.28	-	0.31	0.26	0.32
HAR-RSRK	0.36	0.19	0.31	0.32	-	0.26	0.33
RGARCH	0.16	0.23	0.27	0.26	0.26	-	0.26
GARCH	0.31	0.3	0.32	0.32	0.32	0.27	-

Table 22: This table shows the standard deviations of p-values of the Diebold-Mariano test for respective combinations of models. The values below the diagonal are for rolling window forecast, the values above the diagonal are for the expanding window forecast.

	AR(1)-RV	HAR	HAR-AS	HAR-RSV	HAR-RSRK	RGARCH	GARCH
AR(1)-RV	-	0.64	0.3	0.24	0.24	0.79	0.65
HAR	0.65	-	0	0	0	0.64	0.32
HAR-AS	0.3	0	-	0.11	0.15	0.09	0.1
HAR-RSV	0.22	0	0.11	-	0.16	0.07	0.1
HAR-RSRK	0.22	0	0.16	0.17	-	0.09	0.1
RGARCH	0.8	0.64	0.09	0.09	0.09	-	0.51
GARCH	0.65	0.32	0.1	0.1	0.1	0.51	-

Table 23: This table shows the the percentage for how many stocks the p-value of the Diebold-Mariano test was below 0.05 for respective combinations of models. The values below the diagonal are for rolling window forecast, the values above the diagonal are for the expanding window forecast.

	AR(1)-RV	HAR	HAR-AS	HAR-RSV	HAR-RSRK	RGARCH	GARCH
AAL	0.35	0.58	0.87	0.10	0.25	0.03	0.03
AAPL	0.35	0.58	0.87	0.10	0.25	0.03	0.03
ABBV	0.19	0.58	0.80	0.10	0.49	0.03	0.25
ACN	0.19	0.58	0.80	0.10	0.49	0.03	0.25
ADBE	0.19	0.58	0.80	0.10	0.10	0.10	0.25
AMAT	0.19	0.58	0.80	0.10	0.10	0.10	0.25
AMD	0.02	0.87	0.80	0.49	0.49	0.10	0.49
AMGN	0.02	0.87	0.80	0.49	0.49	0.10	0.49
AMZN	0.02	0.10	0.80	0.10	0.49	0.03	0.49
ANET	0.02	0.10	0.80	0.10	0.49	0.03	0.49
AVGO	0.19	0.87	0.87	0.25	0.49	0.10	0.49
BA	0.19	0.87	0.87	0.25	0.49	0.10	0.49
BAC	0.58	0.87	0.80	0.80	0.49	0.03	0.49
BKNG	0.58	0.87	0.80	0.80	0.49	0.03	0.49
C	0.10	0.87	0.49	0.80	0.87	0.03	0.49
CAT	0.10	0.87	0.49	0.80	0.87	0.03	0.49
CMCSA	0.49	0.80	0.49	0.80	0.03	0.03	0.25
CMG	0.49	0.80	0.49	0.80	0.03	0.03	0.25
COP	0.49	0.87	0.25	0.80	0.00	0.03	0.25
COST	0.49	0.87	0.25	0.80	0.00	0.03	0.25
CRM	0.49	0.10	0.25	0.35	0.00	0.10	0.00
CSCO	0.49	0.10	0.25	0.35	0.00	0.10	0.00
CVS	0.49	0.10	0.25	0.80	0.00	0.49	0.00
CVX	0.49	0.10	0.25	0.80	0.00	0.49	0.00
DIS	0.49	0.87	0.25	0.00	0.00	0.10	0.00
EMR	0.25	0.87	0.25	0.00	0.00	0.10	0.00
FCX	0.25	0.87	0.49	0.03	0.00	0.03	0.00
FTNT	0.25	0.87	0.49	0.03	0.00	0.03	0.00
GE	0.10	0.87	0.25	0.10	0.00	0.10	0.03
GME	0.10	0.87	0.25	0.10	0.00	0.10	0.03
GOOG	0.49	0.87	0.80	0.10	0.00	0.10	0.03
GS	0.49	0.87	0.80	0.10	0.00	0.10	0.03
HD	0.49	0.87	0.58	0.10	0.00	0.10	0.00
HES	0.49	0.87	0.58	0.10	0.00	0.10	0.00
IBM	0.80	0.58	0.58	0.03	0.35	0.10	0.25
INTC	0.80	0.87	0.58	0.03	0.35	0.10	0.25
JNJ	0.80	0.10	0.58	0.25	0.35	0.49	0.87
JPM	0.80	0.10	0.58	0.25	0.35	0.49	0.87
KO	0.10	0.49	0.58	0.03	0.35	0.10	0.87
LRCX	0.10	0.49	0.58	0.03	0.35	0.10	0.87

Table 24: This table shows the p-values of the Kupiec's test on 0.9 VaR computed using expanding forecast values of all 7 models for the first half of stocks.

	AR(1)-RV	HAR	HAR-AS	HAR-RSV	HAR-RSRK	RGARCH	GARCH
LRCX	0.10	0.49	0.58	0.03	0.35	0.10	0.87
MA	0.00	0.49	0.87	0.25	0.35	0.03	0.80
MCD	0.00	0.49	0.87	0.25	0.35	0.03	0.80
MCHP	0.25	0.49	0.58	0.25	0.87	0.03	0.80
MELI	0.10	0.25	0.58	0.25	0.87	0.03	0.80
META	0.49	0.10	0.35	0.25	0.10	0.03	0.80
MRK	0.49	0.10	0.35	0.25	0.10	0.03	0.80
MSFT	0.49	0.87	0.19	0.25	0.00	0.03	0.25
MSTR	0.49	0.87	0.19	0.25	0.00	0.03	0.25
MU	0.80	0.10	0.10	0.49	0.10	0.03	0.10
NEE	0.49	0.10	0.10	0.49	0.10	0.03	0.10
NFLX	0.25	0.80	0.35	0.10	0.10	0.03	0.49
NKE	0.25	0.80	0.35	0.49	0.10	0.03	0.49
NOW	0.49	0.87	0.35	0.10	0.10	0.00	0.49
NVDA	0.10	0.87	0.35	0.10	0.10	0.00	0.49
NXPI	0.25	0.80	0.35	0.03	0.10	0.00	0.80
ORCL	0.25	0.80	0.35	0.03	0.10	0.00	0.80
PANW	0.00	0.87	0.87	0.03	0.10	0.25	0.80
PEP	0.00	0.87	0.87	0.03	0.10	0.25	0.80
PFE	0.03	0.87	0.03	0.03	0.10	0.25	0.80
PG	0.03	0.87	0.03	0.03	0.10	0.25	0.80
PYPL	0.00	0.87	0.10	0.03	0.00	0.25	0.49
QCOM	0.00	0.87	0.25	0.03	0.00	0.25	0.49
SBUX	0.03	0.87	0.10	0.03	0.00	0.25	0.25
SHOP	0.03	0.87	0.25	0.03	0.00	0.25	0.25
SMCI	0.03	0.00	0.10	0.03	0.00	0.03	0.80
SO	0.03	0.00	0.10	0.03	0.00	0.03	0.80
SPGI	0.03	0.49	0.10	0.03	0.00	0.25	0.80
SYK	0.03	0.49	0.10	0.10	0.00	0.25	0.80
TJX	0.03	0.49	0.87	0.80	0.00	0.00	0.80
TMO	0.03	0.49	0.87	0.80	0.00	0.00	0.80
TMUS	0.25	0.49	0.10	0.80	0.00	0.80	0.80
TSN	0.25	0.49	0.10	0.80	0.00	0.80	0.80
TXN	0.49	0.87	0.25	0.80	0.00	0.80	0.49
UNH	0.49	0.87	0.25	0.80	0.00	0.80	0.49
V	0.80	0.80	0.25	0.87	0.25	0.25	0.49
VRTX	0.80	0.80	0.25	0.87	0.25	0.25	0.49
WDAY	0.80	0.49	0.80	0.80	0.49	0.49	0.03
WFC	0.80	0.49	0.80	0.80	0.49	0.49	0.03
WMT	0.87	0.25	0.49	0.25	0.49	0.49	0.25
XOM	0.87	0.25	0.49	0.25	0.49	0.49	0.25

Table 25: This table shows the p-values of the Kupiec's test on 0.9 VaR computed using expanding forecast values of all 7 models for the second half of stocks.

	AR(1)-RV	HAR	HAR-AS	HAR-RSV	HAR-RSRK	RGARCH	GARCH
AAL	0.35	0.58	0.87	0.10	0.25	0.03	0.03
AAPL	0.35	0.58	0.87	0.10	0.25	0.03	0.03
ABBV	0.19	0.58	0.80	0.10	0.49	0.03	0.25
ACN	0.19	0.58	0.80	0.10	0.49	0.03	0.25
ADBE	0.19	0.58	0.80	0.10	0.10	0.10	0.25
AMAT	0.19	0.58	0.80	0.10	0.10	0.10	0.25
AMD	0.02	0.87	0.80	0.49	0.49	0.10	0.49
AMGN	0.02	0.87	0.80	0.49	0.49	0.10	0.49
AMZN	0.02	0.10	0.80	0.10	0.49	0.03	0.49
ANET	0.02	0.10	0.80	0.10	0.49	0.03	0.49
AVGO	0.19	0.87	0.87	0.25	0.49	0.10	0.49
BA	0.19	0.87	0.87	0.25	0.49	0.10	0.49
BAC	0.58	0.87	0.80	0.80	0.49	0.03	0.49
BKNG	0.58	0.87	0.80	0.80	0.49	0.03	0.49
C	0.10	0.87	0.49	0.80	0.87	0.03	0.49
CAT	0.10	0.87	0.49	0.80	0.87	0.03	0.49
CMCSA	0.49	0.80	0.49	0.80	0.03	0.03	0.25
CMG	0.49	0.80	0.49	0.80	0.03	0.03	0.25
COP	0.49	0.87	0.25	0.80	0.00	0.03	0.25
COST	0.49	0.87	0.25	0.80	0.00	0.03	0.25
CRM	0.49	0.10	0.25	0.35	0.00	0.10	0.00
CSCO	0.49	0.10	0.25	0.35	0.00	0.10	0.00
CVS	0.49	0.10	0.25	0.80	0.00	0.49	0.00
CVX	0.49	0.10	0.25	0.80	0.00	0.49	0.00
DIS	0.49	0.87	0.25	0.00	0.00	0.10	0.00
EMR	0.25	0.87	0.25	0.00	0.00	0.10	0.00
FCX	0.25	0.87	0.49	0.03	0.00	0.03	0.00
FTNT	0.25	0.87	0.49	0.03	0.00	0.03	0.00
GE	0.10	0.87	0.25	0.10	0.00	0.10	0.03
GME	0.10	0.87	0.25	0.10	0.00	0.10	0.03
GOOG	0.49	0.87	0.80	0.10	0.00	0.10	0.03
GS	0.49	0.87	0.80	0.10	0.00	0.10	0.03
HD	0.49	0.87	0.58	0.10	0.00	0.10	0.00
HES	0.49	0.87	0.58	0.10	0.00	0.10	0.00
IBM	0.80	0.58	0.58	0.03	0.35	0.10	0.25
INTC	0.80	0.87	0.58	0.03	0.35	0.10	0.25
JNJ	0.80	0.10	0.58	0.25	0.35	0.49	0.87
JPM	0.80	0.10	0.58	0.25	0.35	0.49	0.87
KO	0.10	0.49	0.58	0.03	0.35	0.10	0.87
LRCX	0.10	0.49	0.58	0.03	0.35	0.10	0.87

Table 26: This table shows the p-values of the Kupiec's test on 0.9 VaR computed using rolling forecast values of all 7 models for the first half of stocks.

	AR(1)-RV	HAR	HAR-AS	HAR-RSV	HAR-RSRK	RGARCH	GARCH
LRCX	0.10	0.49	0.58	0.03	0.35	0.10	0.87
MA	0.00	0.49	0.87	0.25	0.35	0.03	0.80
MCD	0.00	0.49	0.87	0.25	0.35	0.03	0.80
MCHP	0.25	0.49	0.58	0.25	0.87	0.03	0.80
MELI	0.10	0.25	0.58	0.25	0.87	0.03	0.80
META	0.49	0.10	0.35	0.25	0.10	0.03	0.80
MRK	0.49	0.10	0.35	0.25	0.10	0.03	0.80
MSFT	0.49	0.87	0.19	0.25	0.00	0.03	0.25
MSTR	0.49	0.87	0.19	0.25	0.00	0.03	0.25
MU	0.80	0.10	0.10	0.49	0.10	0.03	0.10
NEE	0.49	0.10	0.10	0.49	0.10	0.03	0.10
NFLX	0.25	0.80	0.35	0.10	0.10	0.03	0.49
NKE	0.25	0.80	0.35	0.49	0.10	0.03	0.49
NOW	0.49	0.87	0.35	0.10	0.10	0.00	0.49
NVDA	0.10	0.87	0.35	0.10	0.10	0.00	0.49
NXPI	0.25	0.80	0.35	0.03	0.10	0.00	0.80
ORCL	0.25	0.80	0.35	0.03	0.10	0.00	0.80
PANW	0.00	0.87	0.87	0.03	0.10	0.25	0.80
PEP	0.00	0.87	0.87	0.03	0.10	0.25	0.80
PFE	0.03	0.87	0.03	0.03	0.10	0.25	0.80
PG	0.03	0.87	0.03	0.03	0.10	0.25	0.80
PYPL	0.00	0.87	0.10	0.03	0.00	0.25	0.49
QCOM	0.00	0.87	0.25	0.03	0.00	0.25	0.49
SBUX	0.03	0.87	0.10	0.03	0.00	0.25	0.25
SHOP	0.03	0.87	0.25	0.03	0.00	0.25	0.25
SMCI	0.03	0.00	0.10	0.03	0.00	0.03	0.80
SO	0.03	0.00	0.10	0.03	0.00	0.03	0.80
SPGI	0.03	0.49	0.10	0.03	0.00	0.25	0.80
SYK	0.03	0.49	0.10	0.10	0.00	0.25	0.80
TJX	0.03	0.49	0.87	0.80	0.00	0.00	0.80
TMO	0.03	0.49	0.87	0.80	0.00	0.00	0.80
TMUS	0.25	0.49	0.10	0.80	0.00	0.80	0.80
TSN	0.25	0.49	0.10	0.80	0.00	0.80	0.80
TXN	0.49	0.87	0.25	0.80	0.00	0.80	0.49
UNH	0.49	0.87	0.25	0.80	0.00	0.80	0.49
V	0.80	0.80	0.25	0.87	0.25	0.25	0.49
VRTX	0.80	0.80	0.25	0.87	0.25	0.25	0.49
WDAY	0.80	0.49	0.80	0.80	0.49	0.49	0.03
WFC	0.80	0.49	0.80	0.80	0.49	0.49	0.03
WMT	0.87	0.25	0.49	0.25	0.49	0.49	0.25
XOM	0.87	0.25	0.49	0.25	0.49	0.49	0.25

Table 27: This table shows the p-values of the Kupiec's test on 0.9 VaR computed using rolling forecast values of all 7 models for the second half of stocks.

	AR(1)-RV	HAR	HAR-AS	HAR-RSV	HAR-RSRK	RGARCH	GARCH
AAL	0.70	0.07	0.43	0.13	0.13	0.43	0.43
AAPL	0.70	0.07	0.43	0.13	0.13	0.43	0.43
ABBV	0.17	0.17	0.86	0.13	0.01	0.43	0.86
ACN	0.17	0.17	0.86	0.13	0.01	0.43	0.86
ADBE	0.17	0.17	0.17	0.13	0.13	0.43	0.43
AMAT	0.17	0.17	0.17	0.13	0.13	0.43	0.43
AMD	0.17	0.86	0.70	0.13	0.43	0.43	0.01
AMGN	0.17	0.86	0.70	0.13	0.43	0.43	0.01
AMZN	0.17	0.01	0.37	0.01	0.43	0.13	0.13
ANET	0.17	0.01	0.37	0.01	0.43	0.13	0.13
AVGO	0.37	0.86	0.37	0.01	0.43	0.43	0.13
BA	0.37	0.86	0.37	0.01	0.43	0.43	0.13
BAC	0.86	0.86	0.70	0.13	0.43	0.13	0.13
BKNG	0.43	0.86	0.70	0.13	0.43	0.13	0.13
C	0.01	0.86	0.43	0.13	0.01	0.13	0.13
CAT	0.01	0.86	0.43	0.13	0.01	0.13	0.13
CMCSA	0.70	0.86	0.43	0.13	0.01	0.13	0.86
CMG	0.70	0.86	0.43	0.13	0.01	0.13	0.86
COP	0.70	0.43	0.86	0.43	0.01	0.13	0.13
COST	0.70	0.43	0.86	0.43	0.01	0.13	0.13
CRM	0.70	0.01	0.86	0.86	0.01	0.13	0.01
CSCO	0.70	0.01	0.86	0.86	0.01	0.13	0.01
CVS	0.70	0.13	0.70	0.13	0.01	0.43	0.01
CVX	0.70	0.13	0.70	0.13	0.01	0.43	0.01
DIS	0.86	0.43	0.70	0.01	0.01	0.13	0.01
EMR	0.86	0.43	0.70	0.01	0.01	0.13	0.01
FCX	0.86	0.43	0.43	0.13	0.01	0.13	0.01
FTNT	0.86	0.43	0.43	0.13	0.01	0.13	0.01
GE	0.13	0.43	0.43	0.01	0.01	0.43	0.01
GME	0.13	0.43	0.43	0.01	0.01	0.43	0.01
GOOG	0.13	0.43	0.43	0.43	0.01	0.43	0.01
GS	0.13	0.43	0.43	0.43	0.01	0.43	0.01
HD	0.13	0.43	0.37	0.13	0.01	0.43	0.01
HES	0.13	0.43	0.37	0.13	0.01	0.43	0.01
IBM	0.13	0.43	0.37	0.13	0.43	0.43	0.01
INTC	0.13	0.43	0.37	0.13	0.43	0.43	0.01
JNJ	0.13	0.01	0.37	0.01	0.43	0.43	0.43
JPM	0.13	0.01	0.37	0.01	0.43	0.43	0.43
KO	0.01	0.43	0.37	0.01	0.43	0.43	0.43
LRCX	0.01	0.43	0.37	0.01	0.43	0.43	0.43

Table 28: This table shows the p-values of the Kupiec's test on 0.95 VaR computed using expanding forecast values of all 7 models for the first half of stocks.

	AR(1)-RV	HAR	HAR-AS	HAR-RSV	HAR-RSRK	RGARCH	GARCH
LRCX	0.01	0.43	0.37	0.01	0.43	0.43	0.43
MA	0.01	0.43	0.37	0.13	0.43	0.13	0.43
MCD	0.01	0.43	0.37	0.13	0.43	0.13	0.43
MCHP	0.01	0.43	0.43	0.13	0.01	0.13	0.43
MELI	0.01	0.43	0.43	0.13	0.01	0.13	0.43
META	0.86	0.13	0.13	0.13	0.01	0.13	0.43
MRK	0.43	0.13	0.13	0.13	0.01	0.13	0.43
MSFT	0.43	0.13	0.17	0.13	0.13	0.13	0.13
MSTR	0.43	0.13	0.17	0.13	0.13	0.13	0.13
MU	0.86	0.01	0.37	0.43	0.13	0.13	0.13
NEE	0.86	0.01	0.37	0.43	0.13	0.13	0.13
NFLX	0.86	0.01	0.07	0.01	0.13	0.13	0.43
NKE	0.86	0.01	0.07	0.01	0.13	0.13	0.43
NOW	0.01	0.70	0.17	0.13	0.13	0.01	0.43
NVDA	0.13	0.70	0.17	0.13	0.13	0.01	0.43
NXPI	0.13	0.70	0.86	0.43	0.13	0.01	0.43
ORCL	0.13	0.70	0.86	0.43	0.13	0.01	0.43
PANW	0.01	0.70	0.13	0.43	0.13	0.01	0.43
PEP	0.01	0.70	0.13	0.43	0.13	0.01	0.43
PFE	0.13	0.70	0.01	0.43	0.13	0.01	0.13
PG	0.13	0.70	0.01	0.43	0.13	0.01	0.13
PYPL	0.13	0.86	0.13	0.43	0.01	0.01	0.13
QCOM	0.13	0.86	0.13	0.43	0.01	0.01	0.13
SBUX	0.13	0.86	0.13	0.43	0.01	0.01	0.43
SHOP	0.13	0.86	0.13	0.43	0.01	0.01	0.43
SMCI	0.13	0.01	0.13	0.43	0.01	0.01	0.86
SO	0.13	0.01	0.13	0.43	0.01	0.01	0.86
SPGI	0.13	0.13	0.13	0.01	0.01	0.01	0.86
SYK	0.13	0.13	0.13	0.01	0.01	0.01	0.86
TJX	0.13	0.13	0.13	0.43	0.01	0.01	0.86
TMO	0.13	0.13	0.13	0.43	0.01	0.01	0.86
TMUS	0.01	0.43	0.13	0.43	0.01	0.13	0.86
TSN	0.01	0.43	0.13	0.43	0.01	0.13	0.86
TXN	0.86	0.43	0.43	0.13	0.01	0.13	0.86
UNH	0.86	0.43	0.43	0.13	0.01	0.13	0.86
V	0.86	0.43	0.43	0.43	0.13	0.13	0.13
VRTX	0.86	0.43	0.43	0.43	0.13	0.13	0.13
WDAY	0.86	0.13	0.86	0.43	0.86	0.13	0.13
WFC	0.86	0.13	0.86	0.43	0.86	0.13	0.13
WMT	0.86	0.13	0.43	0.01	0.86	0.43	0.43
XOM	0.86	0.13	0.43	0.01	0.86	0.43	0.43

Table 29: This table shows the p-values of the Kupiec's test on 0.95 VaR computed using expanding forecast values of all 7 models for the second half of stocks.

	AR(1)-RV	HAR	HAR-AS	HAR-RSV	HAR-RSRK	RGARCH	GARCH
AAL	0.70	0.07	0.43	0.13	0.13	0.43	0.43
AAPL	0.70	0.07	0.43	0.13	0.13	0.43	0.43
ABBV	0.17	0.17	0.86	0.13	0.01	0.43	0.86
ACN	0.17	0.17	0.86	0.13	0.01	0.43	0.86
ADBE	0.17	0.17	0.17	0.13	0.13	0.43	0.43
AMAT	0.17	0.17	0.17	0.13	0.13	0.43	0.43
AMD	0.17	0.86	0.70	0.13	0.43	0.43	0.01
AMGN	0.17	0.86	0.70	0.13	0.43	0.43	0.01
AMZN	0.17	0.01	0.37	0.01	0.43	0.13	0.13
ANET	0.17	0.01	0.37	0.01	0.43	0.13	0.13
AVGO	0.37	0.86	0.37	0.01	0.43	0.43	0.13
BA	0.37	0.86	0.37	0.01	0.43	0.43	0.13
BAC	0.86	0.86	0.70	0.13	0.43	0.13	0.13
BKNG	0.43	0.86	0.70	0.13	0.43	0.13	0.13
C	0.01	0.86	0.43	0.13	0.01	0.13	0.13
CAT	0.01	0.86	0.43	0.13	0.01	0.13	0.13
CMCSA	0.70	0.86	0.43	0.13	0.01	0.13	0.86
CMG	0.70	0.86	0.43	0.13	0.01	0.13	0.86
COP	0.70	0.43	0.86	0.43	0.01	0.13	0.13
COST	0.70	0.43	0.86	0.43	0.01	0.13	0.13
CRM	0.70	0.01	0.86	0.86	0.01	0.13	0.01
CSCO	0.70	0.01	0.86	0.86	0.01	0.13	0.01
CVS	0.70	0.13	0.70	0.13	0.01	0.43	0.01
CVX	0.70	0.13	0.70	0.13	0.01	0.43	0.01
DIS	0.86	0.43	0.70	0.01	0.01	0.13	0.01
EMR	0.86	0.43	0.70	0.01	0.01	0.13	0.01
FCX	0.86	0.43	0.43	0.13	0.01	0.13	0.01
FTNT	0.86	0.43	0.43	0.13	0.01	0.13	0.01
GE	0.13	0.43	0.43	0.01	0.01	0.43	0.01
GME	0.13	0.43	0.43	0.01	0.01	0.43	0.01
GOOG	0.13	0.43	0.43	0.43	0.01	0.43	0.01
GS	0.13	0.43	0.43	0.43	0.01	0.43	0.01
HD	0.13	0.43	0.37	0.13	0.01	0.43	0.01
HES	0.13	0.43	0.37	0.13	0.01	0.43	0.01
IBM	0.13	0.43	0.37	0.13	0.43	0.43	0.01
INTC	0.13	0.43	0.37	0.13	0.43	0.43	0.01
JNJ	0.13	0.01	0.37	0.01	0.43	0.43	0.43
JPM	0.13	0.01	0.37	0.01	0.43	0.43	0.43
KO	0.01	0.43	0.37	0.01	0.43	0.43	0.43
LRCX	0.01	0.43	0.37	0.01	0.43	0.43	0.43

Table 30: This table shows the p-values of the Kupiec's test on 0.95 VaR computed using rolling forecast values of all 7 models for the first half of stocks.

	AR(1)-RV	HAR	HAR-AS	HAR-RSV	HAR-RSRK	RGARCH	GARCH
LRCX	0.01	0.43	0.37	0.01	0.43	0.43	0.43
MA	0.01	0.43	0.37	0.13	0.43	0.13	0.43
MCD	0.01	0.43	0.37	0.13	0.43	0.13	0.43
MCHP	0.01	0.43	0.43	0.13	0.01	0.13	0.43
MELI	0.01	0.43	0.43	0.13	0.01	0.13	0.43
META	0.86	0.13	0.13	0.13	0.01	0.13	0.43
MRK	0.43	0.13	0.13	0.13	0.01	0.13	0.43
MSFT	0.43	0.13	0.17	0.13	0.13	0.13	0.13
MSTR	0.43	0.13	0.17	0.13	0.13	0.13	0.13
MU	0.86	0.01	0.37	0.43	0.13	0.13	0.13
NEE	0.86	0.01	0.37	0.43	0.13	0.13	0.13
NFLX	0.86	0.01	0.07	0.01	0.13	0.13	0.43
NKE	0.86	0.01	0.07	0.01	0.13	0.13	0.43
NOW	0.01	0.70	0.17	0.13	0.13	0.01	0.43
NVDA	0.13	0.70	0.17	0.13	0.13	0.01	0.43
NXPI	0.13	0.70	0.86	0.43	0.13	0.01	0.43
ORCL	0.13	0.70	0.86	0.43	0.13	0.01	0.43
PANW	0.01	0.70	0.13	0.43	0.13	0.01	0.43
PEP	0.01	0.70	0.13	0.43	0.13	0.01	0.43
PFE	0.13	0.70	0.01	0.43	0.13	0.01	0.13
PG	0.13	0.70	0.01	0.43	0.13	0.01	0.13
PYPL	0.13	0.86	0.13	0.43	0.01	0.01	0.13
QCOM	0.13	0.86	0.13	0.43	0.01	0.01	0.13
SBUX	0.13	0.86	0.13	0.43	0.01	0.01	0.43
SHOP	0.13	0.86	0.13	0.43	0.01	0.01	0.43
SMCI	0.13	0.01	0.13	0.43	0.01	0.01	0.86
SO	0.13	0.01	0.13	0.43	0.01	0.01	0.86
SPGI	0.13	0.13	0.13	0.01	0.01	0.01	0.86
SYK	0.13	0.13	0.13	0.01	0.01	0.01	0.86
TJX	0.13	0.13	0.13	0.43	0.01	0.01	0.86
TMO	0.13	0.13	0.13	0.43	0.01	0.01	0.86
TMUS	0.01	0.43	0.13	0.43	0.01	0.13	0.86
TSN	0.01	0.43	0.13	0.43	0.01	0.13	0.86
TXN	0.86	0.43	0.43	0.13	0.01	0.13	0.86
UNH	0.86	0.43	0.43	0.13	0.01	0.13	0.86
V	0.86	0.43	0.43	0.43	0.13	0.13	0.13
VRTX	0.86	0.43	0.43	0.43	0.13	0.13	0.13
WDAY	0.86	0.13	0.86	0.43	0.86	0.13	0.13
WFC	0.86	0.13	0.86	0.43	0.86	0.13	0.13
WMT	0.86	0.13	0.43	0.01	0.86	0.43	0.43
XOM	0.86	0.13	0.43	0.01	0.86	0.43	0.43

Table 31: This table shows the p-values of the Kupiec's test on 0.95 VaR computed using rolling forecast values of all 7 models for the second half of stocks.

	AR(1)-RV	HAR	HAR-AS	HAR-RSV	HAR-RSRK	RGARCH	GARCH
AAL	0.25	0.25	0.25	0.25	0.25	0.25	0.25
AAPL	0.25	0.25	0.25	0.25	0.25	0.25	0.25
ABBV	0.25	0.25	0.25	0.25	0.25	0.25	0.25
ACN	0.25	0.25	0.25	0.25	0.25	0.25	0.25
ADBE	0.25	0.25	0.25	0.25	0.25	0.25	0.25
AMAT	0.25	0.25	0.25	0.25	0.25	0.25	0.25
AMD	0.70	0.25	0.25	0.25	0.25	0.25	0.25
AMGN	0.70	0.25	0.25	0.25	0.25	0.25	0.25
AMZN	0.70	0.25	0.25	0.25	0.25	0.25	0.25
ANET	0.70	0.25	0.25	0.25	0.25	0.25	0.25
AVGO	0.25	0.25	0.70	0.25	0.25	0.25	0.25
BA	0.25	0.25	0.70	0.25	0.25	0.25	0.25
BAC	0.25	0.25	0.25	0.25	0.25	0.25	0.25
BKNG	0.25	0.25	0.25	0.25	0.25	0.25	0.25
C	0.25	0.25	0.25	0.25	0.25	0.25	0.25
CAT	0.25	0.25	0.25	0.25	0.25	0.25	0.25
CMCSA	0.25	0.25	0.25	0.25	0.25	0.25	0.25
CMG	0.25	0.25	0.25	0.25	0.25	0.25	0.25
COP	0.25	0.25	0.25	0.25	0.25	0.25	0.25
COST	0.25	0.25	0.25	0.25	0.25	0.25	0.25
CRM	0.25	0.25	0.25	0.25	0.25	0.25	0.25
CSCO	0.25	0.25	0.25	0.25	0.25	0.25	0.25
CVS	0.25	0.25	0.25	0.25	0.25	0.25	0.25
CVX	0.25	0.25	0.25	0.25	0.25	0.25	0.25
DIS	0.25	0.25	0.25	0.25	0.25	0.25	0.25
EMR	0.25	0.25	0.25	0.25	0.25	0.25	0.25
FCX	0.25	0.25	0.25	0.25	0.25	0.25	0.25
FTNT	0.25	0.25	0.25	0.25	0.25	0.25	0.25
GE	0.25	0.25	0.25	0.25	0.25	0.25	0.25
GME	0.25	0.25	0.25	0.25	0.25	0.25	0.25
GOOG	0.25	0.25	0.25	0.25	0.25	0.25	0.25
GS	0.25	0.25	0.25	0.25	0.25	0.25	0.25
HD	0.25	0.25	0.25	0.25	0.25	0.70	0.25
HES	0.25	0.25	0.25	0.25	0.25	0.70	0.25
IBM	0.25	0.25	0.25	0.25	0.25	0.70	0.25
INTC	0.25	0.25	0.25	0.25	0.25	0.70	0.25
JNJ	0.25	0.25	0.25	0.25	0.25	0.25	0.25
JPM	0.25	0.25	0.25	0.25	0.25	0.25	0.25
KO	0.25	0.25	0.25	0.25	0.25	0.25	0.25
LRCX	0.25	0.25	0.25	0.25	0.25	0.25	0.25

Table 32: This table shows the p-values of the Kupiec's test on 0.99 VaR computed using expanding forecast values of all 7 models for the first half of stocks.

	AR(1)-RV	HAR	HAR-AS	HAR-RSV	HAR-RSRK	RGARCH	GARCH
LRCX	0.25	0.25	0.25	0.25	0.25	0.25	0.25
MA	0.25	0.25	0.25	0.25	0.25	0.25	0.25
MCD	0.25	0.25	0.25	0.25	0.25	0.25	0.25
MCHP	0.25	0.25	0.25	0.25	0.25	0.25	0.25
MELI	0.25	0.25	0.25	0.25	0.25	0.25	0.25
META	0.25	0.25	0.25	0.25	0.25	0.25	0.25
MRK	0.25	0.25	0.25	0.25	0.25	0.25	0.25
MSFT	0.25	0.25	0.25	0.25	0.25	0.25	0.25
MSTR	0.25	0.25	0.25	0.25	0.25	0.25	0.25
MU	0.25	0.25	0.25	0.25	0.25	0.25	0.25
NEE	0.25	0.25	0.25	0.25	0.25	0.25	0.25
NFLX	0.25	0.25	0.25	0.25	0.25	0.25	0.25
NKE	0.25	0.25	0.25	0.25	0.25	0.25	0.25
NOW	0.25	0.25	0.25	0.25	0.25	0.25	0.25
NVDA	0.25	0.25	0.25	0.25	0.25	0.25	0.25
NXPI	0.25	0.25	0.25	0.25	0.25	0.25	0.25
ORCL	0.25	0.25	0.25	0.25	0.25	0.25	0.25
PANW	0.25	0.25	0.25	0.25	0.25	0.25	0.25
PEP	0.25	0.25	0.25	0.25	0.25	0.25	0.25
PFE	0.25	0.25	0.25	0.25	0.25	0.25	0.25
PG	0.25	0.25	0.25	0.25	0.25	0.25	0.25
PYPL	0.25	0.25	0.25	0.25	0.25	0.25	0.25
QCOM	0.25	0.25	0.25	0.25	0.25	0.25	0.25
SBUX	0.25	0.25	0.25	0.25	0.25	0.25	0.25
SHOP	0.25	0.25	0.25	0.25	0.25	0.25	0.25
SMCI	0.25	0.25	0.25	0.25	0.25	0.25	0.25
SO	0.25	0.25	0.25	0.25	0.25	0.25	0.25
SPGI	0.25	0.25	0.25	0.25	0.25	0.25	0.70
SYK	0.25	0.25	0.25	0.25	0.25	0.25	0.70
TJX	0.25	0.25	0.25	0.25	0.25	0.25	0.70
TMO	0.25	0.25	0.25	0.25	0.25	0.25	0.70
TMUS	0.25	0.25	0.25	0.25	0.25	0.25	0.25
TSN	0.25	0.25	0.25	0.25	0.25	0.25	0.25
TXN	0.25	0.25	0.25	0.25	0.25	0.25	0.25
UNH	0.25	0.25	0.25	0.25	0.25	0.25	0.25
V	0.25	0.25	0.25	0.70	0.25	0.25	0.25
VRTX	0.25	0.25	0.25	0.70	0.25	0.25	0.25
WDAY	0.25	0.25	0.70	0.25	0.25	0.25	0.25
WFC	0.25	0.25	0.70	0.25	0.25	0.25	0.25
WMT	0.25	0.25	0.25	0.25	0.25	0.25	0.25
XOM	0.25	0.25	0.25	0.25	0.25	0.25	0.25

Table 33: This table shows the p-values of the Kupiec's test on 0.99 VaR computed using expanding forecast values of all 7 models for the second half of stocks.

	AR(1)-RV	HAR	HAR-AS	HAR-RSV	HAR-RSRK	RGARCH	GARCH
AAL	0.25	0.25	0.25	0.25	0.25	0.25	0.25
AAPL	0.25	0.25	0.25	0.25	0.25	0.25	0.25
ABBV	0.25	0.25	0.25	0.25	0.25	0.25	0.25
ACN	0.25	0.25	0.25	0.25	0.25	0.25	0.25
ADBE	0.25	0.25	0.25	0.25	0.25	0.25	0.25
AMAT	0.25	0.25	0.25	0.25	0.25	0.25	0.25
AMD	0.70	0.25	0.25	0.25	0.25	0.25	0.25
AMGN	0.70	0.25	0.25	0.25	0.25	0.25	0.25
AMZN	0.70	0.25	0.25	0.25	0.25	0.25	0.25
ANET	0.70	0.25	0.25	0.25	0.25	0.25	0.25
AVGO	0.25	0.25	0.70	0.25	0.25	0.25	0.25
BA	0.25	0.25	0.70	0.25	0.25	0.25	0.25
BAC	0.25	0.25	0.25	0.25	0.25	0.25	0.25
BKNG	0.25	0.25	0.25	0.25	0.25	0.25	0.25
C	0.25	0.25	0.25	0.25	0.25	0.25	0.25
CAT	0.25	0.25	0.25	0.25	0.25	0.25	0.25
CMCSA	0.25	0.25	0.25	0.25	0.25	0.25	0.25
CMG	0.25	0.25	0.25	0.25	0.25	0.25	0.25
COP	0.25	0.25	0.25	0.25	0.25	0.25	0.25
COST	0.25	0.25	0.25	0.25	0.25	0.25	0.25
CRM	0.25	0.25	0.25	0.25	0.25	0.25	0.25
CSCO	0.25	0.25	0.25	0.25	0.25	0.25	0.25
CVS	0.25	0.25	0.25	0.25	0.25	0.25	0.25
CVX	0.25	0.25	0.25	0.25	0.25	0.25	0.25
DIS	0.25	0.25	0.25	0.25	0.25	0.25	0.25
EMR	0.25	0.25	0.25	0.25	0.25	0.25	0.25
FCX	0.25	0.25	0.25	0.25	0.25	0.25	0.25
FTNT	0.25	0.25	0.25	0.25	0.25	0.25	0.25
GE	0.25	0.25	0.25	0.25	0.25	0.25	0.25
GME	0.25	0.25	0.25	0.25	0.25	0.25	0.25
GOOG	0.25	0.25	0.25	0.25	0.25	0.25	0.25
GS	0.25	0.25	0.25	0.25	0.25	0.25	0.25
HD	0.25	0.25	0.25	0.25	0.25	0.70	0.25
HES	0.25	0.25	0.25	0.25	0.25	0.70	0.25
IBM	0.25	0.25	0.25	0.25	0.25	0.70	0.25
INTC	0.25	0.25	0.25	0.25	0.25	0.70	0.25
JNJ	0.25	0.25	0.25	0.25	0.25	0.25	0.25
JPM	0.25	0.25	0.25	0.25	0.25	0.25	0.25
KO	0.25	0.25	0.25	0.25	0.25	0.25	0.25
LRCX	0.25	0.25	0.25	0.25	0.25	0.25	0.25

Table 34: This table shows the p-values of the Kupiec's test on 0.99 VaR computed using rolling forecast values of all 7 models for the first half of stocks.

	AR(1)-RV	HAR	HAR-AS	HAR-RSV	HAR-RSRK	RGARCH	GARCH
LRCX	0.25	0.25	0.25	0.25	0.25	0.25	0.25
MA	0.25	0.25	0.25	0.25	0.25	0.25	0.25
MCD	0.25	0.25	0.25	0.25	0.25	0.25	0.25
MCHP	0.25	0.25	0.25	0.25	0.25	0.25	0.25
MELI	0.25	0.25	0.25	0.25	0.25	0.25	0.25
META	0.25	0.25	0.25	0.25	0.25	0.25	0.25
MRK	0.25	0.25	0.25	0.25	0.25	0.25	0.25
MSFT	0.25	0.25	0.25	0.25	0.25	0.25	0.25
MSTR	0.25	0.25	0.25	0.25	0.25	0.25	0.25
MU	0.25	0.25	0.25	0.25	0.25	0.25	0.25
NEE	0.25	0.25	0.25	0.25	0.25	0.25	0.25
NFLX	0.25	0.25	0.25	0.25	0.25	0.25	0.25
NKE	0.25	0.25	0.25	0.25	0.25	0.25	0.25
NOW	0.25	0.25	0.25	0.25	0.25	0.25	0.25
NVDA	0.25	0.25	0.25	0.25	0.25	0.25	0.25
NXPI	0.25	0.25	0.25	0.25	0.25	0.25	0.25
ORCL	0.25	0.25	0.25	0.25	0.25	0.25	0.25
PANW	0.25	0.25	0.25	0.25	0.25	0.25	0.25
PEP	0.25	0.25	0.25	0.25	0.25	0.25	0.25
PFE	0.25	0.25	0.25	0.25	0.25	0.25	0.25
PG	0.25	0.25	0.25	0.25	0.25	0.25	0.25
PYPL	0.25	0.25	0.25	0.25	0.25	0.25	0.25
QCOM	0.25	0.25	0.25	0.25	0.25	0.25	0.25
SBUX	0.25	0.25	0.25	0.25	0.25	0.25	0.25
SHOP	0.25	0.25	0.25	0.25	0.25	0.25	0.25
SMCI	0.25	0.25	0.25	0.25	0.25	0.25	0.25
SO	0.25	0.25	0.25	0.25	0.25	0.25	0.25
SPGI	0.25	0.25	0.25	0.25	0.25	0.25	0.70
SYK	0.25	0.25	0.25	0.25	0.25	0.25	0.70
TJX	0.25	0.25	0.25	0.25	0.25	0.25	0.70
TMO	0.25	0.25	0.25	0.25	0.25	0.25	0.70
TMUS	0.25	0.25	0.25	0.25	0.25	0.25	0.25
TSN	0.25	0.25	0.25	0.25	0.25	0.25	0.25
TXN	0.25	0.25	0.25	0.25	0.25	0.25	0.25
UNH	0.25	0.25	0.25	0.25	0.25	0.25	0.25
V	0.25	0.25	0.25	0.70	0.25	0.25	0.25
VRTX	0.25	0.25	0.25	0.70	0.25	0.25	0.25
WDAY	0.25	0.25	0.70	0.25	0.25	0.25	0.25
WFC	0.25	0.25	0.70	0.25	0.25	0.25	0.25
WMT	0.25	0.25	0.25	0.25	0.25	0.25	0.25
XOM	0.25	0.25	0.25	0.25	0.25	0.25	0.25

Table 35: This table shows the p-values of the Kupiec's test on 0.99 VaR computed using rolling forecast values of all 7 models for the second half of stocks.

	Mean	SD	p-val < 0.05
AR(1)-RV expanding	0.33	0.27	0.25
AR(1)-RV rolling	0.25	0.23	0.26
HAR expanding	0.62	0.30	0.02
HAR rolling	0.34	0.26	0.17
HAR-AS expanding	0.47	0.28	0.02
HAR-AS rolling	0.36	0.25	0.10
HAR-RSV expanding	0.32	0.31	0.26
HAR-RSV rolling	0.39	0.27	0.10
HAR-RSRK expanding	0.20	0.24	0.42
HAR-RSRK rolling	0.35	0.35	0.40
RGARCH expanding	0.16	0.21	0.45
RGARCH rolling	0.27	0.27	0.40
GARCH expanding	0.42	0.31	0.22
GARCH rolling	0.32	0.29	0.26

Table 36: This table shows the summary statistics of the p-values of the Kupiec's test on 0.9 VaR. The first column shows the mean of p-values, the second column the standard deviation and the third column shows in how many cases the p-value was lower than 0.05, i. e. in how many cases the VaR computation was unsuccesful.

	Mean	SD	p-val < 0.05
AR(1)-RV expanding	0.37	0.34	0.16
AR(1)-RV rolling	0.41	0.35	0.29
HAR expanding	0.40	0.30	0.15
HAR rolling	0.37	0.27	0.12
HAR-AS expanding	0.40	0.25	0.02
HAR-AS rolling	0.31	0.32	0.20
HAR-RSV expanding	0.22	0.20	0.22
HAR-RSV rolling	0.24	0.18	0.12
HAR-RSRK expanding	0.17	0.23	0.50
HAR-RSRK rolling	0.18	0.21	0.30
RGARCH expanding	0.20	0.17	0.22
RGARCH rolling	0.24	0.23	0.32
GARCH expanding	0.33	0.30	0.22
GARCH rolling	0.24	0.20	0.24

Table 37: This table shows the summary statistics of the p-values of the Kupiec's test on 0.95 VaR. The first column shows the mean of p-values, the second column the standard deviation and the third column shows in how many cases the p-value was lower than 0.05, i. e. in how many cases the VaR computation was unsuccesful.

	Mean	SD	p-val < 0.05
AR(1)-RV expanding	0.27	0.10	0.00
AR(1)-RV rolling	0.25	0.05	0.00
HAR expanding	0.25	0.00	0.00
HAR rolling	0.34	0.18	0.00
HAR-AS expanding	0.27	0.10	0.00
HAR-AS rolling	0.26	0.07	0.00
HAR-RSV expanding	0.26	0.07	0.00
HAR-RSV rolling	0.25	0.00	0.00
HAR-RSRK expanding	0.25	0.00	0.00
HAR-RSRK rolling	0.29	0.13	0.00
RGARCH expanding	0.27	0.10	0.00
RGARCH rolling	0.26	0.07	0.00
GARCH expanding	0.27	0.10	0.00
GARCH rolling	0.27	0.13	0.05

Table 38: This table shows the summary statistics of the p-values of the Kupiec's test on 0.99 VaR. The first column shows the mean of p-values, the second column the standard deviation and the third column shows in how many cases the p-value was lower than 0.05, i. e. in how many cases the VaR computation was unsuccesful.