Segment Trees

Diego Perez-Torres

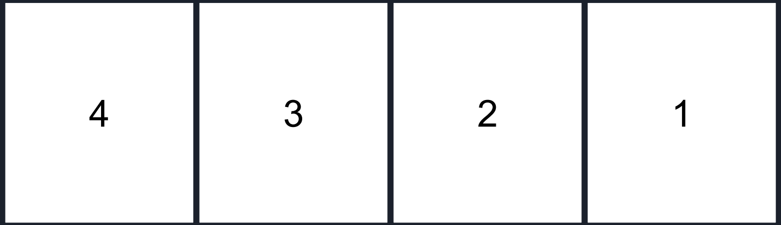
What is a Segment Tree?

A data structure used to answer range queries over an array effectively, while also being flexible enough to modify it if need be.

Queries using the entire segment tree require O(logn) time.

Some simple uses are finding the sum, minimum and maximum of the array of values.

But why use a Segment tree?

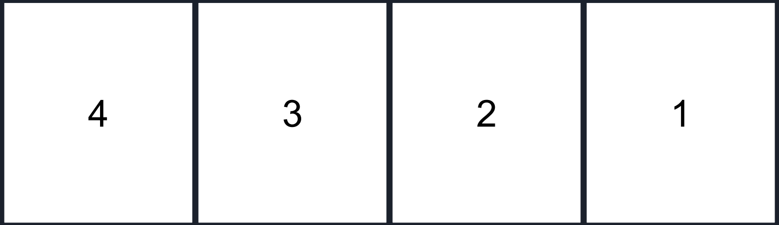


Sum?

Minimum?

Maximum?

Simple so far, no?



Sum:	10
Minimum:	1
Maximum:	4

What about this array of values?

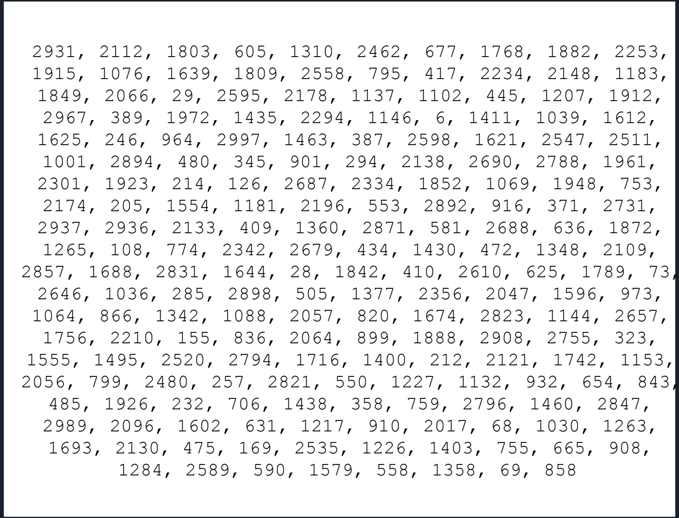
```
 2931, 2112, 1803, 605, 1310, 2462, 677, 1768, 1882, 2253,
 1915, 1076, 1639, 1809, 2558, 795, 417, 2234, 2148, 1183,
  1849, 2066, 29, 2595, 2178, 1137, 1102, 445, 1207, 1912,
   2967, 389, 1972, 1435, 2294, 1146, 6, 1411, 1039, 1612,
  1625, 246, 964, 2997, 1463, 387, 2598, 1621, 2547, 2511,
   1001, 2894, 480, 345, 901, 294, 2138, 2690, 2788, 1961,
  2301, 1923, 214, 126, 2687, 2334, 1852, 1069, 1948, 753,
   2174, 205, 1554, 1181, 2196, 553, 2892, 916, 371, 2731,
  2937, 2936, 2133, 409, 1360, 2871, 581, 2688, 636, 1872,
   1265, 108, 774, 2342, 2679, 434, 1430, 472, 1348, 2109,
2857, 1688, 2831, 1644, 28, 1842, 410, 2610, 625, 1789, 73,
  2646, 1036, 285, 2898, 505, 1377, 2356, 2047, 1596, 973,
 1064, 866, 1342, 1088, 2057, 820, 1674, 2823, 1144, 2657,
   1756, 2210, 155, 836, 2064, 899, 1888, 2908, 2755, 323,
 1555, 1495, 2520, 2794, 1716, 1400, 212, 2121, 1742, 1153,
2056, 799, 2480, 257, 2821, 550, 1227, 1132, 932, 654, 843,
    485, 1926, 232, 706, 1438, 358, 759, 2796, 1460, 2847,
   2989, 2096, 1602, 631, 1217, 910, 2017, 68, 1030, 1263,
    1693, 2130, 475, 169, 2535, 1226, 1403, 755, 665, 908,
          1284, 2589, 590, 1579, 558, 1358, 69, 858
```

Sum?

Minimum?

Maximum?

Tedious labor, even for a machine.

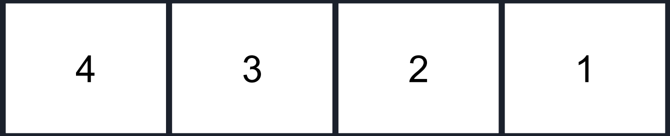


```
 2931, 2112, 1803, 605, 1310, 2462, 677, 1768, 1882, 2253,
 1915, 1076, 1639, 1809, 2558, 795, 417, 2234, 2148, 1183,
  1849, 2066, 29, 2595, 2178, 1137, 1102, 445, 1207, 1912,
   2967, 389, 1972, 1435, 2294, 1146, 6, 1411, 1039, 1612,
  1625, 246, 964, 2997, 1463, 387, 2598, 1621, 2547, 2511,
   1001, 2894, 480, 345, 901, 294, 2138, 2690, 2788, 1961,
  2301, 1923, 214, 126, 2687, 2334, 1852, 1069, 1948, 753,
   2174, 205, 1554, 1181, 2196, 553, 2892, 916, 371, 2731,
  2937, 2936, 2133, 409, 1360, 2871, 581, 2688, 636, 1872,
   1265, 108, 774, 2342, 2679, 434, 1430, 472, 1348, 2109,
2857, 1688, 2831, 1644, 28, 1842, 410, 2610, 625, 1789, 73,
  2646, 1036, 285, 2898, 505, 1377, 2356, 2047, 1596, 973,
 1064, 866, 1342, 1088, 2057, 820, 1674, 2823, 1144, 2657,
   1756, 2210, 155, 836, 2064, 899, 1888, 2908, 2755, 323,
 1555, 1495, 2520, 2794, 1716, 1400, 212, 2121, 1742, 1153,
2056, 799, 2480, 257, 2821, 550, 1227, 1132, 932, 654, 843,
    485, 1926, 232, 706, 1438, 358, 759, 2796, 1460, 2847,
   2989, 2096, 1602, 631, 1217, 910, 2017, 68, 1030, 1263,
    1693, 2130, 475, 169, 2535, 1226, 1403, 755, 665, 908,
           1284, 2589, 590, 1579, 558, 1358, 69, 858
```

Sum:	297,761
Minimum:	6
Maximum:	2,997

Underlying structure

In this implementation of a Segment Tree, I’m using a 1D vector to mimic a node-tree.

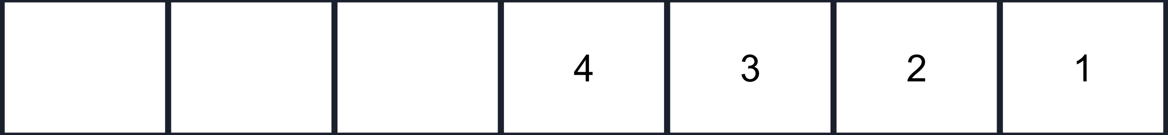


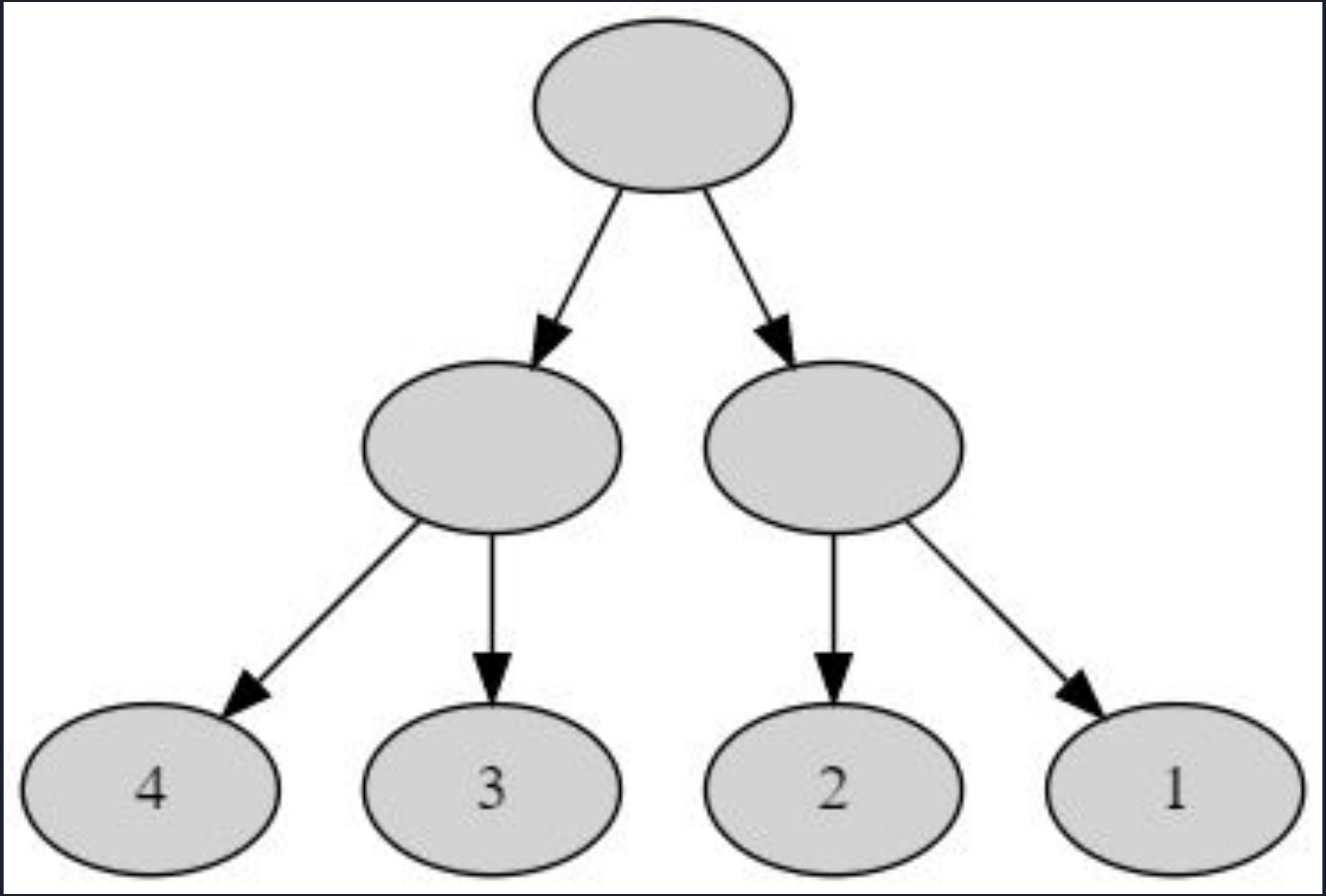
But how does it work?

Creating a Segment Tree

Pseudo Code:

```
Create an object that takes in the vector of data and a string to determine if you are looking for either the sum, min or max.
    Initialize n = size of the vector and create a copy of vector for later modification if need be
    Create a second vector the size of 2n-1 filled from the end to front with the values of the first vector
    The first n-1 values should always be empty or 0
```



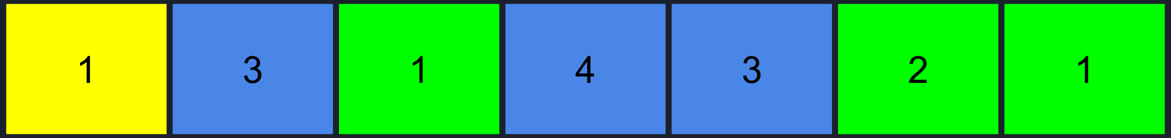

4
3
2
1

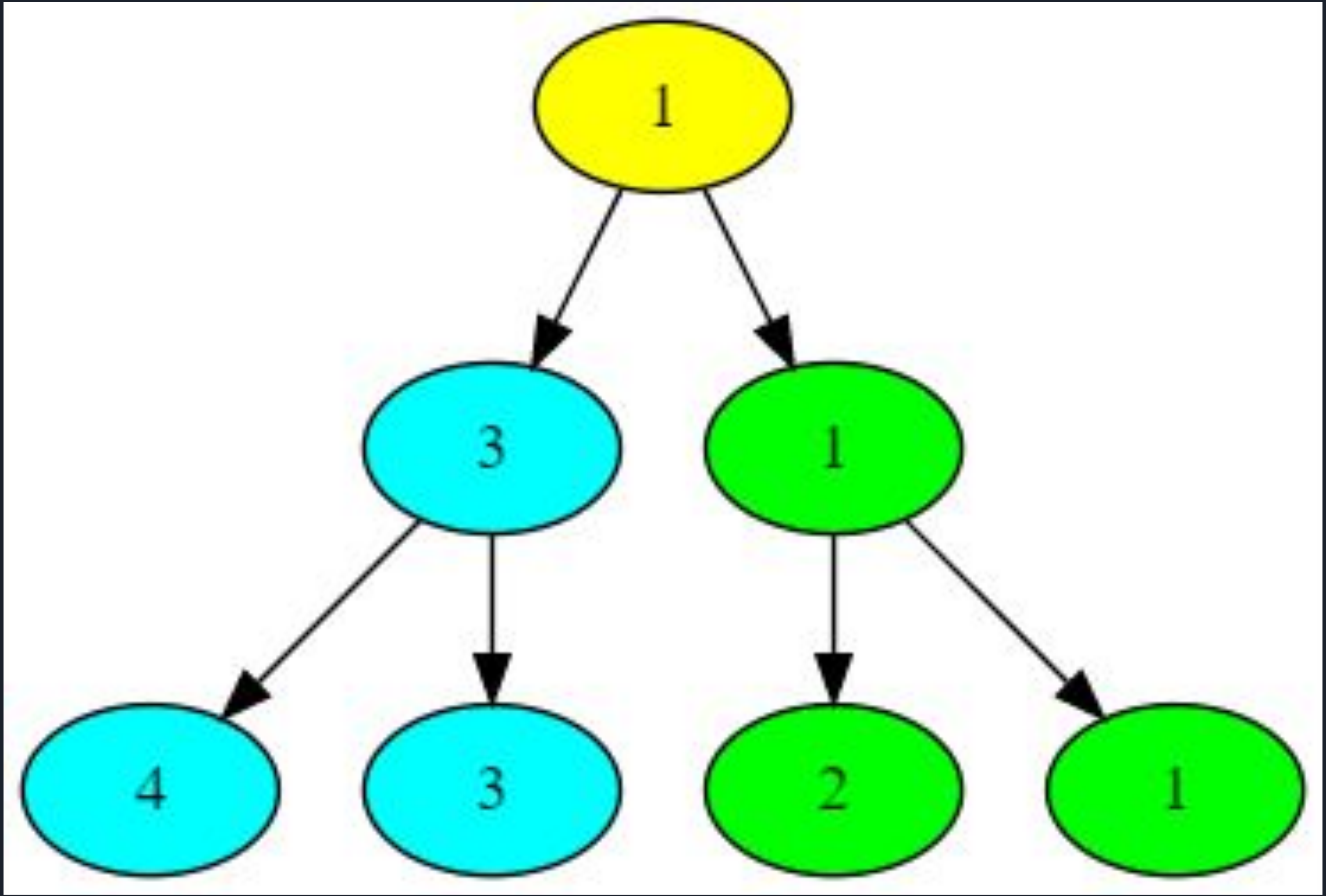
Filling in the rest of the Segment Tree

Check what the string is, and then calculate the min, max or sum using a for loop

For loop loops through the second vector, initializing index = n-1 and stopping when index > 0

Calculates the min, max, or sum by comparing/adding the values at index n*2 and n*2 +1



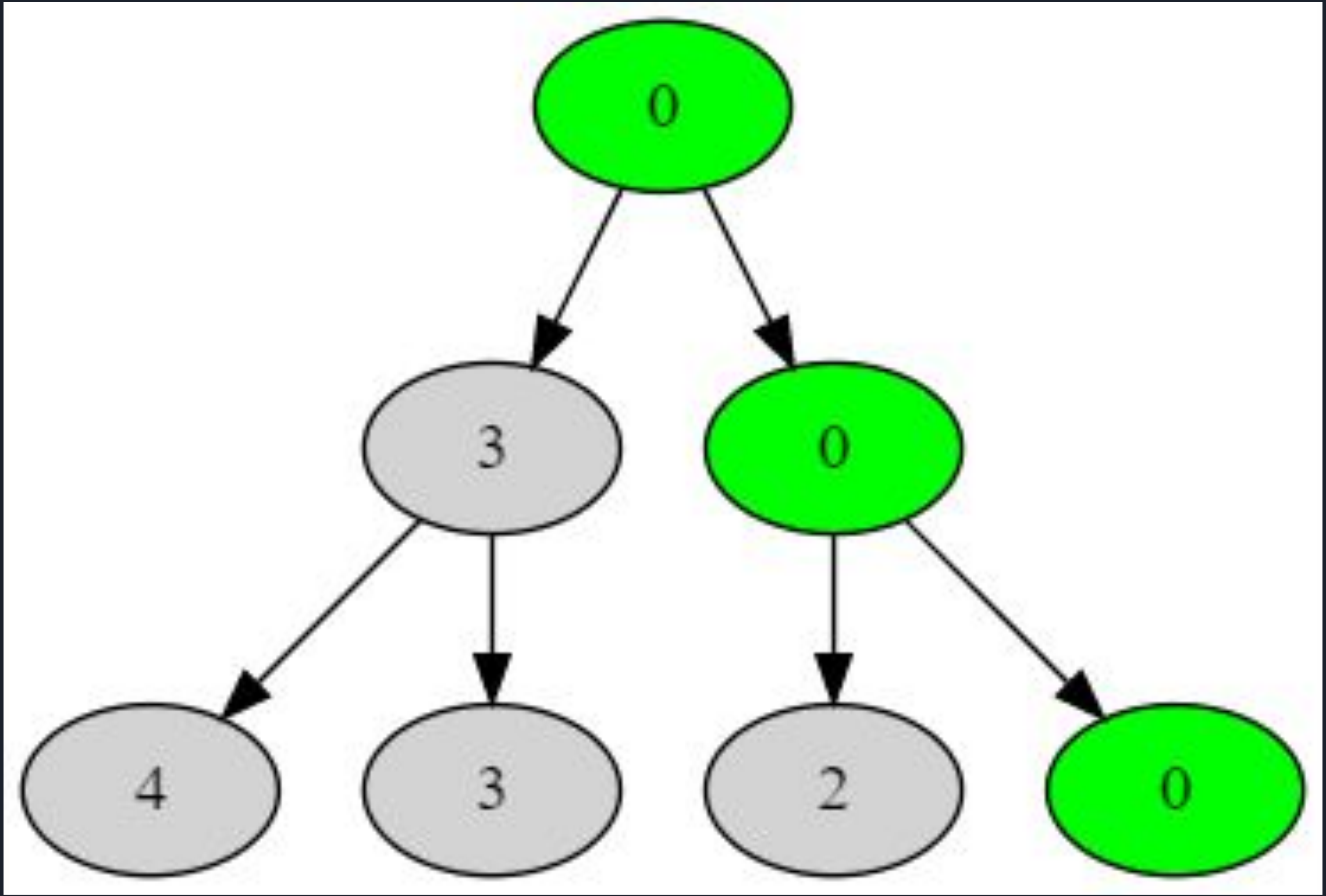

1
3
1
4
3
2
1

Updating the values in the vector

Pseudo Code:

```
Create a function that takes in the index of which value you want to update, and the value itself
    Update the value at the index of the original vector
    Follow the same steps as when creating the completed segment tree with the updated value
```

Let’s say I want to update the value at index 3, replacing the value with 0.

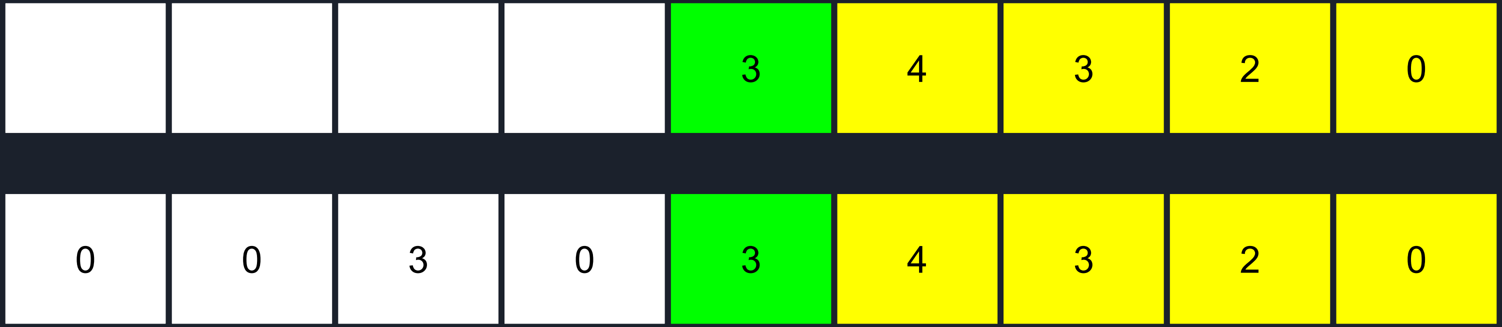

0
3
0
4
3
2
0

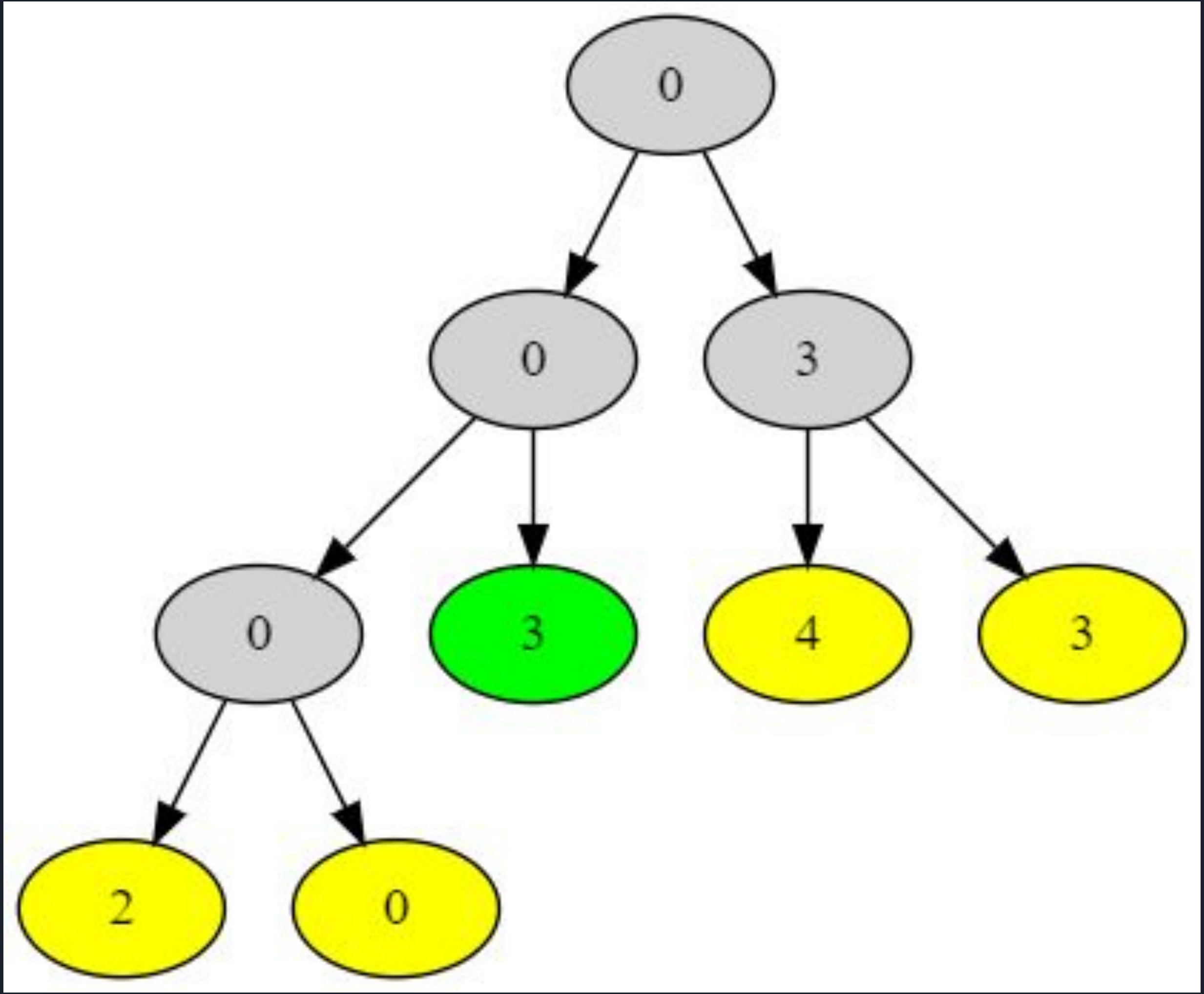
Inserting a value to the vector

Pseudo Code:

```
Create a function that takes in the index of which value you want to insert, and the value itself
    Insert the value into the original vector, pushing all the values after it to the metaphorical left
    Update the value of n with the size of the newly modified vector
    Follow the same steps as when creating the completed segment tree with the updated value
```

Let’s say I want to insert the value 3 at index 0.


3 4 3 2 0
0 0 3 0 3 4 3 2 0


0
0
3
0
3
4
3
2
0

Searching for a value in the Vector

Pseudo Code:

```
Create a function that takes in the value you are looking for
    Create an empty vector that will store in the indexes in which the value is found
    Use a for loop that checks the entire vector for the value, and then stores into into the
    new vector
    If the vector is empty, print out an error message
    Else, use another for loop to loop through the new vector, printing out where it was found
        Print out how many times it was found
```

Output should look like this:

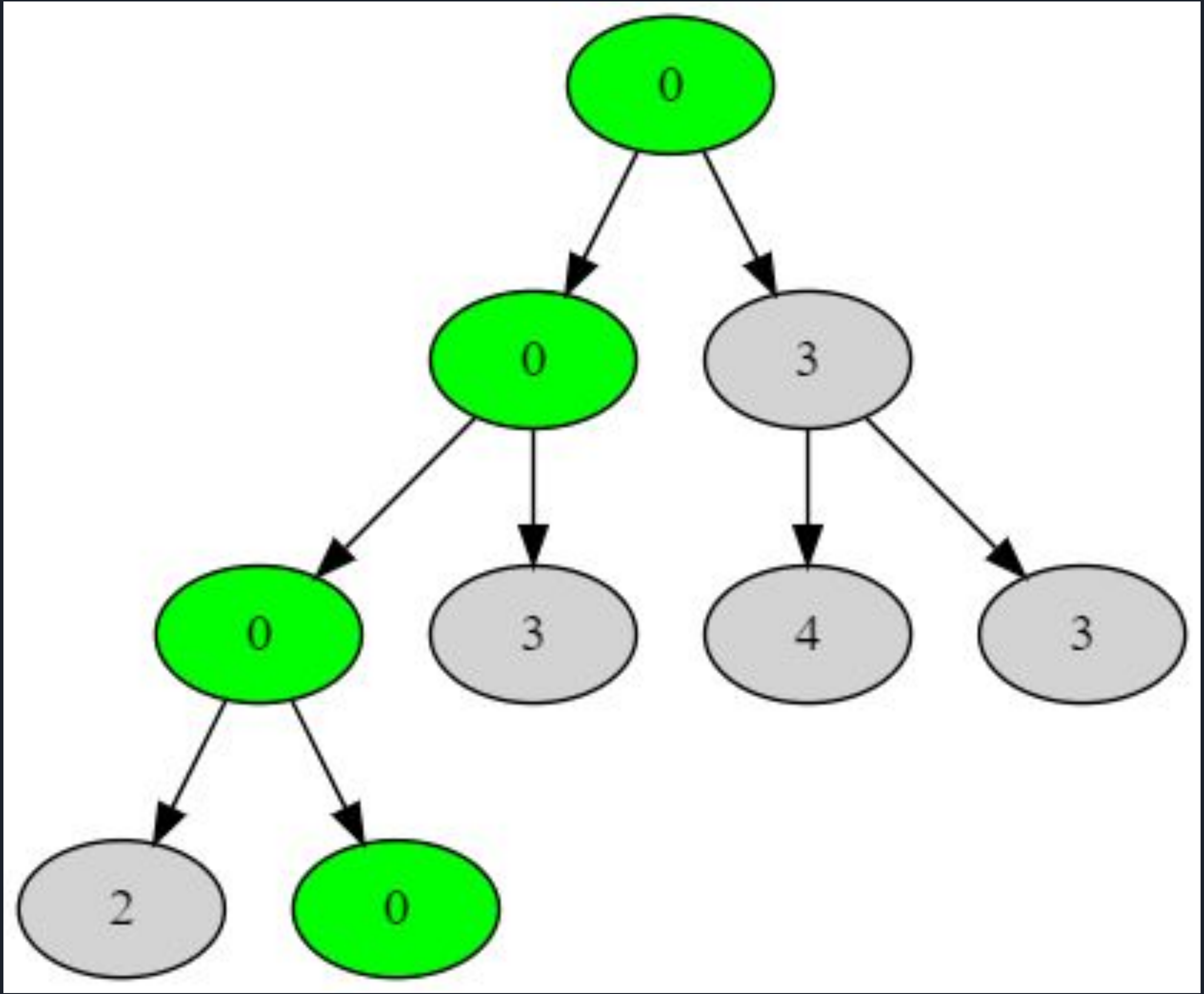
```
0 was found at: 1

0 was found at: 2

0 was found at: 4

0 was found at: 9

Found 4 time(s).
```

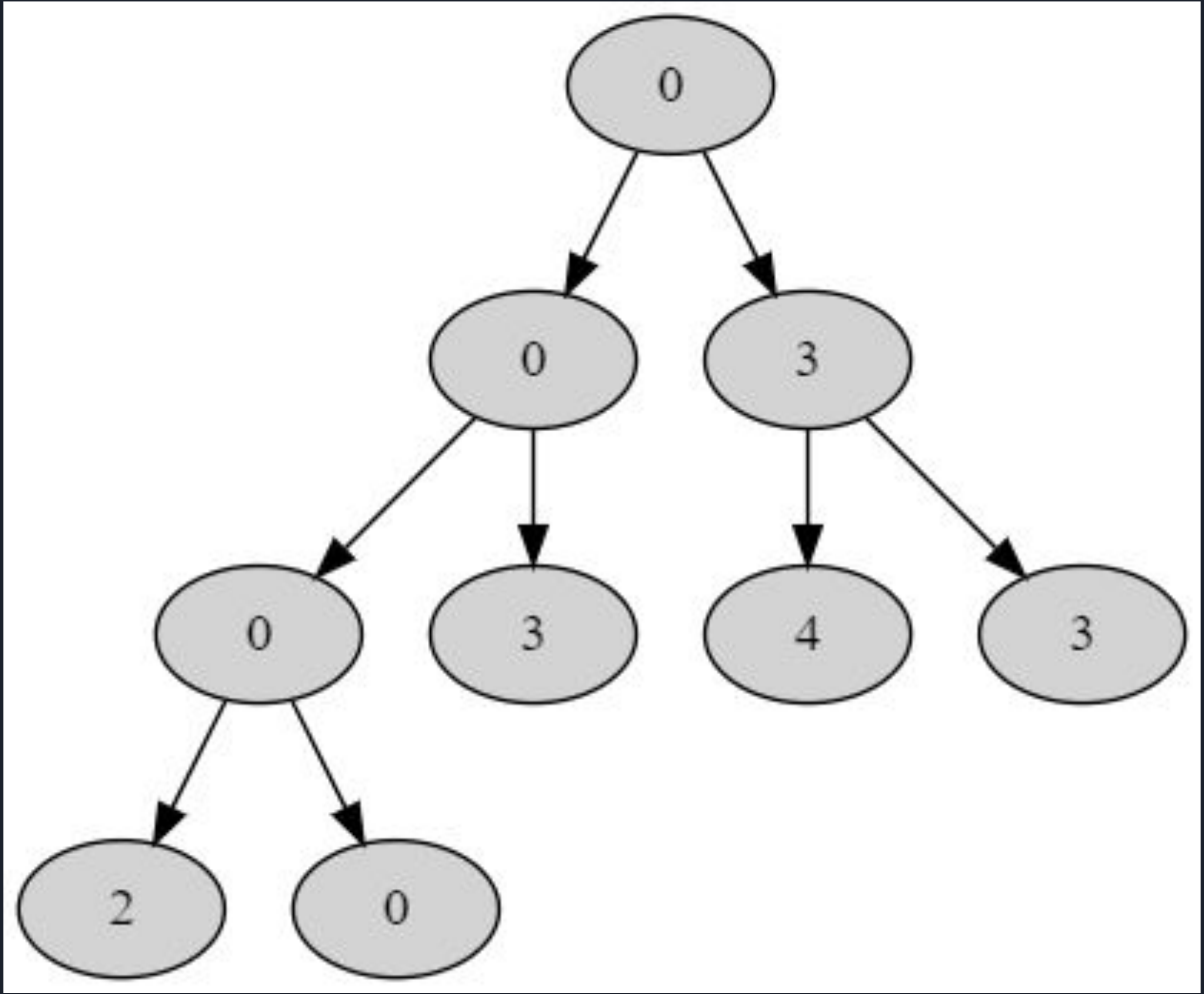

0
0
3
0
3
4
3
2
0

Viewing the entire Segment Tree

Pseudo Code:

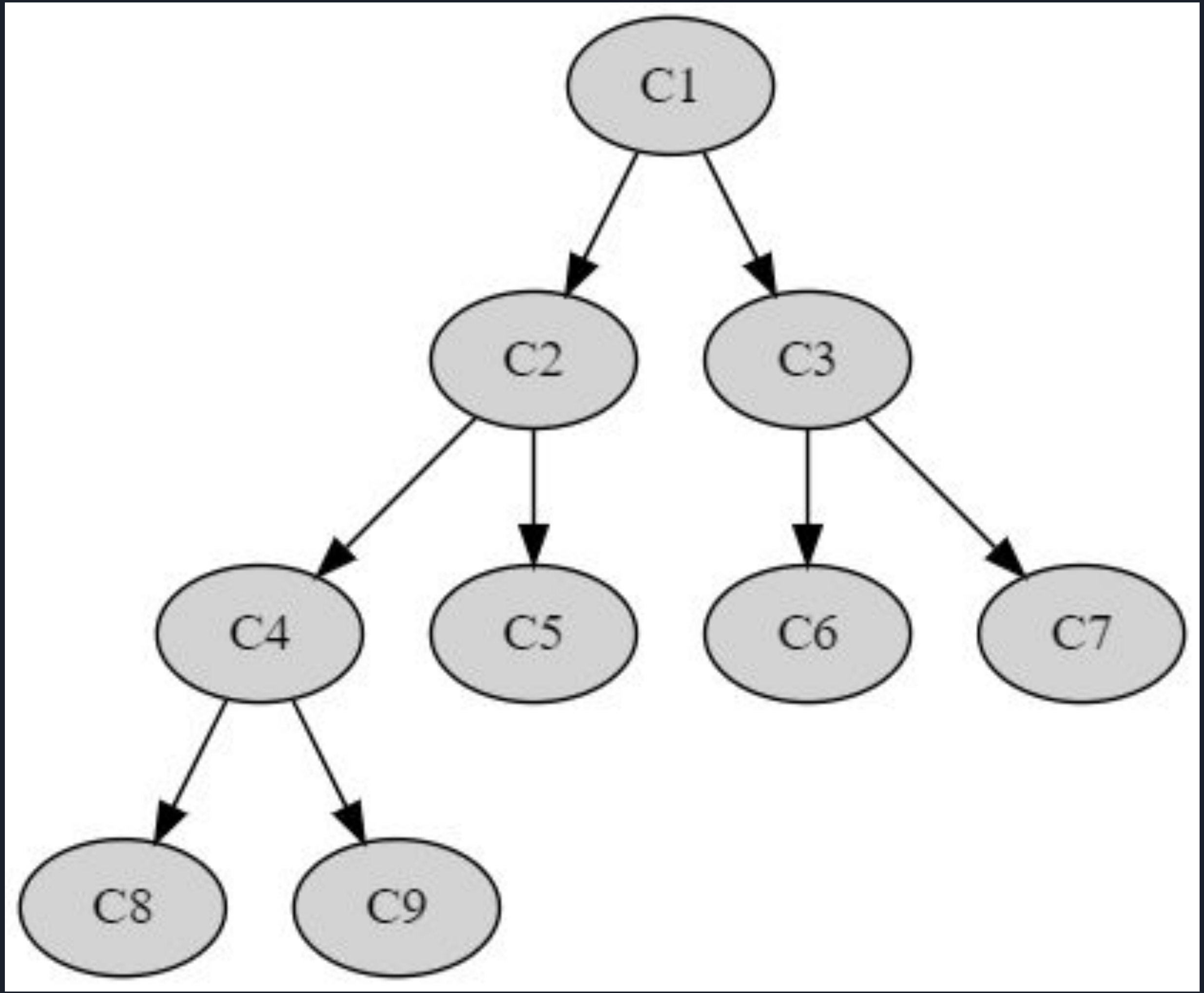
```
Create a function that takes in a string that can be converted into a txt file, ex: “output.txt”
    Initialize the variable “current” to 2
    Use ofstream to input the current index with a label of its stored value in a for loop
        Print out the current index and its value
        If current < n:
        input current index with two pointers to current and current +1
        Current += 2
```

I use https://dreampuf.github.io/GraphvizOnline/ to view my output



```
C1: 0
C2: 0
C3: 3
C4: 0
C5: 3
C6: 4
C7: 3
C8: 2
C9: 0
```

```
 C1 [label=0]
 C1 -> {C2, C3}
 C2 [label=0]
 C2 -> {C4, C5}
 C3 [label=3]
 C3 -> {C6, C7}
 C4 [label=0]
 C4 -> {C8, C9}
 C5 [label=3]
 C6 [label=4]
 C7 [label=3]
 C8 [label=2]
 C9 [label=0]
```



2^0

2^1

2^2

2^4 or when index = n-1

Sources:

https://en.wikipedia.org/wiki/Segment_tree

https://cp-algorithms.com/data_structures/segment_tree.html

https://www.youtube.com/watch?v=Oq2E2yGadnU&t=2s

https://www.geeksforgeeks.org/segment-tree-efficient-implementation/?ref=lbp