Model Checking and Strategy Synthesis for Multi-Agent Systems for Resource Allocation – Proofs

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1 Theorems

Theorem 1 (Model Checking). *Let $[M, s \models_k \langle\langle A, \Sigma\rangle\rangle\, \varphi]$ be a strategic bounded model checking problem and let $[M, \langle\langle A, \Sigma\rangle\rangle\, \varphi, k]$ be its encoding over Vars. Then:*

$$[M, s \models_k \langle\langle A, \Sigma\rangle\rangle\, \varphi] \;\equiv\; \mathbf{sat}\big([M, \langle\langle A, \Sigma\rangle\rangle\, \varphi, k]\big)$$

Lemma 1. *Bla*

Theorem 2 (Strategy Synthesis). *Let $[M, s \models_k \langle\langle A, \Sigma\rangle\rangle\, \varphi]$ be a strategic bounded model checking problem, let $[M, \langle\langle A, \Sigma\rangle\rangle\, \varphi, k]$ be its encoding over Vars and let $\alpha : Vars \to \{\mathbf{0}, \mathbf{1}\}$ with $\alpha\big([M, \langle\langle A, \Sigma\rangle\rangle\, \varphi, k]\big) = \mathbf{1}$. Then for the strategy*

$$\alpha_A = \big(\{(s_a, act^a) \mid s_a \in S_a \wedge act^a \in Act \wedge \alpha([s_a.act^a]) = \mathbf{1}\}_{a \in A}\big)$$

the following holds: $\forall\, \pi \in \Pi(s, \alpha_A, \Sigma) : [M, \pi \models_k \varphi]$.

Thus, from a truth assignment α that satisfies the encoding we can directly derive a corresponding uniform strategy α_A that guarantees φ. The correctness of Theorem 1 and Theorem 2 is closely linked.

Proof Sketch.
It can be shown that every satisfying truth assignment of $[M, k]$ characterises a k-bounded path in the state space of M that is conform with the evolution. Yet, such a path may not be conform with the protocol. $[M, k] \wedge [\varphi, k]$ is satisfied for assignments that characterise paths of M for which the property φ holds. The conjunction of this encoding with $[M, \langle\langle A\rangle\rangle\, , k]$ adds the constraint that the agents in A must follow a uniform strategy that is conform with the protocol. Assuming that β is a protocol-conform strategy for the opposition $B = Agt \backslash A$ and by adding $[\beta, k]$ to the encoding we restrict the paths to those where the opposition adheres to β. This can be generalised to having a set Σ of possible strategies for B. We finally get that the overall propositional formula is satisfiable if and only if the encoded model checking problem holds. Moreover, the strategic decision encodings $[s_a.act^a]$ that evaluate to *true* for a satisfying assignment α are exactly those that characterise the winning strategy for the coalition A. □

References