Assignment - 2

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① $f(x) = Kx^2(1-x^3)$, $0 \le x \le 1$

$$\int_0^1 f(x)dx = 1$$

$$\int_0^1 Kx^2(1-x^3)dx = 1$$

$$K\left[\frac{x^3}{3} - \frac{x^6}{6}\right]_0^1 = 1$$

$$K\left[\frac{1}{6}\right] = 1 \quad \boxed{K = 6}$$

Here, $f(x) = 6x^2(1-x^3)$

Mean $E(X) = \int_0^1 xf(x)dx = 6\int_0^1 x^3(1-x^3)dx$

$$= 6\left[\frac{x^4}{4} - \frac{x^7}{7}\right]_0^1 \Rightarrow 6 \times \frac{3}{28} = \frac{9}{14}$$

Variance $E(X^2) = \int_0^1 x^2 f(x)dx =$

$$= 6\int_0^1 x^4(1-x^3)dx$$

$$\Rightarrow 6\left[\frac{x^5}{5} - \frac{x^8}{8}\right]_0^1 \Rightarrow 6 \times \frac{3}{40} \Rightarrow \frac{9}{20}$$

$$Var(X) = E(X^2) - [E(X)]^2$$

$$= \frac{9}{20} - \left(\frac{9}{14}\right)^2 \Rightarrow \frac{9}{245}$$

② $f(x) = \begin{cases} \frac{3(x+1)}{4} & , 0 \le x \le 1 \\ 0 & , \text{otherwise} \end{cases}$

A function $f(x)$ is PDF if,

→ $f(x) \geq 0$ for all x

→ $\int f(x)\,dx = 1$

So, $f(x)$ $0 \leq x \leq 1$ $f(x) = \frac{3(x+1)}{4} > 0$ ④

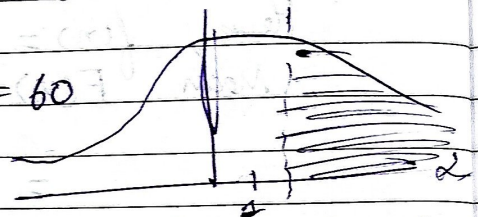
$$\int_0^1 \frac{3(x+1)}{4}\,dx = \frac{3}{4}\int_0^1 (x+1)\,dx \Rightarrow \frac{3}{4}\left[\frac{x^2}{2}+x\right]_0^1$$

$$= \frac{3}{4} \times \frac{3}{2} \Rightarrow 1.125 \neq 1$$

Hence it is not a valid PDF

③ $\mu = 2040$, $\sigma = 60$

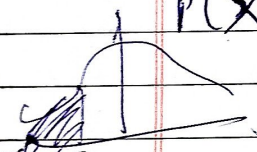
$$z = \frac{x-\mu}{\sigma}$$



(i) $x = 2150 \rightarrow z = \frac{2150 - 2040}{60} \Rightarrow 1.83$

$P(X > 2150) = P(z > 1.83)$

$= 1 - 0.9664 \rightarrow 0.833$



(ii) $x = 1950 \rightarrow z = \frac{1950 - 2040}{60} \Rightarrow -1.5$

$P(X < 1950) = P(z < -1.5) \Rightarrow 0.668$

(iii) $1920 < X < 2160$

$z_1 = \frac{1920 - 2040}{60} \Rightarrow -2.0$

$z_2 = \frac{2160 - 2040}{60} \Rightarrow 2.0$

$P(-2 < z < 2) = 0.9545$

Here, ① 3.36 % bulbs last > 2150 hrs
② 6.68 % last < 1950 hrs
③ 95.45 % last b/w 1920 & 2160 hrs

④ Given,
31 % under 45 → $P(z < z_1) = 0.31$
$z_1 = -0.50$

8 % over 64 → $P(z > z_2) = 0.08$
$P(z < z_2) = 0.92$
$z_2 = 1.41$

$z_1 = \frac{45-\mu}{6} = -0.50 \Rightarrow 45 = \mu - 6(0.50)$ — ①

$z_2 = \frac{64-\mu}{6} = 1.41 \rightarrow 64 = \mu + 6(1.41)$ — ②

$\mu = 50$ $\quad \sigma = 9.95$ Ans

⑤ Given, $\mu = 500$, $\sigma = 50$, $n = 1000$

i) Income > 750 $\quad z = \frac{750-500}{50} \Rightarrow 5$

ii) Top 100 persons ⇒ 10 %
$P(z > z_1) = 0.10$
$z = 1.28$

$x = \mu + z\sigma$
$500 + 1.28(50) = 564$ Ans

(6) $f_{x,y}(x,y) = k(x+y)$, $0<x<1$, $0<y<1$

a) $\int_0^1 \int_0^1 k(x+y)\,dx\,dy = 1$

$k\int_0^1 \left[\frac{1}{2}+y\right]dy = 1$

$k\left[\frac{y}{2}+\frac{y^2}{2}\right]_0^1 = 1$

$\boxed{k=1}$

b) $f_{x,y}(x,y) = (x+y)$; $0<x<1$, $0<y<1$

i) $f(x,y) = (x+y) > 0$ } Since $f_{x,y}(x,y)$ is a valid Joint PDF

ii) $\int_0^1 \int_0^1 (x+y)\,dx\,dy = 1$

(7) $f(x,y) = \frac{2(x+y)}{27}$, $x,y \in \{0,1,2\}$

$f_x(x) = \sum_{y=0} f_{x,y}(x,y)$

$f_x(0) = \frac{0+1+2}{27} = \frac{1}{9}$

$f_x(1) = \frac{(2+0)+(2+1)+(2+2)}{27}$

$= \frac{1}{3}$

(7)

195
180
165
150
135
120

$f(x=2) = \frac{(4+0)+(4+1)+(4+2)}{27} = \frac{5}{9}$

$f(x=0) + f(x=1) + f(x=2) = 1$

$\frac{1}{9} + \frac{1}{3} + \frac{5}{9} = 1$

$(1=1)$

①

Year	2015	2016	2017	2018	2019	2020
Production	120	135	150	165	160	175

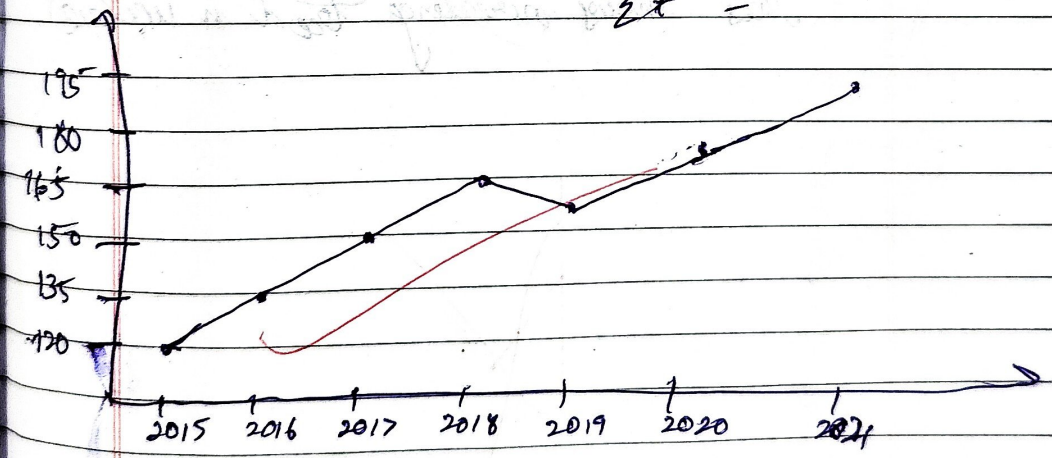
2021
140

$n=7 \quad \Sigma Y = 1095$

Let $t = 1, 2 \ldots$ $\Sigma t = 28$

$\Sigma t^2 = 140$

$\Sigma t =$



This shows an increasing trend

Q7

Month	Sales	3 Month Weighted	3 month moving avg
Jan	40	–	–
Feb	42	–	–
March	45	127	42.33
April	48	135	45
May	50	143	47.66
June	52	150	50
July	53	155	51.66
Aug	55	160	53.33
Sept	54	162	54
Oct	56	165	55
Nov	58	168	56
Dec	60	174	58

This shows increasing trend in value