Economics Letters 193 (2020) 109314

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Chaos control in presence of financial bubbles

Giovanni Bella *, Paolo Mattana

Department of Economics and Business, University of Cagliari Italy, Via S. Ignazio, 17 – 09123 Cagliari, Italy

ARTICLE INFO

Article history:
Received 2 April 2020
Accepted 7 June 2020
Available online 8 June 2020

Keywords:
Shilnikov chaos
Global indeterminacy
Financial bubbles
Liquidity trap

ABSTRACT

The paper shows that in the economy described by Miao and Wang (2015), with financial bubbles emerging in the banking sector, a Shilnikov chaotic attractor giving rise to unpredictable global indeterminacy can appear when the bubble bursts, resulting in a financial crisis. In this case, a suitable policy algorithm to eliminate or control the chaotic dynamics is proposed.

1. Introduction

Within the huge bulk of literature existing on policy implications of chaos in economics, we concentrate here on the Shilnikov (1965) route to chaos mechanism which has been proved to generate a strange attractor spiraling around the homoclinic orbit generated by a three-dimensional saddle focus equilibrium (e.g., Bella et al., 2017). The irregular transitional dynamics generated may consequently undermine the ability of an economy to converge to a long-run stable equilibrium. Instead, it results in a set of multiple (admissible) equilibria, depending on the initial values of the control variables (with infinite saddle limit cycles coexisting at the bifurcation point), which could eventually trap the systems in an undesired low-growth equilibrium.

In this light, it can be of particular interest to reshape the analysis of the economic downturn generated by the recent Great Recession in 2007, as the effect of the burst of a financial bubble in the banking sector. Indeed, the collapse of the financial sector has led the economy to a low-level equilibrium, characterized by less households' deposits in the banking sector, which mean conversely lower lending to producing firms, and therefore lower levels of final output.

In particular, we aim to show the emergence of the aforementioned Shilnikov's chaotic dynamics in the model economy described by Miao and Wang (2015), where financial bubbles are the key mechanism to explain a bank run, and the driving force that may drive the economy towards a low-growth trapping region.

Additionally, we show that the chaotic dynamics generated by the burst of the bubble can be controlled, by applying the algorithm proposed by Ott et al. (1990), henceforth OGY.

* Corresponding author.
E-mail addresses: bella@unica.it (G. Bella), mattana@unica.it (P. Mattana).

https://doi.org/10.1016/j.econlet.2020.109314

The paper proceeds as follows. In Section 2, we present the Miao and Wang (2015) reference model and the implied three-dimensional system of first-order differential equations. We also characterize the parametric conditions at which the model with financial bubbles satisfies the requirements of the Shilnikov (1965) theorem for the emergence of chaotic dynamics. In Section 3, we apply the OGY algorithm for controlling the chaos, and provide a conclusive example.

2. The model

Consider the baseline model described in Miao and Wang (2015), where financial bubbles are faced by a (continuous time) deterministic economy composed of households, non-financial firms and banks. The model assumes that households consume, save and supply labor (normalized to unity). Households own non-financial firms and banks. Workers and bankers are two type of agents belonging to each household. Hence, households deposit their savings in banks that lend thereafter funds to non-financial firms. Financial bubbles arise in the stock market value of the banks.

The standard optimization problem faced by the representative household leads to the following three-dimensional system of differential equations:

$$\dot{Q} = rQ - Q\left[r_k + (r_k - r)\,\xi Q\right] - \theta\,(1 - Q) \tag{1a}$$

$$\dot{B} = rB - Q\,(r_k - r)\,B \tag{1b}$$

$$\dot{N} = \left[r_k - \theta + (r_k - r)\,\xi Q\right] N + (r_k - r)\,B \tag{1c}$$

where Q is the shadow price of bank's net worth N, and B is the bubble component of the stock market value of a bank, given by $QN + B$. Moreover, r represents the deposit rate, whereas r_k is the lending rate. Additionally, $(\xi, \theta) \in (0, 1)^2$ are the degree of financial frictions and the share of banks' dividends, respectively.

Remark 1. If we re-write Eq. (1b) as

$$rB = \dot{B} + Q\,(r_k - r)\,B$$

we obtain a standard asset-pricing equation, where the return on the bubble (rB) is equal to the capital gains ($\dot{B}$) plus the net benefit to the bank (i.e., the dividend) given by $Q\,(r_k - r)\,B$. This means that every additional earning coming from the bubble permits to the bank to attract one additional currency unit in terms of new deposits, and therefore a new unit of more loans to the non-financial firms, thereby increasing the net benefit of the bank, which increases the bubble.

To characterize the solution of system (1.i), we need to resort the result obtained by Miao and Wang (2015) for the full endogenization of the lending rate, r_k.

Definition 1. Following the Assumptions made by Miao and Wang (2015), consider output as given by $Y = K^{\alpha}$, where $\alpha \in (0, 1)$ is the standard share of physical capital in production, and where it is explicitly that $K = N + \xi QN + B$, which defines capital as the sum of Bank's net worth plus the additional net worth coming from financial frictions plus the bubble. Then, the lending rate results as

$$r_k = \alpha\,[(\xi Q + 1)\,N + B]^{\alpha-1} - \delta$$

which is therefore equal to the marginal productivity of capital, $\alpha K^{\alpha-1}$, minus the depreciation rate δ.

Let $P^* \equiv (Q^*, B^*, N^*)$ denote the steady values of (Q, B, N) such that $\dot{Q} = \dot{B} = \dot{N} = 0$. Then, we can prove the following result.

Definition 2. Assume the parametric region defined in Miao and Wang (2015), and calibrate the bubbly stead state by choosing $(\alpha, \delta, r, \theta, \xi) = (0.33, 0.1, 0.015, 0.055, 0.5556)$. Then, the steady state $P^* \equiv (1.5, 2.36, 1.09)$ has one stable eigenvalue (-0.1097) and two complex-conjugate eigenvalues with positive real part $(0.0315 \pm 0, 0126i)$. This implies that the equilibrium is a locally unique saddle-point.

But even if it is proved that P^* is unique, and hence the equilibrium is locally determinate, this information is not enough to exclude the emergence of odd dynamics in the large. The application of the Shilnikov (1965) theorem is used to this end

Proposition 1. *Consider the dynamic system*

$$\frac{dx}{dt} = f(x, \nu), \;\; x \in \mathbb{R}^3, \;\; \nu \in \mathbb{R}^1$$

with f sufficiently smooth. Assume f has a hyperbolic saddle-focus equilibrium point, $x_0 = 0$, at $\nu = 0$, implying that eigenvalues of the Jacobian, $J = Df$, are of the form γ and $\tau \pm \omega i$, where γ, τ, and ω are real constants with $\gamma\tau < 0$. Assume that the following conditions also hold:

(1) *The saddle quantity, $\sigma \equiv |\gamma| - |\tau| > 0$;*

(2) *There exists a homoclinic orbit, Γ_0, based at x_0.*

Then the Shilnikov map, defined in the neighborhood of the homoclinic orbit, exhibits horseshoes chaos.

Proof. Condition (1) can be easily verified, because, within the parametric region calibrated in the Miao and Wang (2015) economy, $\sigma = 0.0782 > 0$. Moreover, following the algorithm implemented in Bella et al. (2017), we can determine the existence of a homoclinic loop doubly asymptotic to the saddle-focus in $\mathbb{R}^3$, when parameters satisfy the following condition

$$\frac{\vartheta_1\varphi^2}{\gamma} + \varphi - (\gamma - 2\tau)\frac{\vartheta_2\phi\psi + \vartheta_3\phi^2 + \vartheta_4\psi^2}{(2\tau-\gamma)^2 + 4\omega^2} = 0$$

where $(\varphi, \phi, \psi) \in (0, 1)^3$ are free constants, and the ϑ_i's are coefficients of the second-order nonlinear terms resulting from the Taylor expansion of the vector field in (1.*i*). ■

The following statement is therefore implied.

Lemma 1. *Assume that the parametric conditions in Definition 1 and 2 are satisfied. Then, given a triplet of initial conditions sufficiently close to the origin, system (1.i) admits perfect-foresight chaotic dynamics, and the equilibrium is globally indeterminate.*

Proof. Following Freire et al. (2002), we can put system (1.*i*) in its hypernormal form, and then obtain the following versal deformation

$$\begin{pmatrix}\dot{w}_1\\ \dot{w}_2\\ \dot{w}_3\end{pmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ \varepsilon_1 & \varepsilon_2 & \varepsilon_3\end{bmatrix}\begin{pmatrix}w_1\\ w_2\\ w_3\end{pmatrix} + \begin{pmatrix}0\\ 0\\ dw_1^2 + kw_1^3\end{pmatrix} \tag{2}$$

where (w_1, w_2, w_3) is the new set of coordinates arising from the near-identity transformation; $\varepsilon_1 = \mathbf{Det}\,(J)$, $\varepsilon_2 = -\mathbf{B}(J)$, $\varepsilon_3 = \mathbf{Tr}(J)$; and d and k are combinations of the coefficients of the non-linear terms (see, Bella et al., 2017, for a detailed computation of these results). ■

3. The chaos control algorithm

What tools should a policy-maker adopt for controlling the chaotic dynamics fueled by the burst of the financial bubble and driving the economy back to the intended stable steady state? To answer this question, we propose here the method proposed by Ott et al. (1990), henceforth OGY. This algorithm needs the perturbation of a control parameter, in our case we take the deposit rate (r), to let that system (1.*i*) may finally exhibit a structure of eigenvalues with all negative real parts.

First, we need to put the linear part of system (2) in the form

$$\dot{w} = Jw + MKw \tag{3}$$

where $M = \left(\frac{\partial \dot{w}_1}{\partial r}, \frac{\partial \dot{w}_2}{\partial r}, \frac{\partial \dot{w}_3}{\partial r}\right)^T$, and $K = (k_1, k_2, k_3)$ is a (1×3) vector. System (3) now needs to be put into its first-companion form,

$$\dot{\omega} = (A - CK)\omega. \tag{4}$$

where $\omega = (\omega_1, \omega_2, \omega_3)^T$ from the transformation $w = T\omega$, and $A = T^{-1}JT$ is given by

$$A = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ \hat{\varepsilon}_1 & \hat{\varepsilon}_2 & \hat{\varepsilon}_3\end{bmatrix} \tag{5}$$

where now $\hat{\varepsilon}_1 = \mathbf{Det}\,(\mathrm{A})$, $\hat{\varepsilon}_2 = -\mathbf{B}(\mathrm{A})$, $\hat{\varepsilon}_3 = \mathbf{Tr}(\mathrm{A})$ are the values that permit three eigenvalues with negative real parts. Moreover, $C = T^{-1}M$. In detail, the transformation matrix T has to be chosen to satisfy the product $T = PW$, with $P = \left[C, JC, J^2C\right]$, and

$$W = \begin{bmatrix}\hat{\varepsilon}_2 & \hat{\varepsilon}_3 & 1\\ \hat{\varepsilon}_3 & 1 & 0\\ 1 & 0 & 0\end{bmatrix} \tag{6}$$

Controllability requires that matrix P has full rank.

We are now ready to prove our algorithm within the set of the parameter space given in Definition 2. If we choose $r \in (0.0405, 0.05)$, the eigenvalues become one real and two complex-conjugate but all with a negative real part $(-0.01093749001, -0.004185763496 \pm 0.06412911656i)$. Therefore, the steady state P^* exhibits a fully stable dynamic behavior, and controllability of system (1.*i*) by means of changes of the parameter r is feasible.

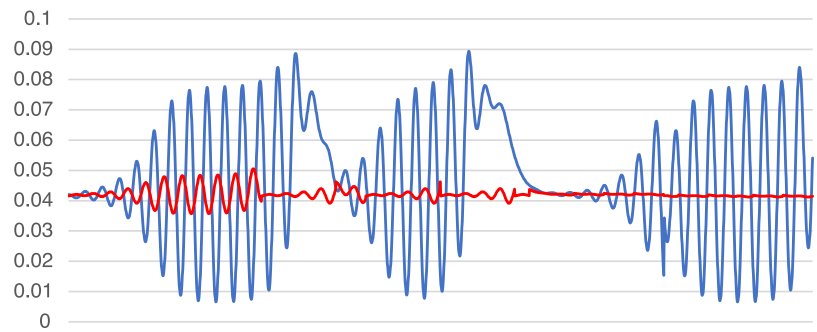

Fig. 1. Uncontrolled and controlled bubble . (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The two different scenarios are reported in Fig. 1, where the evolutionary time path of the bubble, B, is reported in presence of a chaotic dynamics (the blue curve), when $r = 0.015$ as in Miao and Wang (2015), and in the case when we apply the OGY algorithm (red curve), when $r = 0.045$.

As we can easily notice, once the controllability of the systems is implemented, the bubble does not burst, and the variable converges without spikes to the long run steady state solution.

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