Lista1

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4. Prove por definição as seguintes afirmações:

a. $n^2 = O(n^3)$

$$n^2 \leq c * n^3$$

$$1 \leq c * n$$

$c = 1 \mid n > 1$

b. $2 * n^2 + 1 = O(n^2)$

$$2 * n^2 + 1 \leq c * n^2$$

$$2 * n^2 - c * n^2 \leq -1$$

$$n^2 * (2 - c) \leq -1$$

$c > 2 \mid n > 0$

c. $n^2 + 3 * n + 7 = \Omega(6 * n + 7)$

$$n^2 + 3 * n + 7 \geq c * (6 * n + 7)$$

$$n^2 + 3 * n + 7 \geq 6 * c * n + 7 * c$$

$$n^2 + 3 * n - 6 * c * n - 7 * c \geq -7$$

$$n(n + 3 - 6 * c) - 7 * c \geq -7$$

$$c = \frac{1}{2}; n \geq 2$$

d. $1000 * n = o(\frac{n^2}{1000})$

$$1000 * n < c * (\frac{n^2}{1000})$$

$$1000 < \frac{c * n}{1000}$$

$$1000 * 1000 < c * n$$

$$10^6 < c * n$$

$$\frac{10^6}{c} < n$$

e. $\frac{1}{2} * n * (n + 1) = \Theta(n^2)$

$$c1 * n^2 \leq \frac{1}{2} * n * (n + 1)$$

$$c1 * n^2 \leq \frac{1}{2} * n^2 + \frac{1}{2} * n$$

$$c1 * n \leq \frac{1}{2} * n + \frac{1}{2}$$

$$c1 * n - \frac{1}{2} * n \leq \frac{1}{2}$$

$$c1 = \frac{1}{4}; n = 1$$

$$\frac{1}{2} * n * (n + 1) \leq c2 * n^2$$

$$\frac{1}{2} * n^2 + \frac{1}{2} * n \leq c2 * n^2$$

$$\frac{1}{2} * n + \frac{1}{2} \leq c2 * n$$

$$\frac{1}{2} \leq c2 * n - \frac{1}{2} * n$$

$$c2 = 1; n > 1$$

f. $\log^2(n) = \Omega(\log(n^2))$

$$\log^2(n) \geq c * \log(n^2)$$

$$\log(n) * \log(n) \geq 2 * c * \log(n)$$

$$\log(n) \geq 2 * c$$

$c = 1 \mid n \geq 4$

g. $10 * n^2 + 12 * n + 6 = \Theta(2 * n^2 - n)$

$$10 * n^2 + 12 * n + 6 \leq c1 * (2 * n^2 - n)$$

$$10 * n^2 + 12 * n + 6 \leq 2 * c1 * n^2 - c1 * n$$

$$10 * n^2 - 2 * c1 * n^2 + 12 * n + c1 * n \leq -6$$

$$n * (10 * n - 2 * c1 * n + 12 + c1) \leq -6$$

$$c1 * (2 * n^2 - n) \leq 10 * n^2 + 12 * n + 6$$

$$2 * c1 * n^2 - c1 * n \leq 10 * n^2 + 12 * n + 6$$

$$-6 \leq 10 * n^2 - 2 * c1 * n^2 + 12 * n + c1 * n$$

$$-6 \leq n * (10 * n - 2 * c1 * n + 12 + c1)$$

h. $2^{n+1} = O(2^n)$

$$2^{n+1} \leq c * (2^n)$$

$$\frac{2^{n+1}}{2^n} \leq c$$

$$2^{n+1-n} \leq c$$

$$2 \leq c$$

i. $2^{2*n} = O(2^n)$

$$2^{2*n} \leq c * 2^n$$

$$2^n * 2^n \leq c * 2^n$$

$$2^n \leq c$$

$$c = 1$$