On instabilities of the Bitcoin protocol

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Breaking Bitcoin

Paris

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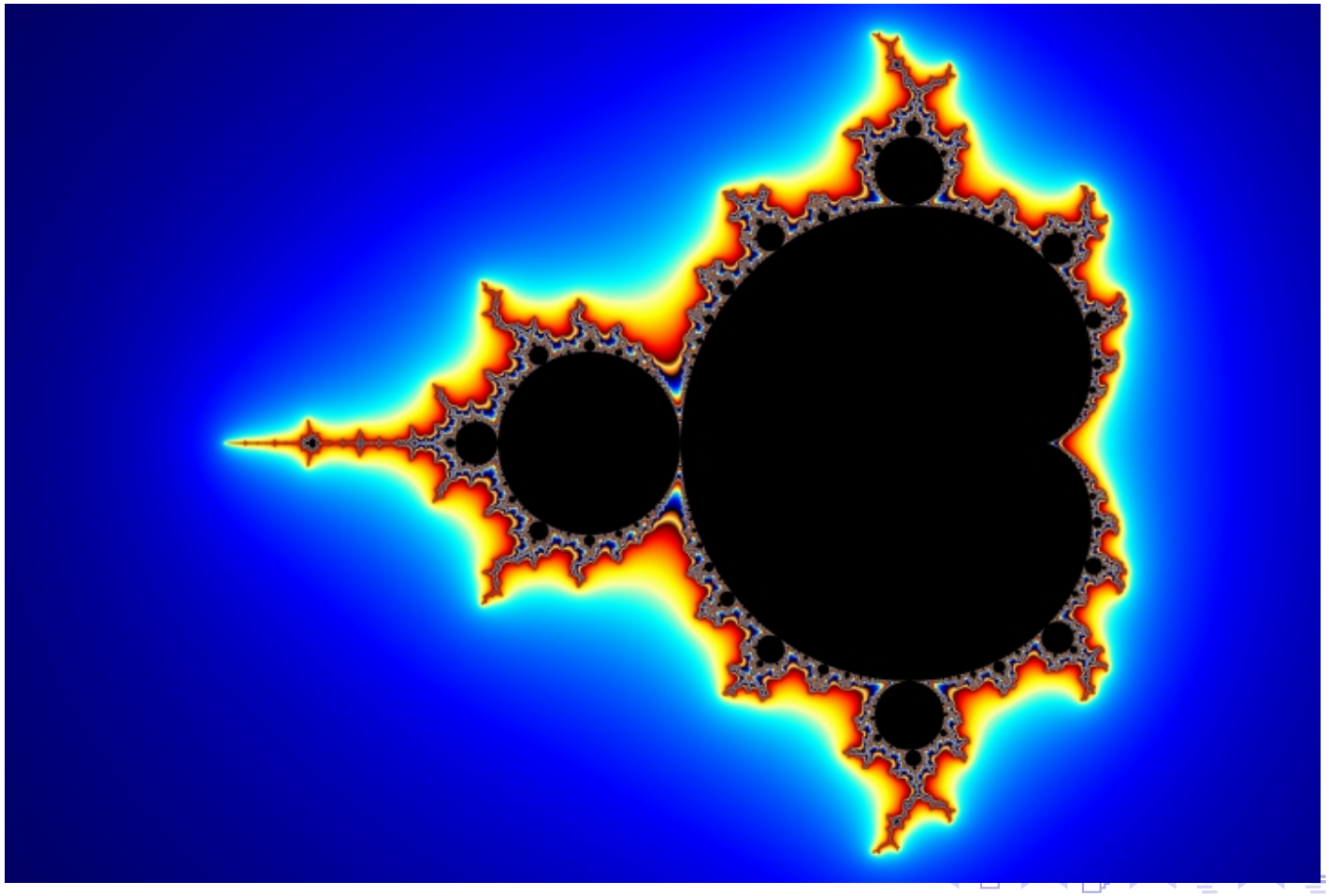
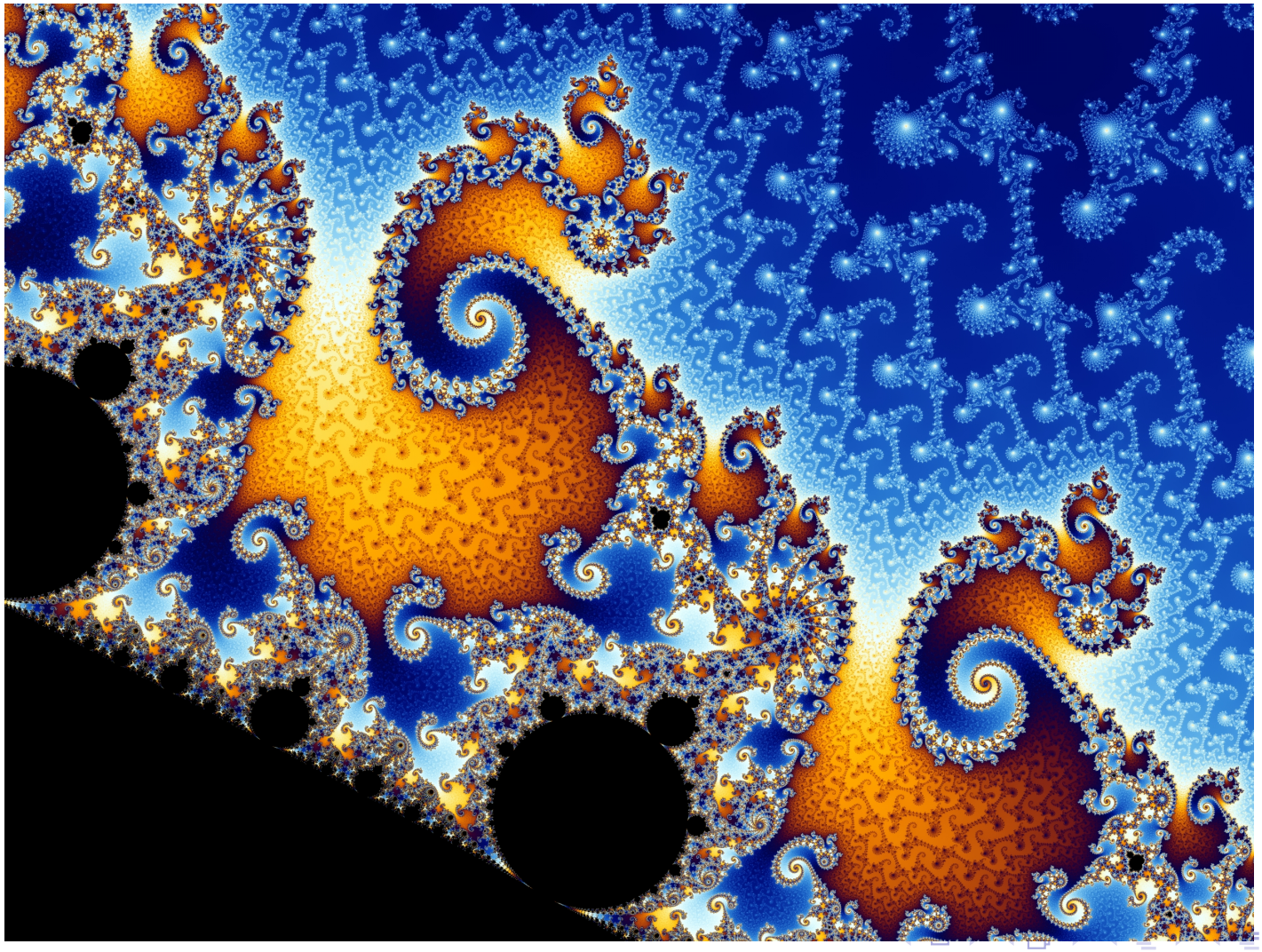
The Mandelbrot set

Iteration of $x \mapsto x^2 + c$

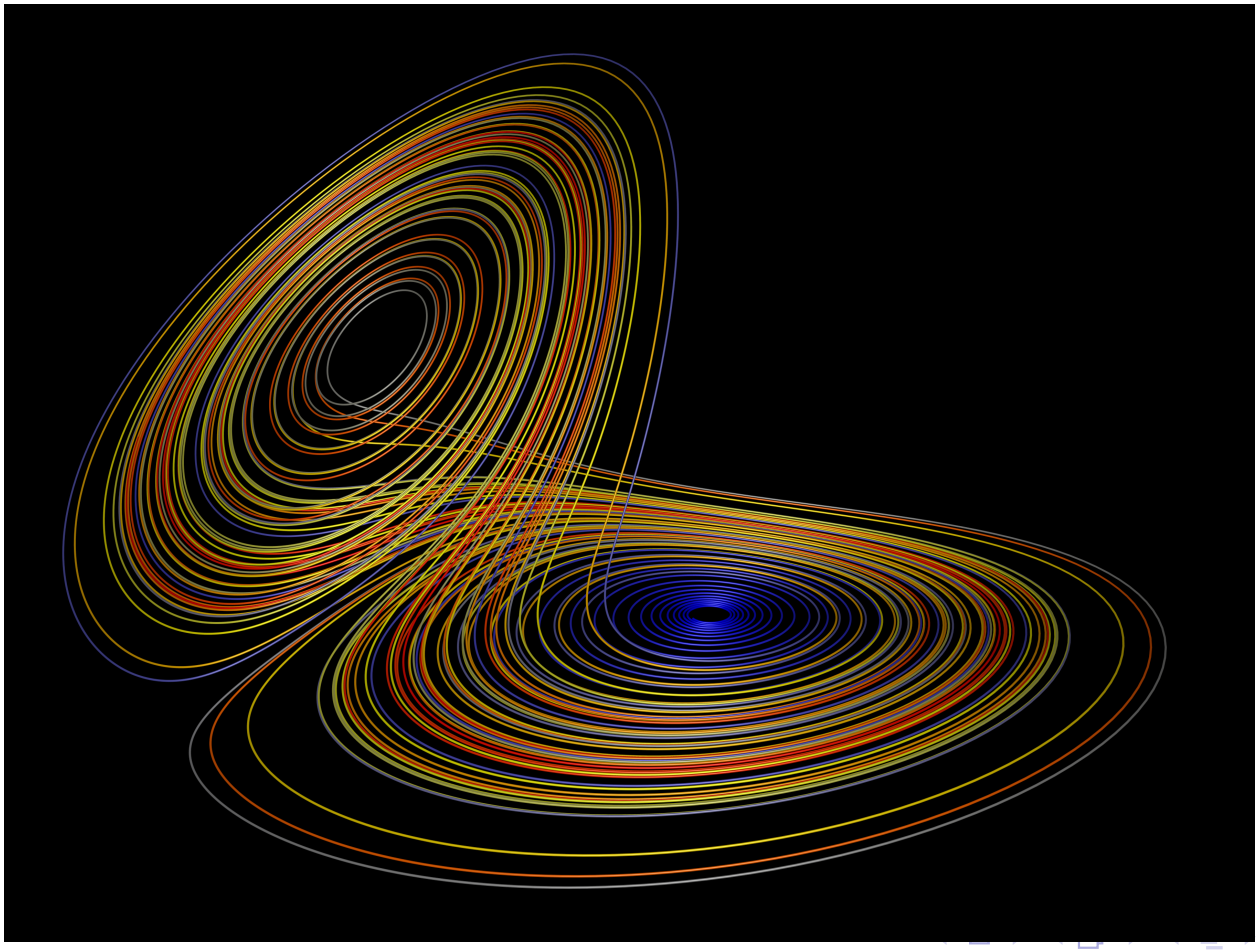


Details of the Mandelbrot set



The Lorentz Attractor

Simple meteorological model.



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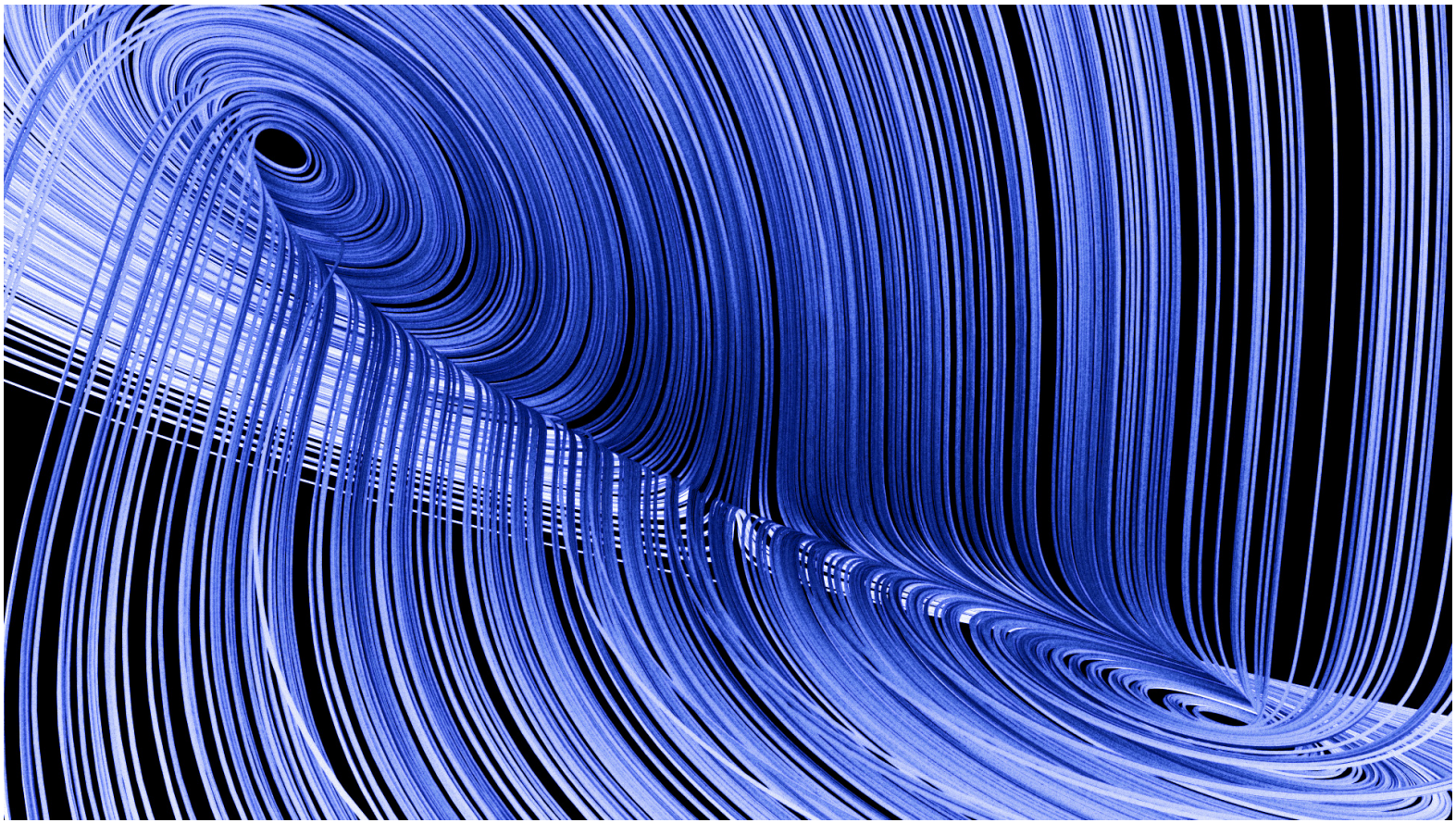

The Butterfly Effect

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A tiny change on initial conditions may produce large deviations on the long term behavior.

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Example

Something as small as the flutter of a butterfly's wing can cause a tornado in another part of the Earth.

Dynamical Instability

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This “change” can have different natures: Catastrophic, mildly disfunctioning, operating abnormaly,...

Some examples

- The stability of the Solar System.

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- Stability of the Financial System.
- In particular, stability of the Monetary System.
- Stability of the Bitcoin Protocol.

The Bitcoin Dynamical System

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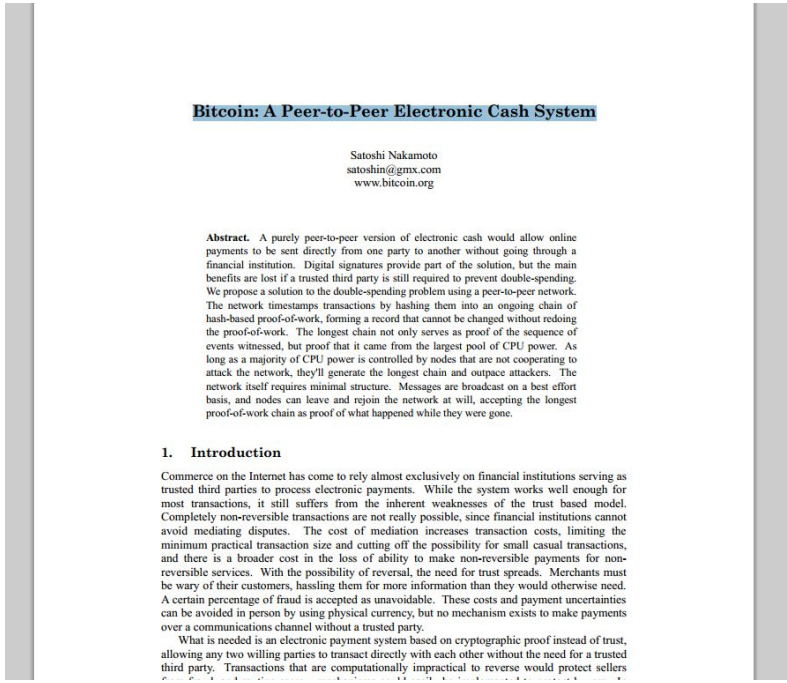
S. Nakamoto, 11/1/2008



Bitcoin: A Peer-to-Peer Electronic Cash System

Satoshi Nakamoto
satoshin@gmx.com
www.bitcoin.org

Abstract. A purely peer-to-peer version of electronic cash would allow online payments to be sent directly from one party to another without going through a financial institution. Digital signatures provide part of the solution, but the main benefits are lost if a trusted third party is still required to prevent double-spending. We propose a solution to the double-spending problem using a peer-to-peer network. The network timestamps transactions by hashing them into an ongoing chain of hash-based proof-of-work, forming a record that cannot be changed without redoing the proof-of-work. The longest chain not only serves as proof of the sequence of events witnessed, but proof that it came from the largest pool of CPU power. As long as a majority of CPU power is controlled by nodes that are not cooperating to attack the network, they'll generate the longest chain and outpace attackers. The network itself requires minimal structure. Messages are broadcast on a best effort basis, and nodes can leave and rejoin the network at will, accepting the longest proof-of-work chain as proof of what happened while they were gone.

1. Introduction

Commerce on the Internet has come to rely almost exclusively on financial institutions serving as trusted third parties to process electronic payments. While the system works well enough for most transactions, it still suffers from the inherent weaknesses of the trust based model. Completely non-reversible transactions are not really possible, since financial institutions cannot avoid mediating disputes. The cost of mediation increases transaction costs, limiting the minimum practical transaction size and cutting off the possibility for small casual transactions, and there is a broader cost in the loss of ability to make non-reversible payments for non-reversible services. With the possibility of reversal, the need for trust spreads. Merchants must be wary of their customers, hassling them for more information than they would otherwise need. A certain percentage of fraud is accepted as unavoidable. These costs and payment uncertainties can be avoided in person by using physical currency, but no mechanism exists to make payments over a communications channel without a trusted party.

What is needed is an electronic payment system based on cryptographic proof instead of trust, allowing any two willing parties to transact directly with each other without the need for a trusted third party. Transactions that are computationally impractical to reverse would protect sellers

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There are some general arguments that give an useful insight.

Lindy Effect

A quantitative version of the “test of time”.

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Goldman (1964), Mandelbrot (1984), Taleb (2007),...

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Example: Bitcoin’s life expectancy is 4 times larger than ETH life expectancy.

Idea for a definite BIP proposal

Corollary

Resilience improves with time.

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Definite BIP

Don't touch the Bitcoin protocol unless there is an obvious bug or a direct threat.

And this is a BIP since Bitcoin improves along with its lifetime...

Antifragility

Antifragile System (Taleb, 2012)

An evolving Dynamical System that increases in capability, resilience, or robustness as a result of stressors, shocks, volatility, noise, mistakes, faults, attacks, or failures

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“What doesn’t kill you makes you stronger”

Good and bad hard forks

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Are hard forks a disruption or a feature of the Bitcoin protocol?

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But things are far more subtle...

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- What's going on?

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- Once the tension is resolved, the uncertainty leaves and the market price adjusts.

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The recipient waits until the transaction has been added to a block and z blocks have been linked after it. He doesn't know the exact amount of progress the attacker has made, but assuming the honest blocks took the average expected time per block, the attacker's potential progress will be a Poisson distribution with expected value:

$$\lambda = z\frac{q}{p}$$

To get the probability the attacker could still catch up now, we multiply the Poisson density for each amount of progress he could have made by the probability he could catch up from that point:

$$\sum_{k=0}^{\infty} \frac{\lambda^k e^{-\lambda}}{k!} \cdot \begin{cases} (q/p)^{(z-k)} & \text{if } k \le z \\ 1 & \text{if } k > z \end{cases}$$

Rearranging to avoid summing the infinite tail of the distribution...

$$1 - \sum_{k=0}^{z} \frac{\lambda^k e^{-\lambda}}{k!} \left(1 - (q/p)^{(z-k)}\right)$$

"assuming the honest blocks took the average expected time per block"

Correct computation

Join work with Cyril Grunspan, ArXiv:1702.02867, 2017.

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Theorem

Let $0 < q < 1/2$ *be the relative hash power of the group of the attackers, and* $p = 1 - q$. *After* z *blocks have been validated by the honest miners, the probability of success of the attackers is*

$$P(z) = I_{4pq}(z, 1/2) \ ,$$

where $I_x(a, b)$ *is the Regularized Incomplete Beta Function*

$$I_x(a, b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \int_0^x t^{a-1}(1-t)^{b-1}\, dt \ .$$

Corollary

A Corollary (that everyone knew)

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The probability of a double spend decreases exponentially with the number of confirmations.

A more precise probability

The parameter κ measures the deviation from average time validation of honest blocks ($\kappa = 1$ is Satoshi's assumption).

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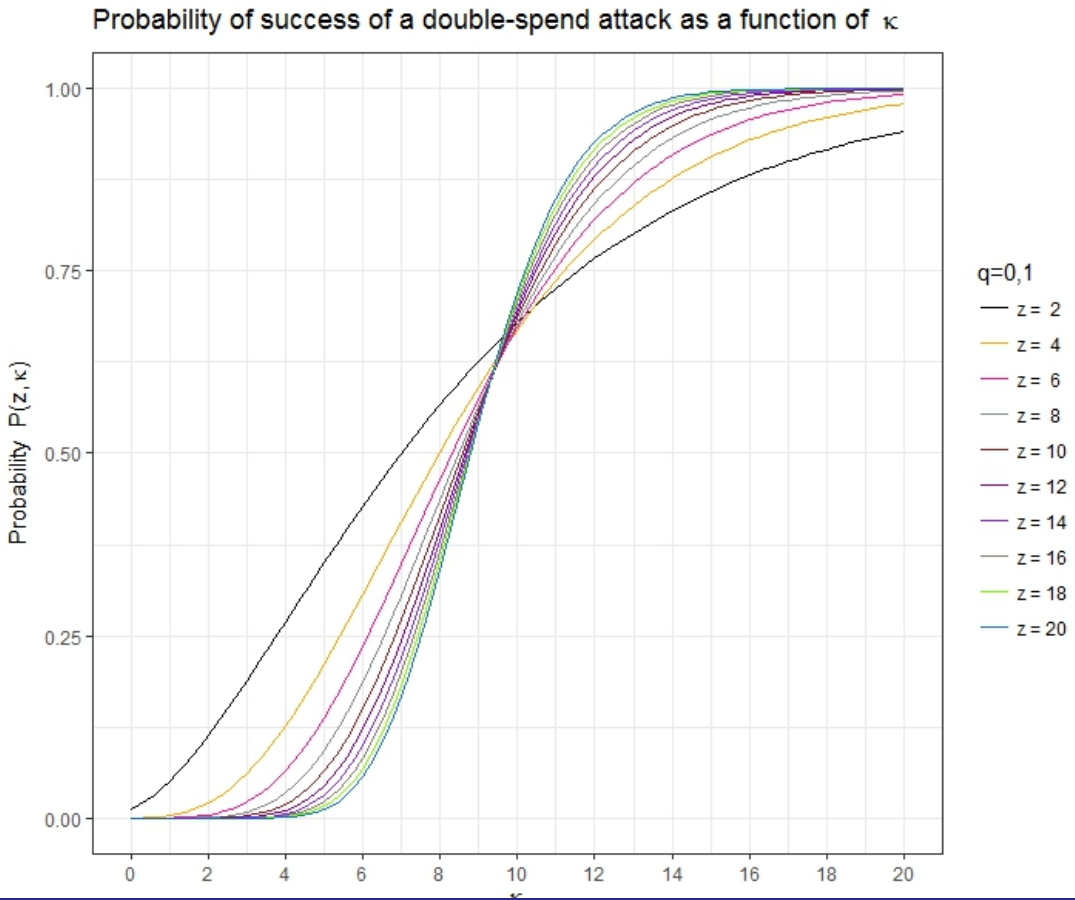

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- Example: A miner must mine on top of the largest work blockchain.
- If a miner just mined a block, but he realizes than just before another block has propagated on the network, will he start mining on top of the network blockchain?
- The answer depends on the miner's hashrate.

Catch-up mining

- Let $E_n^m(q, v)$ be the EV (Expected Value) of the optimal strategy to overcome m blocks delay in n steps for a reward v (includes block rewards, plus fees, plus a possible double spend). When $E_n^m(q, v) > 0$ the strategy is profitable.

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• If $v > 3b$, considering block fees or possible rewards for double spends, the minimal value for q is lower.

Why this happens?

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and thank you for your attention!!