

# Fundamental Algorithms, String Algorithms, HW - 9

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# 1. Different Pattern

Let's say length of Pattern, P is m

Given all characters in p are different.

Algorithms would be as following

First, we would iterate the text, T and look for index, i in T such that it matches the first character of P. then we would increase both T and P index by one if it matches we would repeat this step until we complete and if it did not matches then we would set P's index to first element and T to next index. In the question, it is mentioned that characters of P are different hence when we found a mismatch we do not have to start from very next element of which P's first index matches. As characters occurred in between can not first character of P.

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**Algorithm 1** DifferentPattern

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**function** DIFFERENTPATTERN( $P, T$ )

▷ where P pattern, T text

$m = P.length;$

$n = T.length;$

$matchCount = 0;$

$j = 1;$

**for**  $i = 1$  **to**  $n$  **do**

**if**  $T[i] == P[j]$  **then**

$j++;$

**if**  $j == m+1$  **then**

$matchCount++;$

**Match found at shift**  $i - m$

$j = 1;$

**end if**

**else**

$j = 1;$

**end if**

$i++;$

**end for**

**end function**

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## 2. Gap Character

First step: We would split the Pattern,  $P$  with gap character. Let's say of gap character is  $n$ . then we would get  $n + 1$  smaller pattern with no occurrence on gap character.

Now, we would try to find first occurrence of first pattern (out of  $n + 1$ ) in text,  $T$ . If this is match then we would move to match next pattern without starting from beginning in Text,  $T$ . Doing this, in single iteration of text,  $T$  we can find the first occurrence of the pattern,  $P$ . Total running time of this algorithms is  $O(mn)$ . Where  $n$  is the length of text,  $T$  and  $m$  is the length of pattern,  $P$  after removing gap characters.

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**Algorithm 2** GapCharacter

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```
function GAPCHARACTER( $T, P$ ) ▷ where  $T$  text,  $P$  pattern
    SubPattern [] =  $P$ .split("<>");
     $j = 0$ ;
     $n = T$ .length;
    for  $i = 1$  to  $n$  do
        if  $j == \text{SubPattern.length}$  then
            return true;
        end if
        if SubPattern[ $j$ ][0] ==  $T[i]$  then
            try matching SubPattern[ $j$ ] in from  $T[i+1]$ 
             $k = \text{SubPattern}[j].\text{length} - 1$ ;
            if SubPattern[ $j$ ] ==  $T[i..i + k]$  then
                 $j = j + 1$ ;
                 $i = i + k$ ;
            end if
        end if
    end for
    return false;
end function
```

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# 3. Working Modulo

T = 3141592653589793, q = 11, P = 26:

| Pattern | Mod |
|---------|-----|
| 31      | 9   |
| 14      | 3   |
| 41      | 8   |
| 15      | 4   |
| 59      | 4   |
| 92      | 4   |
| 26      | 4   |
| 65      | 10  |
| 53      | 9   |
| 35      | 2   |
| 58      | 3   |
| 89      | 1   |
| 97      | 9   |
| 79      | 2   |
| 93      | 5   |

Spurious hit is = 15, 59, 92  
26 is correct

## 4. String-matching Automaton

P = aabab, T = aaababaabaababaab State-transition

| State | a | b |
|-------|---|---|
| 0     | 1 | 0 |
| 1     | 2 | 0 |
| 2     | 2 | 3 |
| 3     | 4 | 0 |
| 4     | 2 | 5 |
| 5     | 1 | 0 |

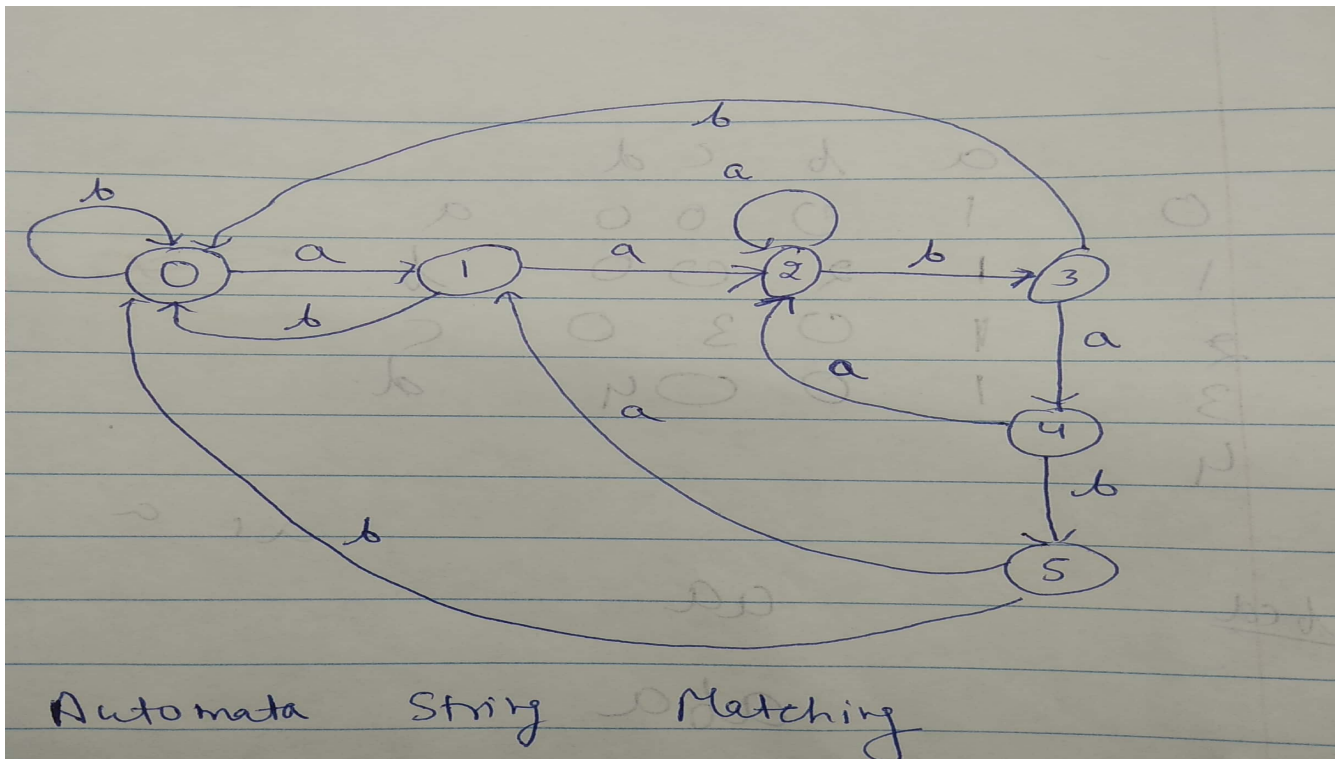


Figure 4.1: Second Approach

State-transition diagram

$0 \rightarrow 1 \rightarrow 2 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \text{ (match)} \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \text{ (match)} \rightarrow 1 \rightarrow 2 \rightarrow 3$

## 5. Non-overlappable Pattern

Given pattern is non-overlappable. Given its conditions to be non-overlappable we can infer that all the characters are different in Pattern, P. This state transition function would go back to either next stage or go back to either initial vertex or first element. If we go to next stage it means suffix of what we have traversed is prefix of what we are trying to find out.

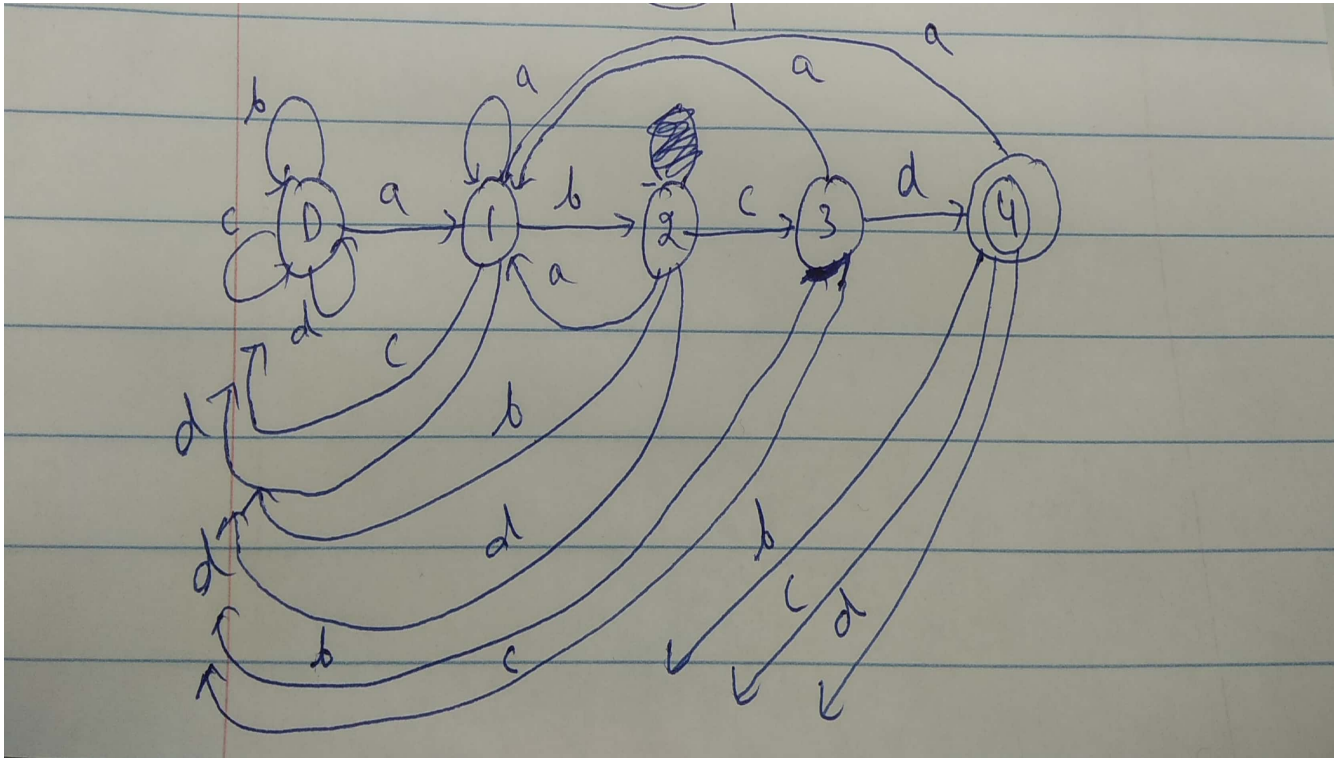


Figure 5.1: Second Approach

## 6. Cyclic Rotation

If length of  $T$  is not equal to  $T'$  it means  $T'$  is not cyclic rotation of  $T$ . If length is equal then lets make a new string,  $TT$  by concatenating  $T$  with  $T$ . If  $T'$  is cyclic rotation it should occur in  $TT$ . This can be done in linear time using **Knuth-Morris-Pratt algorithm**.

Preprocessing would take  $O(n)$  to build suffix prefix matching of Pattern,  $T'$ . That is  $1 \leq q \leq n$  suffix of  $T'_q$  is also a prefix of  $T'_q$ . Then, we can linearly traverse the text,  $T$  when we found a mismatch at  $T_k, P_q$  then we would find maximum suffix,  $M_s$  in  $P[1..q]$  which is also a prefix. Next we would again start matching  $T_k P[|M_s|+1]$  and soon. This only takes a linear traversal.

Below mentioned algorithms are referenced from CLRS, chapter 32. Arguments to the KMP-MATCHER is  $TT$  and  $T'$ .

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**Algorithm 3** COMPUTE-PREFIX-FUNCTION

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```
function COMPUTE-PREFIX-FUNCTION( $P$ )                                ▷ where  $P$  pattern
     $m = P.length$ 
    let  $\pi [1...m]$  be a new array
     $\pi[1] = 0;$ 
     $k = 0;$ 
    for  $q = 2$  to  $m$  do
        while  $k > 0$  and  $P[k+1] \neq P[q]$  do
             $k = \pi[k];$ 
            if  $P[k+1] == P[q]$  then
                 $k = k + 1;$ 
            end if
             $\pi[q] = k;$ 
        end while
    end for
end function
```

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**Algorithm 4** KMP-MATCHER

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**function** KMP-MATCHER( $T, P$ )▷ where  $T$  text,  $P$  pattern     $m = P.\text{length};$      $n = T.\text{length};$      $\pi = \text{COMPUTE-PREFIX-FUNCTION}(P);$      $q = 0;$      $k = 0;$     **for**  $i = 1$  *to*  $n$  **do**        **while**  $q > 0$  *and*  $P[k+1] \neq T[i]$  **do**             $q = \pi[q];$             **if**  $P[q+1] == T[i]$  **then**                 $q = q + 1;$             **end if**            **if**  $q == m$  **then**

return true;

 $q = \pi[2];$             **end if**        **end while**    **end for**

return false;

**end function**

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