SEQUENCE MODELING
INTRODUCTION TO RNNS

Deep Neural Networks

Session 18

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Agenda

- Sequence Modeling
- Introduction to RNN
- Different Architectures
- Language Modelling
- Image Captioning

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Examples – Sequence Modelling

Domain	Data Type	Output type
Speech Recognition	Audio	Words (text)
Music Creation	Nodes (∅)	Audio 
Sentiment classification	… an enjoyable one-time-watch for the funny punchlines, far-out characters and performances. But the unconvincing story and the temperate screenplay prevent it from reaching its full potential …	Integers (Stars ratings from 1 to 5)
Machine Translation	डीएनएन व्याख्यानमाला आपले स्वागत आहे|	Welcome to DNN Lecture.
Named Entity Recognition	Mohan was driving a Maruti	Mohan was driving a Maruti
Video activity recognition	Sequence of Video Frames	Identify activity say running

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Sequence Modeling – Named Entity Recognition

- x : Mohan was driving a Maruti

- y: 1 0 0 0 1

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Sequence Modeling – Named Entity Recognition

- x : <Mohan Sharma> was driving a <Maruti 800>

- y: 1 0 0 0 1

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Sequence Modeling – Named Entity Recognition

- x : Mohan was driving a Maruti

x_1 x_2 x_3 x_4 x_5 $\Rightarrow x_t$

- y: 1 0 0 0 1

y_1 y_2 y_3 y_4 y_5 $\Rightarrow y_t$

- T_x is length of input and T_y is length of output

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Representing Words

- Vocabulary = [a, aakash, aamaan… to zulu, zyzzogeton]
  - Also referred as corpus
  - Two more tokens <UNK> and <EOS>

- Can be converted to one hot encoding

- x : Mohan was driving a Maruti

Mohan	was	driving	a	Maruti
0	0	0	1	0
0	0	0	0	0
–	–	1	–	–
–	–	–	–	–
–	–	–	–	–
–	–	–	0	1
1	–	–	–	–
–	1	–	–	–
0	0	0	0	0

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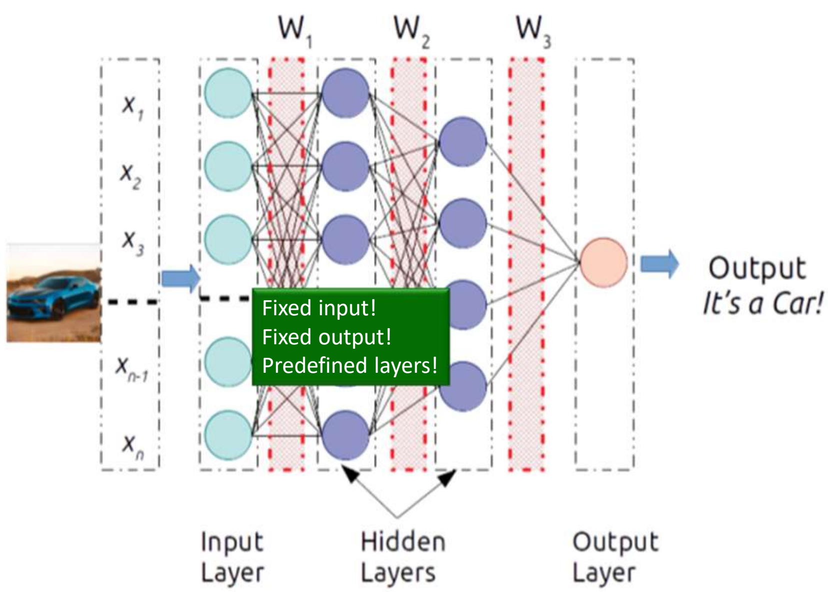
Using Standard Architecture



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To Summarize….

- Not all problems can be converted into one with fixed length inputs and outputs
- Problems such as Speech Recognition or Time-series Prediction require a system to store and use context information
- Hard/Impossible to choose a fixed context window
- There can always be a new sample longer than anything seen

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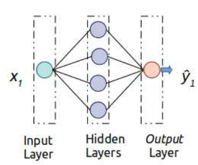
What is Recurrent Neural Network…



- Remember our little Neural Network…
- Let's simplify the layout a little

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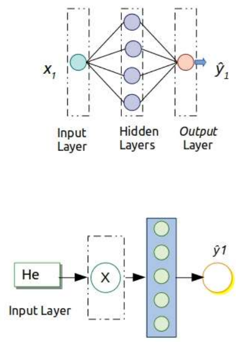
What is Recurrent Neural Network…



Simple Feed- Forward Network

- It takes one value and gives probability of it being a word or character or a value

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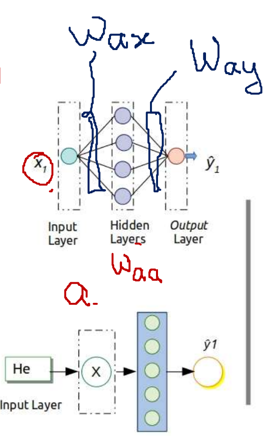
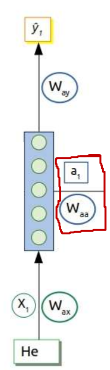
What is Recurrent Neural Network…

$(X.W_{aa} + b_{aa})$



Simple Feed- Forward Network



- Let's also calculate activations a_1 and weights W_{aa}
- Assume that we have some method of calculating them
- At the moment both W_{ax} and W_{aa} would seem to be same

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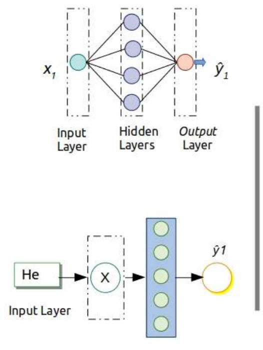
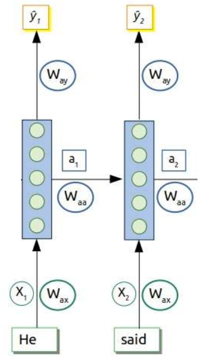
What is Recurrent Neural Network…



Simple Feed- Forward Network



- Using the weights and activations, read X_2 and process it through the network
- Calculation of $\hat{y}_2$ will be based on X_2, W_{ax}, W_{aa} and a_1,

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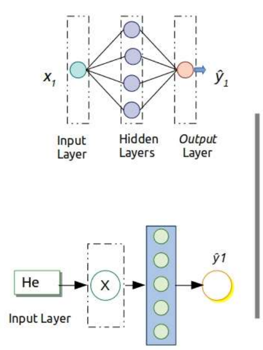
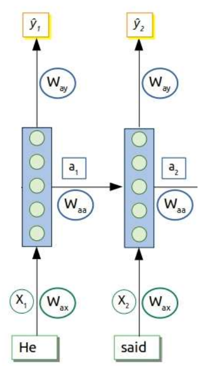
What is Recurrent Neural Network…



Simple Feed- Forward Network



- But it makes two set of calculations
- Using different formulae
- To make it consistent let's initialize a_0 with weights W_{aa}

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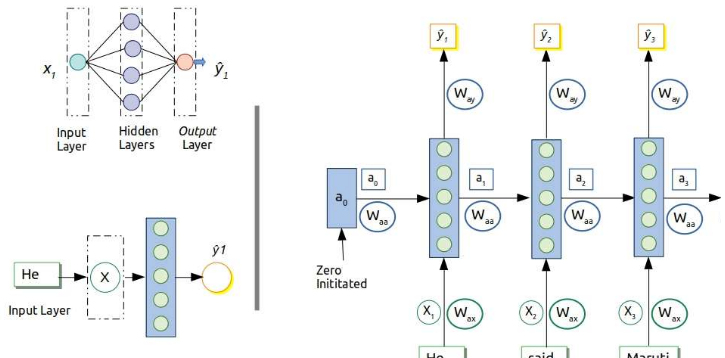
What is Recurrent Neural Network…

- Similarly we can calculate $\hat{y}_3$



Simple Feed- Forward Network

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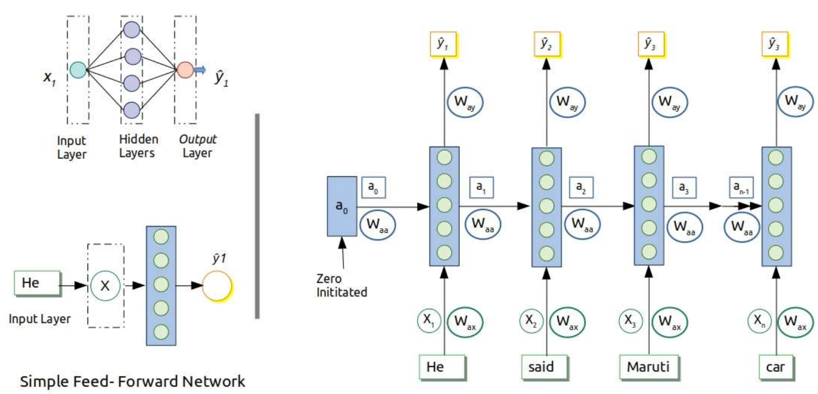
What is Recurrent Neural Network…

- And continue till end,
- Some literatures represent it with a loop,



Simple Feed- Forward Network

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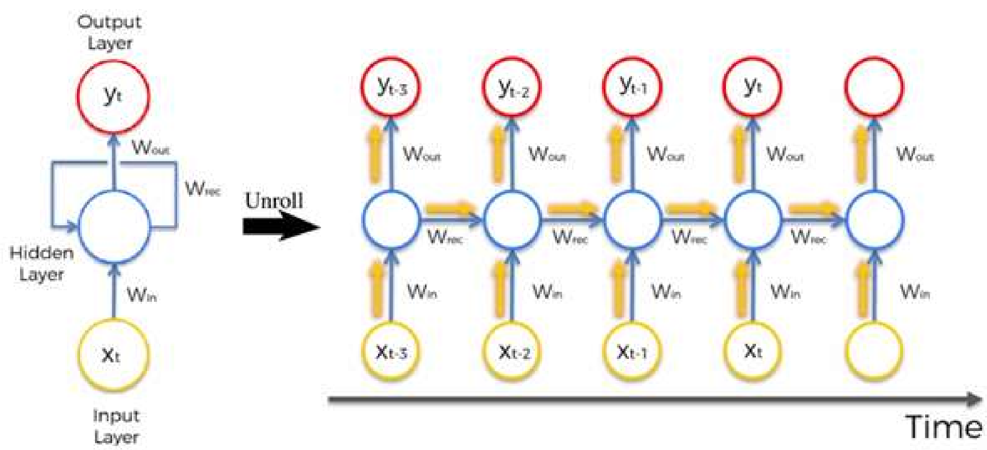
Alternate Representations



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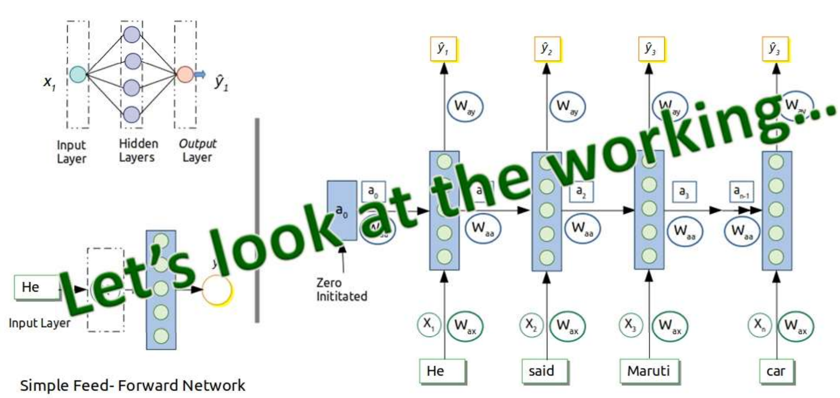
What is Recurrent Neural Network…



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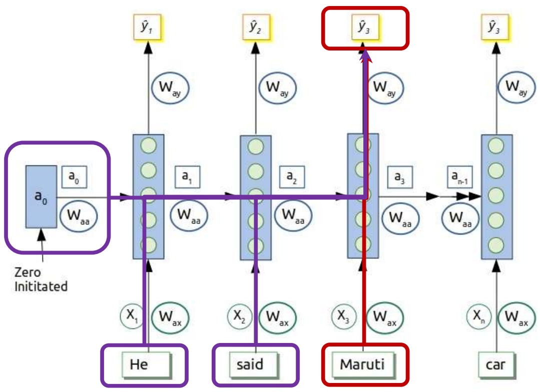
What is Recurrent Neural Network…



Recurrent Neural Network

- Taking activations from previous time step also
- The W_{ax} and W_{aa} are shared parameters across all time steps
- So, for calculation of $\hat{y}_3$ would be influenced by those for $\hat{y}_2$ and $\hat{y}_1$

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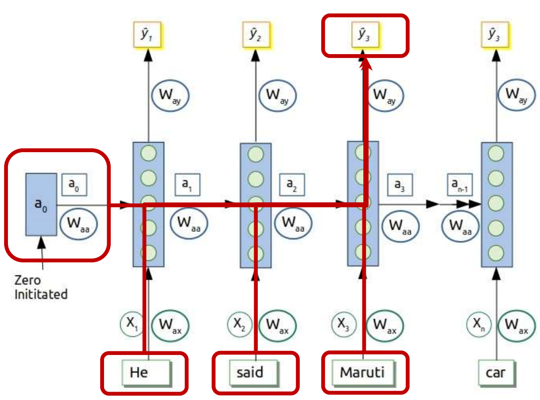
What is Recurrent Neural Network…



Recurrent Neural Network

- It is using the information till time step 3.
  - He said “*Maruti…*
- However, it has no clue what comes next!!!
  - He said “*Maruti is most fuel efficient car*”
  - He said “*Maruti is most expensive shop*”
  - He said “*Maruti is strongest*”

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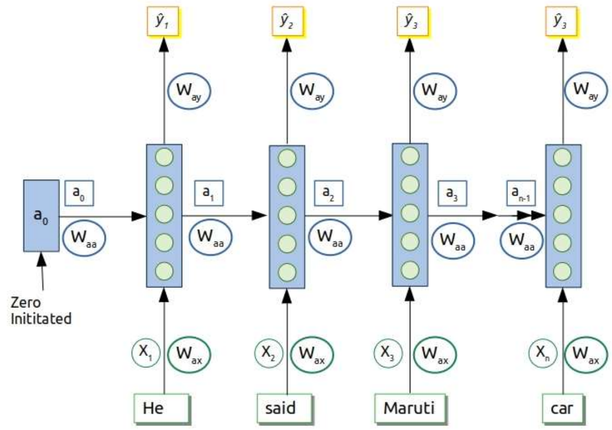
That's is Recurrent Neural Network…



Recurrent Neural Network

- Its it great!
- All done… sealed, signed, and delivered…
- Wait… let's do some math too….

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What We Know So Far….

- Recurrent Neural Networks take the previous output or hidden states as inputs.
- The composite input at time 't' has some historical information about the happenings at time 'T' < 't'.
- RNNs are useful as their intermediate values (state) can store information about past inputs for a time that is not fixed a priori
- Note that the weights are shared over time
- Essentially, copies of the RNN cell are made over time (unrolling/unfolding), with different inputs at different time steps

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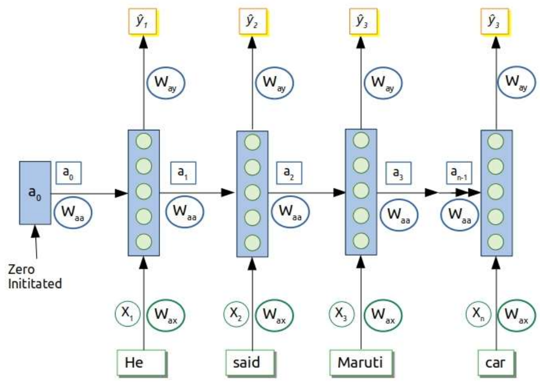
Forward Propagation



Recurrent Neural Network

- Let's work on equations

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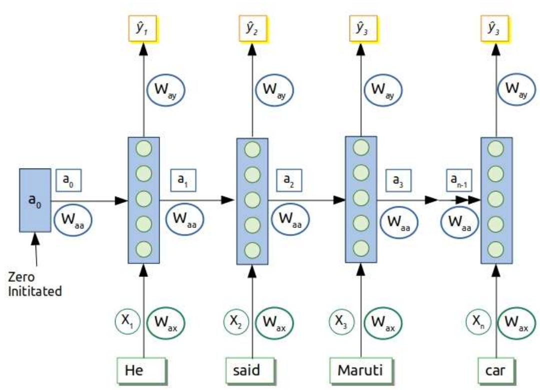
Forward Propagation



Recurrent Neural Network

- To start with; a_0 is vector of all zeros
  - $a_1 = g_1\ (a_0\ .\ W_{aa} + X_1\ .\ W_{ax} + b_a)$ ➔ Tanh / ReLU
  - $\hat{y}_1 = g_2\ (a_1 .\ W_{ay} + b_y)$ ➔ Sigmoid/Softmax (for classification)
- Tanh Activation function is more prevalent in RNN
  - Sometime ReLU too is used
- For output layers, the activation function will depend on type of output
- Generally, at 't' we can write
  - $a_t = g_1\ (a_{t-1}\ .\ W_{aa} + X_t\ .\ W_{ax} + b_a)$
  - $\hat{y}_t = g_2\ (a_t .\ W_{ay} + b_y)$

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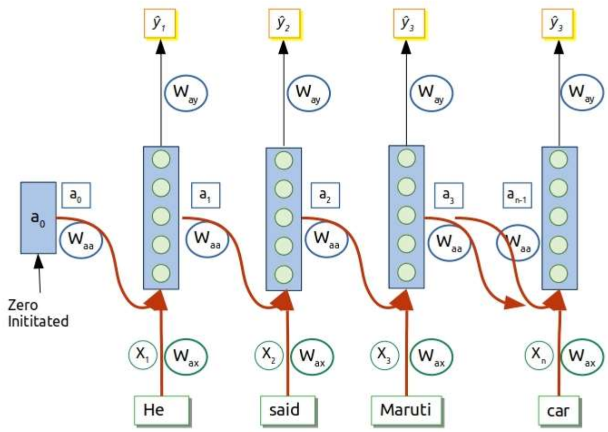
Forward Propagation



Recurrent Neural Network

- Our equations

  $a_t = g_1(a_{t-1}.W_{aa} + x_t.W_{ax} + b_a)$

  $\hat{y}_t = g_2\big(a_t.W_{ya} + b_y\big)$

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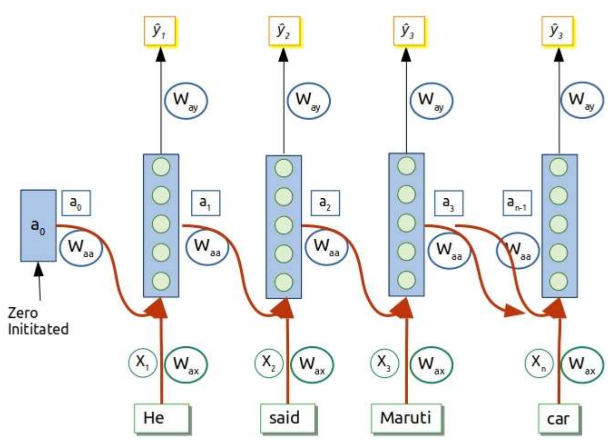
Forward Propagation



Recurrent Neural Network

- Our equations

  $a_t = g_1(a_{t-1}.W_{aa} + x_t.W_{ax} + b_a)$

  $\hat{y}_t = g_2\big(a_t.W_{ay} + b_y\big)$

- Can be written as:

  $a_t = g_1([a_{t-1}, x_t]\,.\,W_a + b_a)$

  $\hat{y}_t = g_2\big(a_t.W_y + b_y\big)$

where W_a will be stacked matrix of W_{aa} and W_{ax}

$$W_a = \begin{bmatrix} W_{aa} \\ W_{ax} \end{bmatrix}$$

Similarly ,

$[a_{t-1}, x_t]$ = $[a_{t-1} | \, x_t]$

We know that :

$[a_{t-1} \mid x_t] \; . \; [\begin{matrix} \mathrm{W_{aa}} \\ \mathrm{W_{ax}} \end{matrix}] = a_{t-1}.W_{aa} + x_t.W_{ax}$

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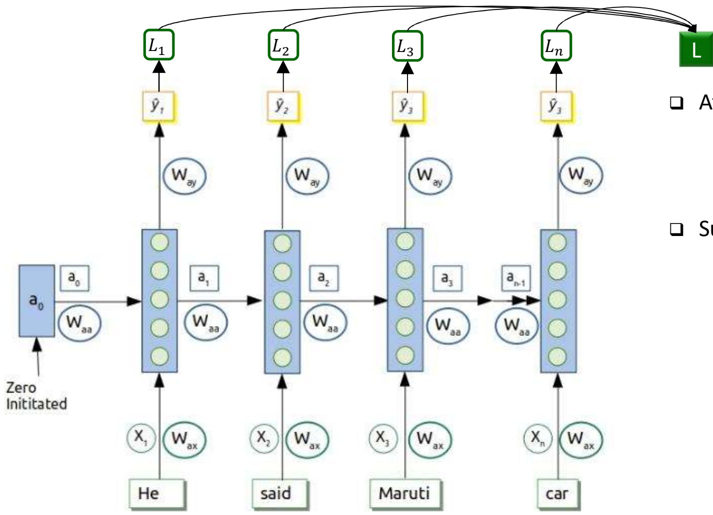
Back Propagation



Recurrent Neural Network

- At time step 't'; Loss Function for single prediction
  - $L_t(\hat{y}_t, y) = - y_t \,.\, \log(\hat{y}_t) - (1- y_t) \,.\, \log(1 - \hat{y}_t)$
- Sum of losses at all time steps:
  - $L(\hat{y}, y) = \sum_{t=1}^{T_x} L_t(\hat{y}_t, y)$

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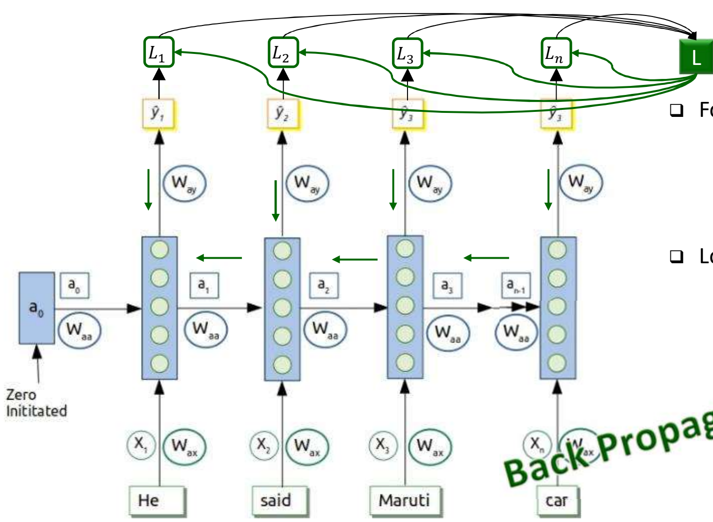
Back Propagation



Recurrent Neural Network

- Forword propagation:

$$a_t = g_1([a_{t-1}, x_t] \,.\, W_a + b_a)$$
$$\hat{y}_t = g_2(a_t . W_y + b_y)$$

- Loss Function
  - $L_t(\hat{y}, y) = - y_t \,.\, \log(\hat{y}_t) - (1- y_t) \,.\, \log(1 - \hat{y}_t)$

Back Propagation through Time.

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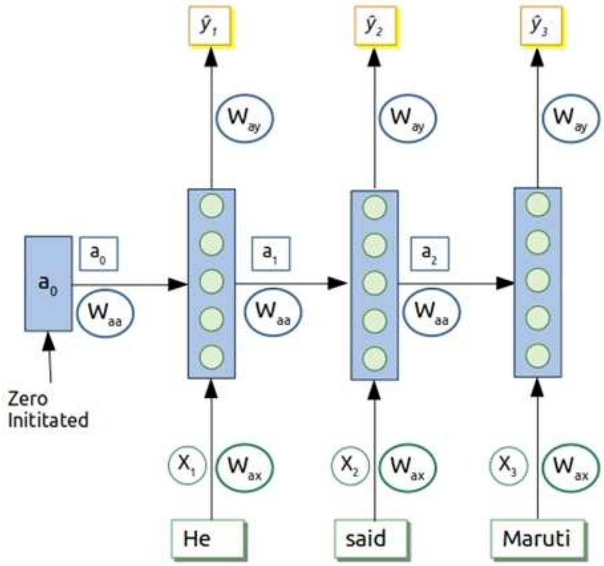
Back Propagation Through Time…



Forword propagation:

$$a_t = g_1([a_{t-1}, x_t].W_a + b_a)$$
$$\hat{y}_t = g_2(a_t.W_y + b_y)$$

Loss Function :

$$L_t(\hat{y}, y) = -y_t.\log(\widehat{y_t}) - (1 - y_t).\log(1 - \widehat{y_t})$$

Step 3:

$$\frac{dL_3}{dw_y} = \frac{dL_3}{d\hat{y}_3}.\frac{d\hat{y}_3}{dw_y}$$

$$\frac{dL_3}{dw_a} = \frac{dL_3}{d\hat{y}_3}.\frac{d\hat{y}_3}{da_3}.\frac{da_3}{dw_a} + \frac{dL_3}{d\hat{y}_3}.\frac{d\hat{y}_3}{da_3}.\frac{da_3}{da_2}.\frac{da_2}{dw_a} + \frac{dL_3}{d\hat{y}_3}.\frac{d\hat{y}_3}{da_3}.\frac{da_3}{da_2}.\frac{da_2}{da_1}.\frac{da_1}{dw_a}$$

There is a pattern here!

$$\frac{dL_3}{dw_x} = \frac{dL_3}{d\hat{y}_3}.\frac{d\hat{y}_3}{da_3}.\frac{da_3}{dw_x} + \frac{dL_3}{d\hat{y}_3}.\frac{d\hat{y}_3}{da_3}.\frac{da_3}{da_2}.\frac{da_2}{dw_x} + \frac{dL_3}{d\hat{y}_3}.\frac{d\hat{y}_3}{da_3}.\frac{da_3}{da_2}.\frac{da_2}{da_1}.\frac{da_1}{dw_x}$$

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Quickly check the dimension….

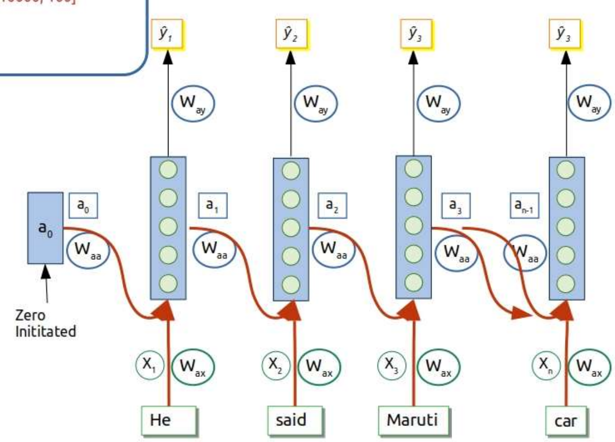
$$a_t = g^1(\underset{[100,100]}{a_{t-1}} . \underset{[100,\ 100]}{W_{aa}} + \underset{[100,10000]}{X_t} . \underset{[10000,\ 100]}{W_{ax}} . + b_a)$$

$$\hat{y}_t = g^2(a_t . W_{ya}. + b_y)$$

$$a_t = g^1([a_{t-1} : X_t].W_a.+ b_a)$$
$$\hat{y}_t = g^2(a_t .W_y + b_y)$$

$$\underset{[100 \times 10100]}{[a_{t-1}, X_t]} = [\underset{[100]}{a_{t-1}} : \underset{[10000]}{X_t}]\ [100]$$

$$W_a = \overset{[100]}{\begin{pmatrix} W_{aa} \\ W_{ax} \end{pmatrix}} \begin{matrix} [100] \\ [10000] \end{matrix}$$



Recurrent Neural Network

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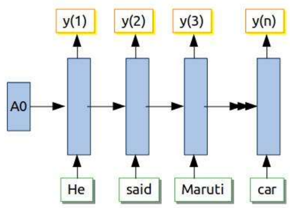
Type of Architectures



Many-to-Many Architecture

Many to many mapping. T_x input parameters are same as T_y output parameters

Named entity Recognition:
Mohan was driving a Maruti
➔ 1 0 0 0 1

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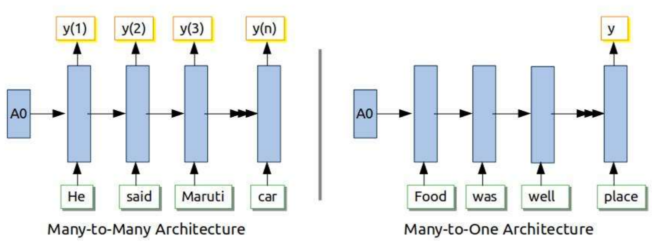
Type of Architectures



Many-to-One Architecture

Many to one architecture.

Input is the 'review' written by a patron and output is an integer (star rating)

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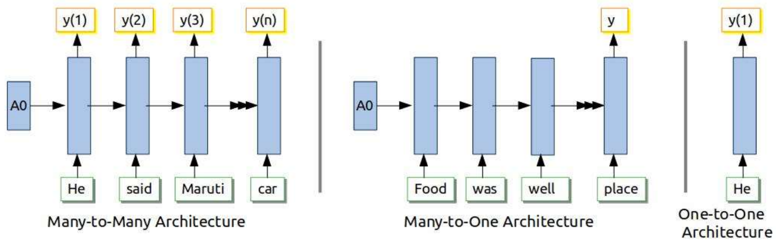
36 Type of Architectures



Of course there is one to one. i.e. Basic neural network…

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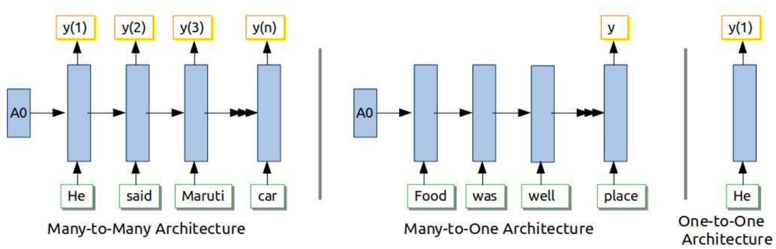
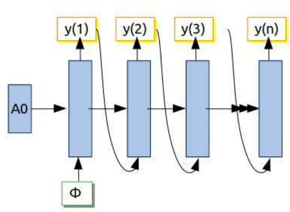
37 Type of Architectures





One-to-Many Architecture

We also have one to many architecture, mapping one input to multiple outputs. Output from each layer becomes input for next time step.

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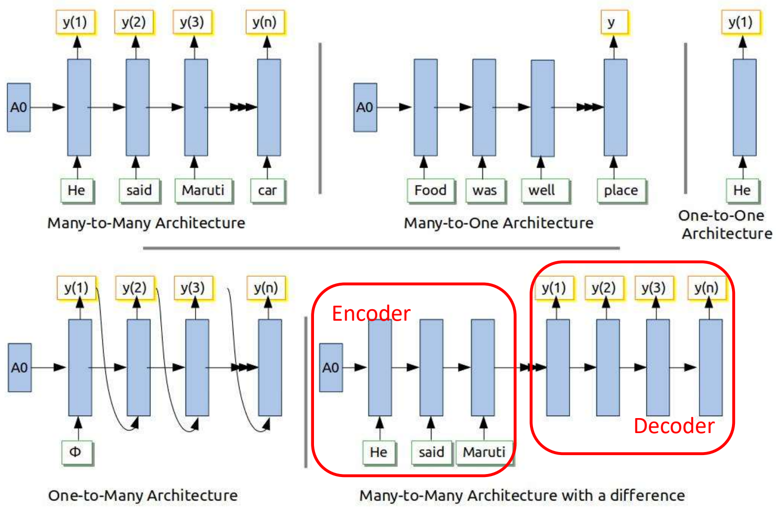
Type of Architectures



डीएनएन व्याख्यानमाला आपले स्वागत आहे।
➔ Welcome to DNN Lecture

In this Architecture, we have two completely different parts. One side reading sentences in one language, and other side translating in different language. We can have $T_x$and T_y different which is a case in machine translations

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Language Modelling

Speech Recognition

- Toad met Pit….
- Todd met Pete…
- Given any sentence, what is the probability of that being a valid sentence
- So what language model would do is to calculate probability of a sentence with that combination of words
  - P(Toad met Pit) = $4.6\ x\ 10^{-1}$
  - P(Todd met Pete) = $9.3\ x\ 10^{-9}$
- Mathematically P(sentence) = P($y_1, y_2, y_3, \ldots\ y_n$)

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How to Model?

- Training set : Large corpus of English text
  - Adults need eight hours of sleep a day!

Adults	need	eight	hours	of	sleep	a	day	!	<EOS>
y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	-	y_9

- First step is to tokenize the sentence
- Add a token at end and at the beginning <EOS> (y_9)
- Remember we have limited tokens (say we only have 10,000 tokens).
- Unknown words will be given a token <unk>

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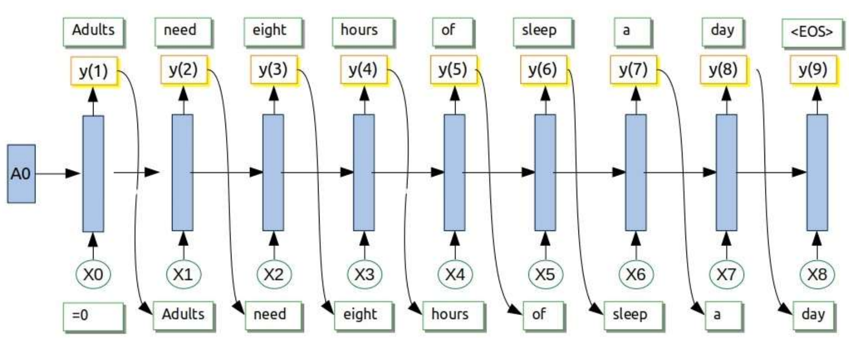
RNN Model

- At the onset RNN tries to predict probabilities of each word in the corpus of being first word in this sentence.
- i.e. P[a], P[aakash], P[aamaan]… to P[zulu], P[zyzzogeton]
  - This would be an array of 10002 elements



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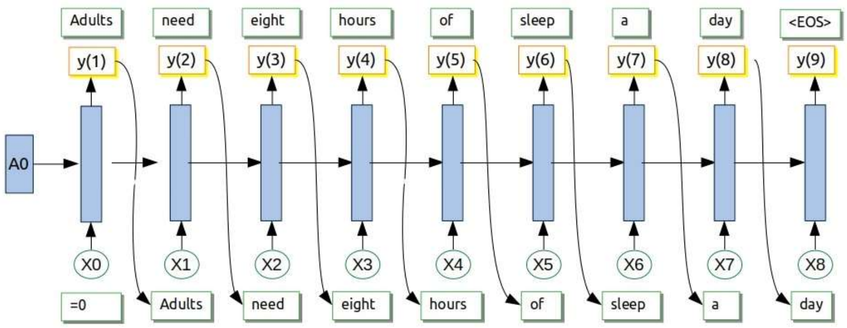
RNN Model

- Thus we can calculate error between $\hat{y}_1$ and "Adults"
- Given first word "Adults", again RNN predicts the probabilities for second word, thus combined probability, and it continues…
  - i.e. P[a|Adult], P[aakash|Adult], P[aamaan |Adult]… to P[zulu |Adult], P[zyzzogeton |Adult]
- Somewhere in that bunch there will be a probability P[need|Adult]



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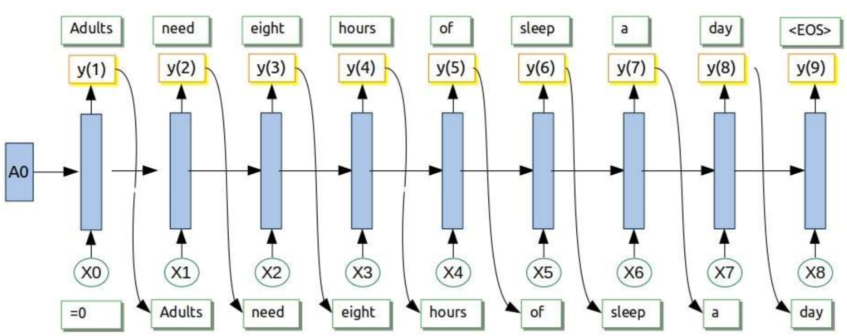
RNN Model

- At third step we can calculate error between $\hat{y}_2$ and "need".
- Given first word "Adults", and second word as "need", again RNN predicts the probabilities for third word
- i.e. P[a | Adult, need], P[aakash | Adult , need], P[aamaan | Adult , need]… to P[zulu |Adult , need], P[zyzzogeton |Adult , need]
- Somewhere in that bunch there will be a probability P[eight|Adult, need]



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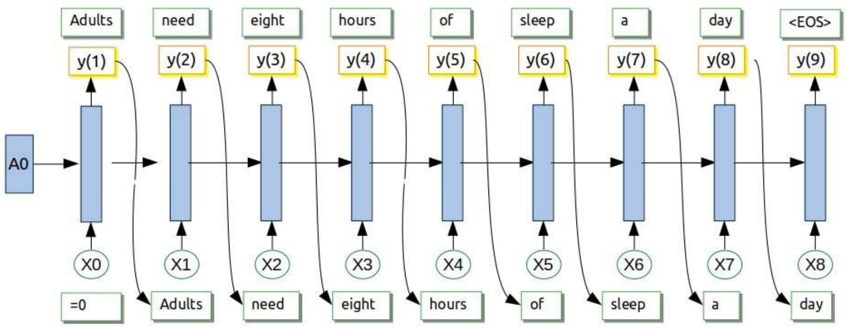
RNN Model

- Thus we can calculate error between $\hat{y}_3$ and “eight”.
- It continues from left to right till end, X_8
- Given all previous words, what is the probability of this word being <EOS>.



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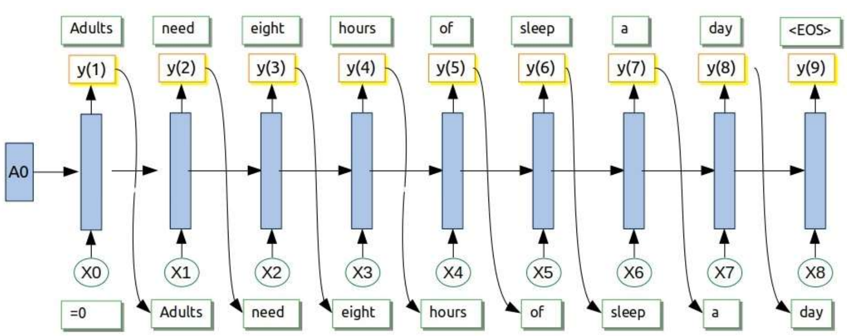
RNN Model

- RNN is trying to predict one word at a time from left to right.
- Given that we are going to use logits and subsequently softmax for loss function, our loss function will be
- ℓ (ŷ, y) = -y * log (ŷ) as ŷ is very close to 0 for all other words
  - since its remaining part [(1 – y) * log (1 – ŷ)] is insignificantly small we can ignore it.



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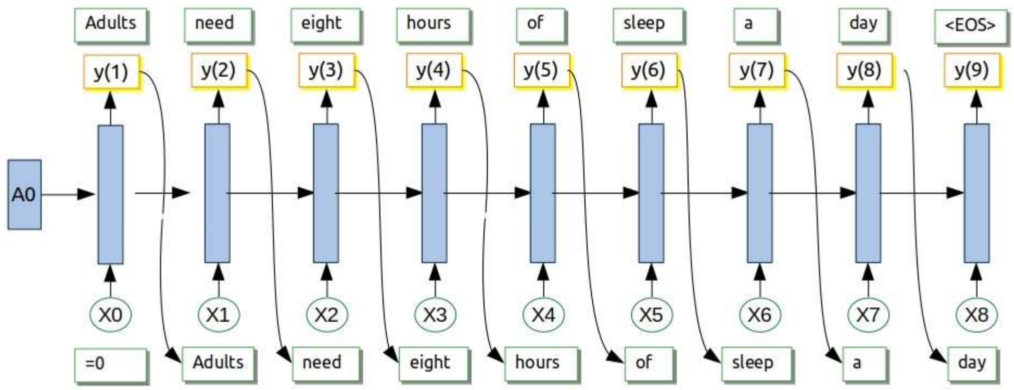
RNN Model

- Thus for overall sentence, Cost will be
  - J(ŷ, y) = Σ ℓ (ŷ, y)
  - J(ŷ, y) = - $\frac{1}{m}$Σ y * log(ŷ)
  - Which we will be minimizing.



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RNN Model

- Suppose you have sentence with 3 words
- You want to know probability of it being a sentence
- Given a sentence y_1, y_2, y_3
- P(y_1, y_2, y_3) = P[y_1] * P[y_2| y_1] * P[y_3 | y_1, y_2]

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Word representation

- Vocabulary = [a, aakash, aamaan… to zulu, zyzzogeton]
  - Also referred as corpus
  - Two more tokens <UNK> and <EOS>

- Can be converted to one hot encoding

Man	Women	King	Queen	Apple	Oranges
(5468)	(8701)	(4823)	(7157)	(56)	(7259)
0	0	0	0	0	0
0	0	0	0	1	0
—	—	1	—	0	—
—	—	—	—	—	—
1	—	—	—	—	—
—	—	—	1	—	1
—	1	—	—	—	—
—	—	—	—	—	—
0	0	0	0	0	0

This representation is treating words independently….

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Featured Representation

	Man (5468)	Women (8701)	King (4823)	Queen (7157)	Apple (56)	Oranges (7259)
Gender	-1	1	-0.95	0.97	0	0.001
Royal	0.01	0.02	0.90	0.98	0.05	-0.01
Age	0.05	0.02	0.7	0.68	0.001	-0.4
Food	0.001	0.002	0.0001	0.0002	0.95	0.90

Feature representing a huge corpus can drastically be reduced…

- Man → Women ≈ King → ????

- In terms of algorithm, we can use this using Similarity Coefficients
  - Find a word W : argmax ($e_w, \;\; e_{king} - e_{man} + e_{women}$)
  - Cosine sim (u, v) = $\frac{(u^T.v)}{||u||_2.||v||_2}$
  - Euclidian distances or Manhattan distances can also be used

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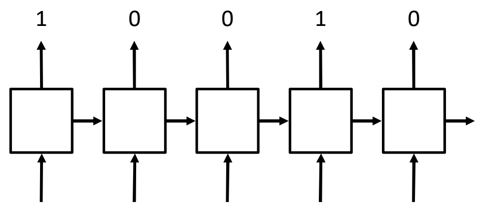
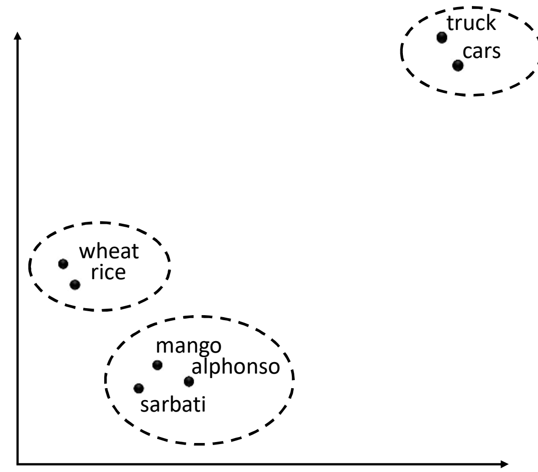
Named entity and word embedding



Mohan	is	a	wheat	farmer
Sudesh	is	a	mango	______
Ravi	is	a	sarbati	cultivator



Words	→	Embedding Layer	→	Dense Layer W_d, b_d	→	Softmax Layer W_o, b_0
5 words		5 x 300				10000 probabilities

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Sampling a Sequence from a Well Trained Model

- Imagine we have super trained RNN network
- We ask it to predict first word,
  - which results in probability words in corpus to be first word,
- Pick a word from the probabilities to be first word (np.random.choice())
- Enter this word as input to timestamp '2' to generate second word, again pick a word at random and pass it to third time stamp.
- and you will generate a sentence till you reach a <EOS>
- Alternatively, you can limit the sentence to say 20 words
- Voila!!!
- Remember 2016 US Election, someone fabricated how Trump would have answered questions during press conference
- Obviously it would not make exact sense. But in general it will be same.

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RNN Model

- In some cased, it is advantageous to have character based RNN instead of word based RNN.
- Both formats have their own advantages.

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Sequence to sequence : Image Captioning

- Given an image, produce a sentence describing its contents
- Inputs: Image feature (from a CNN)
- Outputs: Multiple words (let's consider one sentence)

: The dog is hiding

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Image Captioning

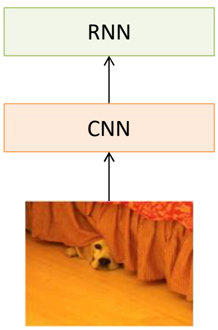

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Image Captioning

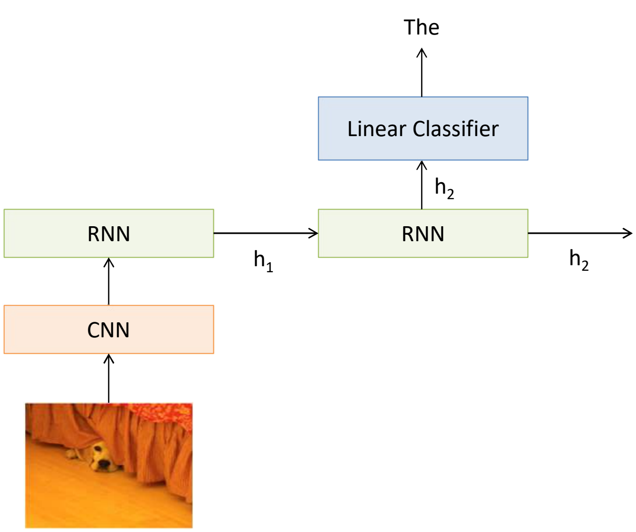

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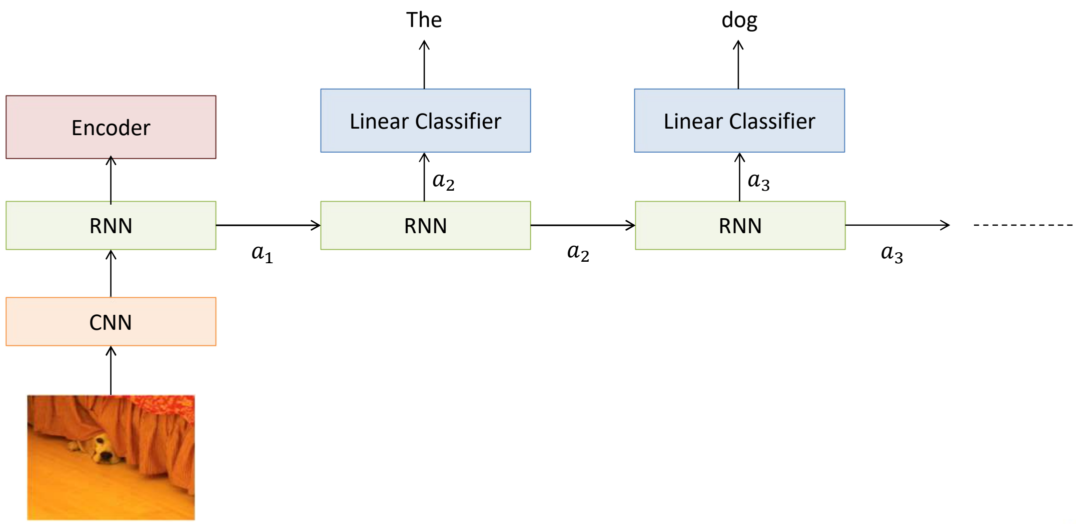
Image Captioning



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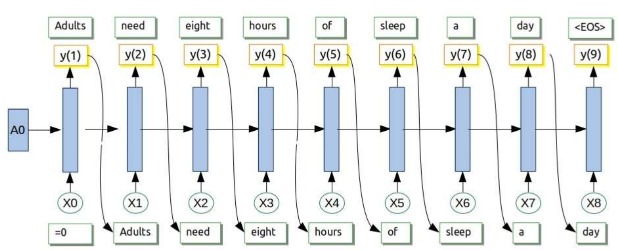
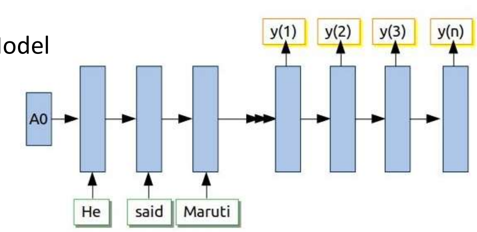
Machine translation



Language Model

- Conditional language Model



Machine translation

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Sequence to sequence : Bleu Score

- 'Dog' , 'bed', 'hiding'
- Le chien est sous le lit
- कुत्ता बिस्तर के नीचे है.
- कुत्रा पलंगाच्या खाली आहे.

: The dog is hiding

- Reference 1: The Dog is hiding under the bed
- Reference 2: There is a dog under the bed

- MT Output : The dog the dog hiding under the bed

"BLEU: a Method for Automatic Evaluation of Machine Translation" By Kishore Papineni, Salim Roukos, Todd Ward, Wei Jing Zhu.

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RNN Outputs: Image Captions

Show and Tell: A Neural Image Caption Generator, CVPR 15

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