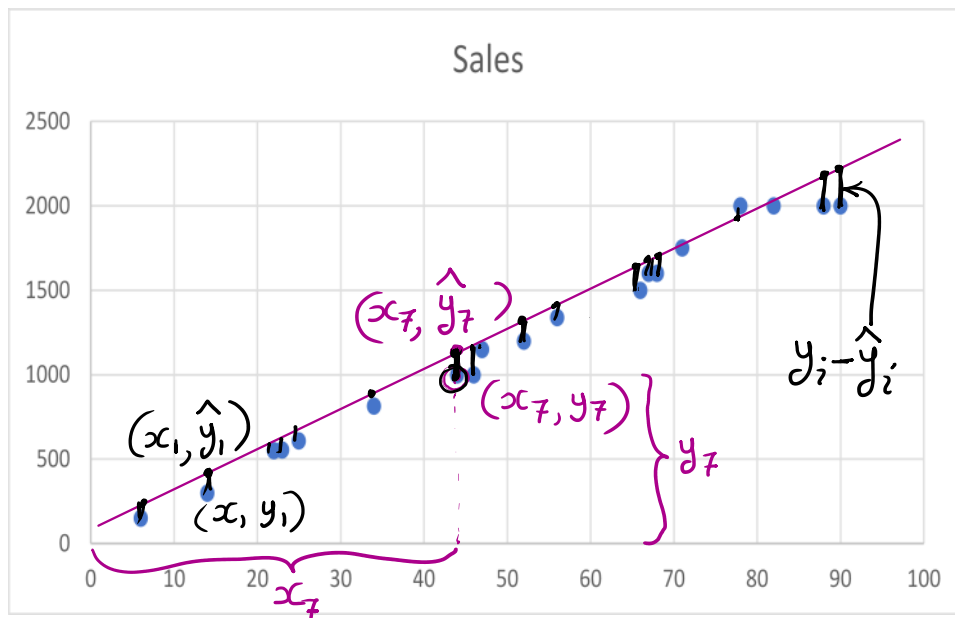
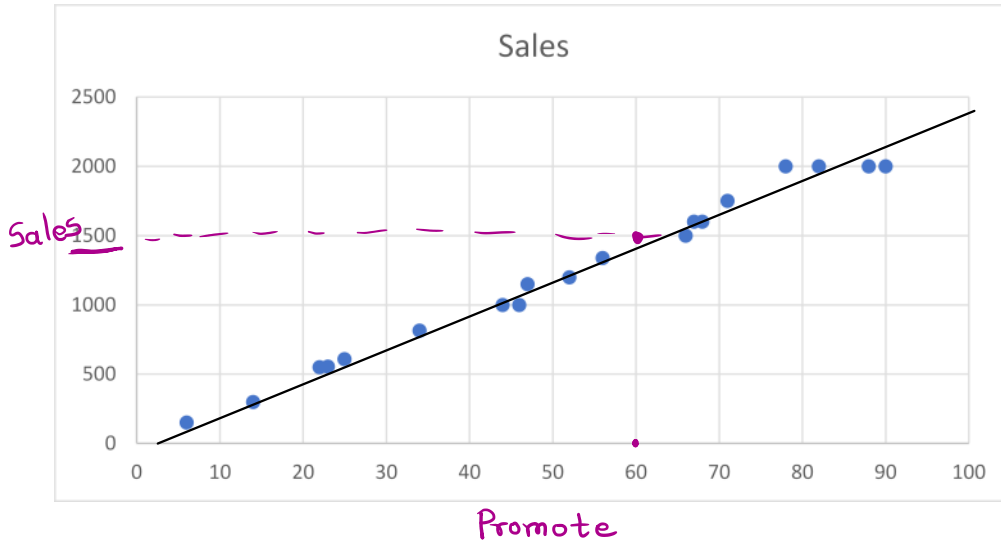


# Least Squares

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$$\text{Sales} = b_0 + b_1 \text{ Promote}$$



$$y = b_0 + b_1 x$$

$$b_0 = ?$$

$$b_1 = ?$$

$(x_i, y_i)$  are data points

$(x_i, \hat{y}_i)$  are corresponding points on the line

Residual  $\frac{y_i - \hat{y}_i}{\text{error}}$  ;  $(x_i, \hat{y}_i)$  is on line  $y = b_0 + b_1 x$   
 $\Rightarrow \hat{y}_i = b_0 + b_1 x_i$

$$y_i - (b_0 + b_1 x_i)$$

$$Z = \text{Loss f}^n = \sum_{i=1}^n (y_i - \hat{y}_i)^2 = \text{Residual Sums of Squares}$$

To minimize

$$\therefore Z = \sum_{i=1}^n [y_i - (b_0 + b_1 x_i)]^2$$

$$\frac{\partial Z}{\partial b_0} = 0 \Rightarrow -2 \left[ \sum_i (y_i - b_0 - b_1 x_i) \right] = 0$$

$$\frac{\partial Z}{\partial b_1} = 0 \Rightarrow -2 \sum x_i (y_i - b_0 - b_1 x_i) = 0$$

$$\sum_{i=1}^n y_i - \sum_{i=1}^n b_0 - b_1 \sum_{i=1}^n x_i = 0$$

$$\sum y_i - n b_0 - b_1 \sum x_i = 0$$

$$\bar{y} - b_1 \bar{x} = b_0$$

$$b_0 = \bar{y} - b_1 \bar{x} \quad \text{--- (1)}$$

$$\sum x_i y_i - b_0 \sum x_i - b_1 \sum x_i^2 = 0$$

$$\frac{\sum x_i y_i}{n} = \cancel{\frac{b_0 \sum x_i}{n}} + b_1 \frac{\sum x_i^2}{n} \quad \text{--- (2)}$$

$$\bar{x} \times \bar{y} = b_0 + b_1 \bar{x}$$

$$\bar{x} \bar{y} = \cancel{b_0 \bar{x}} + b_1 \bar{x}^2$$

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$$\frac{\sum x_i y_i}{n} - \bar{x} \bar{y} = b_1 \frac{\sum x_i^2}{n} - b_1 \bar{x}^2$$

$$\frac{\frac{\sum x_i y_i}{n} - \bar{x} \bar{y}}{\frac{\sum x_i^2}{n} - \bar{x}^2} = b_1 \quad \begin{array}{l} x_i : \text{Promote} \\ y_i : \text{Sales} \end{array}$$

$$b_0 = \bar{y} - b_1 \bar{x}$$

$$y_i = b_0 + b_1 x_{1i} + b_2 x_{2i}$$

$$Z = \sum \left[ y_i - (b_0 + b_1 x_{1i} + b_2 x_{2i}) \right]^2$$

$$\frac{\partial Z}{\partial b_0} = -2 \sum (y_i - b_0 - b_1 x_{1i} - b_2 x_{2i}) = 0$$

$$\frac{\partial Z}{\partial b_1} = -2 \sum x_{1i} (y_i - b_0 - b_1 x_{1i} - b_2 x_{2i}) = 0$$

$$\frac{\partial Z}{\partial b_2} = -2 \sum x_{2i} (y_i - b_0 - b_1 x_{1i} - b_2 x_{2i}) = 0$$

$$\bar{y} - n b_0 - b_1 \bar{x}_1 - b_2 \bar{x}_2 = 0$$

$$\therefore b_0 = \bar{y} - b_1 \bar{x}_1 - b_2 \bar{x}_2$$

$$(2) \rightarrow \sum x_{1i} y_i - b_0 \sum x_{1i} - b_1 \sum x_{1i}^2 - b_2 \sum x_{2i} x_{1i} = 0$$

$$(3) \rightarrow \sum x_{2i} y_i - b_0 \sum x_{2i} - b_1 \sum x_{1i} x_{2i} - b_2 \sum x_{2i}^2 = 0$$

$$(2) \rightarrow b_0 \sum x_{1i} + b_1 \sum x_{1i}^2 + b_2 \sum x_{2i} x_{1i} = \sum x_{1i} y_i$$

$$(3) \rightarrow b_0 \sum x_{2i} + b_1 \sum x_{1i} x_{2i} + b_2 \sum x_{2i}^2 = \sum x_{2i} y_i$$

$$(1) \rightarrow b_0 + b_1 \bar{x}_1 + b_2 \bar{x}_2 = \bar{y}$$