

## 6.4 Bivariate Normal Distribution

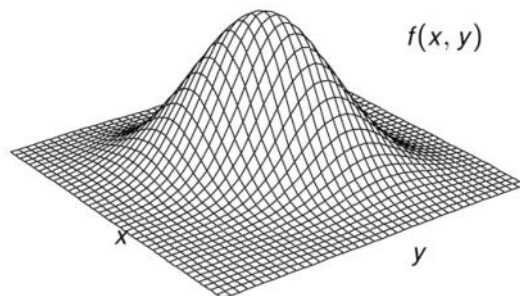
The continuous random variables  $X$  and  $Y$  with joint pdf

$$f(x, y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} e^{-\frac{1}{2(1-\rho^2)} \left[ \left( \frac{x-\mu_X}{\sigma_X} \right)^2 - 2\rho \left( \frac{x-\mu_X}{\sigma_X} \right) \left( \frac{y-\mu_Y}{\sigma_Y} \right) + \left( \frac{y-\mu_Y}{\sigma_Y} \right)^2 \right]}$$

on  $\mathcal{A} = \{-\infty < x < \infty, -\infty < y < \infty\}$  with parameter space

$$\Omega = \{(\mu_X, \mu_Y, \sigma_X, \sigma_Y, \rho) | -\infty < \mu_X < \infty, -\infty < \mu_Y < \infty, \sigma_X > 0, \sigma_Y > 0, -1 < \rho < 1\}$$

are *bivariate normal random variables* with parameters  $\mu_X$ ,  $\mu_Y$ ,  $\sigma_X$ ,  $\sigma_Y$ , and  $\rho$



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Joint Distributions

Probability

## Multivariate normal distribution

**Multivariate normal distribution:** Joint pdf

$$f(x_1, x_2, \dots, x_n) = \frac{1}{(2\pi)^{n/2} |\Sigma|^{1/2}} e^{-\frac{1}{2}(x-\mu)' \Sigma^{-1} (x-\mu)} \quad (x_1, x_2, \dots, x_n) \in \mathcal{A}$$

where

- random vector  $X = (X_1, X_2, \dots, X_n)'$
- vector of means  $\mu = (\mu_1, \mu_2, \dots, \mu_n)'$
- variance-covariance matrix  $\Sigma$
- $X = (x_1, x_2, \dots, x_n)'$
- $\Sigma^{-1}$  is the inverse of the variance-covariance matrix
- $|\Sigma|$  is the determinant of the variance-covariance matrix
- the support is

$$\mathcal{A} = \{(x_1, x_2, \dots, x_n) | -\infty < x_i < \infty, \text{ for } i = 1, 2, \dots, n\}$$

- the parameter space is

$$\Omega = \{(\mu, \Sigma) | \mu \in \mathcal{R}^n, \Sigma \text{ is an } n \times n \text{ positive-semidefinite matrix}\}$$

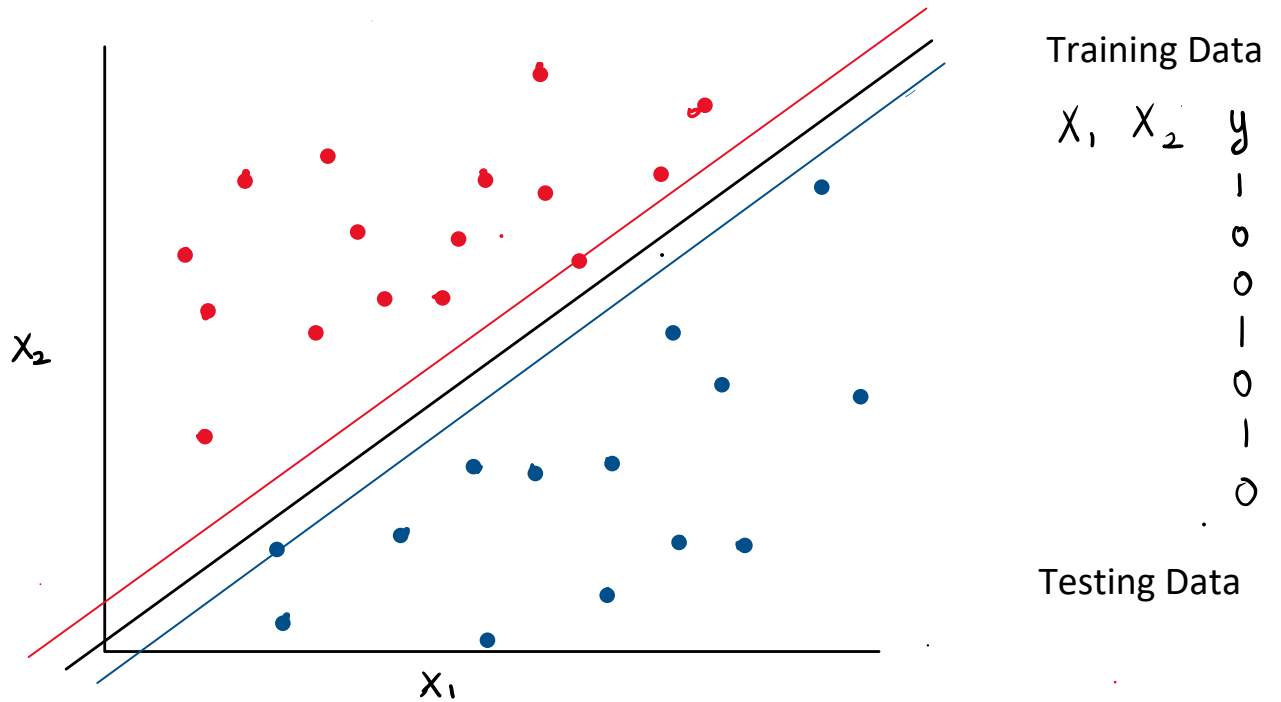
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Joint Distributions

Probability

$$\delta_i(x) = x \frac{\mu_i}{\sigma^2} - \frac{\mu_i^2}{2\sigma^2} + \log(P(C_i))$$

$$\delta_i(\bar{x}) = x^T \Sigma^{-1} \mu_i - \frac{1}{2} \mu_i^T \Sigma^{-1} \mu_i + \log(P(C_i))$$



params	plit0_test_score	plit1_test_score	plit2_test_score	plit3_test_score	plit4_test_score	mean_test_score	std_test_score	rank_test_score
{'C': 0.1}	0.705882	0.8125	0.875	0.875	0.6875	0.791176	0.0806615	5
{'C': 1}	0.705882	0.8125	0.875	0.875	0.75	0.803676	0.0673748	1
{'C': 0.5}	0.705882	0.8125	0.875	0.875	0.75	0.803676	0.0673748	1
{'C': 2}	0.705882	0.8125	0.875	0.875	0.75	0.803676	0.0673748	1
{'C': 3}	0.705882	0.8125	0.875	0.875	0.75	0.803676	0.0673748	1