$$\hat{y}^{(i)} = w^T x^{(i)} + b$$

$$L(y, \hat{y}) = \begin{cases} \frac{1}{2}(y - \hat{y})^2 & si \ |y - \hat{y}| \leq \delta \\ \delta(|y - \hat{y}| - \frac{1}{2}\delta) & si \ |y - \hat{y}| > \delta \end{cases}$$

$$\hat{y} = w_1 x_1 + w_2 x_2 + \dots + w_n x_n + b$$

$$\frac{\partial \hat{y}}{\partial w} = \frac{\partial}{\partial w_1} w_1 x_1 + \frac{\partial}{\partial w_2} w_2 x_2 + \dots + \frac{\partial}{\partial w_n} w_n x_n + 0$$

$$= x_1 + x_2 + \dots + x_n = x$$

$$\frac{\partial \hat{y}}{\partial b} = 1$$

ya que sabemos esto:

$$\frac{\partial L}{\partial w_j} = \frac{1}{2}\left[y - \hat{y}\right]^2 = \frac{2}{2}[y - \hat{y}]\frac{\partial \hat{y}}{\partial w}$$

$$= [y - \hat{y}]\, x \quad \text{para } |y - \hat{y}| \leq \delta$$

$$L(y, \hat{y}) = \delta(|y - \hat{y}| - \tfrac{1}{2}\delta)$$

Sabemos que:

$$\frac{d}{dx}|x| = \text{sign}(x) = \begin{cases} 1 & si \ x > 0 \\ -1 & si \ x < 0 \end{cases}$$

$$\frac{\partial L}{\partial w_j} = \frac{\partial}{\partial w_j}\delta|y - \hat{y}| - 0 = \delta \frac{\partial}{\partial w_j}|y - \hat{y}| = \delta\, \text{sign}(y - \hat{y}) \cdot \frac{\partial \hat{y}}{\partial w_j}$$

$$= \delta\, \text{sign}(y - \hat{y}) \cdot x \quad \text{para } |y - \hat{y}| > \delta$$

$$\frac{\partial L(y, \hat{y})}{\partial w_j} = \begin{cases} (y - \hat{y})\, x & si \ |y - \hat{y}| \leq \delta \\ \delta\, \text{sign}(y - \hat{y}) \cdot x \end{cases}$$

$$\frac{\partial L}{\partial b} = (y - \hat{y}) \quad \text{si } |y - \hat{y}| \leq \delta$$

$$\frac{\partial L}{\partial b} = \delta \, \text{sign}(y - \hat{y}) \quad \text{si } |y - \hat{y}| > \delta$$

$$\frac{\partial}{\partial b} L(y, \hat{y}) = \begin{cases} y - \hat{y} & \text{si } |y - \hat{y}| \leq \delta \\ \delta \, \text{sign}(y - \hat{y}) & \text{si } |y - \hat{y}| > \delta \end{cases}$$