probabilidad a Posteriori — Prob a priori — Verosimilitud

$$P(Y=y \mid X=x) = \frac{P(Y=y)\, P(X=x \mid Y=y)}{P(X=x)}$$

| | | |
|---|---|---|
| .3 | → | .3/2.7 |
| .9 | → | .9/2.7 |
| .8 | → | .8/2.7 |
| .7 | → | .7/2.7 |
| 2.7 | ↑ | 1 |

Normalizar

Si son estocásticamente independientes

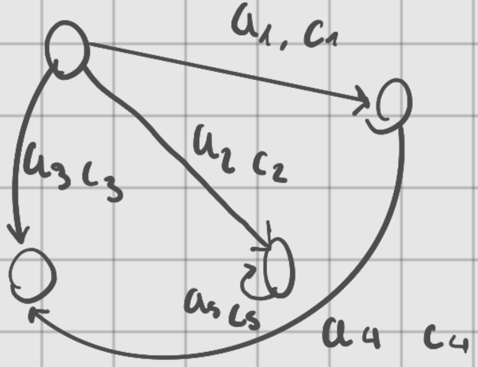
$$P(Y=y \mid X=x) = P(Y=y)$$

## Ley de la cadena

$$P(Y=y \mid X=x_1, x_2 \dots x_n)$$

$$P(X_1, X_2 \dots X_n) = P(X_1=x_1)\, P(X_2=x_2 \mid X_1=x_1, X_2=x_2) \dots P(X_n \mid X_{n-1}=x_{n-1} \dots X_1=x_1)$$

$$S_t = \text{transición}(S_{t-1}, a_t)$$




$S_{0:T} \to S_0, S_1 \dots S_T$

$A_{1:T} \to a_1, a_2 \dots a_T$

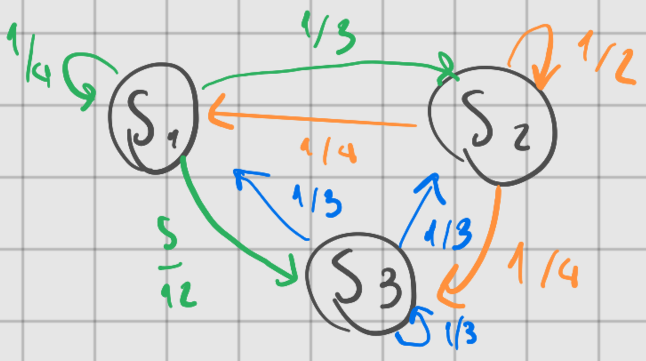
X es una V.A. donde
$Val(x) = \{S_1, S_2 \dots S_n\}$

$X_{0:T} \to X_0, X_1 \dots X_T$

No puedo calcularlo

$$P_r(x) = \begin{bmatrix} P[X_0=S_1] \\ \vdots \\ P[X_0=S_T] \end{bmatrix} \begin{bmatrix} P[X_1=S_1] \\ \vdots \\ P[X_1=S_T] \end{bmatrix} \dots \begin{bmatrix} P[X_T=S_1] \\ \vdots \\ P[X_T=S_T] \end{bmatrix}$$

Ya no podemos decir que estamos en un estado, pero podemos decir la probabilidad de pertenecer a un estado y podemos calcular la probabilidad de cambiar a un estado




$$\underset{P(X_0)}{\begin{bmatrix}1\\0\\0\end{bmatrix}} \qquad \underset{P(X_1)}{\begin{bmatrix}1/4\\1/3\\5/12\end{bmatrix}} \qquad \underset{P(X_2)}{\begin{bmatrix}1/4\cdot 1/4 + 1/4\cdot 1/3 + 1/3\cdot 5/12\\ 1/3\cdot 1/4 + 1/2\cdot 1/3 + 1/3\cdot 5/12\\ 5/12\cdot 1/4 + 1/4\cdot 1/3 + 1/3\cdot 5/12\end{bmatrix}}$$

$$P(X_1) = P(X_0)\,P(X_1|X_0)$$

$$P(X_2) = P(X_0)\,P(X_1|X_0)\,P(X_2|X_0,X_1) = P(X_1)\,P(X_2|X_1)$$

Proceso de Markov de primer orden

$$P(X_{t+1}|X_t, X_{t-1}, \dots, X_0) \equiv P(X_{t+1}|X_t)$$

$$P(R_{t+1}|R_t \cdots R_0)$$

No reducible

$$A_{t+1} = \begin{bmatrix}R_{t+1}\\R_t\\R_{t-1}\end{bmatrix} \quad A_t = \begin{bmatrix}R_t\\R_{t-1}\\R_{t-2}\end{bmatrix} \quad A_{t-1} = \begin{bmatrix}R_{t-1}\\R_{t-2}\\R_{t-3}\end{bmatrix}$$

Cualquier problema de Markov de orden k se puede hacer de orden 1

$$P(X_{t+1} = s_i \mid X_t = s_j) = P_{ij}$$

$$\begin{bmatrix} P_{11} = 1/4 & P_{12} = 1/3 & P_{13} = 5/12 \\ P_{21} = 1/4 & P_{22} = 1/2 & P_{23} = 1/4 \\ P_{31} = 5/12 & P_{32} = 1/3 & P_{33} = 1/3 \end{bmatrix}$$

$$P$$

$$P(X_1) = P \cdot P(X_0)$$

$$P(X_2) = P(X_1) = P^2 \, P(X_0)$$

$$\vdots$$

$$P(X_T) = P \cdot P(X_T) = P^T \, P(X_0)$$