Optimal Portfolio Using Factor Graphical Lasso

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Abstract

Graphical models are a powerful tool to estimate a high-dimensional inverse covariance (*precision*) matrix, which has been applied for portfolio allocation problem. The assumption made by these models is a sparsity of the precision matrix. However, when the stock returns are driven by the common factors, this assumption does not hold. Our paper develops a framework for estimating a high-dimensional precision matrix which combines the benefits of exploring the factor structure of the stock returns and the sparsity of the precision matrix of the factor-adjusted returns. The proposed algorithm is called *Factor Graphical Lasso* (FGL). We study a high-dimensional portfolio allocation problem when the asset returns admit the approximate factor model. In high dimensions, when the number of assets is large relative to the sample size, the sample covariance matrix of the excess returns is subject to the large estimation uncertainty, which leads to unstable solutions for portfolio weights. To resolve this issue, we consider the decomposition of low-rank and sparse components. This strategy allows us to consistently estimate the optimal portfolio in high dimensions, even when the covariance matrix is ill-behaved. We establish consistency of the portfolio weights in a high-dimensional setting *without assuming sparsity on the covariance or precision matrix of stock returns*. Our theoretical results and simulations demonstrate that FGL is robust to heavy-tailed distributions, which makes our method suitable for financial applications. The empirical application uses daily and monthly data for the constituents of the S&P500 to demonstrate superior performance of FGL compared to the equal-weighted portfolio, index and some prominent precision and covariance-based estimators.

Keywords: High-dimensionality, Portfolio optimization, Graphical Lasso, Approximate Factor Model, Sharpe Ratio, Elliptical Distributions

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