Question 2:

Manual Calculation

given probability density function

$$f(x) = \frac{1}{2\sqrt{2\pi}} e^{-\frac{(x-\theta)^2}{8}}$$

given measurements = 10, 13, 15 and 20

The likelihood function is the product of the individual probabilities:

$$L(\theta) = \prod_{i=1}^{n} f(x_i)$$

Taking the log-likelihood function:

$$\log L(\theta) = \sum_{i=1}^{n} \log f(x_i)$$

Substituting the given PDF:

$$\log L(\theta) = \sum_{i-1}^{n} \left(- \frac{(x_i - \theta)^2}{8} \right) + \sum_{i-1}^{n} \log \left(\frac{1}{2\sqrt{2\pi}} \right)$$

Since the second term does not depend on θ, maximizing $\log L(\theta)$ is equivalent to minimizing.

$$\sum_{i=1}^{n} (x_i - \theta)^2$$

This is minimized when:

$$\theta = \frac{1}{n} \sum_{i=1}^{n} x_i$$

Substituting the values:

$$\theta = \frac{10+13+15+20}{4} = \frac{58}{4} = 14.5$$

Thus the MLE estimate for θ is 14.5