

Q1) Manual Calculations

Given Data

$$x_1 = (1, 2, 0)^T \quad y_1 = +1$$

$$x_2 = (1, 2, 2)^T \quad y_2 = +1$$

$$x_3 = (1, 0, 2)^T \quad y_3 = -1$$

initial assignment of w is $[0, 6, 6]^T$

Iteration 1: Checking x_1

$$\begin{aligned} w^T x_1 &= (0, 6, 6) \cdot (1, 2, 0) \\ &= 0(1) + 6(2) + 6(0) \\ &= 12 \end{aligned}$$

$$\text{sign}(12) = +1 \quad \left(\begin{array}{l} \text{value is greater than threshold} \\ \text{and no need to update} \\ \text{since } y_1 = +1 \end{array} \right)$$

Iteration 2: Checking x_2

$$\begin{aligned} w^T x_2 &= (0, 6, 6) \cdot (1, 2, 2) \\ &= 0(1) + 6(2) + 6(2) \\ &= 24 \end{aligned}$$

$$\text{sign}(24) = +1 \quad \left(\begin{array}{l} \text{no need to update} \\ \text{since } y_2 = +1 \end{array} \right)$$

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$$x_1 = (1, 2, 0)^T \quad y_1 = +1$$

$$x_2 = (1, 2, 2)^T \quad y_2 = +1$$

$$x_3 = (1, 0, 2)^T \quad y_3 = -1$$

initial assignment of w is $[0, 6, 6]^T$

Iteration 1: Checking x_1

$$w^T x_1 = (0, 6, 6) \cdot (1, 2, 0)$$

$$= 0(1) + 6(2) + 6(0)$$

$$= 12$$

$\text{sign}(12) = +1$ (value is greater than threshold
and no need to update)
since $y_1 = +1$

Iteration 2: Checking x_2

$$w^T x_2 = (0, 6, 6) \cdot (1, 2, 2)$$

$$= 0(1) + 6(2) + 6(2)$$

$$= 24$$

$\text{sign}(24) = +1$ (no need to update)
since $y_2 = +1$

Iteration 3: Checking x_3

$$w^T x_3 = (0, 6, 6) \cdot (1, 0, 2)$$

$$= 0(1) + 6(0) + 6(2)$$

$$= 12$$

$$\text{sign}(12) = +1 \quad (\text{need to update since } y_3 = -1)$$

updating w

$$w = w + y_3 x_3$$

$$= (0, 6, 6) + (-1)(1, 0, 2)$$

$$= (0, 6, 6) + (-1, 0, -2)$$

$$= (0 - 1, 6 - 0, 6 - 2)$$

$$= (-1, 6, 4)$$

Iteration 4: Checking x_1 again

$$w^T x_1 = (-1, 6, 4) \cdot (1, 2, 0)$$

$$= -1(1) + 6(2) + 4(0)$$

$$= 11$$

$$\text{sign}(11) = +1 \quad (\text{no update needed, } y_1 = +1)$$

Iteration 5: Checking x_2 again

$$\omega^T x_2 = (-1, 6, 4) \cdot (1, 2, 2)$$

$$= -1(1) + 6(2) + 4(2)$$

$$= -1 + 12 + 8 = 19$$

$\text{sign}(19) = +1$ (no update needed $y_2 = +1$)

Iteration 6: Checking x_3 again

$$\omega^T x_3 = (-1, 6, 4) \cdot (1, 0, 2)$$

$$= -1(1) + 6(0) + 4(2)$$

$$= -1 + 0 + 8 = 7$$

$\text{sign}(7) = +1$ (incorrect, because $y_3 = -1$)

updating

$$\omega = \omega + y_3 x_3 = (-1, 6, 4) + (-1)(1, 0, 2)$$

$$= (-1-1, 6-0, 4-2)$$

$$= (-2, 6, 2)$$

Iteration 7: Checking x_1 again

$$\begin{aligned}w^T x_1 &= (-2, 6, 2) \cdot (1, 2, 0) \\&= (-2)(1) + (6)(2) + 2(0) \\&= -2 + 12 + 0\end{aligned}$$

$$= 10$$

$$\text{sign}(10) = +1 \quad (\text{no update needed})$$

Iteration 8: Checking x_2 again

$$\begin{aligned}w^T x_2 &= (-2, 6, 2) \cdot (1, 2, 2) \\&= (-2)(1) + 6(2) + 2(2)\end{aligned}$$

$$= -2 + 12 + 4$$

$$= 14$$

$$\text{sign}(14) = +1 \quad (\text{no update needed})$$

Iteration 9: Checking x_3 again

$$\begin{aligned}w^T x_3 &= (-2, 6, 2) \cdot (1, 0, 2) \\&= -2(1) + 6(0) + 2(2)\end{aligned}$$

$$= -2 + 0 + 4 = 2$$

$$\text{sign}(2) = +1 \quad (\text{need to update } y_3 = -1)$$

updating w

$$w = w + y_3 x_3$$

$$= (-2, 6, 2) + (-1)(1, 0, 2)$$

$$= (-2 - 1, 6 - 0, 2 - 2)$$

$$= (-3, 6, 0)$$

Iteration 10: Checking x_1 again

$$w^T x_1 = (-3, 6, 0) \cdot (1, 2, 0) = -3(1) + 6(2) + 0(0)$$

$$\text{sign}(q) = +1 \text{ (no update needed)}$$

Iteration 11: Checking x_2 again

$$w^T x_2 = (-3, 6, 0) \cdot (1, 2, 2) = -3(1) + 6(2) + 0(2)$$

$$\text{sign}(q) = +1 \text{ (no update needed)}$$

Iteration 12: Checking x_3 again

$$w^T x_3 = (-3, 6, 0) \cdot (1, 0, 2) = -3(1) + 6(0) + 0(2)$$

$$\text{sign}(-3) = -1 \text{ (no update needed)}$$

hence the final weight vector, $w = (-3, 6, 0)$