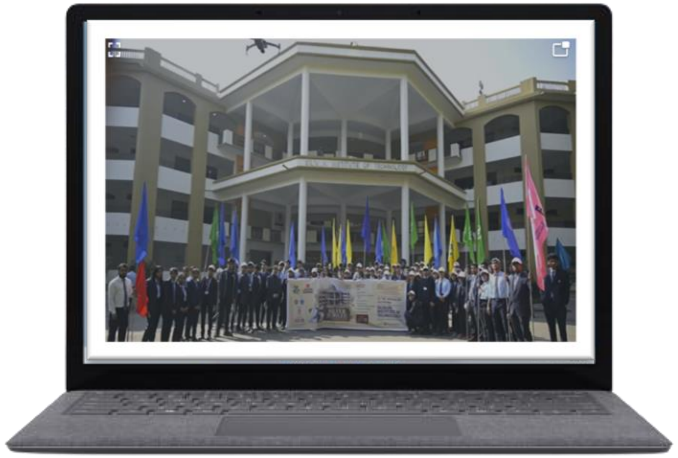
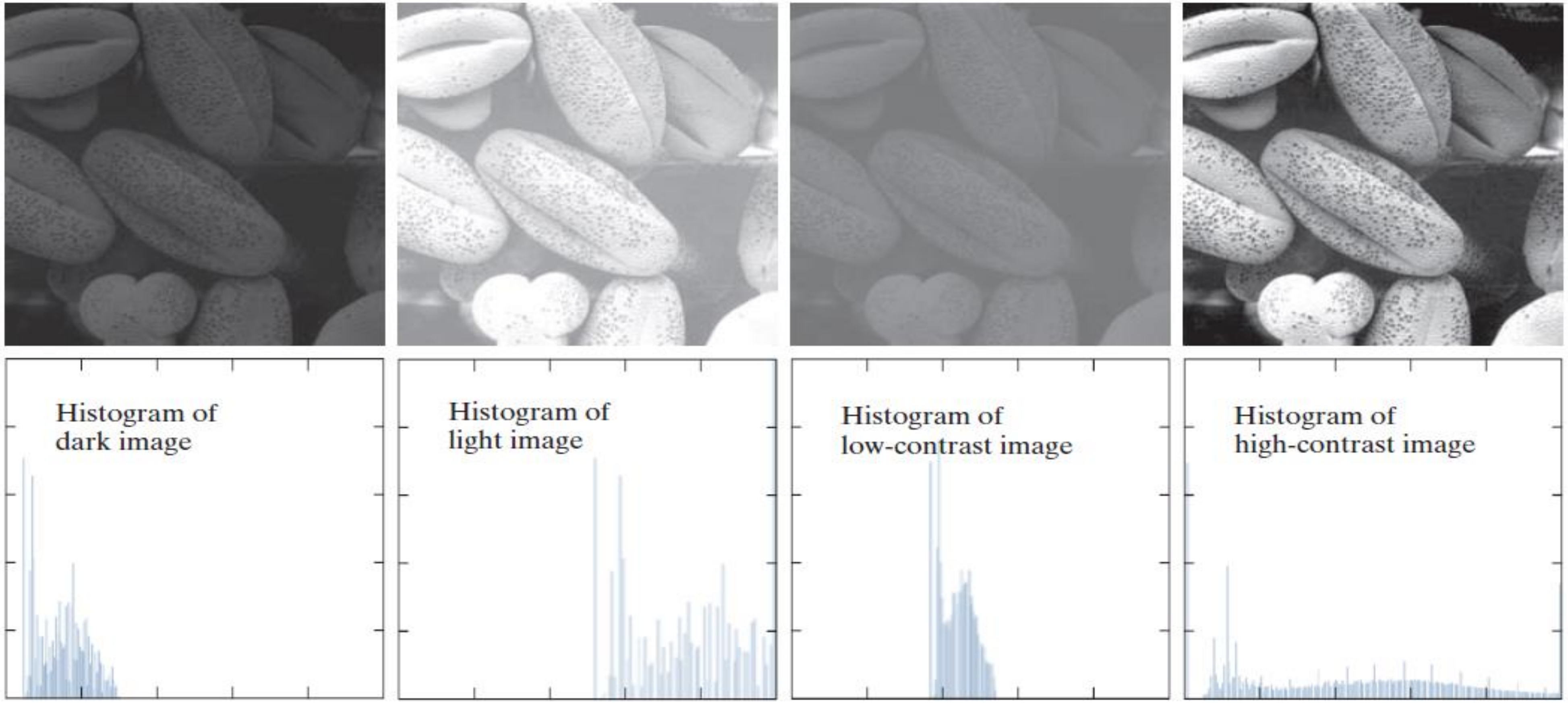


Histogram Processing, Equalization, Matching

Image Histogram

- Let r_k, for $k = 0,1,2, \dots . L - 1$ denote the intensities of an L level digital image $f(x, y)$.
- For 4-level digital image we get r_0, r_1, r_2, r_3.
- The unnormalized histogram of f is defined as $h(r_k) = n_k$, where n_k is the number of pixels in f with the intensity r_k, and the subdivision of the intensity scale are called histogram bins.
- The normalized histogram of f is defines as $p(r_k) = \frac{n_k}{MN}$, where M and N are the number of rows and columns in the image.

Image Histogram



Abcd Four image types and their corresponding histograms. (a) dark; (b) light; (c) low contrast; (d) high contrast. The horizontal axis of the histograms are values of r_k and the vertical axis are values of $p(r_k)$.

Image Histogram

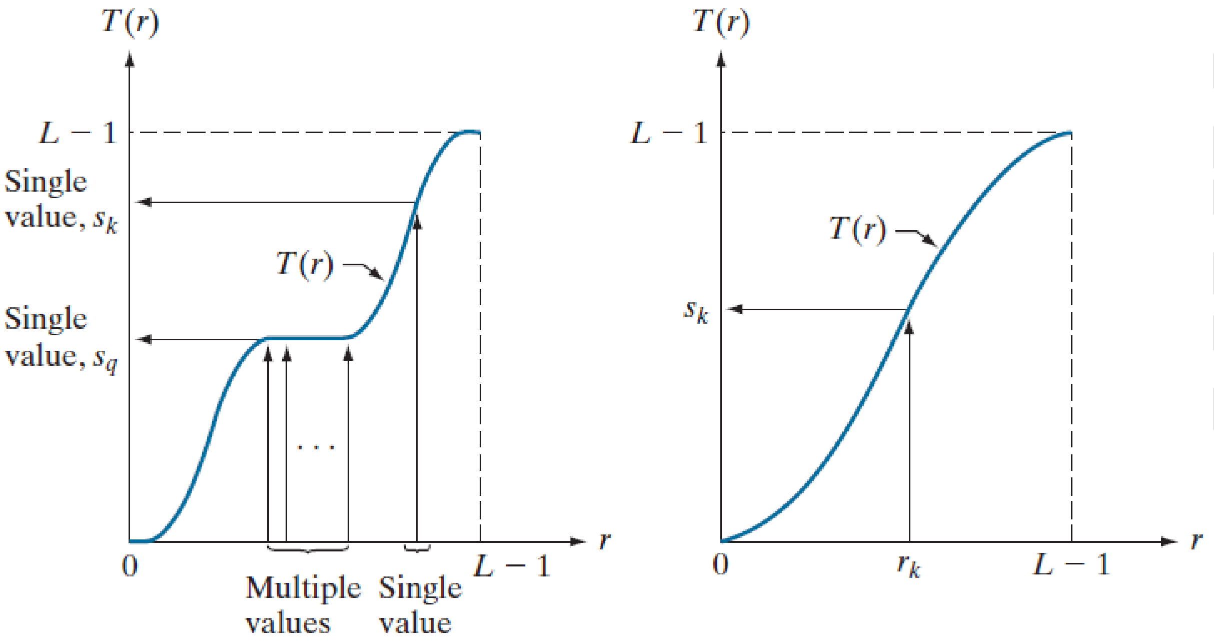
- Image histogram refers normalized histogram
- Histogram shape is related to image appearance
- High contrast image cover a wide range of the intensity scale and distribution of pixels is not too far from uniform.
- Developing an transformation function that can achieve this effect automatically, using only the histogram of an image will help to enhance image.

Histogram Equalization

- Assuming initially continuous intensity values, let the variable r denote the intensities of an image to be processed. As usual, we assume that r is in the range $[0, L-1]$, with $r = 0$ representing black and $r = L - 1$ presenting white.
- The transformations (intensity mappings) function of the form $s = T(r), 0 \leq r \leq L-1$, produce an output intensity value s, for a given intensity value r in the input image.

Histogram Equalization

- $T(r)$ is a monotonic increasing function in the interval $0 \le r \le L-1$
- $0 \le T(r) \le L-1$ for $0 \le r \le L-1$
- If we required inverse transformation then the function $T(r)$ should be strictly monotonic increasing function in the interval $0 \le r \le L-1$



Histogram Equalization

- Let $p_r(r)$ and $p_s(s)$ denote the PDF of intensity values r and s in two different image.
- A fundamental result from probability theory is that if $p_r(r)$ and $T(r)$ known, and $T(r)$ is continuous and differentiable over the range of values of interest, then the PDF of transformation variable s can be obtained as $p_s(s) = p_r(r)\frac{dr}{ds}$
- The PDF of the output intensity variable, s, is determined by the PDF of the input intensities and the transformation function used.

Histogram Equalization

- A transformation function of particular importance in image processing is $s = T(r) = (L-1)\int_0^r p_r(w)dw$
- The integral on the right side is the CDF of the random variable r.
- Leibniz rule says that the derivative of a definite integral with respect to its upper limit is the integrated evaluates at the limit

$$\frac{dr}{ds} = \frac{dT(r)}{dr} = (L-1)\frac{d}{dr}\left(\int_0^r p_r(w)dw\right) = (L-1)p_r(r)$$

Histogram Equalization

- For discrete values $p_r(r_k) = \frac{n_k}{MN}$ it refers to the normalized histogram
- The discrete form of transformation

$$s_k = T(r_k) = (L-1)\sum_{\{j=0\}}^{k} p_r(r_j), k = 0,1,2.3, \dots, L$$

A processed image is obtained by using above equation to map each pixel in the input image with intensity r_k into a corresponding pixel with level s_k in the output image.

This is this is called histogram equalization or histogram linearization.

Histogram Equalization

- Take an image
- Calculate pdf of the intensity values
- Calculate cdf of the intensity values
- Calculate the discrete form of the cdf
- Round off discrete values and map to the new intensity values

Histogram Equalization

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6	6	7	7	6
5	2	2	3	4
3	3	4	4	5
5	7	3	6	2
7	6	5	5	4

↓

6	6	7	7	6
4	1	1	2	3
2	2	3	3	5
4	7	2	6	1
7	6	4	4	3

Intensity values	frequencies	Pdf	cdf	(L-1)*cdf	Histogram equal level
0	0	0	0	0	0
1	0	0	0	0	0
2	3	0.12	0.12	0.84	1
3	4	0.16	0.28	1.96	2
4	4	0.16	0.44	3.08	3
5	5	0.20	0.64	4.48	4
6	5	0.20	0.84	5.88	6
7	4	0.16	1	7	7

Histogram Equalization

4	4	4	4	4
3	4	5	4	3
3	5	5	5	3
3	4	5	4	3
4	4	4	4	4

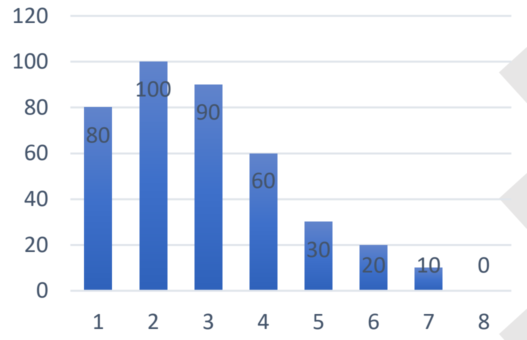
5	5	5	5	5
2	5	7	5	2
2	7	7	7	2
2	5	7	5	2
5	5	5	5	5

Intensity values	frequencies	Pdf	cdf	(L-1)*cdf	Histogram equal level
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	6	0.24	0.24	1.68	2
4	14	0.56	0.80	5.6	5
5	5	0.20	1	7	7
6	0	0	1	7	7
7	0	0	1	7	7

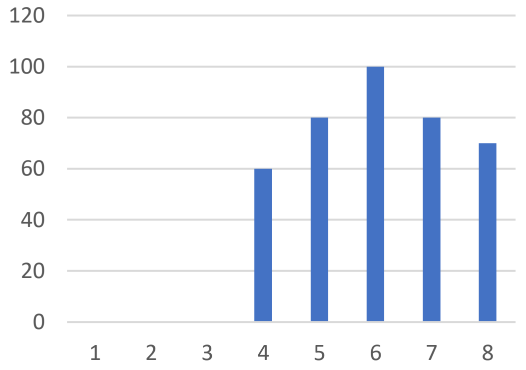
Histogram Matching (Specification)

- Given below are two histograms (i) and (ii). Modify the histogram (i) as given by histogram (ii).

Gray Level	0	1	2	3	4	5	6	7
No of pixel	80	100	90	60	30	20	10	0



Gray Level	0	1	2	3	4	5	6	7
No of pixel	0	0	0	60	80	100	80	70



Histogram Matching (Specification)

Gray levels	m_k	$p(r_k) = m_k/n$	s_k	$s_k * (L-1)$	Histogram Equalization	New
0	80	0.20	0.20	1.4	1	80
1	100	0.25	0.45	3.15	3	100
2	90	0.23	0.68	4.76	5	90
3	60	0.15	0.83	5.81	6	90
4	30	0.07	0.90	6.3	6	-
5	20	0.05	0.95	6.65	7	30
6	10	0.02	0.97	6.79	7	-
7	0	0	0.97	6.79	7	-

N=390

Histogram Matching (Specification)

Gray levels	m_k	$p(r_k) = m_k/n$	s_k	$s_k * (L-1)$	Histogram Equalization	New
0	0	0	0	0	0	
1	0	0	0	0	0	
2	0	0	0	0	0	
3	60	0.15	0.15	1.05	1	
4	80	0.20	0.35	2.45	2	
5	100	0.25	0.6	4.2	4	
6	80	0.20	0.8	5.6	6	
7	70	0.17	0.97	6.79	7	

Histogram Matching (Specification)

Desired Histogram

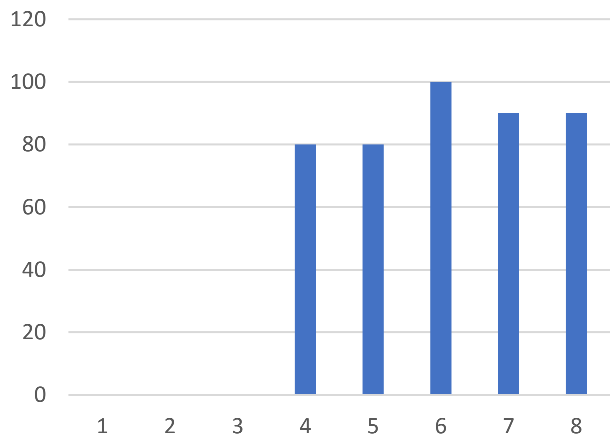
Gray Level	n_k
0	0
1	0
2	0
3	1
4	2
5	4
6	6
7	7

Equalized Histogram

Gray Level	n_k
0	0
1	80
2	0
3	100
4	0
5	90
6	90
7	30

Histogram Matching (Specification)

Gray Level	0	1	2	3	4	5	6	7
No of pixel	0	0	0	80	80	100	90	90



Reference

- E Woods, Richard, and Rafael C Gonzalez. "Digital image processing." (2008).