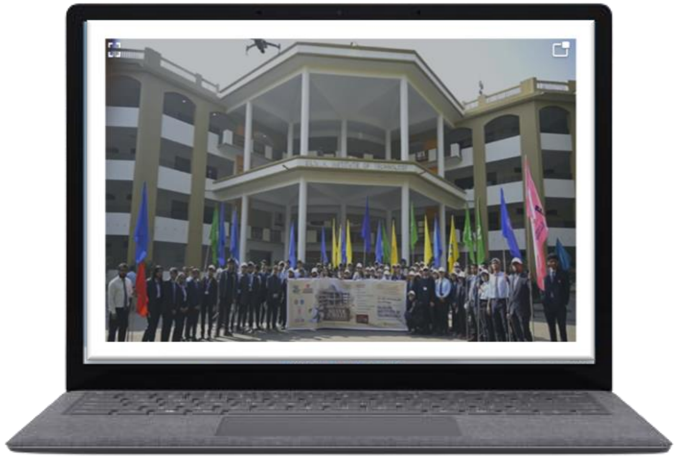


# Image Enhancement in Frequency Domain

Fourier Series, Fourier transform, Inverse Fourier Transform, Filtering, Low Pass Filtering, High Pass filtering

# FOURIER SERIES AND TRANSFORM

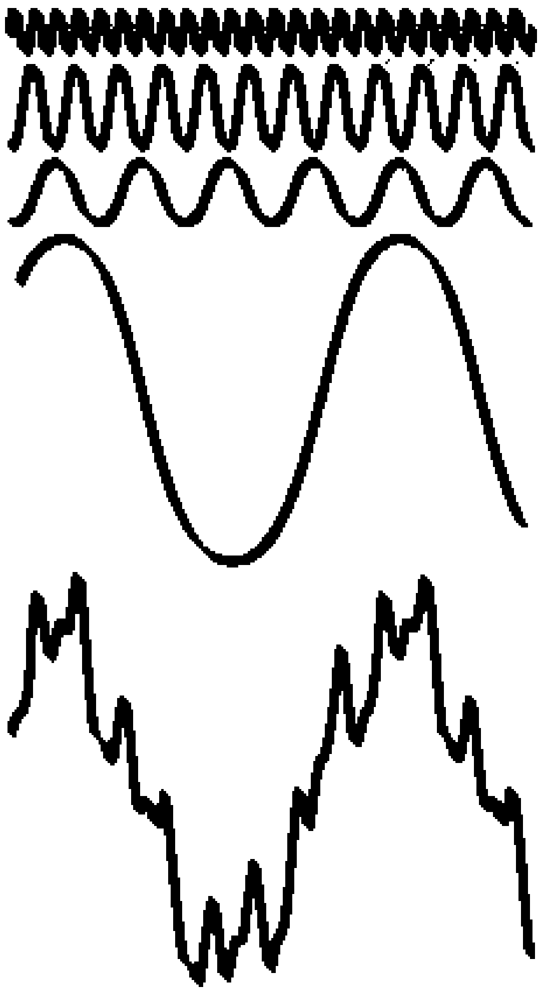
- The French mathematician Jean Baptiste Joseph Fourier was born in 1768 in the town of Auxerre, about midway between Paris and Dijon.
- The contribution for which he is most remembered was outlined in a memoir in 1807, and later published in 1822 in his book, La Théorie Analitique de la Chaleur (The Analytic Theory of Heat).

# FOURIER SERIES AND TRANSFORM

- Fourier's contribution state that any periodic function can be expressed as the sum of sines and cosines of different frequencies, each multiplied by a different co-efficient. it is called sum of series.
- Functions that are not periodic (but areas under the curve is finite) can be expressed as the integral of sines and cosines multiplied by a weighting function. It is called Fourier transform.
- both share important characteristics that a function can be expressed in either Fourier series or transform and can be reconstructed completely via an inverse process with no loss of information.
- this allows us to work in Fourier(frequency) domain and return back to the original domain.

# FOURIER SERIES AND TRANSFORM

The function at the bottom is the sum of the four functions above it. Fourier's idea in 1807 that periodic functions could be represented as a weighted sum of sines and cosines was met with skepticism.

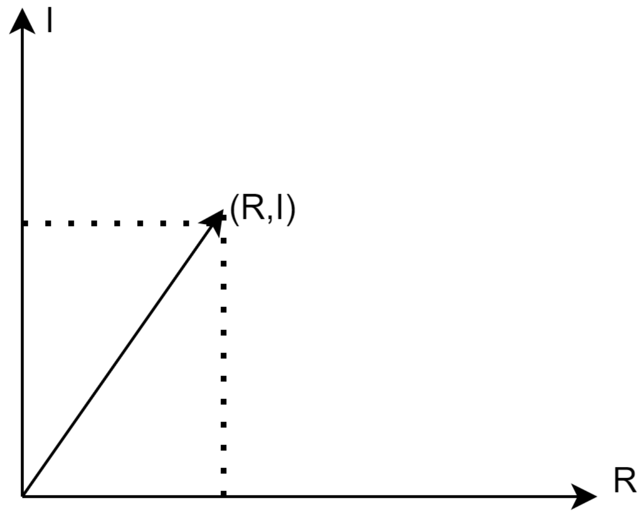


# Complex Numbers

- A complex number $C$ is defined as $C = R + jI$
- Where $R$ and $I$ are the real numbers and $j = \sqrt{-1}$ . Here $R$ denotes the real part and $I$ is the imaginary part of the complex number.
- Real numbers are the subset of complex number numbers with imaginary part $I = 0$.
- The conjugate of $C$ is denoted by $C^*$, is defined as $C = R - jI$

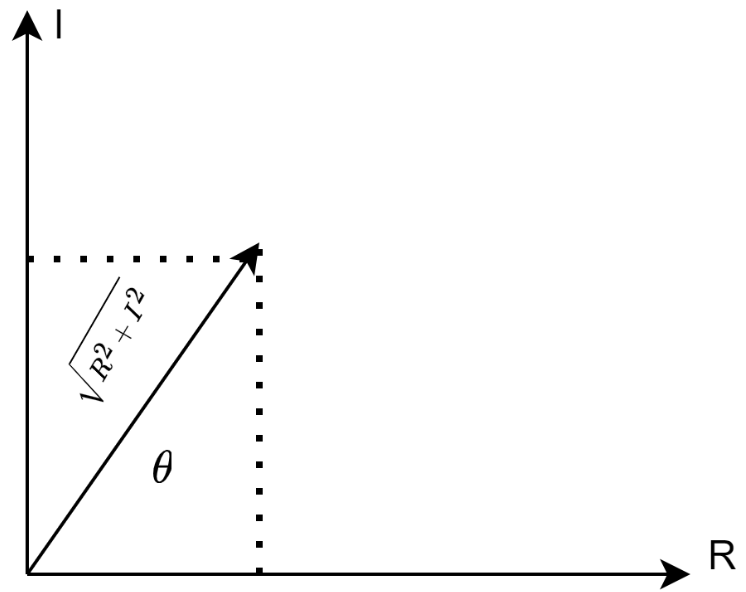
# Complex Number

- A complex number can be viewed geometrically as a point on plane (called complex plane) whose abscissa is the real axis and imaginary part is the real axis.
- The complex number is a point $(R, I)$ in the coordinate system of the complex plane.



# Polar Coordinate

- Sometimes it is useful to represent number in polar coordinates $C = |C|(cos\theta + j\ sin\theta)$
- Where $|C| = \sqrt{R^2 + I^2}$, is the length of the vector extending from the origin of the complex plane to the point $(R, I)$, and $\theta$is the angle between the real axis and the vector.



# Complex number

- Using Euler formula $e^{j\theta} = cos\theta + j\sin\theta$ where $e = 2.71828$
- Gives the following familiar representation of complex numbers in polar coordinates $C = |C|e^{j\theta}$ where $|C|$ and $\theta$ defined as earlier.
- Example: A complex number $1 + j2$, the polar representation of the complex number is $\sqrt{5}e^{j\theta}$ or $\sqrt{5}(cos\theta + jsin\theta)$

# Complex Function

- A complex function $F(u)$ of a real number $u$ can be expressed as the sum $F(u) = R(u) + jI(u)$, where $R(u)$ and $I(u)$ are real and imaginary components of the function $F(u)$
- The complex conjugate is $F^*(u) = R(u) - jI(u)$

# Fourier Series

- A function $f(t)$ of a continuous variable $t$ that is periodic with a period $T$, can be expressed as the sum of cosines and sines multiplied by appropriate co-efficients.
- This sum is known as the Fourier series has the form
- $f(t) = \sum_{n=-\infty}^{\infty} C_n e\hat{}(\frac{j2\pi n}{T} t)$
- Where $C_n = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{\frac{j2\pi n}{T} t} dt$ for $n = 0, \pm 1, \pm 2, \ldots\ldots$

# Fourier Transform

- The Fourier transform of a continuous function $f(t)$ of a continuous variable $t$, denoted as $F(u)$, is defined by the equation
- $\mathrm{F}(u) = \int_{-\infty}^{\infty} f(t)e^{-j2\pi ut}dt$
- Conversely, given $F(u)$, we obtain $f(t)$ back using the inverse Fourier transform, written as
- $f(t) = \int_{-\infty}^{\infty} F(u)e^{(j2\pi ut)}du$
- $f(t) \leftrightarrow F(u), F(u)$ is the Fourier transform of the function $f(t)$ and $f(t)$ can be return back after applying inverse Fourier transform of $F(u)$

# Fourier Transform

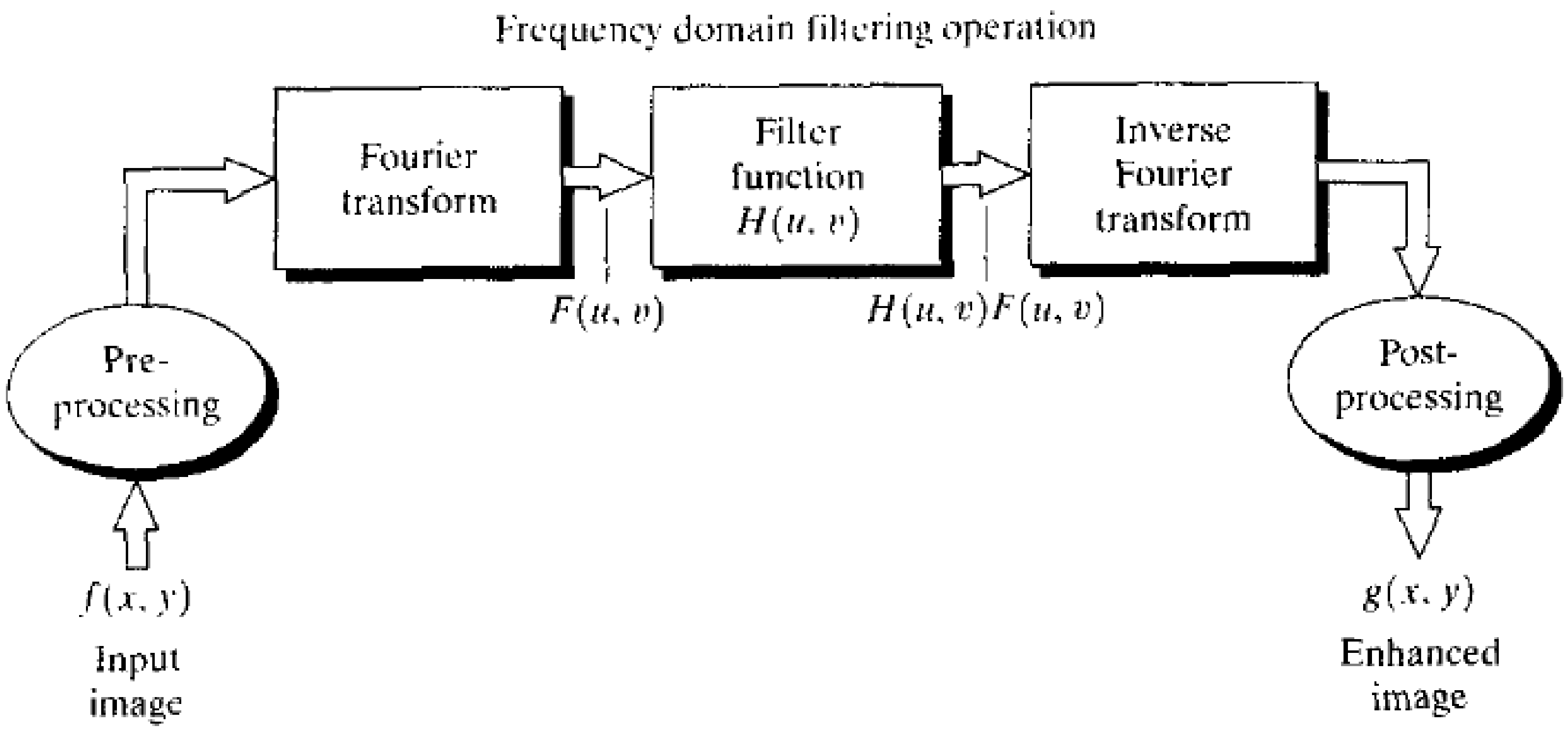
- The Fourier transform equation can be extended for two variables $u$ and $v$:
- $F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t, z) e^{(-j2\pi(ut+vz))} dtdz$
- And the inverse transform function is
- $\mathrm{f}(t, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{(j2\pi(ut+vz))} dudv$

# Basics of Filtering in the frequency domain

- Filtering in the frequency domain is straightforward. It consists of the following steps:

1. multiply the input image by $(-1)^{x+y}$ to center the transform.
2. Computed $F(u, v)$, the DFT of the image from (1)
3. Multiply $F(u, v)$ by a filter function $H(u, v)$
4. Compute the inverse DFT of the result in (3)
5. Obtain the real part of the result in (4)
6. Multiply the result in (5) by $(-1)^{x+y}$.

# Filtering in the frequency domain



# Characteristics of frequency domain

- Each term of $F(u, v)$ contains all values of $f(x, y)$, modified by the values of the exponential terms.
- It is impossible to make direct associations between specific component of an image and its transform.
- It can be shown that slowest varying frequency component is proportional to the average intensity of an image.
- The low frequencies corresponds to the slowly varying intensities of an image
- The high frequencies begin to correspond to faster and faster intensity.

# Low Pass Filter and High Pass filter

- A function that attenuates high frequencies while passing low frequencies (called low pass filter) would blur an image.
- A filter with opposite properties would enhance sharp details.
- Three types of filters are considered: ideal, butterworth and Gaussian
- These three categories cover the range from very sharp(ideal) to very smooth (Gaussian) filtering.

# Low Pass Filter and High Pass filter

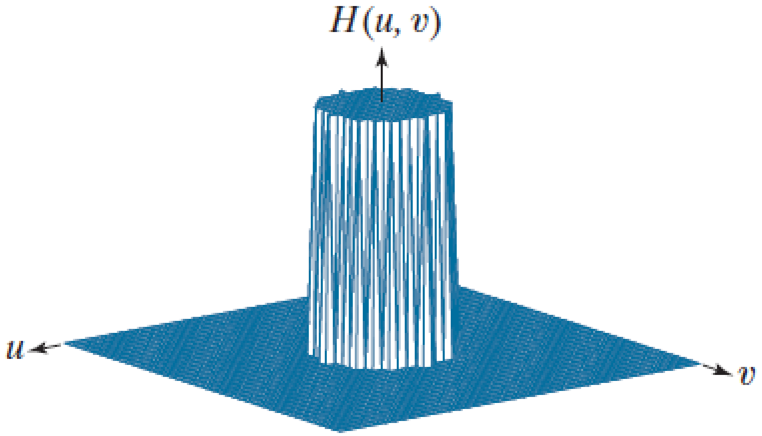
- The shape of butterworth filter is controlled by a parameter is called the filter order.
- For large value of this parameter, the butterworth filter approaches to ideal filter
- For lower value of this parameter, butterworth filter is more like a Gaussian filter

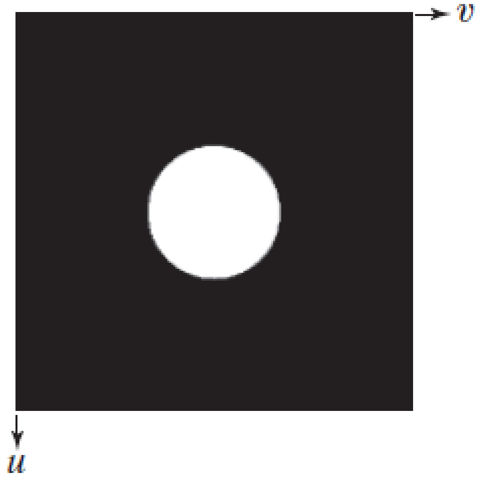
# Ideal Lowpass Filter

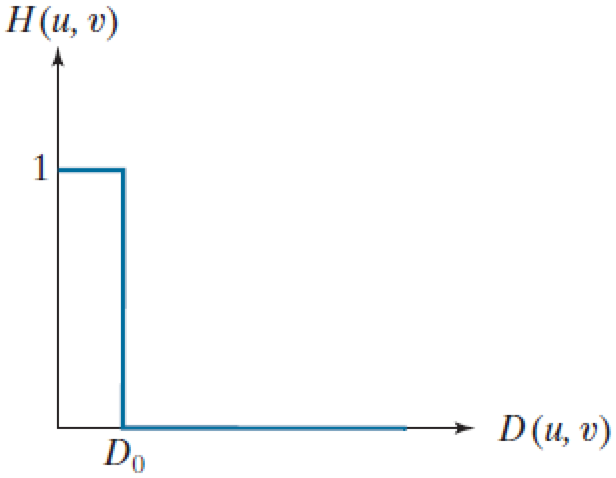
- Cutoff all high frequency components of the Fourier Transform that are at a distance greater than a specified distance $D_0$ from the origin of the transform.
- The transfer function of two dimensional transfer function is
- $H(u, v) = \begin{cases} 1 \ \ if \ D(u, v) \leq D_0 \\ 0 \ \ if \ D(u, v) > D_0 \end{cases}$
- $D_0$ is a specified nonnegative quantity and $D(u, v)$ is the distance from the point $(u, v)$ to the origin of the frequency rectangle.
- The distance from any point $(u, v)$ to the center of the Fourier transform is given by $D(u, v) = [(u - M/2)^2 + (v - N/2)^2]^{1/2}$

# Ideal Lowpass Filter

- The name ideal filter indicates that all frequencies inside a circle of radius $D_0$ are passed with no attenuation, whereas all frequencies outside the circle are completely attenuated.



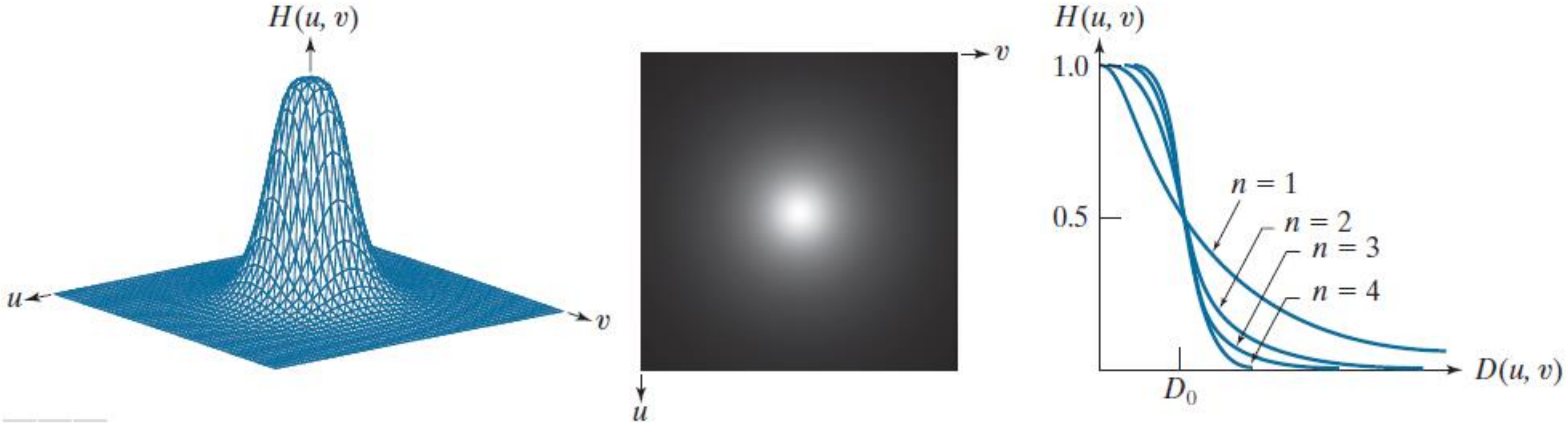



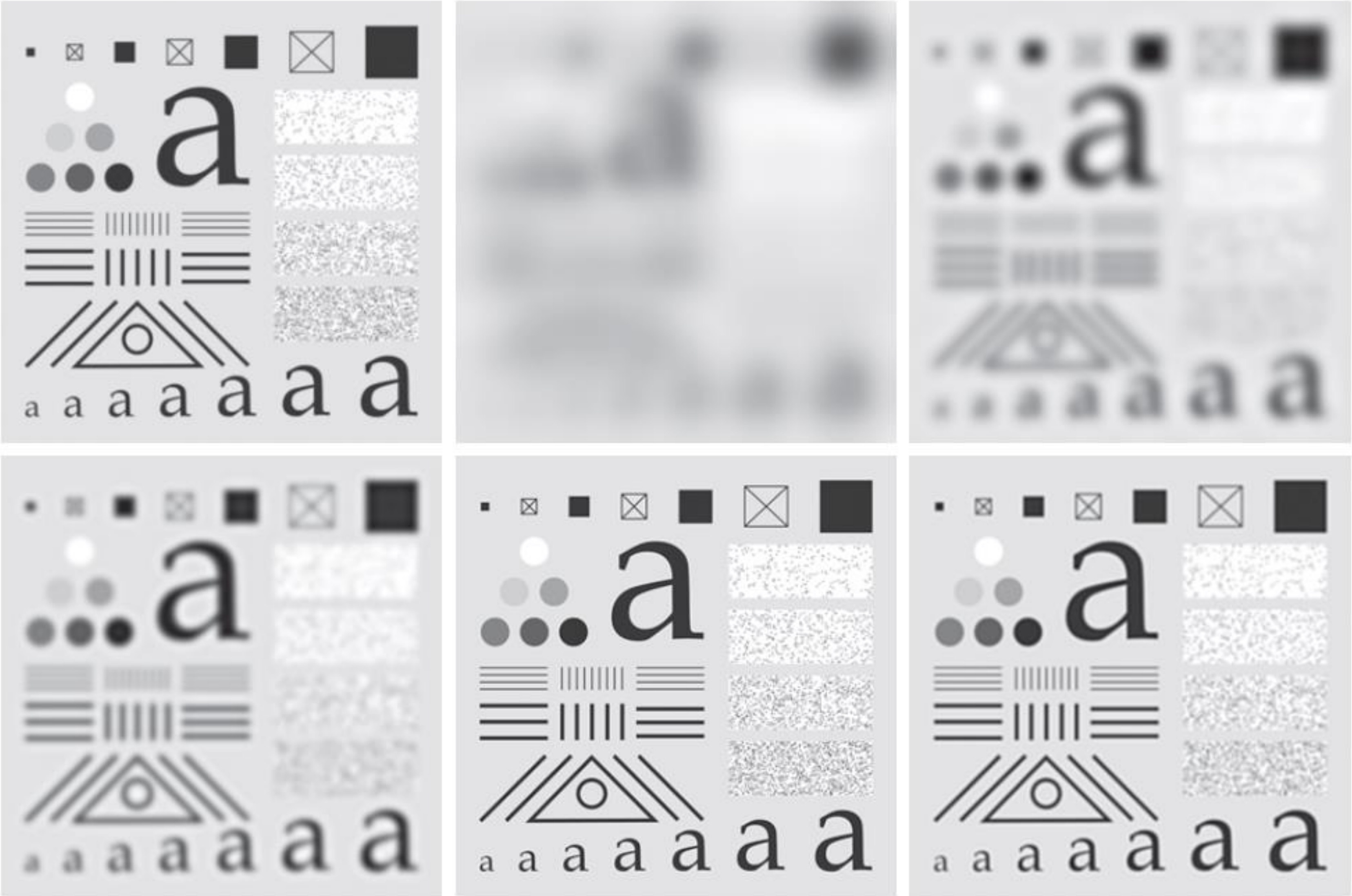

abc
def

(a) Original image of size 688 × 688 pixels. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Figure. The power removed by these filters was 13.1, 7.2, 4.9, 2.4, and 0.6% of the total, respectively. We used mirror padding to avoid the black borders characteristic of zero padding

# Butterworth Low Pass Filter

- The transfer function of butterworth low pass filter of order $n$, and with cutoff frequency at a distance $D_0$ from the origin, is defined as
- $H(u,v) = \frac{1}{1+[D(u,v)/D_0]^{2n}}$

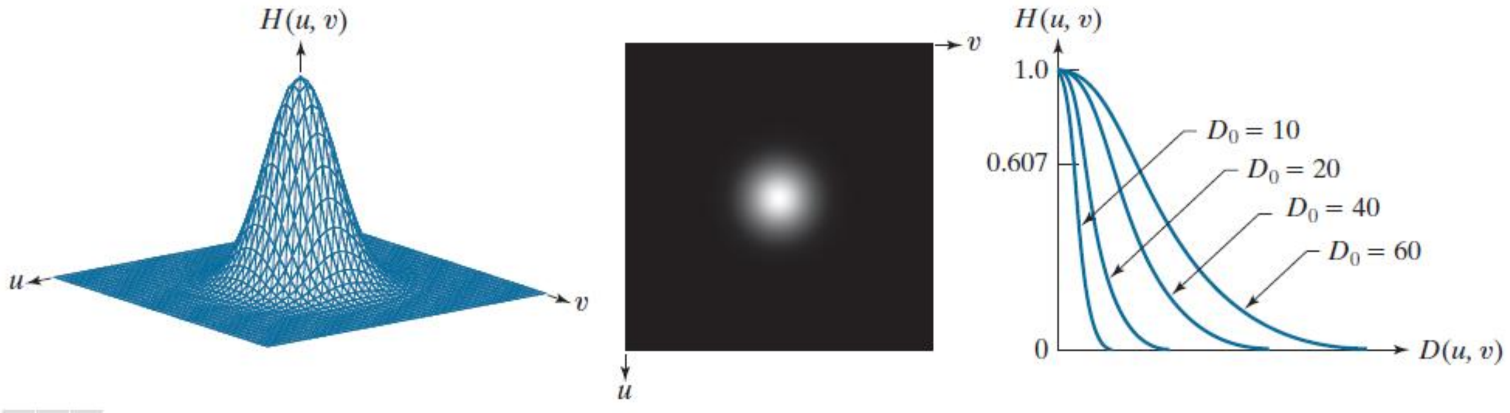


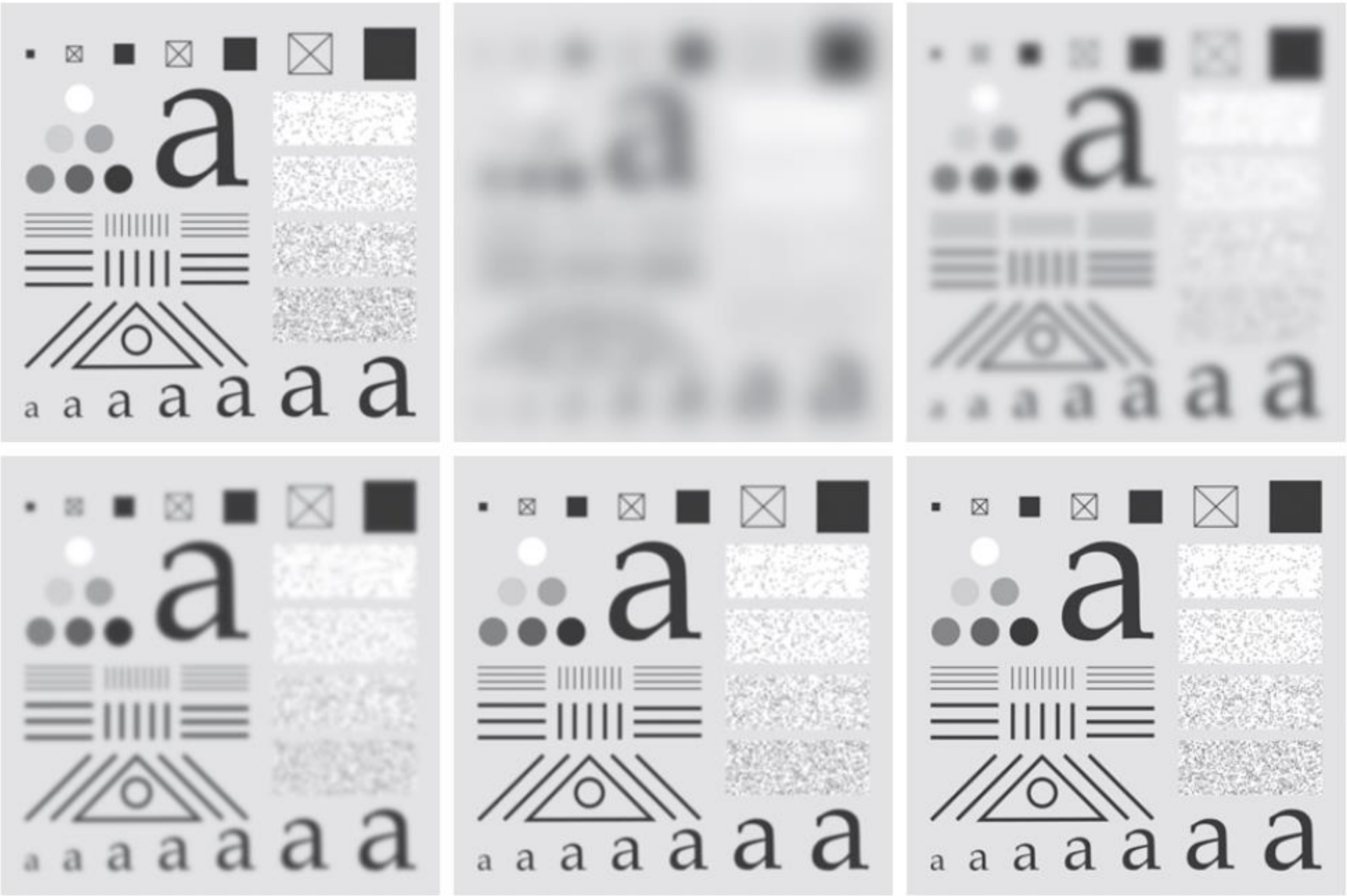


abc def Original image of size 688 × 688 pixels. (b)–(f) Results of filtering using BLPFs with cutoff frequencies at the radii shown in Figure IDLP and n = 2.25.

# Gaussian Lowpass Filter

- The transition function of Gaussian Lowpass filter is defined as
- $H(u,v) = e^{-D^2(u,v)/2\sigma^2}$
- Where $\sigma$ is the measure of spread of the Gaussian curve. By letting $\sigma = D_0$, the transition function can be written as
- $H(u,v) = e^{-D^2(u,v)/2D_0^2}$

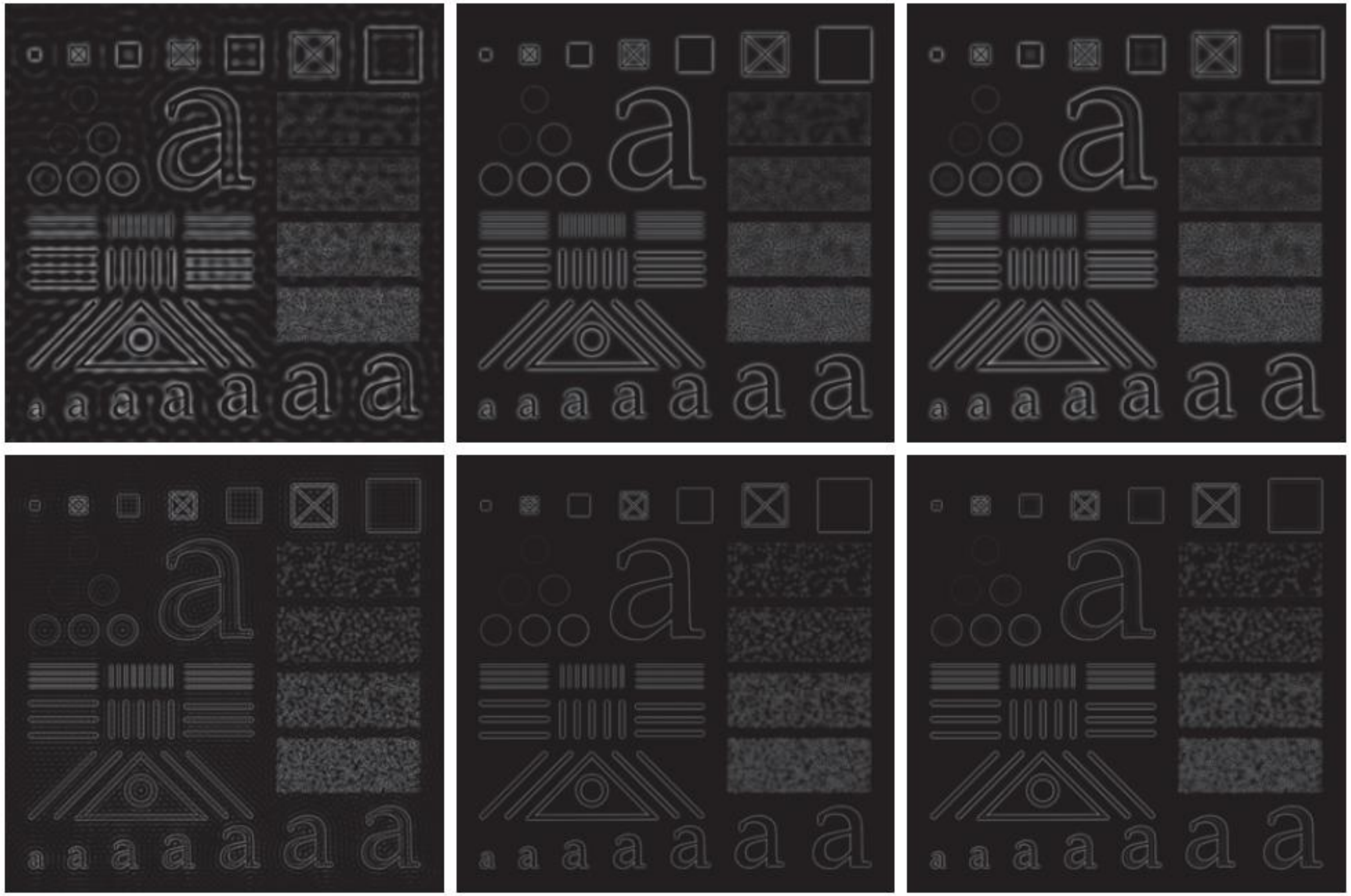




abc (a) Original image of size 688 · 688 pixels. (b)–(f) Results of filtering using GLPFs with cutoff frequencies
def at the radii shown in Figure of IDLP

# Image Sharpening using Highpass Filter

- Subtracting the lowpass transfer function from 1 yields the corresponding highpass filter transfer function in the frequency domain:
- $H_{HP}(u,v) = 1 - H_{LP}(u,v)$



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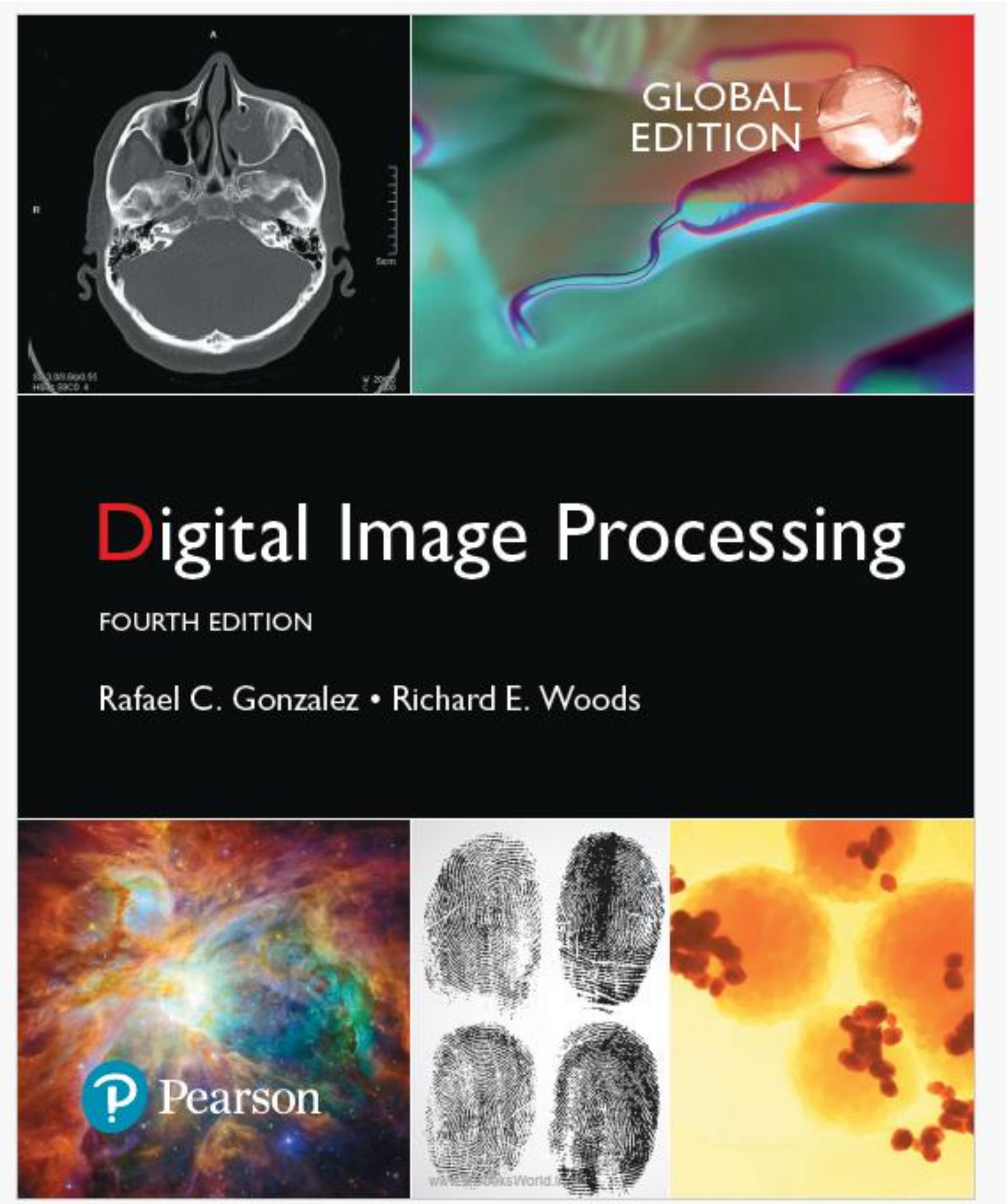
Top row: The image from IDLF filtered with IHPF, GHPF, and BHPF transfer functions using D0 = 60 in all cases (n = 2 for the BHPF). Second row: Same sequence, but using D0 = 160

# Reference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