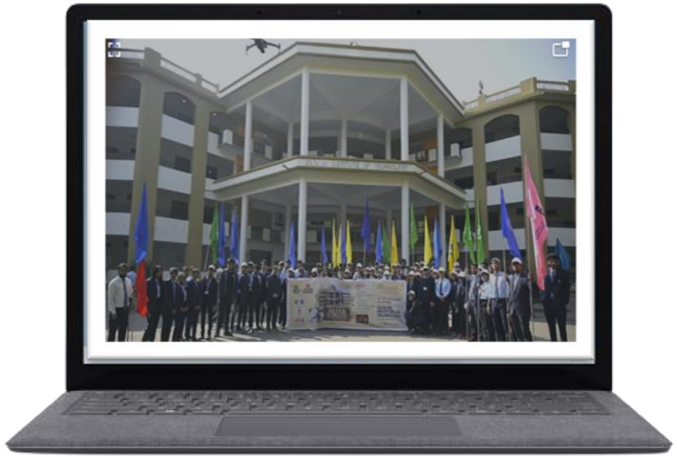
  

Spatial Filtering

Mechanism of Linear Spatial Filtering, Correlation, Convolution, Box filter, Gaussian Filter, Order Statistics Non Linear Filter

Fundamentals of Spatial Filtering

- Spatial filtering is used in a broad spectrum of image processing applications
- a solid understanding of filtering principles is important.
- the filtering examples in this section deal mostly with image enhancement.
- Other applications of spatial filtering are discussed in later chapters.

Fundamentals of Spatial Filtering

- In frequency domain processing "filtering" refers to passing, modifying, or rejecting specified frequency components of an image.
- For example, a filter that passes low frequencies is called a lowpass filter.
- The net effect produced by a lowpass filter is to smooth an image by blurring it.
- similar smoothing can be directly accomplished on the image itself by using spatial filters.
- Spatial filtering modifies an image by replacing the value of each pixel by a function of the values of the pixel and its neighbors.

The Mechanics Of Linear Spatial Filtering

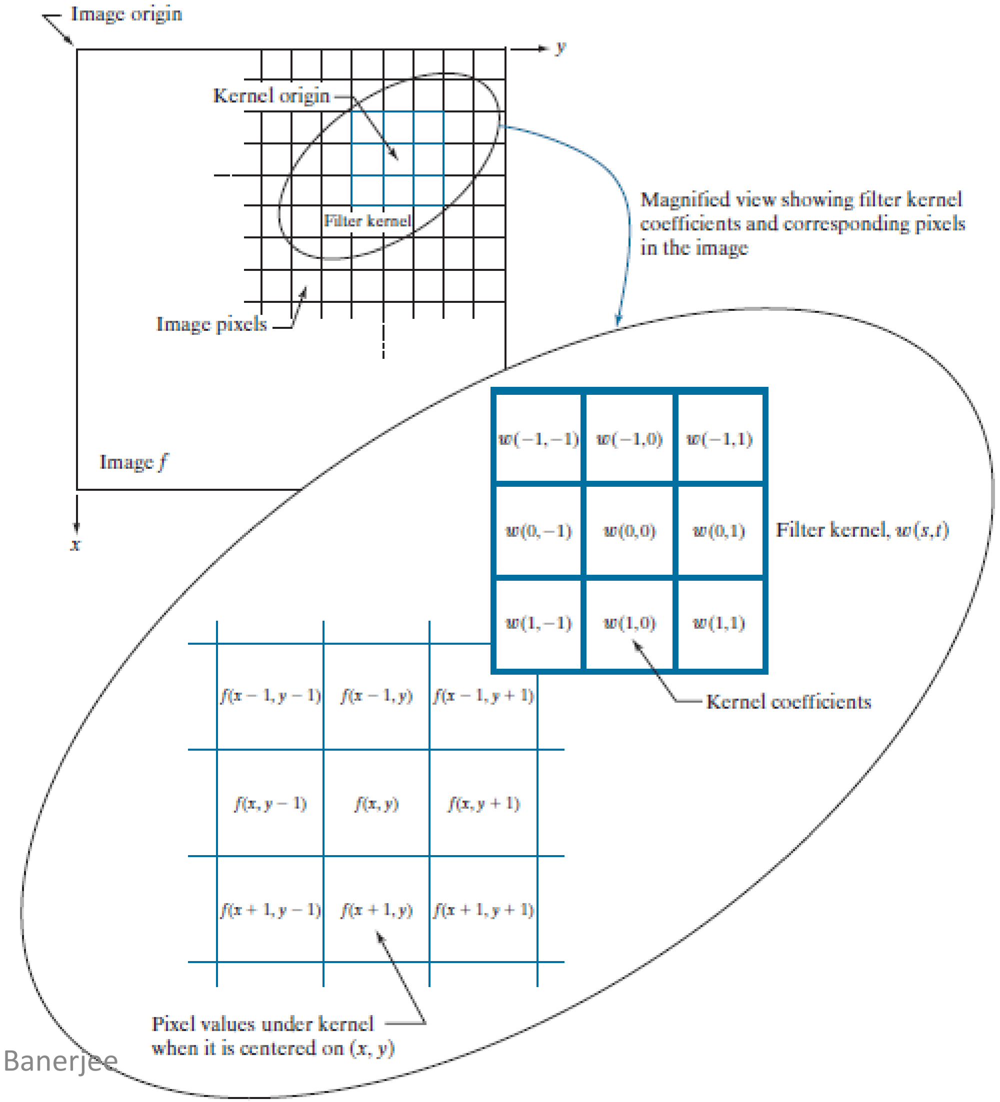
- A linear spatial filter performs a sum-of-products operation between an image f and a filter kernel, w.
- The kernel is an array whose size defines the neighborhood of operation, and whose coefficients determine the nature of the filter.
- Other terms used to refer to a spatial filter kernel are mask, template, and window.

The Mechanics Of Linear Spatial Filtering

- At any point (x, y) in the image, the response, $g(x, y)$, of the filter is the sum of products of the kernel coefficients and the image pixels encompassed by the kernel
- $g(x,y) = w(-1,-1)f(x-1,y-1) + w(-1,0)f(x-1,y) + \cdots + w(0,0)f(x,y) + \cdots + w(1,1)f(x+1,y+1)$
- For a mask of size $m \times n$, we assume that $m = 2a + 1$ and $n = 2b + 1$, where a and b are nonnegative integers.
- linear spatial filtering of an image of size $M \times N$ with a kernel of size $m \times n$ is given by the expression
- $g(x,y) = \sum_{\{s=-a\}}^{\{a\}} \sum_{\{t=-b\}}^{b} w(s,t) f(x+s, y+t)$
- All this says is that our focus in the following discussion will be on masks of odd sizes, with the smallest meaningful size being 3×3 .

The Mechanics Of Linear Spatial Filtering

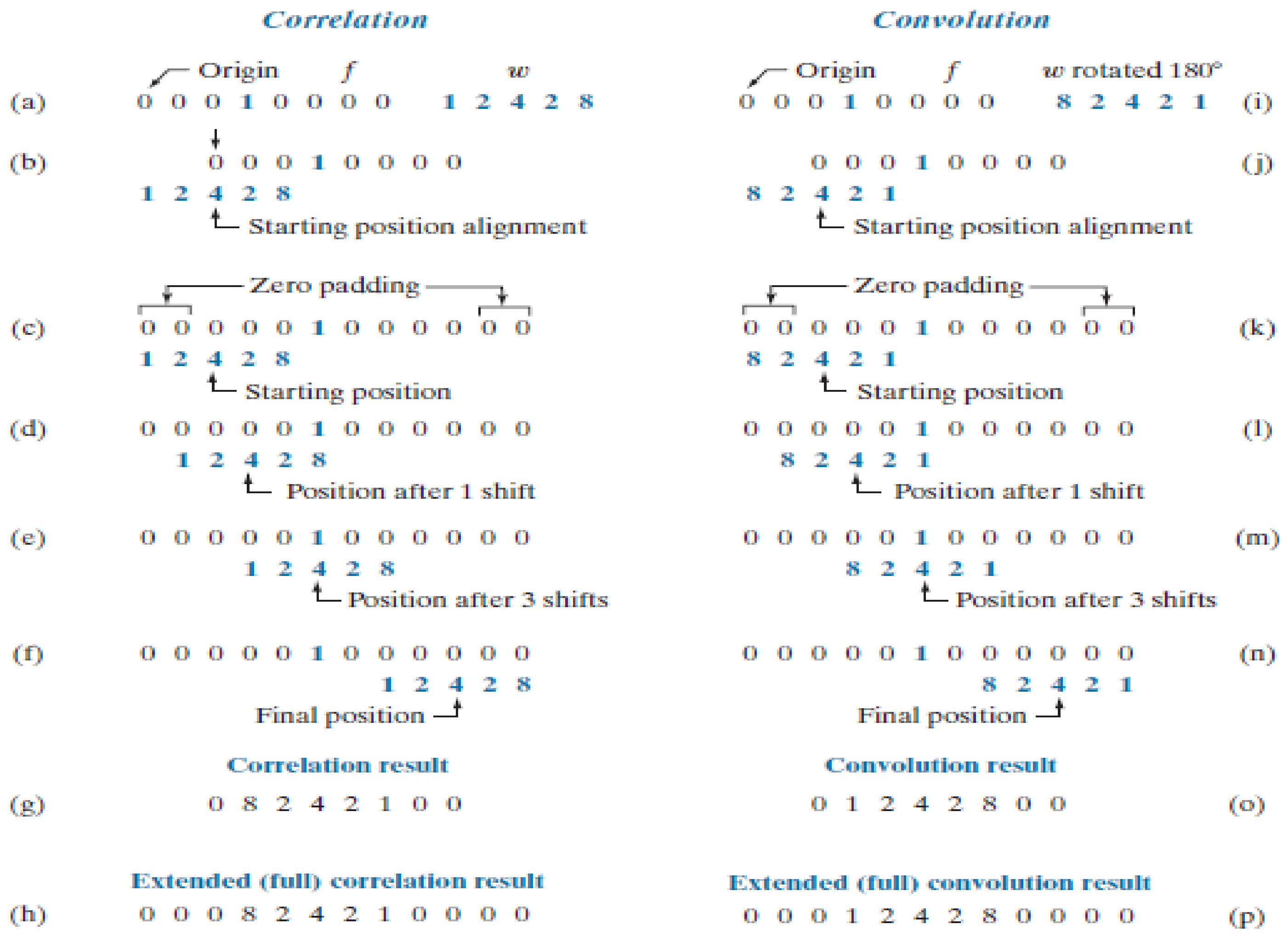
- The mechanics of linear spatial filtering using a 3 × 3 kernel. The pixels are shown as squares to simplify the graphics.
- Note that the origin of the image is at the top left, but the origin of the kernel is at its center.
- Placing the origin at the center of spatially symmetric kernels simplifies writing expressions for linear filtering.



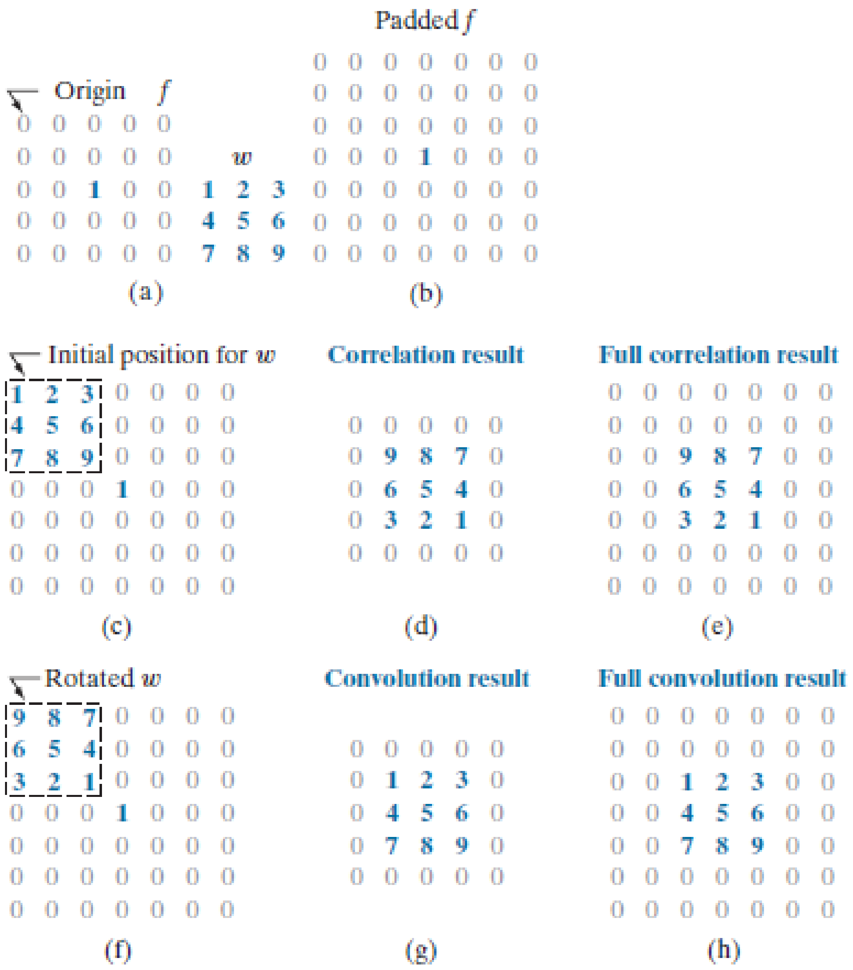
Spatial Correlation And Convolution

- Spatial correlation is described by the previous equation.
- Correlation consists of moving the center of a kernel over an image, and computing the sum of products at each location.
- The mechanics of spatial convolution are the same, except that the correlation kernel is rotated by 180°.
- Thus, when the values of a kernel are symmetric about its center, correlation and convolution yield the same result.

Spatial Correlation And Convolution



Spatial Correlation And Convolution



Separable Filter Kernels

- a $2-D$ function $G(x,y)$ is said to be separable if it can be written as the product of two $1-D$ functions, $G_1(x)$ and $G_2(x)$; that is, $G(x,y) = G_1(x,y)G_2(y)$
- A spatial filter kernel is a matrix, and a separable kernel is a matrix that can be expressed as the outer product of two vectors.
- $w = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ $c = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, and\ r = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$
- $cr^T = w$

Smoothing Spatial Filters

- Smoothing (also called averaging) spatial filters are used to reduce sharp transitions in intensity.
- Smoothing is used to reduce irrelevant detail in an image, where “irrelevant” refers to pixel regions that are small with respect to the size of the filter kernel.

Smoothing linear filters

- linear spatial filtering consists of convolving an image with a filter kernel.
- Convolving a smoothing kernel with an image blurs the image, with the degree of blurring being determined by the size of the kernel and the values of its coefficients. lowpass filters are fundamental
- The output (response) of a smoothing, linear spatial filter is simply the average of the pixels contained in the neighborhood of the filter mask. These filters sometimes are called averaging filters also are referred to a lowpass filters.
- other important filters, including sharpening (highpass), bandpass, and bandreject filters, can be derived from lowpass filters

Box Filter Kernels

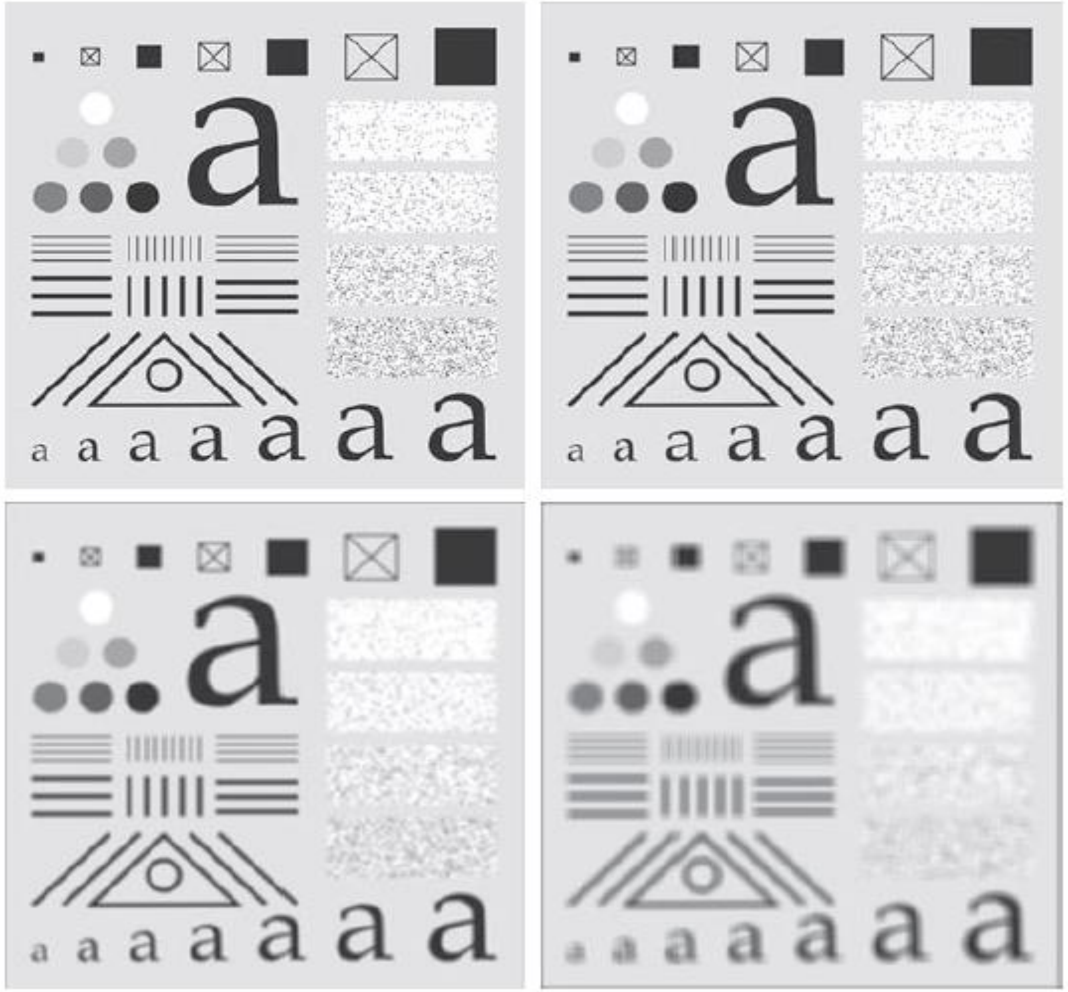
- The simplest, separable lowpass filter kernel is the box kernel, whose coefficients have the same value (typically 1).
- The name “box kernel” comes from a constant kernel resembling a box when viewed in 3-D.
- Purpose of this filter
  - ✓ First, the average value of an area of constant intensity would equal that intensity in the filtered image, as it should.
  - ✓ Second, normalizing the kernel in this way prevents introducing a bias during filtering; that is, the sum of the pixels in the original and filtered images will be the same.

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

Box Filter Kernels

(a) Test pattern of size 1024 × 1024 pixels.

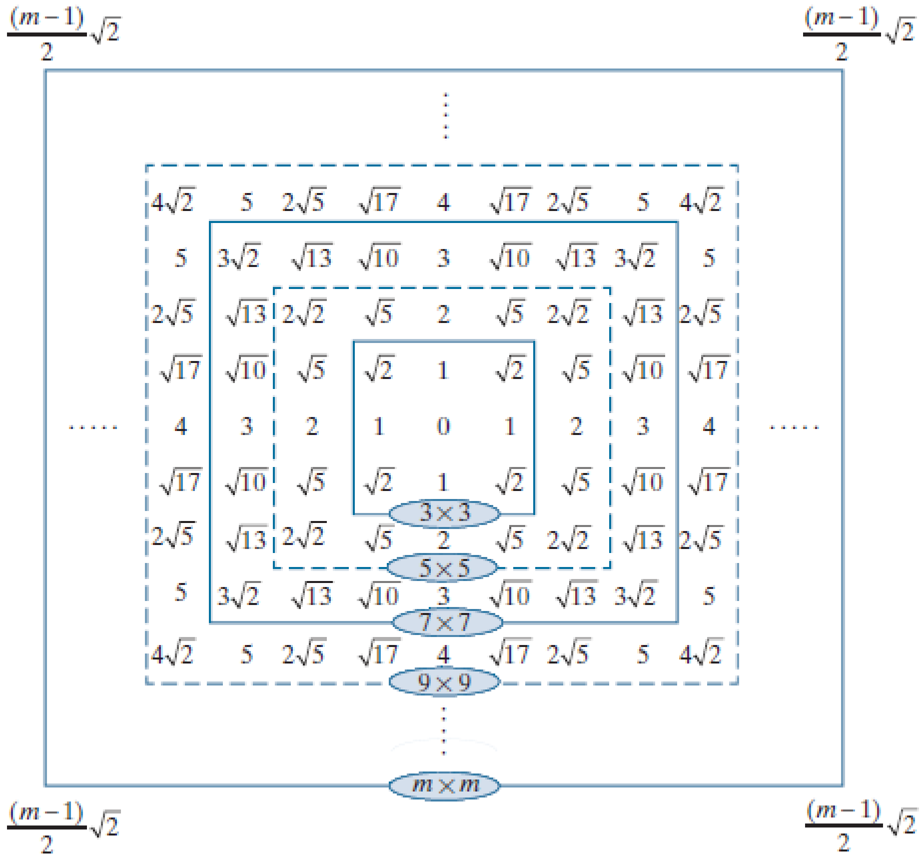
(b)-(d) Results of lowpass filtering with box kernels of sizes 3 × 3, 11 × 11, and 21 × 21, respectively.



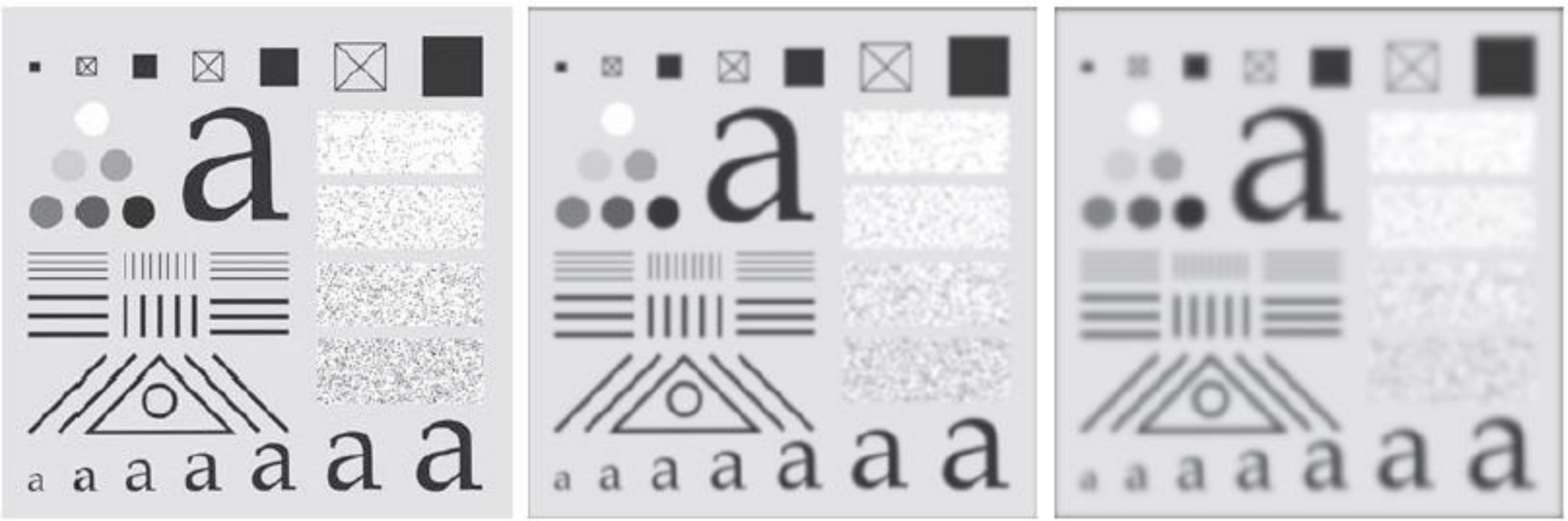
Lowpass Gaussian Filter Kernels

- The kernels of choice in applications such as those just mentioned are circularly symmetric (also called isotropic, meaning their response is independent of orientation)

$$\frac{1}{4.8976} \times \begin{array}{|c|c|c|} \hline 0.3679 & 0.6065 & 0.3679 \\ \hline 0.6065 & 1.0000 & 0.6065 \\ \hline 0.3679 & 0.6065 & 0.3679 \\ \hline \end{array}$$



Lowpass Gaussian Filter Kernels



(a)A test pattern of size 1024 × 1024. (b) Result of lowpass filtering the pattern with a Gaussian kernel of size 21 × 21, with standard deviations s = 3.5. (c) Result of using a kernel of size 43 × 43, with s = 7. This result is comparable to Fig. 3.33(d). We used K = 1 in all cases

Order-statistic (Nonlinear) Filters

- Order-statistic filters are nonlinear spatial filters whose response is based on ordering (ranking) the pixels contained in the region encompassed by the filter. Smoothing is achieved by replacing the value of the center pixel with the value determined by the ranking result.
- The best-known filter in this category is the median filter, which replaces the value of the center pixel by the median of the intensity values in the neighborhood of that pixel.
- Median filters provide excellent noise reduction capabilities for certain types of random noise, with considerably less blurring than linear smoothing filters of similar size.

Order-statistic (Nonlinear) Filters

- Median filters are particularly effective in the presence of impulse noise (sometimes called salt-and-pepper noise, when it manifests itself as white and black dots superimposed on an image).
- The median represents the 50th percentile of a ranked set of numbers, but ranking lends itself to many other possibilities.
- For example, using the 100th percentile results in the so-called max filter, which is useful for finding the brightest points in an image or for eroding dark areas adjacent to light regions.
- The 0th percentile filter is the min filter, used for the opposite purpose.