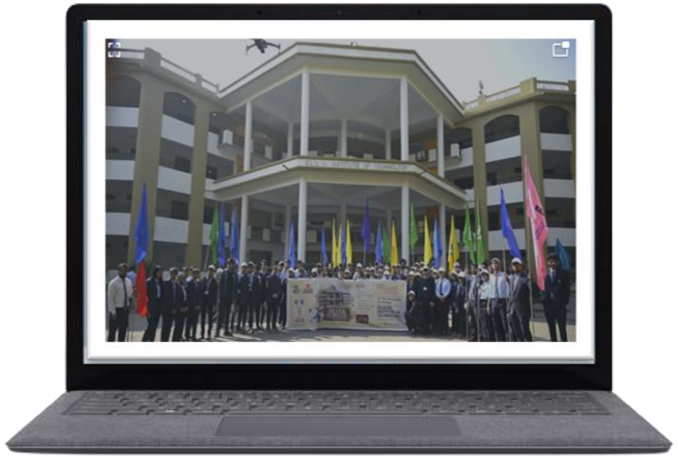
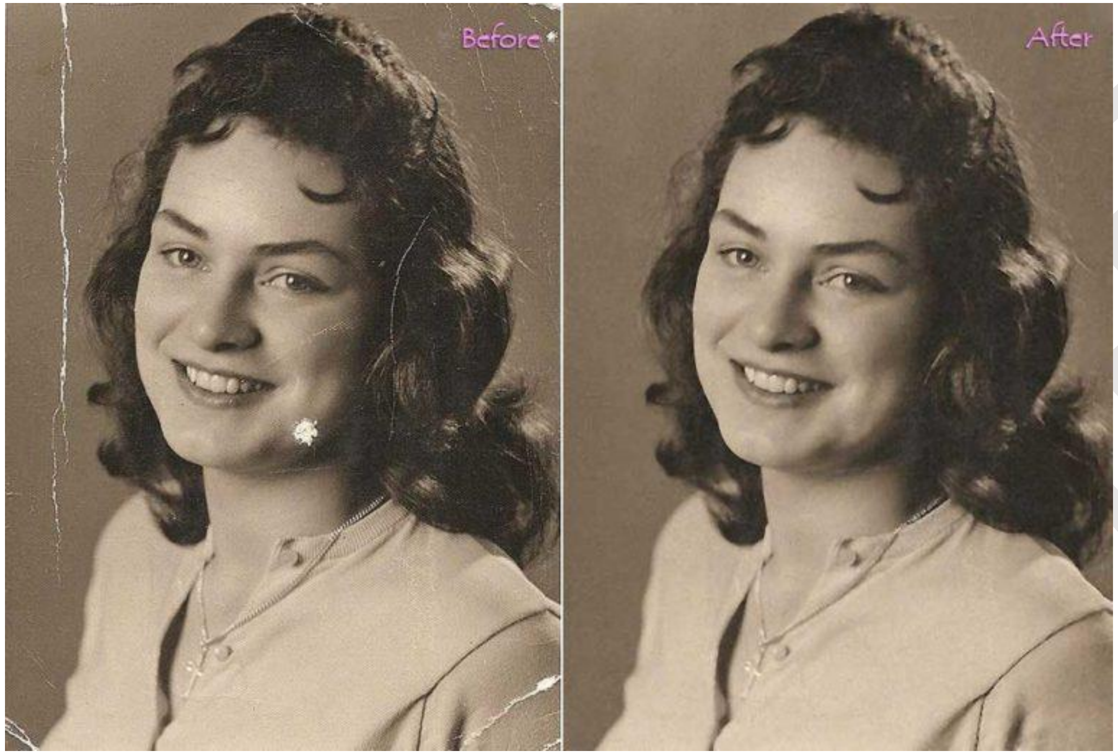


Image Restoration and Reconstruction

Table of Content

Image Restoration

- the principal goal of restoration techniques is to improve an image in some predefined sense.
- Restoration attempts to recover an image that has been degraded by using a priori knowledge of the degradation phenomenon
- Image enhancement is subjective whereas image restoration is objective



Causes of Image Degradation

- Noise due to image sensor
- Blurring due to miss focus
- Blurring due to motion
- Noise from transmission channel

Image Degradation/Reconstruction Process Model

- Image degradation model use operator $\mathcal{H}$ with an additive noise term, operate on an image $f(x, y)$ to produce an degraded image $g(x, y)$
- Image restoration model use degraded image $g(x, y)$ to estimate the original image $\bar{f}(x, y)$, the restoration requires the knowledge of degradation operator $\mathcal{H}$ and additive noise $\eta(x, y)$

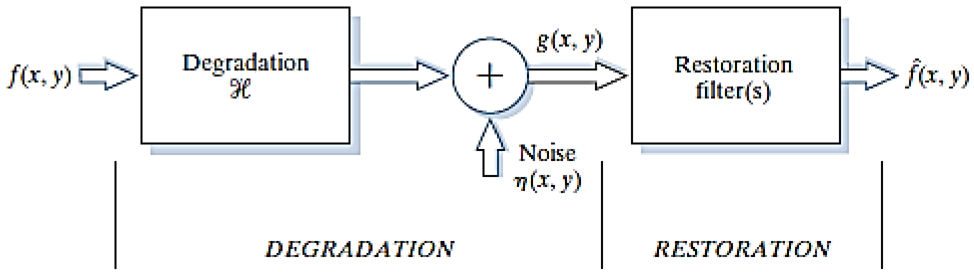


Image Degradation/Reconstruction Process Model

- If $\mathcal{H}$ is a linear, position- invariant operator then the degraded image is given in the spatial domain by
- $g(x, y) = (h * f)(x, y) + \eta(x, y)$
- Where $h(x, y)$ is the spatial representation of the degradation function, and $*$ indicate convolution
- Frequency domain representation is
- $G(u, v) = H(u, v)F(u, v) + N(u, v)$
- Where the capital letters are the Fourier Transform of the corresponding terms.

Noise Models with Examples

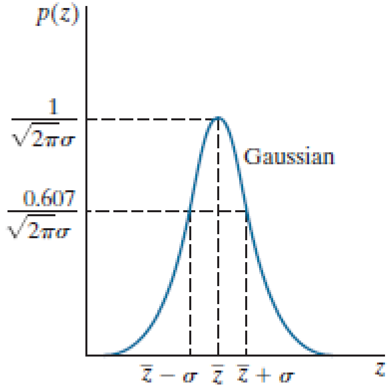
- The source of noise in digital images arises during acquisition(digitization) and transmission
  - ✓ image sensors can be affected by ambient condition
  - ✓ inference can be added to an image during transmission
- to understand the restoration process, it is important to understand the noise function $\eta(x, y)$
- Noise can not be predicted but can be approximately described in a statistical way using the probability density function(pdf)

Noise Types

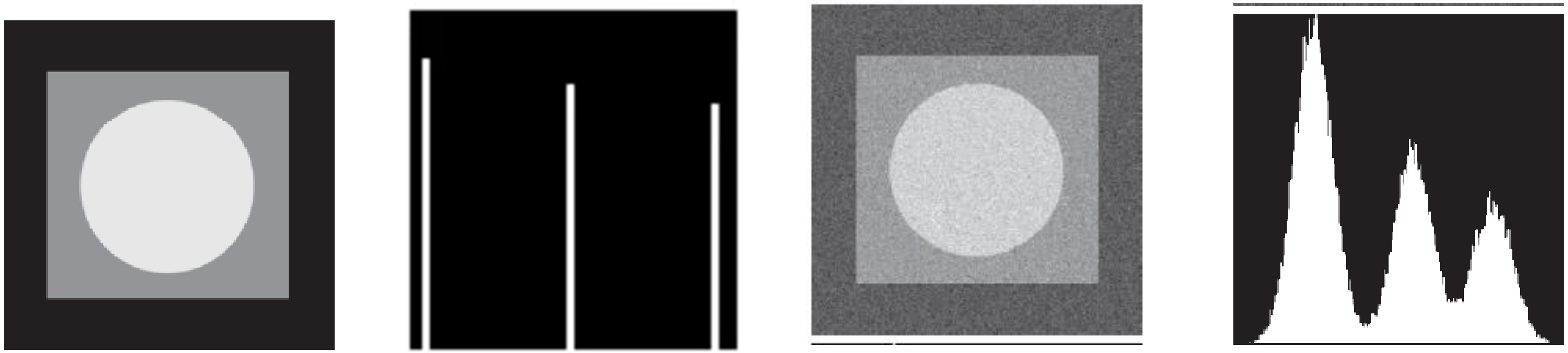
There are many different models for the image noise term $\eta(x, y)$

- ✓ Gaussian
- ✓ Rayleigh
- ✓ Erlang
- ✓ Erlang(Gamma)
- ✓ Uniform
- ✓ Impulse

Gaussian Noise



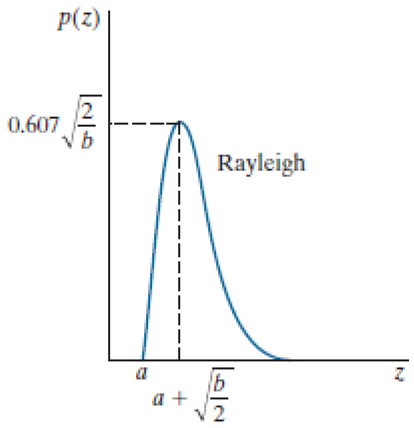
- The pdf of a Gaussian random variable z, is defined by the following familiar expression:
- $p(z) = \frac{1}{2\pi\sigma} e^{-\frac{(z-\bar{z})^2}{2\sigma^2}} \quad -\infty < z < \infty$

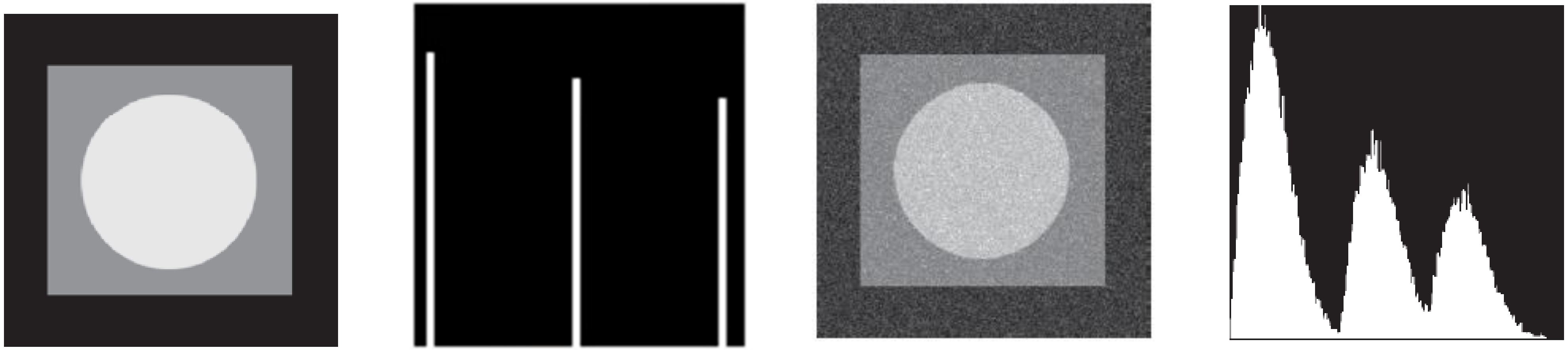


abcd (a) Original Image (b) Histogram of original image © image degraded by the Gaussian image (d) Histogram of image ©

Rayleigh Noise

- The pdf of Rayleigh noise is given by
- $p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-\frac{(z-b)^2}{b}} & z \geq a \\ 0 & z < a \end{cases}$



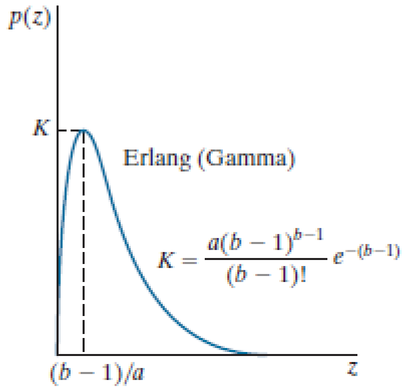


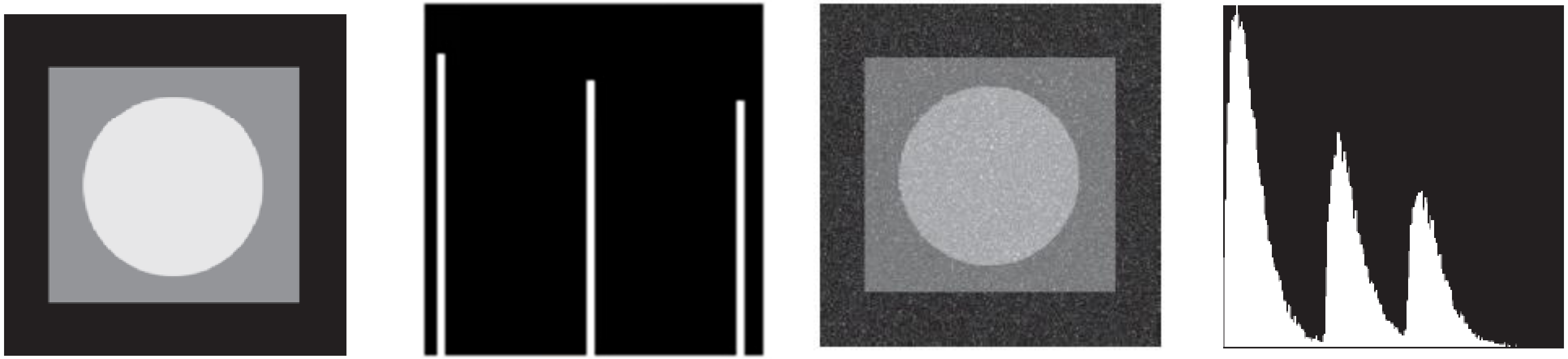
abcd (a) Original Image (b) Histogram of original image © image degraded by the Rayleigh Noise (d) Histogram of image ©

Erlang Noise

- The pdf of Erlang noise is

$$p(z) = \begin{cases} \dfrac{a^z z^{b-1}}{(b-1)!} e^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

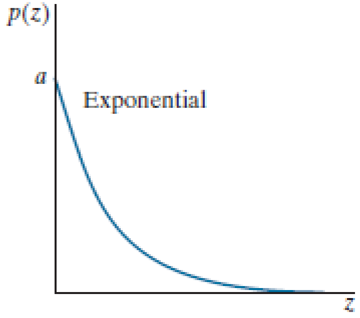


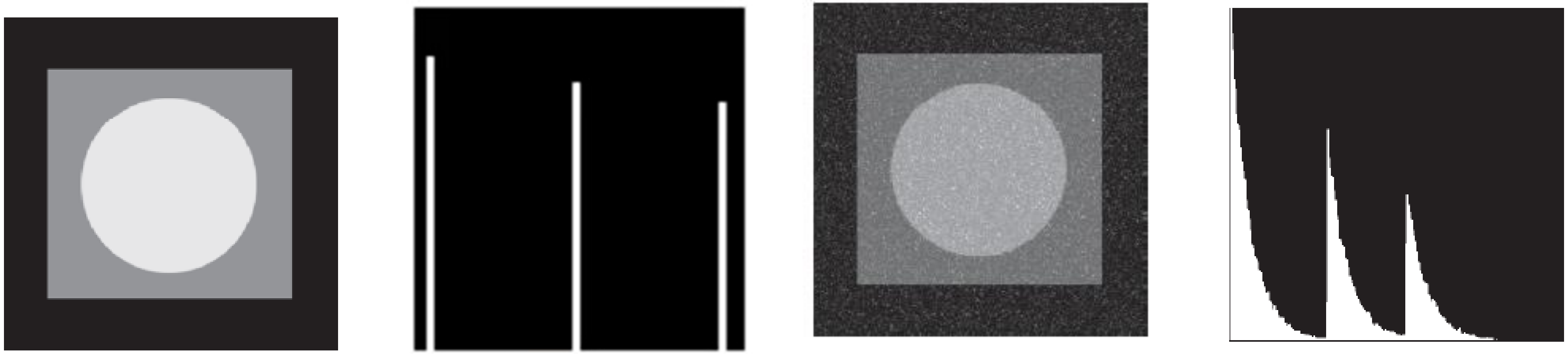


abcd (a) Original Image (b) Histogram of original image © image degraded by the Rayleigh Noise (d) Histogram of image ©

Exponential Noise

- The pdf of Exponential Noise is $p(z) = \begin{cases} ae^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$



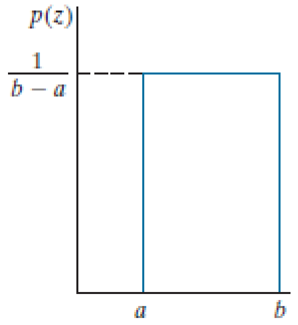


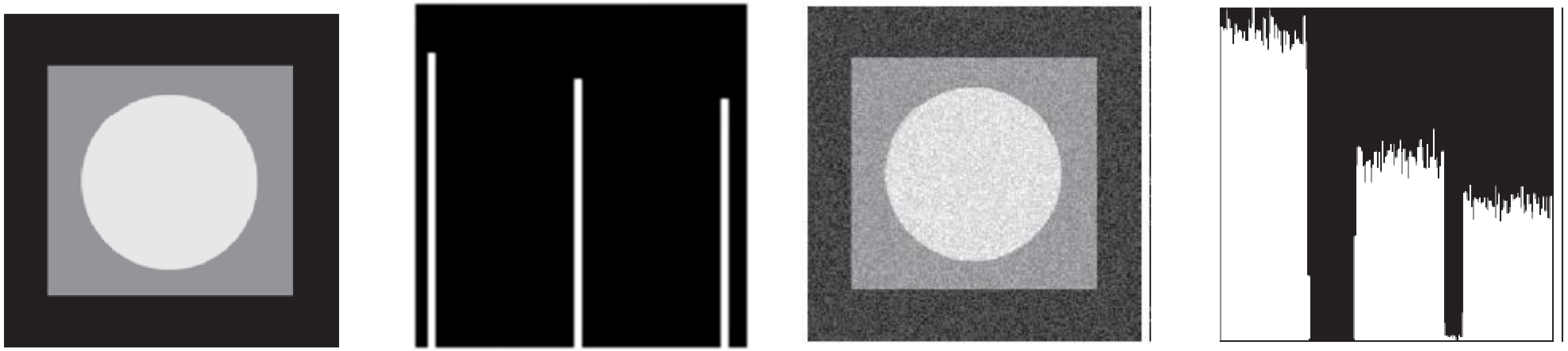
abcd (a) Original Image (b) Histogram of original image © image degraded by the exponential Noise (d) Histogram of image ©

Uniform Noise

- The pdf of uniform noise is

$$p(z) = \begin{cases} \dfrac{1}{b-a} & a \le z \le b \\ 0 & Otherwise \end{cases}$$

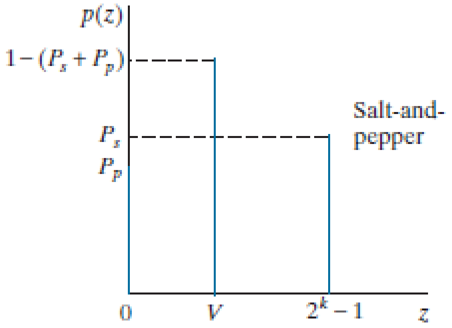


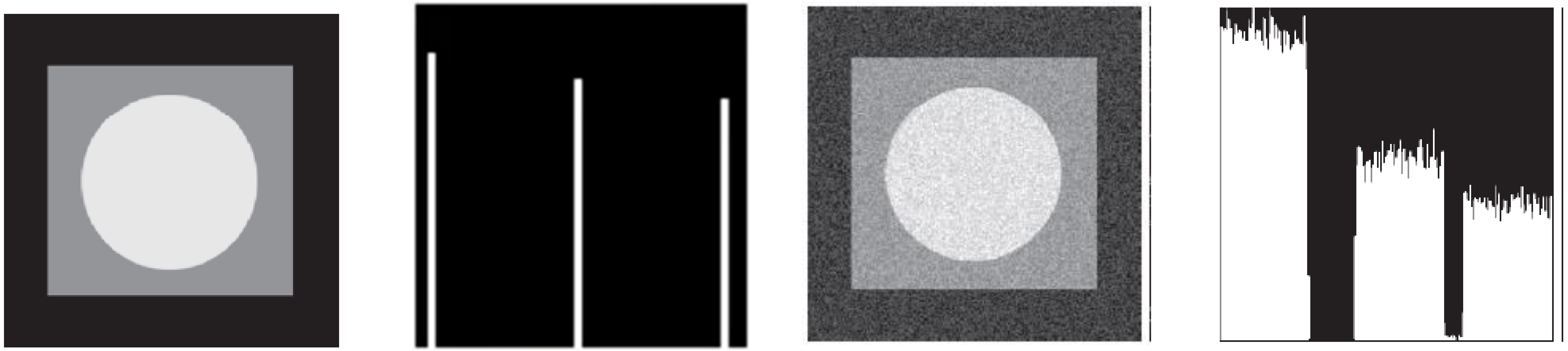


abcd (a) Original Image (b) Histogram of original image © image degraded by the uniform Noise (d) Histogram of image ©

Salt and Pepper Noise

- The pdf of salt and pepper noise
- $p(z) = \begin{cases} P_s & for\ z = 2^k - 1 \\ P_p & for\ z = 0 \\ 1 - (P_s + P_p) & for\ z = V \end{cases}$





abcd (a) Original Image (b) Histogram of original image © image degraded by the Salt and Pepper Noise (d) Histogram of image ©

Restoration in the presence of Noise only

- When an image is degraded only by additive noise then the degraded image becomes
- $g(x, y) = f(x, y) + \eta(x, y)$
- And $G(u, v) = F(u, v) + N(u, v)$
- The noise terms are unknown, subtracting noise from degraded image to obtain original image is not the option.
- when only additive noise is present then spatial filtering is the option for estimating $f(x, y)$[i.e denoising image $g(x, y)$]

Mean Filters

- Arithmetic mean filter: $\hat{f}(x,y) = \frac{1}{mn}\sum_{r,c\in s_{xy}} g(r,c)$
- Where s_{xy} represents set of co-ordinate in an rectangular sub-image of size $m \times n$, centred on point (x,y)
- Noise is reduced due to blurring.

Geometric Filter

- Geometric Mean filter: $\hat{f}(x, y) = [\prod_{(r,c)\in s_{xy}} g(r, c)]^{\frac{1}{mn}}$
- Loses less image details
- Noise is reduced
- Remove Gaussian noise
- Not good for salt and pepper noise

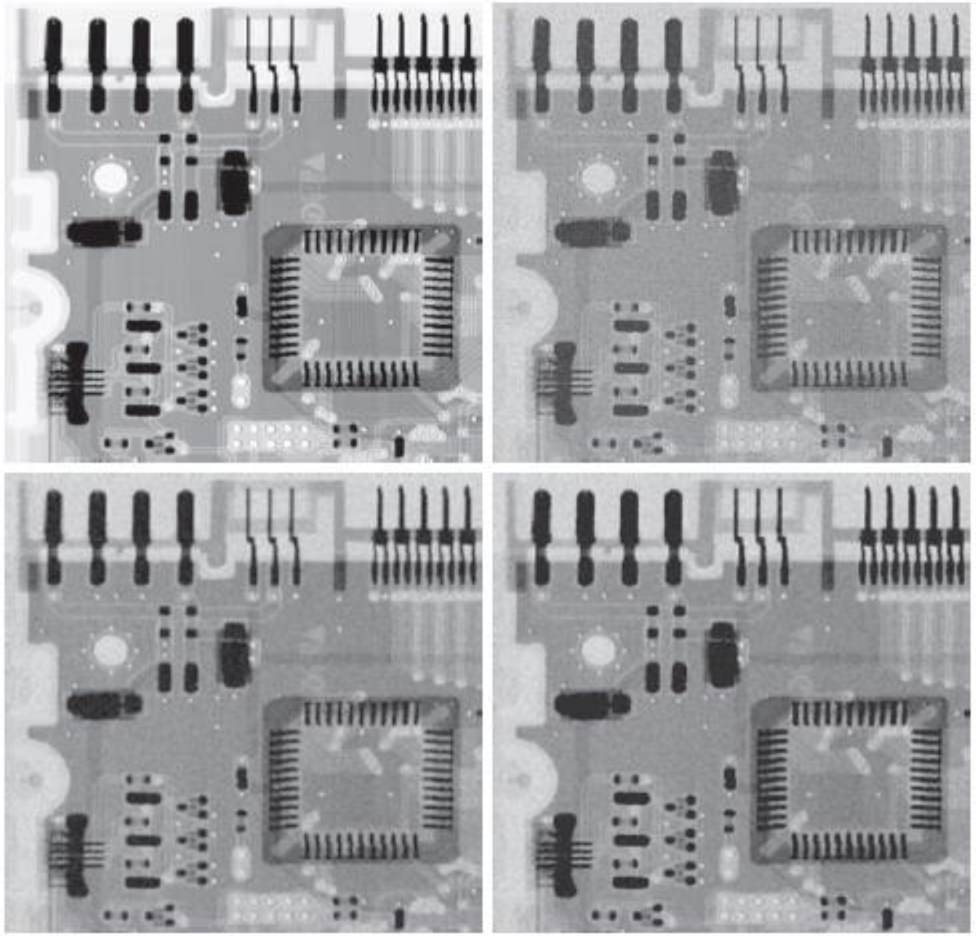
Harmonic Mean Filter

- Harmonic Mean filter:$\hat{f}(x, y) = \dfrac{mn}{\sum(1/g(r,c))}$
- Remove Gaussian noise
- Works well for salt noise fail for pepper noise

Contra harmonic Filter

- $\hat{f}(x,y) = \frac{\sum g(s,t)^{Q+1}}{\sum g(s,t)^{Q}}$
- Positive Q suitable for eliminating pepper noise
- Negative charge Q suitable for salt noise
- Can not do simultaneously

ab
cd
(a) X-ray image of circuit board. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3 × 3. (d) Result
of filtering with a geometric mean filter of the same size.



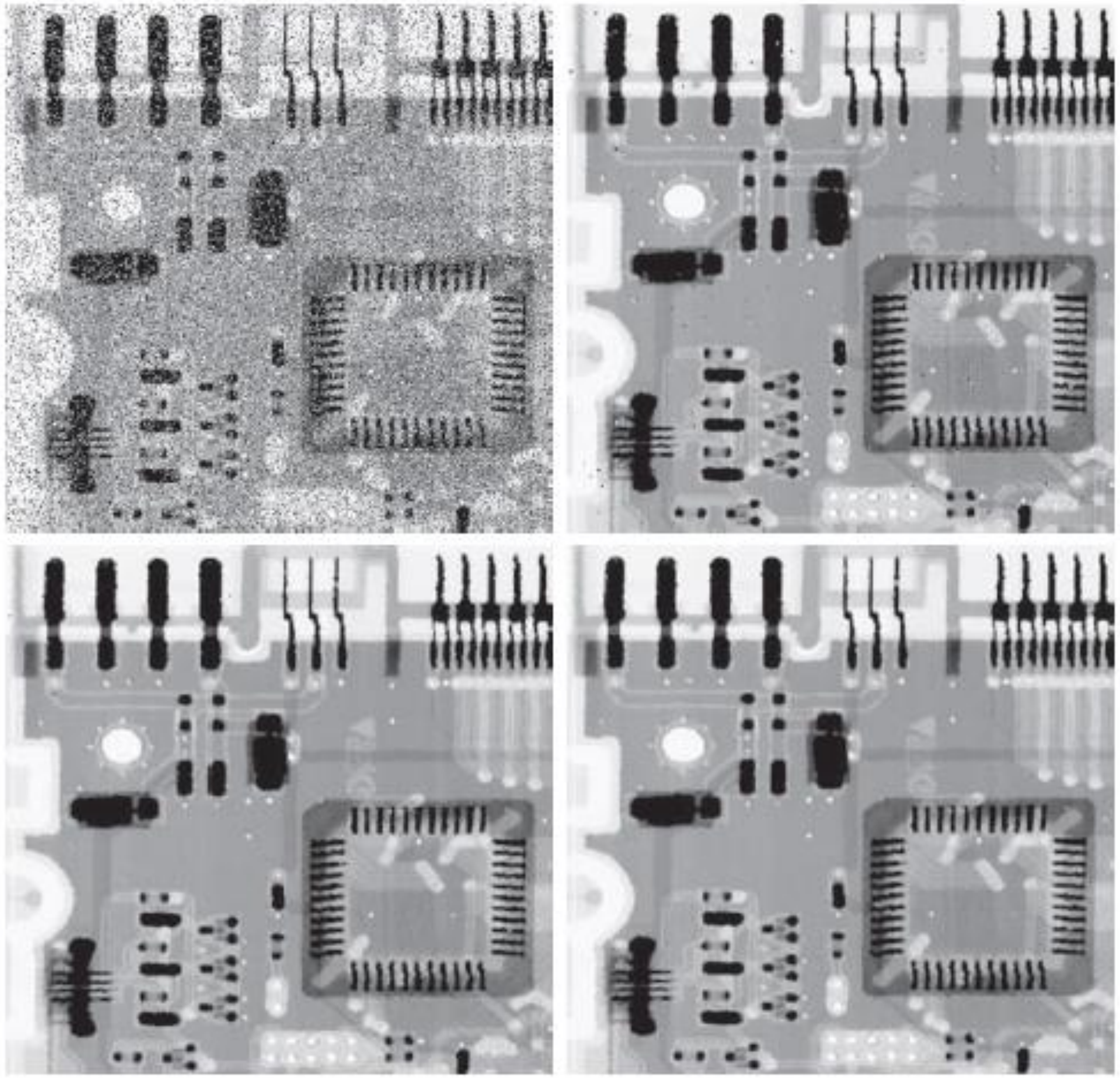
Order Statistic Filter

- Median Filter $\hat{f}(x,y) = media_{(r,c)\in S_{xy}}\{g(r,c)\}$
- Max Filter
- Min Filter
- Mid point filter

Order Statistic Filter

- Alpha-trimmed filter:
- Suppose that we delete the $\frac{d}{2}$ lowest and the $\frac{d}{2}$ highest intensity values of $g(r, c)$ in the neighborhood S_{xy}.
- Let $g_R(r, c)$ represent the remaining $mn - d$ pixels in S_{xy}.
- A filter formed by averaging these remaining pixels is called an alpha-trimmed mean filter.
- If $d = 0$, it becomes arithmetic mean filters
- If $d = mn - 1$, it becomes median filters

(a) Image corrupted by saltand-pepper noise with probabilities (b) Result of one pass with a median filter of size 3 × 3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.



Estimation Degradation Function

Popular ways to estimate degradation function are

1. Estimation by observation
2. Estimation by experimentation
3. Estimation by modeling

Estimation by observation

- One way to estimate the degradation function is to gather information from the image itself with the assumption that the image was degraded by a linear, position-invariant process.
- Identify a good portion of the image in which the signal content is strong
- Lets observed the sub-image be $g_s(x, y)$ and let the processed subimage be $\hat{f}_s(x, y)$
- Assume the noise is negligible $H(u, v) = \frac{G(u,v)}{\hat{F}_s(u,v)}$
- Now apply the intense Fourier transform to the above equation

Estimation by Experimentation

- Identify the instrument used for obtaining the image and study the device settings
- Obtain the impulse response of the degradation by imaging as small white using the same system settings
- But the Fourier transform of an impulse is constant, so $H(u,v) = G(u,v)/A$
- Now as the effect of noise on impulse is negligible, we can apply the inverse Fourier transform to the equation and estimate the degradation function.

Estimation by modeling

- Using mathematical models, the degraded functions like motion blur, atmospheric turbulence etc. can be estimated
- Now there are three possibilities:
- complete knowledge about the blur is available. then we can apply Inverse filtering
- Partial knowledge about the blur is available. Weiner filter can be used in this case
- No knowledge about the blur is available. Blind deconvolution is applied in this case.

Constrained, Unconstrained Method

- Complete Knowledge Available: Inverse Filter deconvolution
- Partial Knowledge Available: Weiner Filter
- No Prior knowledge available: Blind restoration, blind deconvolution
- Categorization has been done on the basis of the knowledge available for the blurring function
- Algebric methods are very popular for image restoration because it use the concept of matrices and linear algebra instead of integral

➢ Unconstrained Method

➢ Constrained Method

Constrained Least Square filtering

- The least squared method is very sensitive to noise. to reduce noise sensitivity a measure of smoothness like the second derivative is minimized. The criterion is
- $C = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left(\nabla^2 f(x,y)\right)^2$
- subject to constraint $\left|\left|g - H\hat{f}\right|\right|^2 = ||n||^2$
- The frequency domain solution to this optimization is given by the expression
- $\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma |P(u,v)|^2}\right] G(u,v)$

Constrained Least Square filtering

- $P(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$
- We recognize this function as a Laplacian kernel

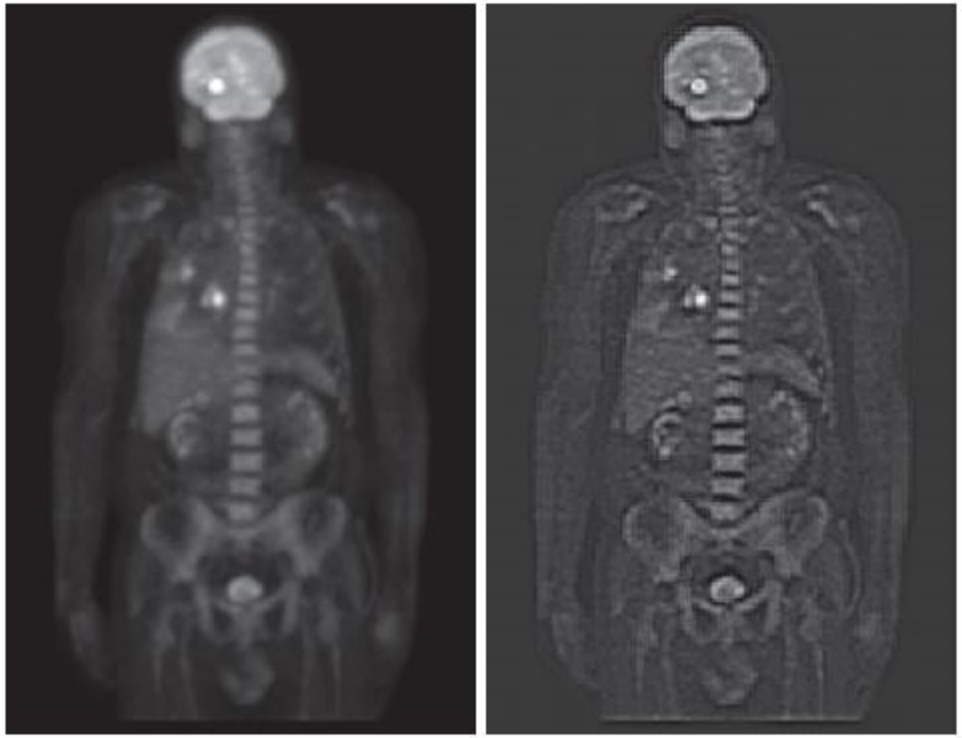
Homomorphic Filtering

TECHNO INDIA SILIGURI INSTITUTE OF TECHNOLOGY
A Satyam Roychowdhury Initiative

- Homomorphic filtering is a generalized technique for signal and image processing
- Involve a nonlinear mapping to a different domain in which linear filter techniques are applied
- Followed by a mapping to the original domain
- Homomorphic filtering simultaneously normalize the brightness across an image and increase contrast

Application

- Removing multiplicative noise that has certain characteristics
- Correcting non uniform illumination in image
- Improving the appearance of a grey scale image



Illumination-reflection Model

- An image can be modeled as the product of an illumination function and the reflection function at every point

$$f(x, y) = i(x, y) * r(x, y)$$

- This model is use to address the problem of improving the quality of an image that has been acquired under poor conditions.
- For many images, the illumination is the primary contributor to the dynamic range and varies slowly in space.
- While the reflectance concept represent the details of object edge and varies rapidly in place.

Illumination-reflection Model

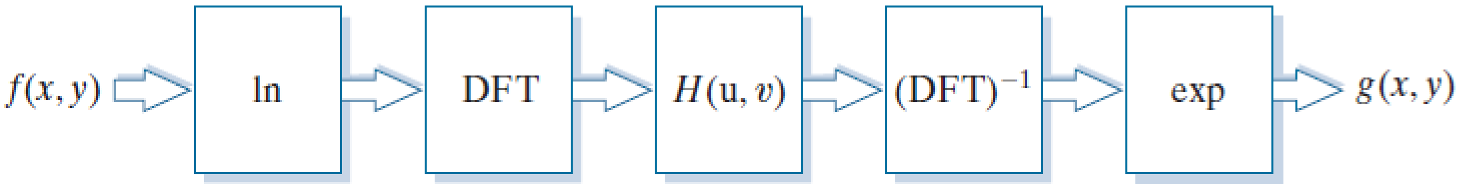
- The characteristic lead to associating a low frequency of the Fourier transform of an logarithm of an image with illumination the high frequencies with reflectance
- The idea of homomorphic is to separate these components and apply two different transfer function to have more control
- The problem with Fourier transform is that the product of two functions is not seperable
- F$[f(x,y)] \neq F[i(x,y)] + F[r(x,y)]$

Procedure for Applying Homomorphic Filter

- Step 1: logarithm(log function) is applied to the image so that it can be expressed as a sum of its illumination and reflectance components.
- $\ln \quad z(x, y) = \ln i(x, y) + \ln r(x, y)$
- Step 2: Apply Fourier transform
- $F[\ln z(x, y)] = F[\ln i(x, y)] + F[\ln r(x, y)]$
- $Z(u, v) = F_i(u, v) + F_r(u, v)$
- Step 3: Apply transfer function $H(u, v)$
- $S(u, v) = H(u, v)Z(u, v) = H(u, v)F_i(u, v) + H(u, v)F_r(u, v)$

Procedure for Applying Homomorphic Filter

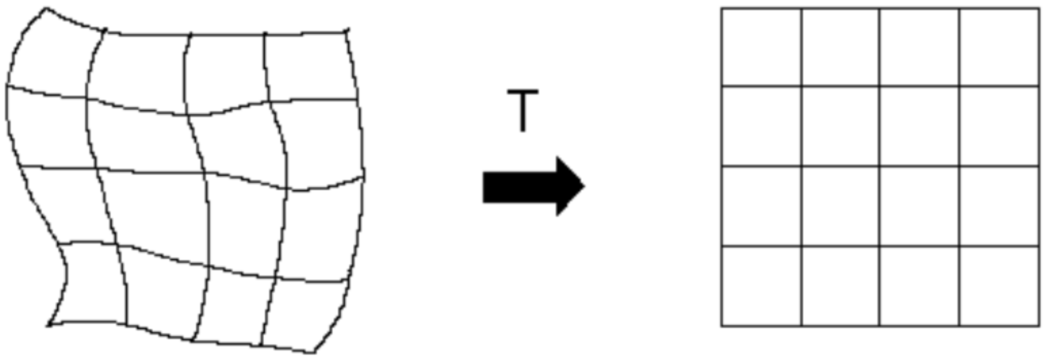
- Step 4: Convert the filtered image into spatial domain
- $S(x,y) = F^{-1}[S(u,v)] = F^{-1}[H(u,v)F_i(u,v)] + F^{-1}[H(u,v)F_r(u,v)]$
- $\mathrm{S}(x,y) = i'(x,y) + r'(x,y)$
- Step 5: reverse the logarithm process by taking exponential of the filtered result to form the output image
- $g(x,y) = e^{S(x,y)} = e^{i'(x,y)+r'(x,y)} = \mathrm{i}_0(\mathrm{x},\mathrm{y})\mathrm{r}_0(\mathrm{x},\mathrm{y})$

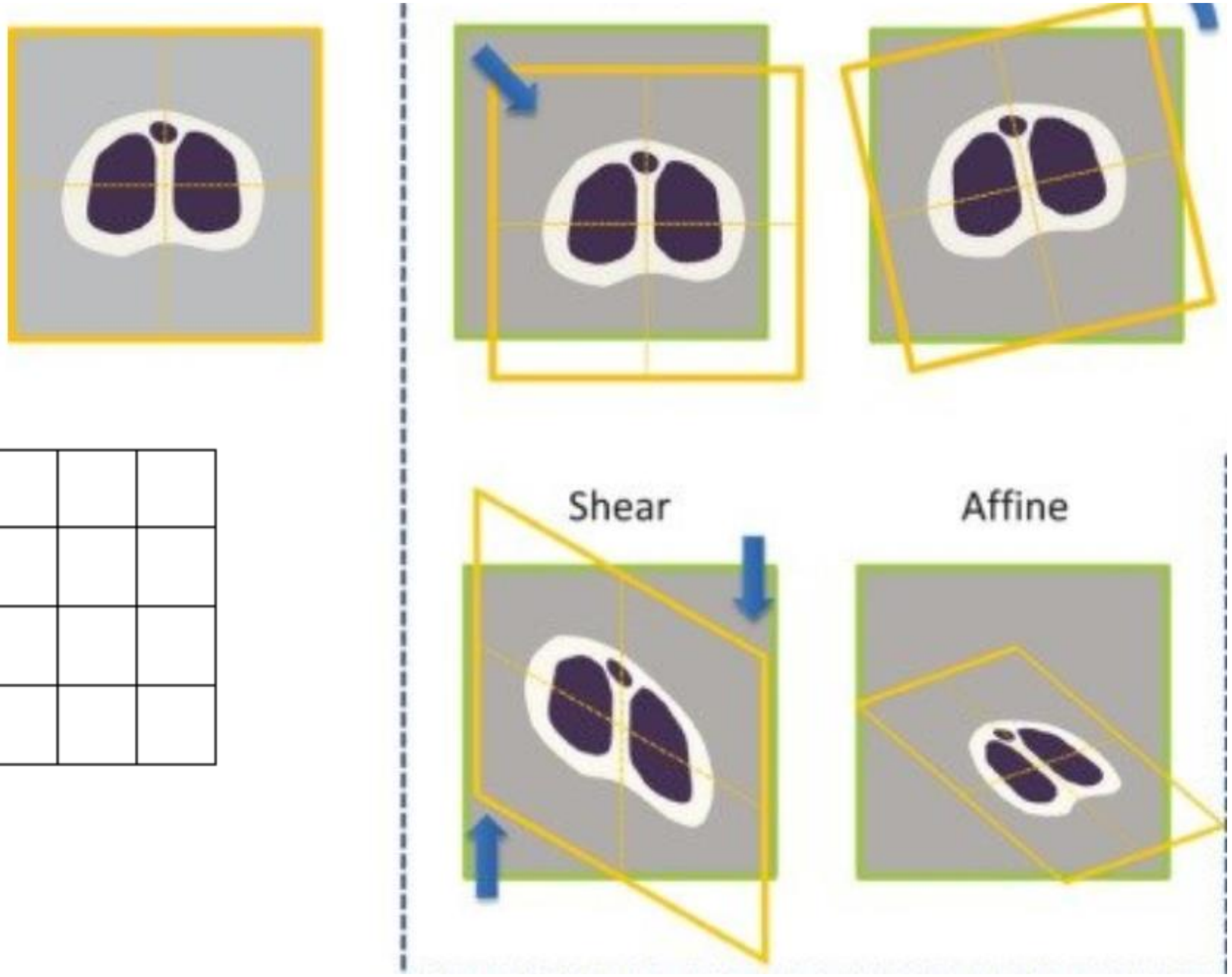


Geometric Transformation

- Modifies the spatial relationships between pixels in an image
- Often called rubber sheet transformations because they may be viewed as the power of printing an image on a sheet of rubber and then stretching this sheet according to some predefined set of values.

Geometric Transformation





Geometric Transformation

- A geometric transform consists of two basic steps

1. determining the pixel co-ordinate transformation
   - mapping of the co-ordinates of the input image pixel to the point in the output image.
   - the output point co-ordinates should be computed as continuous values (real numbers) as the position does not necessarily match the digital grid after the transform.

2. finding the point in the digital raster which matches the transformed point and determining its brightness.
   - brightness is usually computed as an interpolation of the brightnesses of several points in the neighborhood.

Geometric Transformation

- Type of Geometric Transformation:

➢ Spatial transformation: defines rearrangement of pixels on the image plane

➢ Gray level interpolation: which deals with the assignment of gray levels to pixels in the spatially transformed image

Spatial Transformation

- Suppose that an image f with pixel coordinate (x, y) undergoes geometric distortion to produce an image g with coordinates (x', y').
- This transformation may be expressed as $x' = r(x, y)$ and $y' = s(x, y)$
- Where $r(x, y)$ and $s(x, y)$ are spatial transformation that produced geometrically distorted image $g(x', y')$
- If $r(x, y)$ and $s(x, y)$ were known analytically, recovering $f(x, y)$ from the distorted image $g(x', y')$ by applying the transformation in reverse might be possible theoretically.

Spatial Transformation

- In practice, however formulating a single set of analytical function $r(x, y)$ and $s(x, y)$that describe the geometric distortion process over the entire image plane generally is not possible.
- The most frequently used method to overcome this difficulty is to formulate the spatial relocation of pixels whose location in the input (distorted) and output (corrected) image is known precisely.

Spatial Transformation

- Suppose that the geometric distortion process within the quadrilateral regions is modelled by a pair of bilinear equation so that
- $r(x, y) = c_1x + c_2y + c_3xy + c_4$
- $s(x, y) = c_5x + c_6y + c_7xy + c_8$
- Then the equation
- $x' = c_1x + c_2y + c_3xy + c_4$
- $y' = c_5x + c_6y + c_7xy + c_8$
- Since there are a total of eight known known tiepoints these equations can be solved for the eight coefficients $c_i, i = 0,1,2,3, \ldots$

Important Geometric Transformation

- Rotation by the angle ϕ about the origin
  - $x' = xcos\phi + ysin\phi$
  - $y' = -xsin\phi + ycos\phi$
- Change of scale a in the x $-$axis and b in the y $-$axis
  - $x' = ax$
  - $y' = by$
- Skewing by angle ϕ in the x $-$axis
  - $x' = x + ytan\phi$
  - $y' = y$

Grey Level Interpolation

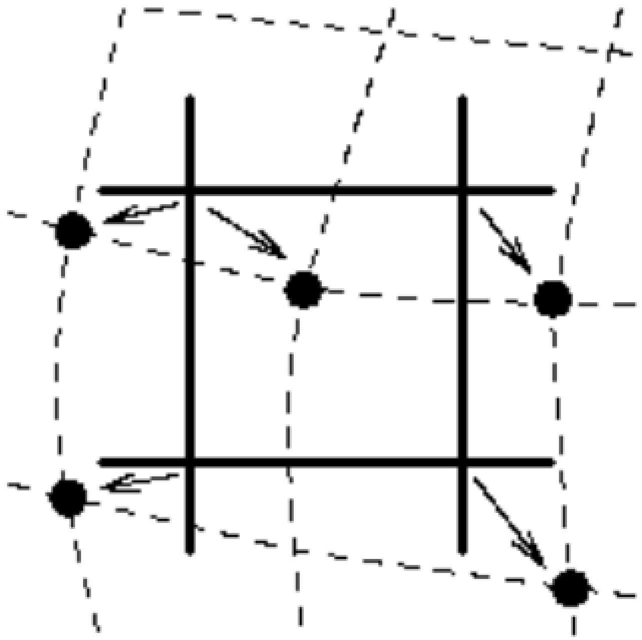
- The method discussed in spatial transformation steps through integer values of the coordinates (x, y) to yield the restorted image $\hat{f}(x, y)$.
- However, depending on the values of the coefficients equations can yield noninteger values for x' and y'
- Because, the distorted image g is digital, its pixel values are defined at image coordinate. Thus using noninteger values for (x', y') causes mapping into locations of g for which no gray levels are defined.
- Inferring what gray level values at those location should be , based only on the pixel values at integer coordinate coordinate locations, then become necessary.

Grey Level Interpolation

- Nearest neighbor approach: zero level interpolation
- Cubic convolution interpolation:
- Bilinear interpolation:

Nearest Neighbor Interpolation

- assigns to the point (x, y) the brightness value of the nearest point g in the discrete raster
- $f(x, y) = g\big(round(x'), round(y')\big)$
- The position error of the nearest neighborhood interpolation is at most half a pixel.
- This error is perceptible on objects with straight line boundaries that may appear step-like after the transformation.



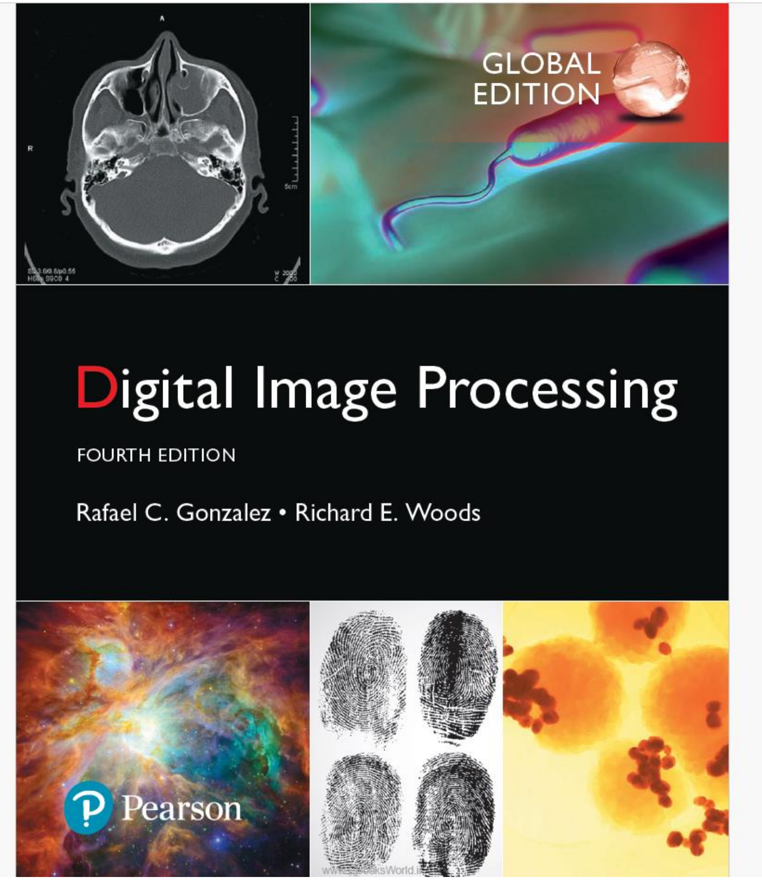
Cubic convolution interpolation

- explores four points neighboring the point (x,y), and assumes that the brightness function is linear in this neighborhood.
- Linear interpolation can cause a small decrease in resolution and blurring due to its averaging nature.
- The problem of step like straight boundaries with the nearest neighborhood interpolation is reduced.

Cubic convolution interpolation

- improves the model of the brightness function by approximating it locally by a bicubic polynomial surface; sixteen neighboring points are used for interpolation.
- Bicubic interpolation does not suffer from the step-like boundary problem of nearest neighborhood interpolation, and copes with linear interpolation blurring as well.
- Bicubic interpolation is often used in raster displays that enable zooming with respect to an arbitrary point -- if the nearest neighborhood method were used, areas of the same brightness would increase.

References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