A GIRL HAS NO NAME
Computer Science 81
Homework 2
1/31/17

1, 2, 3, 4, 5

1 [20 points] Give box-style proofs for each of the following:

A. [4 points] $F \to G \ \vdash \ ((F \wedge G) \to F) \wedge (F \to (F \wedge G))$.

B. [4 points] $(F \to G) \wedge (F \to H) \ \vdash \ F \to (G \wedge H)$.

C. [4 points] $F \vee G, \ \neg F \ \vdash \ G$.

D. [4 points] $\neg (F \to G) \ \vdash \ G \to F$.

E. [4 points] $\neg (F \wedge G) \ \vdash \ \neg F \vee \neg G$.

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2 [6 points] Pierce's Law is the formula $((F \to G) \to F) \to F$.

A. [2 points] By using truth tables, show that Pierce's law is a tautology (i.e., that $\vDash ((F \to G) \to F) \to F$), or equivalently that this formula is true under every valuation.

B. [4 points] Prove Pierce's Law using natural deduction.

Hint: As mentioned in class, *constructive* logic rejects the use of indirect arguments. Consequently, in constructive logic one cannot use the excluded middle axiom ($\vdash \phi \vee \neg\phi$), double-negation elimination rule ($\neg\neg\phi \vdash \phi$), or the proof by contradiction rule (if $\neg\phi \vdash \bot$, then $\vdash \phi$). In constructive logic, Pierce's Law is not provable! Therefore, your proof has to involve at least one use of at least one of these three classical rules.

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3 [24 points] **Proof or No Proof?** For each of the following, either show there is no proof (by finding a valuation/model that makes the assumptions true and the conclusion false), or provide a natural deduction (box) proof.

A. [4 points] $\neg F \vee (G \to F) \ \vdash \ \neg F \wedge G$.

B. [4 points] $F \to G, \ \neg F \to G \ \vdash \ G$.

C. [4 points] $F \to G \to H \ \vdash \ G \to F \to H$.

D. [4 points] $F \to G \to H \ \vdash \ F \to H \to G$.

E. [4 points] $(F \to G) \to H \ \vdash \ F \to G \to H$.

F. [4 points] $F \to G, \ C \to D \ \vdash \ F \vee C \to G \wedge D$.

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4 [9 points] **OS Wars:** Consider the following argument:

- Premise 1: If my computer runs Windows, then Microsoft got my money.
- Premise 2: If my computer runs Mac OS X, then Apple got my money.
- Conclusion: At least one of the following statements is true:
  - If my computer runs Mac OS X, then Microsoft got my money.
  - If my computer runs Windows, then Apple got my money.

A. Define propositions (e.g., F, G, etc.) to stand for relevant statements in this proof, and show how to symbolically represent the premises and conclusion using these propositions.

B. Either provide a formal proof that this argument holds, or find a valuation where the assumptions are true but the conclusion is false.

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5 [1 easy point] Please wait until you're done with the rest of the assignment to answer this quick survey:

A. How long (in hours) did you spend working on this assignment?

B. What was the most interesting thing you learned while answering these problems? (We're sure there was *something* you learned.)

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