A GIRL HAS NO NAME
Computer Science 81
Homework 11
4/18/17

1, 2, 3, 4, 5, 6

1. [2 points] Consider the language

$$\textbf{Reach} = \{< M, x, s > | M \text{ eventually reaches control state } s \text{ when started on input } x\}.$$

Is **Reach** recognizable, co-recognizable, both, or neither? Justify your answers.

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2. [6 points] Define a language to have "prime nature" iff every string in it has a prime number of symbols. In other words, the language is a subset of

$$\textbf{Primes} = \{1^p \mid p \text{ is prime}\} = \{11, 111, 11111, 1111111, \ldots\}$$

Any language containing a string not of prime length, such as the language $\{111, 1111\}$ does not have prime nature. Define language **PN** as follows:

$$\textbf{PN} = \{< M > \mid M \text{ is a Turing machine and } L(M) \text{ has prime nature}\}$$

Show that **PN** is undecidable. Note that whether **PN** itself has prime nature is not at issue. (It is unlikely that it would.)

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3. [2 points] In the previous problem, is **PN** recognizable? Is **PN** co-recognizable? Justify.

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4. [2 points] Show that if L is a recognizable language, and $L \leq_m L^c$ (i.e. L is *mapping reducible* to its own complement) then L is decidable.

$\blacksquare$

5. [10 points] Show that

$$\textbf{Infinite}_{\mathrm{TM}} = \{< M > \mid M \text{ is a Turing machine and } L(M) \text{ is infinite}\}$$

is neither recognizable nor co-recognizable. (This requires two separate proofs, using different techniques described in the lecture slides.)

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6. [8 points] For each of the following questions for arbitrary Turing machine codes $< M >$, is the question rendered unrecognizable or un-corecongizable by Rice's theorem? If so, state which (unrecognizable or un-corecongizable). If Rice's theorem doesn't apply, so state and indicate whether the question is decidable, giving the best justification that you can. You may use the Church-Turing thesis in describing decidable cases.

a. M accepts more than 81 different strings.

b. M has no rejecting control states.

c. M uses more than 81 steps on some input without halting.

d. $L(M)$ is decidable by some finite-state machine.

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