1, 2, 3

1) In Example 3.35(b), Rudin writes that "$\lim \sqrt[n]{a_n} = \frac{1}{2}$" when the associated series is

$$\frac{1}{2} + 1 + \frac{1}{8} + \frac{1}{4} + \frac{1}{32} + \frac{1}{16} + \frac{1}{128} + \frac{1}{64} + \cdots .$$

Prove this fact.

2) In the proof of Theorem 3.33, Rudin writes that "$\alpha = 1$" for the series $\sum \frac{1}{n}$ and $\sum \frac{1}{n^2}$. Prove this fact.

■

3) Let $a_n = \dfrac{n^n}{n!}$. Prove that $\displaystyle\lim_{n\to\infty} \dfrac{a_{n+1}}{a_n} = e$, and then determine

$$\lim_{n\to\infty} \frac{n}{(n!)^{1/n}}.$$

Hint: Find and then use some helpful theorems in our textbook.

■