Math 131
Homework 2
2/6/17

1, 2, 3

(1) Prove that the irrational numbers are dense in $\mathbb{R}$. In other words, show that if $a, b \in \mathbb{R}$ and $a < b$, then there exists an irrational number x such that $a < x < b$.

$\blacksquare$

(2) Suppose $a, b \in \mathbb{R}$ and $a < b$. Prove that the open interval (or "segment" in Rudin)

$$(a, b) = \{c \in \mathbb{R} \ : \ a < c < b\}$$

has the same cardinality as $\mathbb{R}$.

■

(3) Let S be a set of positive real numbers with the property that the sum of any finite subset of S is always less than or equal to 1. What can you say about the cardinality of S?

■