1, 2, 3

For all three of the problems below, view $\mathbb R$ as a metric space with the usual metric.

(1) Prove or disprove the following: There is only one open set in \mathbb{R} that contains all of \mathbb{Q} , and it is the set \mathbb{R} .

(2) How many subsets of \mathbb{R} are both open and closed?

(3) Rudin uses Theorem 1.20 to say that " \mathbb{Q} is dense in \mathbb{R} ." But is \mathbb{Q} still dense in \mathbb{R} if we instead use the definition for "dense" given in Definition 2.18? What is the relationship between these two definitions of "dense" when applied to \mathbb{Q} in \mathbb{R} ? Does each one imply the other?