1, 2, 3

1) Let X be a metric space. Prove or disprove the following statement: Every compact subset of X is bounded.

■

2) Let X be a finite metric space. What can you say about the open subsets, closed subsets, and compact subsets of X?

3) Let a and b be irrational real numbers such that $a < b$, and consider the following subset of the rational numbers:

$$A = \{q \in \mathbb{Q} : a < q < b\}.$$

View $\mathbb{Q}$ as a metric space with the usual metric. Is A an open subset of $\mathbb{Q}$? Is A a closed subset of $\mathbb{Q}$? Is A a compact subset of $\mathbb{Q}$?

■