Differential Equations Computational Practicum

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1 Introduction

Given the initial value problem:

$$\begin{cases} y' = 1 + 2y/x \\ y(1) = 2 \\ x \in (1, 10) \end{cases}$$

Needed to solve it analytically and using 3 numerical methods. Solutions should be analyzed and compared to each other. To provide data visualization I chose JavaFX and used OOP-design in code structure. Repository is available here. But first, let us find the exact solution for this equation.

2 Analytical solution

$$y' = 1 + \frac{2y}{x}$$

$$y' - \frac{2y}{x} = 1$$

A nonhomogeneous equation of form y' + f(x)y = g(x) and can be solved using the integrating factor:

$$\mu(x) = e^{\int \frac{-2}{x} \partial x} = x^{-2}$$

$$\frac{\frac{\partial y}{\partial x}}{x^2} - \frac{2y}{x^3} = \frac{1}{x^2}$$

$$\frac{\frac{\partial y}{\partial x}}{x^2} - \frac{\partial}{\partial x}\left(\frac{1}{x^2}\right) y = \frac{1}{x^2}$$

Apply the reverse product rule:

$$\frac{\partial}{\partial x}\left(\frac{y}{x^2}\right) = \frac{1}{x^2}$$

Integrate both sides with respect to x:

$$\int \frac{\partial}{\partial x}\left(\frac{y}{x^2}\right) \partial x = \int \frac{1}{x^2} \partial x$$

$$\frac{y}{x^2} = -\frac{1}{x} + c_1$$

$$y = x(c_1 x - 1)$$

Using $y(1) = 2$:

$$2 = c_1 - 1$$

$$c_1 = 3$$

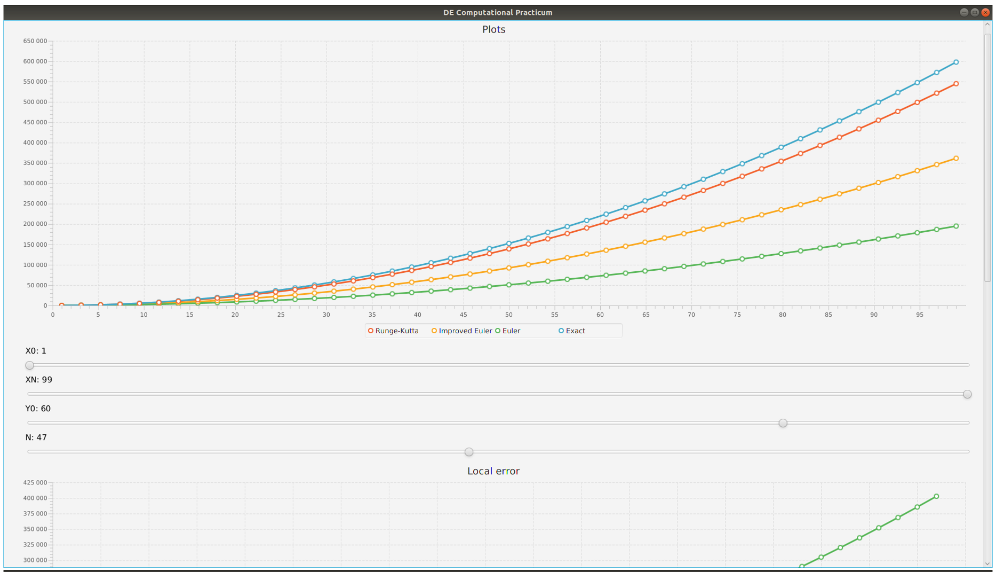
The solution is:

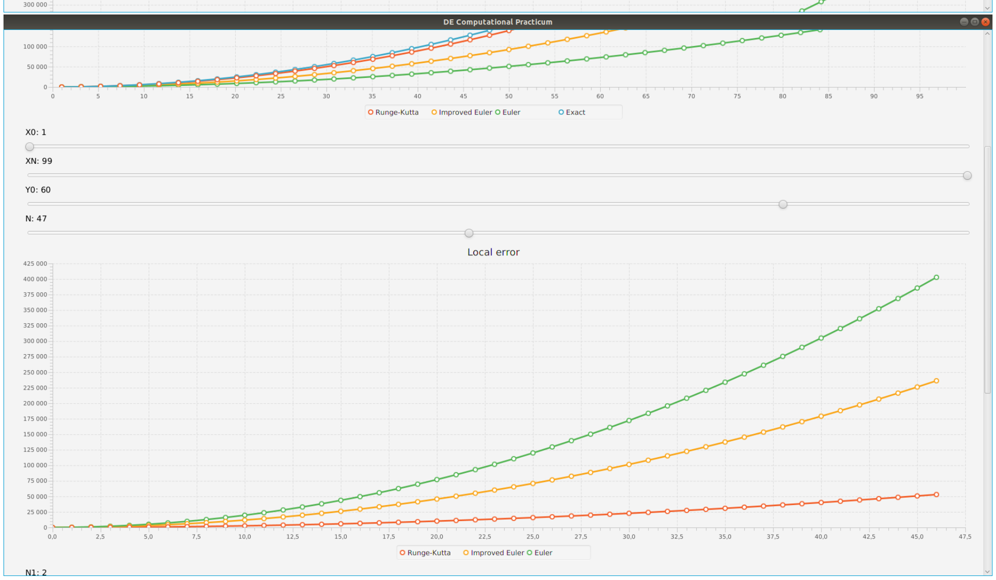
$$y = x(3x - 1), \quad x \neq 0$$

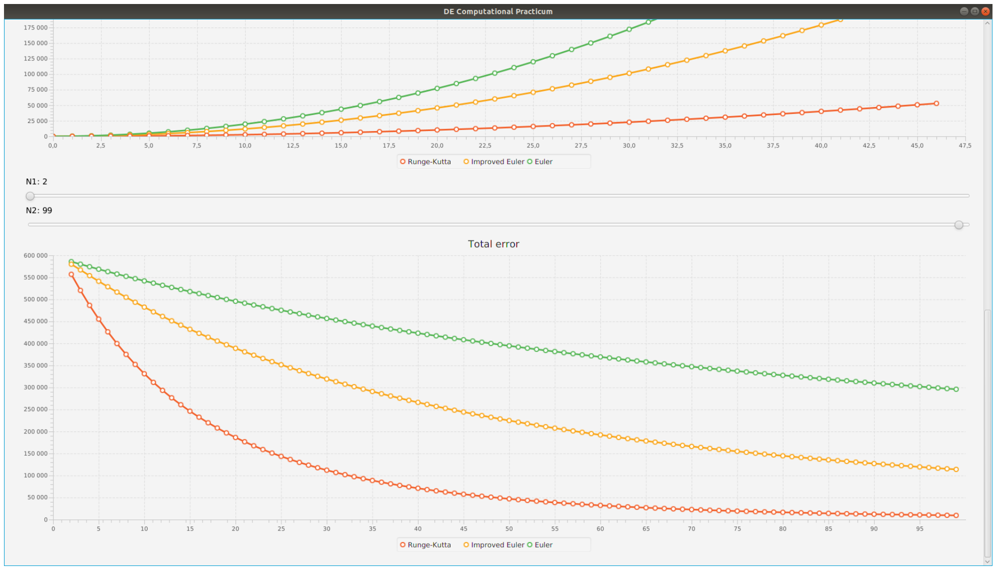
3 Numerical solutions

3.1 Application

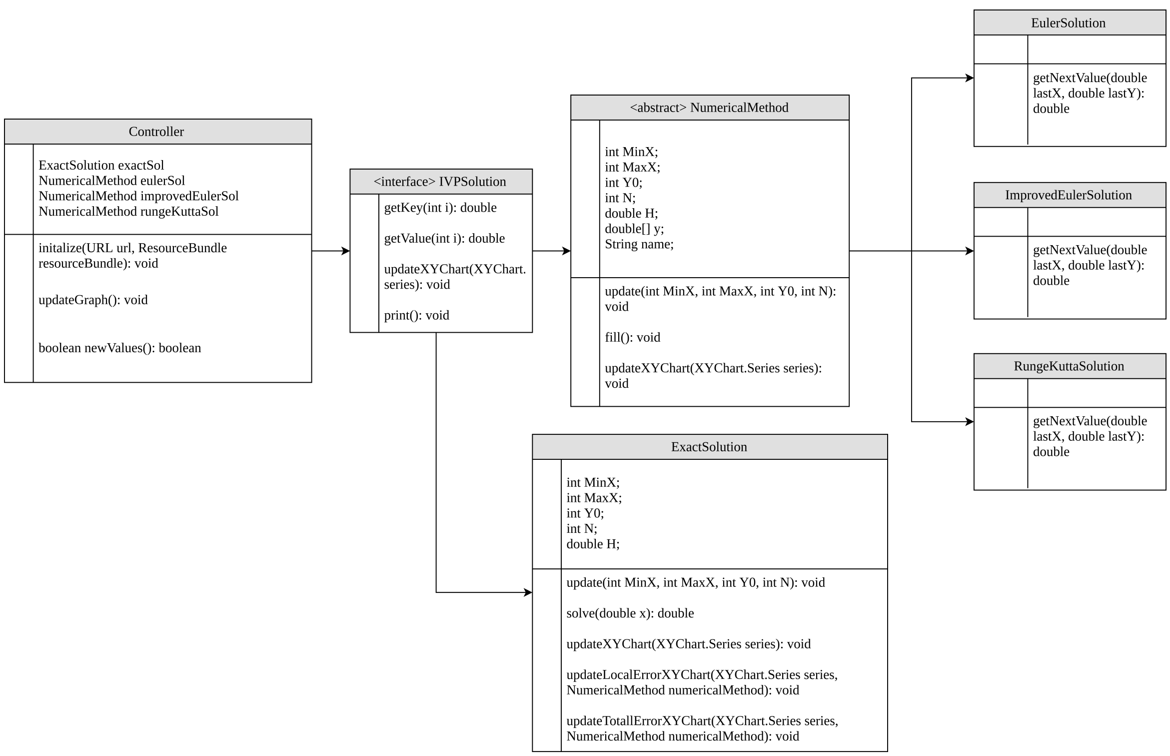
There is some screenshots of application:







3.2 UML-diagram



3.3 Code

To avoid same code for different `NumericalMethod`s when we need to calculate local and total errors I wrote function that updates this graph in `ExactSolution` class. This is code I got with function that just updates solution for comparation:

```
public void updateXYChart(XYChart.Series series) {
series.getData().clear();
for (int i = 0; i < N; ++i) {
series.getData().add(new XYChart.Data(getKey(i), getValue(i)));
}
}

public void updateLocalErrorXYChart(XYChart.Series series,
NumericalMethod numericalMethod) {
series.getData().clear();
for (int i = 0; i < N; ++i) {
series.getData().add(new XYChart.Data(i, getValue(i)
- numericalMethod.getValue(i)));
}
}
```

The `NumericalMethod` unites all 3 methods because they are exactly the same but with different formulas to get next y_i. There is part of the abstract class that shows it:

```
private void fill() {
y = new double[N];
y[0] = Y0;
for (int i = 1; i < N; i++) {
y[i] = getNextValue(getKey(i - 1), y[i - 1]);
}
}

abstract double getNextValue(double lastX, double lastY);
```

3.4 Conclution

That way we can see how Eule's, improved Euler's and Runge-Kutta methods works compare to each other and Exact solution on given IVP.