Indefinite Integrals

- Recall the fundamental theorem of calculus:

  $g(x) = \int_a^x f(t)\,dt$ or $\int_a^b f(t)\,dt = F(b) - F(a)$

  in either case, $g(x)$ and $F(x)$ are antiderivatives of $f(x)$.

  This leads to our definition of an indefinite integral:

  $$\int f(x)\,dx = F(x) + C$$

  where $F'(x) = f(x)$ and C is a constant.

  Ex: $\int 4x^3\,dx = x^4 + C$

- Definite vs. Indefinite Integrals

  definite integrals are a number (area under a curve over a defined interval), whereas an indefinite integral is a family of functions.

- List of common integrals:

  - $\int a f(x)\,dx =$
  - $\int k\,dx =$
  - $\int x^n\,dx =$
  - $\int \frac{1}{x}\,dx =$
  - $\int e^x\,dx =$
  - $\int a^x\,dx =$
  - $\int \sin x\,dx =$
  - $\int \cos x\,dx =$
  - $\int \tan x\,dx =$
  - $\int \sec^2 x\,dx =$
  - $\int \sec x \tan x\,dx =$
  - $\int \csc^2 x\,dx =$

Problems:

1) $\int e^2\,dx$
2) $\int \sin x + 2\,dx$
3) $\int \frac{5}{x}\,dx$
4) $\int 2^x + 1\,dx$
5) $\int_0^{\pi/4} \sec^2 x\,dx$
6) $\int_0^{\pi/3} \sec x \tan x + x\,dx$
7) $\int_{-1}^{3} (3x^3 + 4x - 5)\,dx$

Applications for the Definite Integral

Net change theorem:

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

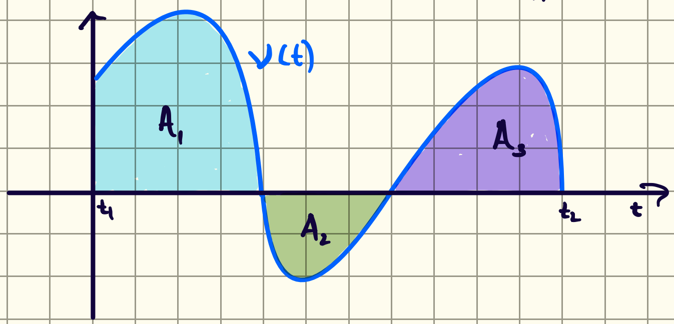
Ex: Given a velocity function $v(t) = 2t - 7$, what is the net displacement from $t=0$ to $t=6$?

$$\Rightarrow \int_0^6 2t-7\,dt = t^2-7t\Big|_0^6 = 36-42-0 = -6$$

What is the difference between displacement and distance?

Calculating displacement: $\int_{t_1}^{t_2} v(t)\,dt$

Calculating distance: $\int_{t_1}^{t_2} |v(t)|\,dt$



displacement: $\int_{t_1}^{t_2} v(t)\,dt = A_1 - A_2 + A_3$

distance: $\int_{t_1}^{t_2} |v(t)|\,dt = A_1 + A_2 + A_3$

From this, what issues do you see in calculating distance?

Ex: Given the velocity function $v(t) = 2t^2 - 4$, find the displacement and distance from $t=0$ to $t=3$.

displacement: $\int_0^3 (2t^2-4)\,dt = \frac{2}{3}t^3 - 4t\Big|_0^3 = 18-12-0 = 6$

distance: $2t^2-4=0 \Rightarrow t = \pm\sqrt{2}$

$$\Rightarrow \int_0^{\sqrt{2}} -(2t^2-4)\,dt + \int_{\sqrt{2}}^3 (2t^2-4)\,dt$$

$$= 4t - \frac{2}{3}t^3\Big|_0^{\sqrt{2}} + \frac{2}{3}t^3 - 4t\Big|_{\sqrt{2}}^3$$

$$= 4\sqrt{2} - \frac{4}{3}\sqrt{2} + 18 - 12 - \frac{4}{3}\sqrt{2} + 4\sqrt{2}$$

$$= \frac{16}{3}\sqrt{2} + 6 \approx 13.54\ldots$$

Problems: find the displacement and distance of the following

1) $\int_0^{\pi} \cos t \, dt$

2) $\int_{-2}^{1} 5t - 3 \, dt$

3) $\int_{-1}^{2} e^t - 1 \, dt$

4) $\int_{1}^{4} \frac{1}{t} - \frac{1}{2} \, dt$

5) $\int_{0}^{4} t^3 - 4t^2 + 3t \, dt$

6) $\int_{0}^{\pi/3} \sec^2 t - 2 \, dt$

7) $\int_{-\pi/3}^{\pi/3} \tan t - 1 \, dt$