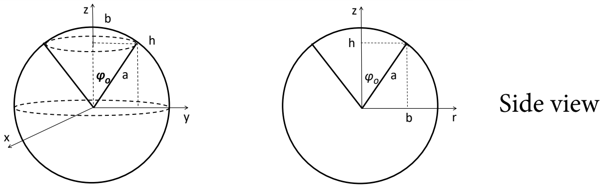
QUESTION. The region (domain) D is defined by the cone $z = \sqrt{x2 + y2}$ and the sphere with radius one centered at the origin of the coordinate system.

Write the integral of $f(x, y) = x^2 + y^2$ in D by a. spherical, b. cylindrical coordinates. (Reduce the triple integral to a single integral, but No need to calculate it.)



SOLUTION. a. In spherical coordinates: $f(x, y) = x^2 + y^2 = r^2 = R^2 \sin^2 \phi$

$$dV = R^2 \ \sin\phi \ dR \ d\phi \ d\theta \qquad I = \int_{\theta=0}^{\theta=2\pi} \int_{\phi=0}^{\phi=\pi/4} \int_{R=0}^{R=1} R^4 \ \sin^3\phi \ dR \ d\phi \ d\theta$$

I is Separable: $I = \left[\int_{\theta=0}^{\theta=2\pi} d\theta\right]\left[\int_{R=0}^{R=1} R^4 dR\right]\left[\int_{\phi=0}^{\phi=\pi/4} \sin^3\phi\right] d\phi = \dfrac{2\pi}{5}\int_{\phi=0}^{\phi=\pi/4} \sin^3\phi \, d\phi$

b. In cylindrical coordinates: $f(x, y) = x^2 + y^2 = r^2 \quad dV = r \ dr \ dz \ d\theta$

Eq. of the cone: $z = \sqrt{(1 - r^2)}$ $\qquad I = \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=1/\sqrt{2}} \int_{z=r}^{z=\sqrt{1-r^2}} r^3 dz \ dr \ d\theta$

Separable in θ, integrable in z: $\qquad I = 2\pi \int_{r=0}^{r=1/\sqrt{2}} \left[\sqrt{1 - r^2} - r\right] r^3 dr$