KOÇ UNIVERSITY

COLLEGE OF ENGINEERING

ENG 200 - PROBABILITY AND STATISTICAL METHODS FOR ENGINEERS

FALL 2022

MIDTERM EXAM 2

TIME ALLOWED: 120 MINUTES

MAKE SURE THAT YOU JUSTIFY ALL STEPS IN YOUR ANSWERS.

Name:
Student Number:

I PLEDGE ON MY HONOR THAT I HAVE NEITHER GIVEN NOR RECEIVED UNAUTHORIZED ASSISTANCE OF THE EXAM.

SIGNATURE:

Question	**Maximum points**	**Grade**
1	25	
2	25	
3	25	
4	25	
Total	100	

Question 1 (25 points) Let X and Y be two independent discrete random variables with the following PMFs:

$$P_X(k) = \begin{cases} 1/4, & \text{for } k=1 \\ 1/8, & \text{for } k=2 \\ 1/8, & \text{for } k=3 \\ 1/2, & \text{for } k=4 \\ 0, & \text{otherwise} \end{cases} \qquad P_Y(k) = \begin{cases} 1/6, & \text{for } k=1 \\ 1/6, & \text{for } k=2 \\ 1/3, & \text{for } k=3 \\ 1/3, & \text{for } k=4 \\ 0, & \text{otherwise} \end{cases}$$

(a) (5 points) Find $P(X > 2 | Y > 2)$.

Answer:

(b) (5 points) Find $E[XY]$.

Answer:

(c) (10 points) Find $Var(2X + Y)$.

Answer:

(d) (5 points) Let event A be the event when the sum of X and Y is less than or equal to 3, so $A = \{X + Y \leq 3\}$. Are X and Y independent conditional on event A? Prove your claim.

Answer:

Question 2 (25 points) The life time, X, in days, of a battery has a pdf $f(x)$ given by:

$$f(x) = \begin{cases} \alpha(2x - x^2) & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) (3 points) Find α.

Answer:

(b) (4 points) Find the CDF of X, $F_X(x)$.

Answer:

(c) (3 points) Find the following probabilities: (i) $P(X \ge 1.2)$, (ii) $P(0.5 < X \le 1.5)$, (iii) $P(X = 1)$.

Answer:

(d) (5 points) Find $E[X]$.

Answer:

(e) (5 points) Find $Var(X)$.

Answer:

(f) (5 points) A camping lantern runs on 4 batteries, all of which must be working. If four new batteries are put into the lantern find the probability that the lantern will still be working after 1.2 days.

Answer:

Question 3 (25 points) A transmitter broadcasts a constant signal X to two faulty recipients, Recipient A and B, which receive the signal with an added random noise. More explicitly, the received signal at Recipient A and B are $Y_1 = X + N_1$ and $Y_2 = X + N_2$, respectively, where N_1 and N_2 are normally distributed random variables with zero mean ($\mu_1 = \mu_2 = 0$) and variances $\sigma_1 = 1.5$ and $\sigma_2 = 2$, respectively.

(a) (5 points) Find the value of X, if Recipient A receives a signal of greater than 1 with a probability of 0.25, i.e., $P(Y_1 > 1) = 0.25$.

Answer:

(b) (5 points) The probability that the Recipient A receives a signal of less than 1 is equal to the probability that Recipient B receives a signal of less than 2, i.e., $P(Y_1 < 1) = P(Y_2 < 2)$. Find the value of X.

Answer:

(c) (7 points) The system administrator wants to claim that the probability that the difference between the received signals at Recipient A and B is less than a certain value, δ, is 90%, i.e., $P(|Y_1 - Y_2| < \delta) \leq 0.9$. Find δ.

Answer:

(d) (8 points) If $X = 1$, find the probability that only one of the received signals is between 0.5 and 2.

Answer:

Question 4 (25 points)

Let X and Y be jointly distributed random variables whose joint probability density function is given by:

$$f_{X,Y}(x,y) = \begin{cases} \frac{x}{4}, & 1 < x < 2, 0 < xy < 4, \\ 0, & \text{otherwise.} \end{cases}$$

(a) (3 points) Draw the region where $f_{X,Y}(x,y) > 0$

Answer:

(b) (7 points) Find $P(Y > X)$. *Hint: Drawing will help.*

Answer:

(c) (8 points) Find the marginal probability density function of Y, i.e., $f_Y(y)$. *Hint: You need to differentiate two cases.*

Answer:

(d) (7 points) Find $E[Y \mid Y < 2]$.

Answer: