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NUMERICAL METHODS FOR SMOKING DYNAMICS

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ABSTRACT. This study analyzes the dynamics of smoking behavior using three numerical methods: Euler, Modified Euler, and second-order Runge-Kutta. The impact of model parameters is explored, and the performance of each method is compared through numerical simulations. The results highlight the accuracy and applicability of these methods in modeling smoking behavior.

1. INTRODUCTION

Smoking poses a profound challenge to public health, contributing to a substantial burden of preventable diseases, including cardiovascular conditions, lung cancer, and chronic respiratory disorders. Despite rigorous global anti-smoking campaigns, smoking prevalence remains persistently high, underscoring the necessity for innovative intervention strategies to address this ongoing crisis effectively.

A critical component of understanding smoking behavior is the phenomenon of relapse among individuals who have quit smoking. Behavioral studies, such as those by Rahman et al. (2020), have revealed that relapse is influenced by complex interactions, including social contact with smokers and psychological factors. Empirical data suggest that approximately 15% of relapses can be attributed to sustained interactions with current smokers, highlighting the need for a nuanced approach to smoking cessation modeling.

Mathematical modeling offers a robust tool for capturing the multifaceted dynamics of smoking behavior. Early frameworks by Castillo-Garsow et al. provided foundational models, which have since been expanded to incorporate relapse rates, population demographics, and dynamic incidence rates. This research advances these developments by introducing a comprehensive model for smoking behavior that integrates state transitions, behavioral feedback mechanisms, and relapse dynamics within a delayed system of differential equations.

The model delineates the population into four key subgroups: potential smokers $P(t)$, light smokers $L(t)$, heavy smokers $S(t)$, and quitters $Q(t)$. The interactions and transitions between these subgroups are governed by a set of nonlinear differential equations, accounting for recruitment, cessation, relapse, and mortality rates. The simulation of this model employs three numerical techniques—Euler's method, the Modified Euler method, and the second-order Runge-Kutta method—to explore the system's dynamics and evaluate the accuracy and efficiency of these approaches.

This paper is structured as follows: Section 3 introduces the mathematical framework and the parameters defining the model. Section 4 elaborates on the numerical methods implemented to solve the equations. Section 5 discusses the simulation results, providing insights into the implications of the model, while Section 6 concludes with a synthesis of findings and directions for future research.
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2. Model Parameters

The parameters used in the smoking model are listed below:

- $\beta = 0.03$: The transmission rate of non-smokers into light smokers.
- $\mu = 0.008$: The natural death rate.
- $d = 0.004$: Smoking-related death rate.
- $\varepsilon = 0.02$:: The rate at which light smokers transition into persistent smoking.
- $\delta = 0.074$: The rate at which smokers temporarily quit smoking.
- $\alpha = 0.15$: The quitting rate affected by external factors.
- $b = 0.4$: The fraction of quitters that return to the light smoker population.
- $A = 0.25$: The recruitment rate of potential smokers into the population.

3. Mathematical Model

The smoking dynamics model is described by the following system of differential equations:

$$
\begin{aligned}
\frac{dP(t)}{dt} &= A - \frac{2\beta P(t)L(t)}{P(t)+L(t)} - (d+\mu)P(t),\\
\frac{dL(t)}{dt} &= \frac{2\beta P(t)L(t)}{P(t)+L(t)} - (\varepsilon+d+\mu)L(t) + b\alpha Q(t),\\
\frac{dS(t)}{dt} &= \varepsilon L(t) - \delta S(t) - (d+\mu)S(t) - (1-b)\alpha Q(t),\\
\frac{dQ(t)}{dt} &= \delta S(t) - (\mu+d)Q(t) - \alpha Q(t).
\end{aligned}
\tag{3.1}
$$

where P, L, S, Q represent potential smokers, light smokers, persistent smokers, and quit smokers, respectively.

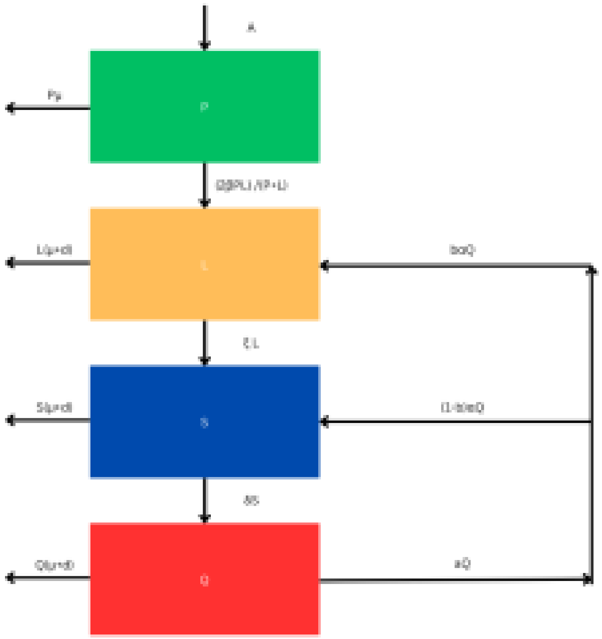

Figure 1. Flow chart diagram of the model.

From	To	Transition Rate
Potential Smokers (P)	Light Smokers (L)	$\frac{2\beta PL}{P+L}$
Light Smokers (L)	Smokers (S)	ξL
Smokers (S)	Quit Smokers (Q)	δS
Quit Smokers (Q)	Light Smokers (L)	$b\alpha Q$
Quit Smokers (Q)	Smokers (S)	$(1-b)\alpha Q$
Potential Smokers (P)	Death	$(\mu)P$
Light Smokers (L)	Death	$(\mu+d)L$
Smokers (S)	Death	$(\mu+d)S$
Quit Smokers (Q)	Death	$(\mu+d)Q$

TABLE 1. Transitions between states in the smoking model.

4. Numerical Methods

Three numerical techniques are used to analyze the model dynamics: Euler's method, Modified Euler method, and the second-order Runge-Kutta method.

4.1. **Euler Method.** The Euler method is a basic first-order numerical technique for solving ordinary differential equations. It approximates the solution incrementally by the formula:

$$x_{n+1} = x_n + hf(t_n, x_n), \quad (4.1) \tag{4.1}$$

where h represents the step size, t_n is the current time point, and $f(t_n, x_n)$ is the derivative evaluated at t_n. The simplicity of the Euler method makes it computationally efficient. However, it suffers from significant numerical errors, particularly when applied to stiff systems or problems requiring high precision. As h becomes smaller, the accuracy improves, but the computational cost increases.

4.2. **Modified Euler Method.** The Modified Euler method, also known as Heun's method, refines the basic Euler approach by employing a predictor-corrector mechanism. The steps are as follows:

$$x_{\text{predictor},n+1} = x_n + hf(t_n, x_n), \quad (4.2) \tag{4.2}$$

$$x_{n+1} = x_n + \frac{h}{2}\left[f(t_n, x_n) + f(t_{n+1}, x_{\text{predictor},n+1})\right], \quad (4.3) \tag{4.3}$$

Here, the predicted value $x_{\text{predictor},n+1}$ is used to correct the final value x_{n+1}. By averaging the slopes at the current and predicted points, this method improves the stability and accuracy of the solution while maintaining moderate computational demands.

4.3. **Second-Order Runge-Kutta Method.** The second-order Runge-Kutta (RK2) method represents a class of intermediate methods between the Euler methods and higher-order Runge-Kutta schemes. It introduces an intermediate slope, k_2, to achieve greater precision:

$$k_1 = f(t_n, x_n), \quad (4.4) \tag{4.4}$$

$$k_2 = f\left(t_n + \frac{h}{2}, x_n + \frac{h}{2}k_1\right), \quad (4.5) \tag{4.5}$$

$$x_{n+1} = x_n + hk_2, \quad (4.6) \tag{4.6}$$

The intermediate slope k_2 incorporates information about the system's behavior halfway through the interval h. This additional step enhances the method's ability to capture nonlinear dynamics and achieve higher accuracy compared to both the Euler and Modified Euler methods.

Each method has distinct advantages and limitations, making the choice dependent on the problem's specific requirements, such as precision and computational constraints.

5. Results and Discussion

Numerical simulations are conducted to evaluate the performance of each method. Separate figures illustrate the dynamics of the four subpopulations (P, L, S, Q).

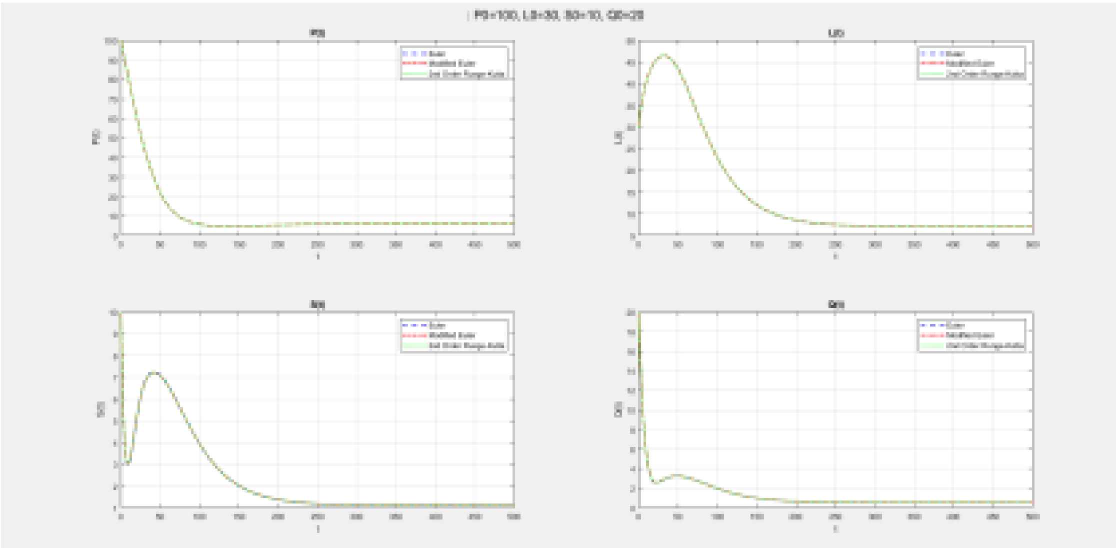

FIGURE 2. Dynamics of Potential Smokers $P(t)$,Light Smokers $L(t)$, Smokers $S(t)$, Quit Smokers $S(t)$ using Euler, Modified Euler, and Runge-Kutta methods. (P0 = 100, L0 = 30, S0 = 10,Q0 = 20)

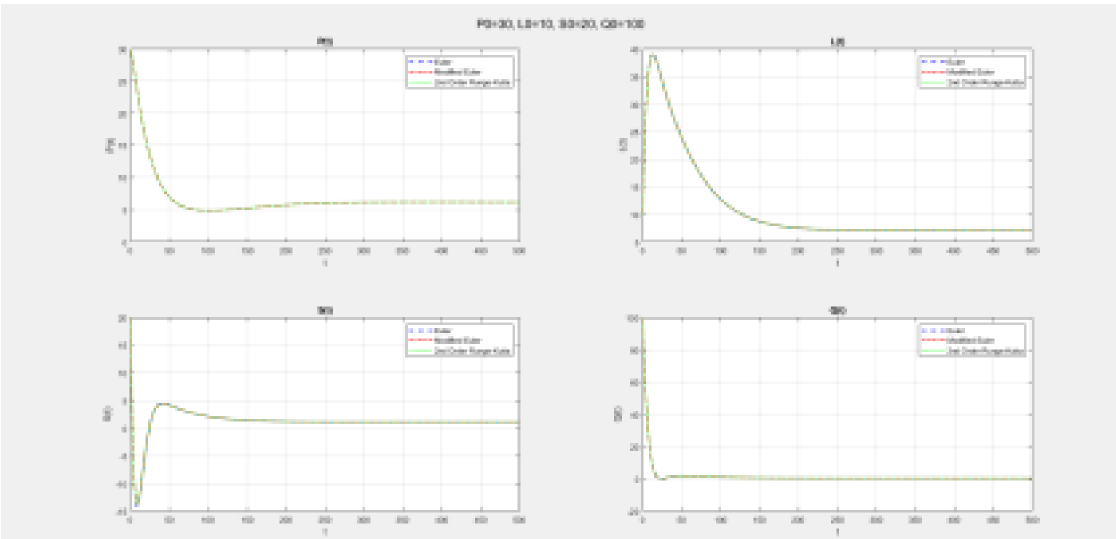

FIGURE 3. Dynamics of Potential Smokers $P(t)$,Light Smokers $L(t)$, Smokers $S(t)$, Quit Smokers $S(t)$ using Euler, Modified Euler, and Runge-Kutta methods. (P0 = 30, L0 = 10, S0 = 20,Q0 = 100)

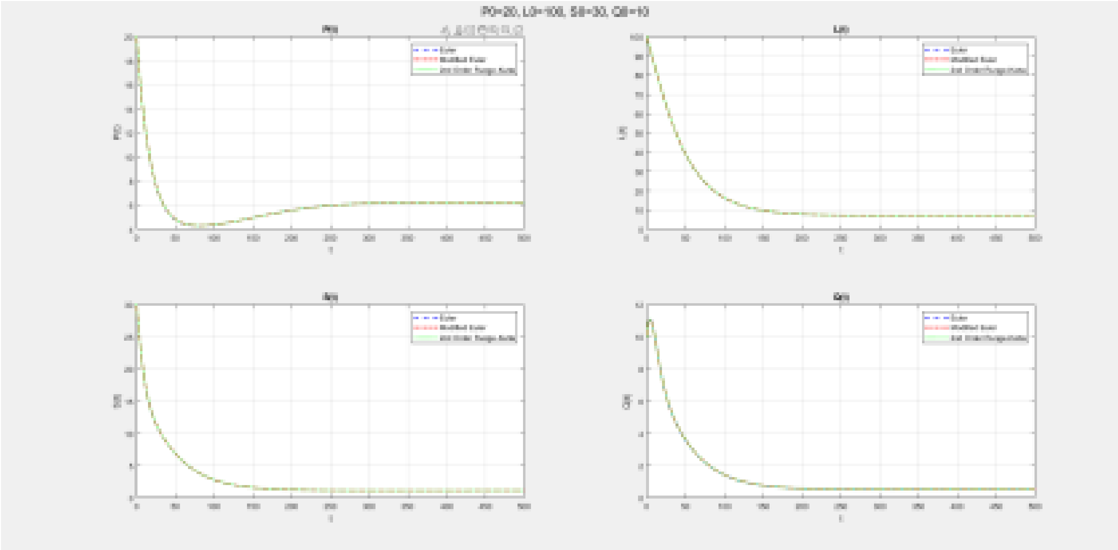

FIGURE 4. Dynamics of Potential Smokers $P(t)$,Light Smokers $L(t)$, Smokers $S(t)$, Quit Smokers $S(t)$ using Euler, Modified Euler, and Runge-Kutta methods. (P0 = 20, L0 = 100, S0 = 30,Q0 = 10)

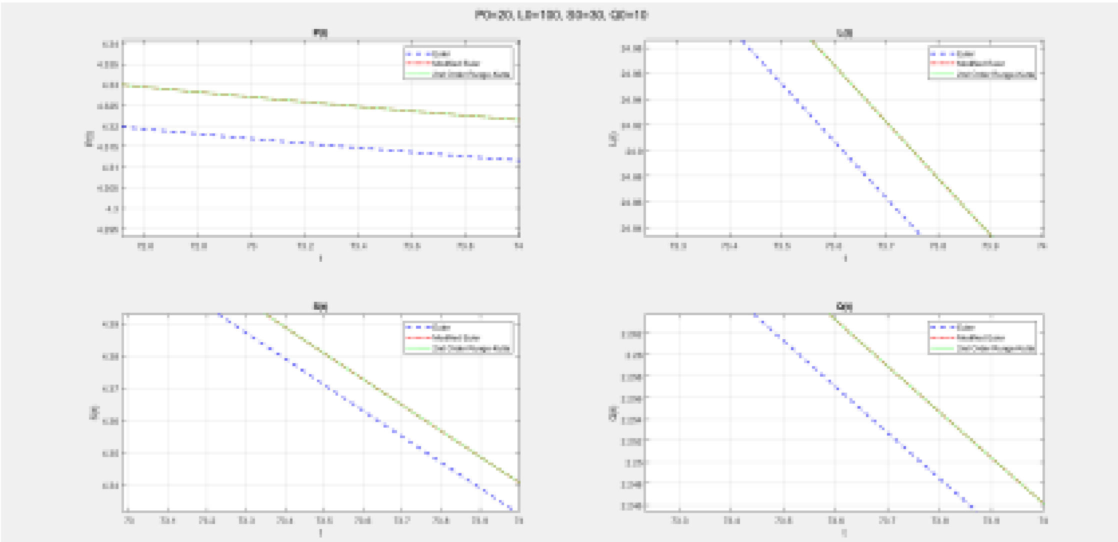

FIGURE 5. Dynamics of Potential Smokers P(t),Light Smokers L(t), Smokers S(t), Quit Smokers S(t) using Euler, Modified Euler, and Runge-Kutta methods.(In order to easily observe the differences in the graphs, h (number of steps) was enlarged and t (time) was reduced.) .

As observed in the graphs, the second-order Runge-Kutta method provides significantly more accurate and stable results compared to the other numerical methods analyzed. In contrast, the Euler method demonstrates noticeable deviations from the expected trajectories, particularly in regions where the system dynamics exhibit rapid changes. This deviation arises from the Euler method's simplistic approach, which only considers the slope at the current point, leading to accumulated errors over time.

The second-order Runge-Kutta method, on the other hand, achieves greater precision by incorporating an intermediate slope, allowing it to better capture the nonlinear behavior of the model. For instance, in (reference the relevant graph, e.g., Figure 4), while the Euler method diverges significantly from the expected trajectory, the second-order Runge-Kutta method closely follows it, demonstrating its superiority in modeling complex interactions. These observations highlight the limitations of the Euler method and the effectiveness of the Runge-Kutta approach in addressing the challenges posed by systems with intricate dynamics.

5.1. **Discussion.** The results from the three numerical methods show that the Runge-Kutta method provides the most accurate and stable solutions, especially for smaller step sizes. The Modified Euler method offers a reasonable balance between computational efficiency and accuracy. However, the Euler method demonstrates significant errors in modeling rapid changes, particularly in the dynamics of persistent smokers (S(t)). The discrepancies observed with the Euler method suggest that it might not be the best choice for modeling systems with fast-changing or highly nonlinear behaviors, as it tends to introduce large errors over time.

Moreover, the second-order Runge-Kutta method, with its increased accuracy, proves to be more effective in capturing these rapid changes, making it a better tool for modeling such dynamic systems. The ability of the Runge-Kutta method to handle these changes ensures that it can more reliably model complex phenomena in systems like the smoking behavior model, where interactions evolve rapidly and nonlinearly.

6. Conclusion

The results of the smoking model provide valuable insights into the behaviors of smokers and how these behaviors evolve over time. For individuals who smoke consistently, the model indicates that their behavior generally remains constant, making it difficult for them to quit smoking and leading to a longer duration of smoking. This suggests that individuals with long-term smoking habits have strong dependencies, requiring stronger and long-term interventions to encourage them to quit smoking. Long-term smoking has serious health implications, and the process of quitting is more complex for these individuals.

Individuals who smoke occasionally, on the other hand, show more variable behavior in the model, transitioning between smoking and non-smoking. This group is more sensitive to external factors such as social environment, stress, or environmental influences, which can cause fluctuations in their decision to continue smoking or quit. Targeted preventive measures for this group are critical in reducing the initiation rate of smoking and preventing continued smoking.

For potential smokers who have not yet started smoking, the model shows that the risk of smoking initiation increases due to factors like environmental influences or peer pressure. This highlights the importance of early interventions and preventive health policies for individuals who have not smoked. Such measures play a significant role in preventing the initiation of smoking.

For individuals who decide to quit smoking, the model shows that this decision can be sustained over time and their behavior can stabilize. This emphasizes the importance of time and ongoing programs in the smoking cessation process. Long-term cessation requires constant and careful monitoring, making it essential for smoking cessation programs to be effective not only in the short term but also in the long term.

The methods used in the model analyze smoking behaviors in different ways. The Euler method, due to its simplicity, provides quick results but fails to accurately reflect the complexity of the system, leading to significant deviations, especially in situations where rapid changes occur. On the other hand, the second-order Runge-Kutta method increases the accuracy of the model and better simulates the complex interactions. This method provides more stable and precise results, reflecting the dynamics of smoking more realistically. These differences are crucial in determining which method should be used in various scenarios.

In conclusion, the findings of this model provide valuable insights into smoking behaviors and contribute to the development of more effective health policies for different groups. As smoking behavior varies from person to person, tailored intervention strategies are required for each group. Specifically, strategies to support individuals who are new to smoking or those who smoke occasionally in quitting smoking are of critical importance in public health policies.

Future work could include incorporating delayed terms into the model to reflect time lags in behavioral responses. Such an extension would improve the model's accuracy and better capture the real-world dynamics of smoking behaviors, where changes in habits often occur with some delay.

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