PHYS2114 Mid-term 2024

Question 1

a) i) There is no electric field inside a conductor. If there were the free charges ~~AO~~ would feel a force and move until they came to the edge and could hence not move any more. There must be free charges on a conductor to cancel out any external electric fields. As these can not be inside the conductor ($\nabla \cdot \underset{\sim}{E} = \rho/\varepsilon_0$ and $\underset{\sim}{E}$ is $0 \Rightarrow \rho$ is 0) the only place they can be is the surface.

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ii) The free charges are free to move tangentially along the surface. They can not move perpendicularily out of the surface (or in as they are on the surface). Hence, they will move (because $F = \underset{\sim}{E}q$) to cancel out any tangential component of $\underset{\sim}{E}$. They can not move perpendicularily to cancel the perpendicular component of $\underset{\sim}{E}$.

b) $V = A\sin(kx)e^{-ky}$

i) $\frac{\partial V}{\partial x} = -Ak\cos(kx)e^{-ky}$ $\qquad$ $\frac{\partial V}{\partial y} = -Ak\sin(kx)e^{-ky}$

$\frac{\partial^2 V}{\partial x^2} = -Ak^2\sin(kx)e^{-ky} = -k^2V$ $\qquad$ $\frac{\partial^2 V}{\partial y^2} = Ak^2\sin(kx)e^{-ky} = k^2V$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = -k^2V + k^2V = 0$$

hence Laplace's eqn is satisfied.

ii) Maximums and minimums must be on the boundary. In this case they must be along the line $y=-b$ as this is when the magnitude of V will be largest. Maximum values will be found where $\sin(kx)=1$ and minimums at $\sin(kx)=-1$. For example, a maximum at $x=\frac{\pi}{2k}, y=-b$ and a minimum at $x=\frac{3\pi}{2k}, y=-b$.

c) The magnetic force does no work and hence can't change the kinetic energy and hence speed.

distance travelled in time dt.

$dW_{mag} = \underset{\sim}{F}_{mag} \cdot d\underset{\sim}{s} = q(\underset{\sim}{v} \times \underset{\sim}{B}) \cdot (\underset{\sim}{v}dt) = 0$

$\underset{\sim}{v} \times \underset{\sim}{B}$ is $\perp$ to $\underset{\sim}{v}$ an hence $(\underset{\sim}{v} \times \underset{\sim}{B}) \cdot \underset{\sim}{v} = 0$.

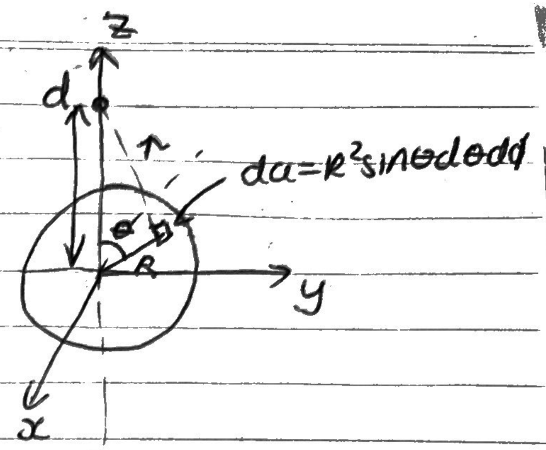
Question 2

a) $dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$

$= \frac{1}{4\pi\varepsilon_0}\frac{k|z|\,da}{(R^2+d^2-2Rd\cos\theta)^{1/2}}$

at $|z| = h$

$dV = \frac{1}{4\pi\varepsilon_0}\frac{kh\,da}{(R^2+d^2-2Rd\cos\theta)^{1/2}}$.



$r^2 = R^2+d^2-2Rd\cos\theta$

b) Need to integrate over the sphere. For $0<\theta<\frac{\pi}{2}$: $|z| = z = R\cos\theta$

$\frac{\pi}{2}<\theta<\pi$: $|z| = -z = -R\cos\theta$.

$$V = \frac{k}{4\pi\varepsilon_0}\left[\int_0^{2\pi}\int_0^{\pi/2}\frac{R\cos\theta \cdot R^2\sin\theta\, d\theta\, d\phi}{(R^2+d^2-2Rd\cos\theta)^{1/2}} + \int_0^{2\pi}\int_{\pi/2}^{\pi}\frac{-R\cos\theta \cdot R^2\sin\theta\, d\theta\, d\phi}{(R^2+d^2-2Rd\cos\theta)^{1/2}}\right]$$

$$= \frac{\cancel{2\pi} kR^3}{2\cancel{4\pi}\varepsilon_0}\left[\int_0^{\pi/2}\frac{\cos\theta\sin\theta\, d\theta}{(R^2+d^2-2Rd\cos\theta)^{1/2}} - \int_{\pi/2}^{\pi}\frac{\cos\theta\sin\theta\, d\theta}{(R^2+d^2-2Rd\cos\theta)^{1/2}}\right]$$

$$= \frac{kR^3}{2\varepsilon_0}\cdot\frac{1}{3d^2R^2}\left(\left[(R^2+dR\cos\theta+d^2)\sqrt{R^2-2dR\cos\theta+d^2}\right]_0^{\pi/2} - \left[(R^2+dR\cos\theta+d^2)\sqrt{R^2-2dR\cos\theta+d^2}\right]_{\pi/2}^{\pi}\right)$$

$$= \frac{kR}{6\varepsilon_0 d^2}\Big((R^2+d^2)\sqrt{R^2+d^2} - (R^2+dR+d^2)\underbrace{\sqrt{R^2-2dR+d^2}}_{\sqrt{(d-R)^2}!} - (R^2-dR+d^2)\underbrace{\sqrt{R^2+2dR+d^2}}_{\sqrt{(R+d)^2}!} + (R^2+d^2)\sqrt{R^2+d^2}\Big)$$

$$= \frac{kR}{6\varepsilon_0 d^2}\left(2(R^2+d^2)^{3/2} - (d-R)(R^2+dR+d^2) - (R+d)(R^2-dR+d^2)\right)$$

$$= \frac{kR}{6\varepsilon_0 d^2}\left(2(R^2+d^2)^{3/2} - dR^2 - d^2R - d^3 + R^3 + dR^2 + Rd^2 - R^3 + dR^2 - Rd^2 - dR^2 + d^2R - d^3\right)$$

$$= \frac{kR}{6\varepsilon_0 d^2}\left(2(R^2+d^2)^{3/2} - 2d^3\right)$$

$$= \frac{kR\left((R^2+d^2)^{3/2} - d^3\right)}{3\varepsilon_0 d^2}.$$

c) The potential should be continuous. To check find V when $d=R$

outside: $V = \frac{kR\left((R^2+R^2)^{3/2}-R^3\right)}{3\varepsilon_0 R^2} = \frac{k(2^{3/2}R^3-R^3)}{3\varepsilon_0 R} = \frac{kR^2(2^{3/2}-1)}{3\varepsilon_0}$

inside: $V = \frac{kR\left((R^2+R^2)^{3/2}-R^3\right)}{3\varepsilon_0 R^2} = \frac{kR^2(2^{3/2}-1)}{3\varepsilon_0}$

these are the same so V is continuous on the z axis.

d) $\underline{E}$ is not continuous normal to the surface when there is a surface charge density so I would not expect E_z to be continuous at $z=d=R$.

$$\underline{E} = -\nabla V \Rightarrow E_z = -\frac{\partial V}{\partial z} = -\frac{\partial V}{\partial d}$$

outside: $$E_z = -\frac{kR}{3\varepsilon_0}\frac{\partial}{\partial d}\left(((R^2+d^2)^{3/2} - d^3)d^{-2}\right)$$

$$= -\frac{kR}{3\varepsilon_0}\left(-\frac{2}{d^3}((R^2+d^2)^{3/2}-d^3) + \frac{1}{d^2}\left(\frac{3}{\cancel{2}}\cdot\cancel{2}d\cdot(R^2+d^2)^{1/2} - 3d^2\right)\right)$$

$$= -\frac{kR}{3\varepsilon_0}\left(\frac{-2((R^2+d^2)^{3/2}-d^3)}{d^3} + \frac{(3d^2(R^2+d^2)^{1/2} - 3d^3}{d^3}\right)$$

$$= \frac{-kR}{3\varepsilon_0 d^3}\left(-2(R^2+d^2)^{3/2} + 3d^2(R^2+d^2)^{1/2} - d^3\right)$$

at $d=R$ $$= -\frac{k\cancel{R}}{3\varepsilon_0 R^{\cancel{3}2}}\left(-2(2R^2)^{3/2} + 3R^2(2R^2)^{1/2} - R^3\right)$$

$$= \frac{-k}{3\varepsilon_0 R^2}\left(-4\sqrt{2}R^3 + 3\sqrt{2}R^3 - R^3\right)$$

$$= \frac{+k}{3\varepsilon_0}(R+\sqrt{2}R)$$

Inside: $$E_z = -\frac{kR}{3\varepsilon_0}\frac{\partial}{\partial d}\left(((R^2+d^2)^{3/2} - R^3)d^{-2}\right)$$

$$= -\frac{kR}{3\varepsilon_0}\left(-\frac{2}{d^3}((R^2+d^2)^{3/2}-R^3) + \frac{1}{d^2}\left(\frac{3}{\cancel{2}}\cdot\cancel{2}d\cdot(R^2+d^2)^{1/2}\right)\right)$$

$$= -\frac{kR}{3\varepsilon_0 d^3}\left(-2((R^2+d^2)^{3/2} - R^3) + 3d^2(R^2+d^2)^{1/2}\right)$$

at $d=R$ $$= -\frac{kR}{3\varepsilon_0 R^3}\left(-2((2R^2)^{3/2} - R^3) + 3R^2(2R^2)^{1/2}\right)$$

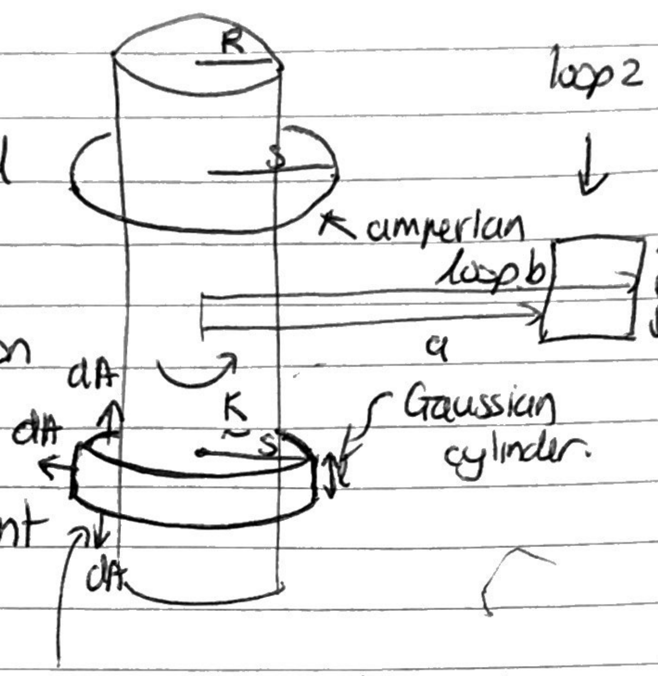
$$= -\frac{k}{3\varepsilon_0 R^2}\left(-4\sqrt{2}R^3 + 2R^3 + 3\sqrt{2}R^3\right) = \frac{k}{3\varepsilon_0}(\sqrt{2}R - 2R)$$

These are not the same so discontinuous as expected.

Question 3

a) Radially $B_s = 0$.

Can either use Griffith's argument: Suppose $\underset{\sim}{B}$ was radially outwards. If you turned the solenoid upside down it would still be radially outwards. But from any point this would look the same as reversing the direction of the current and this would make the field radially inwards. Since these must be the same this can only be true if radial component is zero.



OR

Use Gauss's law for magnetism with surface shown. As it is infinite and symmetric ~~flux~~ B in top surface must be same as B in bottom surface but dA in opposite directions so these cancel. Radial dA must then give:

$$\oint B \cdot d\underset{\sim}{A} = B_s 2\pi s l = 0$$

$\Rightarrow B = 0$ radially.

Circumferentially $B_\phi = 0$

Around Amperian loop.

no enclosed current.

$$\oint \underset{\sim}{B} \cdot d\underset{\sim}{l} = B_\phi 2\pi s = \mu_0 I_{enc} = 0.$$

Vertically $B_z = 0$.

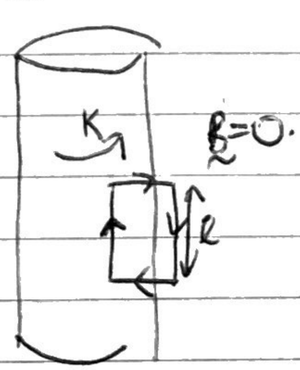
Around loop 2 we can also use Ampere's law. In radial direction $B_s = 0$.

$$\oint B \cdot dl = B(a) l - B(b) l = \mu_0 I_{enc} = 0.$$

$$\Rightarrow B(a) = B(b).$$

$\Rightarrow$ Field does not depend on distance so as it must go to 0 at ∞ must be zero everywhere.

b) Around loop shown only inner vertical side contributes, $\underset{\sim}{B} = 0$ outside, as it is infinite at upper and lower sides B must be the same and dl in opposite directions $\Rightarrow$ they cancel. OR already shown $B_s = 0$



$$\Rightarrow \oint B \cdot dl = I_{enc} \mu_0$$

$$K = \frac{dI}{dl_\perp} = \frac{I}{l} \Rightarrow I = Kl.$$

$$\Rightarrow B \cdot l = Kl\mu_0.$$

$$B = K\mu_0 \,\hat{\underset{\sim}{z}}$$

Alternative argument for (a) and direction on (b).
Biot-Savart law gives direction:

$$d\underset{\sim}{B} = \underset{\sim}{I} \times \hat{\underset{\sim}{r}}$$ so must be ⊥ to $\hat{\underset{\sim}{\phi}}$

in $\underset{\sim}{\phi}$ direction

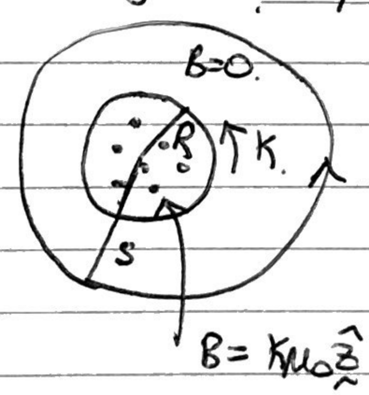
c) $\oint \underset{\sim}{A} \cdot d\underset{\sim}{l} = \int (\nabla \times \underset{\sim}{A}) \cdot d\underset{\sim}{a}$ using stoke's theorem
$= \int \underset{\sim}{B} \cdot d\underset{\sim}{a}$ as $\nabla \times \underset{\sim}{A} = \underset{\sim}{B}$.

d) Outside tube:

$\oint \underset{\sim}{A} \cdot d\underset{\sim}{l} = A \cdot 2\pi s = \int \underset{\sim}{B} \cdot d\underset{\sim}{a}$
$= K\mu_0 \pi R^2.$

$\Rightarrow A = \frac{K\mu_0 R^2}{2s} \hat{\underset{\sim}{\phi}}$ same direction as current.

Looking from on top.



Inside tube:

$$\oint \underset{\sim}{A} \cdot d\underset{\sim}{l} = 2\pi s A = \int \underset{\sim}{B} \cdot d\underset{\sim}{a} = K\mu_0 \pi s^2.$$

$$\Rightarrow \underset{\sim}{A} = \frac{K\mu_0 s}{2} \hat{\underset{\sim}{\phi}}$$

e) Use curl for cylindrical coordinates, $A_s = 0$, $A_z = 0$ $A_\phi = \frac{K\mu_0 R^2}{2s}$

$$\nabla \times \underset{\sim}{A} = -\frac{\partial\left(\frac{K\mu_0 R^2}{2s}\right)}{\partial z} \hat{\underset{\sim}{s}} + \frac{1}{s}\frac{\partial s\left(\frac{K\mu_0 R^2}{2s}\right)}{\partial s} \hat{\underset{\sim}{z}}$$

const wrt z

constant wrt s.

$$= 0.$$

As was shown in part a.