UNSW SCHOOL OF PHYSICS

PHYS2111 – Quantum Mechanics

Tutorial 5

Question 1

(Shankar 1.8.4) An arbitrary $n \times n$ matrix need not have n eigenvectors. Consider as an example

$$\Omega = \begin{pmatrix} 4 & 1 \\ -1 & 2 \end{pmatrix}$$

(a) Show that $\omega_1 = \omega_2 = 3$.

(b) By feeding in this value show we get only one eigenvector of the form

$$\frac{1}{\sqrt{2|a|^2}} \begin{pmatrix} a \\ -a \end{pmatrix}.$$

We cannot find another linearly independent eigenvector.

Question 2

Consider the operator

$$\hat{T} = \begin{pmatrix} 1 & 1-i \\ 1+i & 0 \end{pmatrix}.$$

(a) Is T Hermitian?

(b) Find the two eigenvalues of $\hat{T}$, which we label t_1 and t_2, and show that they are real. Find the corresponding eigenvectors $|t_1\rangle$ and $|t_2\rangle$.

(c) Use the eigenvectors to find the matrix U which diagonalises the matrix $\hat{T}$. That is, find U such that $U^\dagger \hat{T} U = \hat{D}_T$ where $\hat{D}_T$ is a diagonal matrix. What is $\hat{D}_T$?

(d) Check that

$$\sum_i t_i \, |t_i\rangle \langle t_i| = \hat{T}.$$

Question 3

Show that the commutator $\hat{C} = i[\hat{A}, \hat{B}]$ is Hermitian if $\hat{A}$ and $\hat{B}$ are both Hermitian. *Hint:* Remember that $(\hat{A}\hat{B})^\dagger = \hat{B}^\dagger \hat{A}^\dagger$.

Question 4

The Pauli matrices are defined as

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \hat{\sigma}_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \hat{\sigma}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(a) Show that $[\hat{\sigma}_1, \hat{\sigma}_2] = 2i\hat{\sigma}_3$.

(b) Use the generalised uncertainty principle

$$\Delta A . \Delta B \geq \left| \frac{1}{2i} \langle [\hat{A}, \hat{B}] \rangle \right|$$

to obtain $\Delta\sigma_1 \Delta\sigma_2$ for:

i. the state $|u\rangle$;

ii. the state $|d\rangle$;

iii. the state $\frac{1}{\sqrt{2}}(|u\rangle + |d\rangle)$.

Explain the physical meaning.

Question 5

An observable $\hat{A}$ and a particle with a normalised wavefunction $|\psi\rangle$ are represented in some basis by

$$\hat{A} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad |\psi\rangle = \frac{1}{2}\begin{pmatrix} i \\ 1 \\ 1-i \end{pmatrix}.$$

(a) Find the eigenvalues, a_i, and eigenvectors, $|a_i\rangle$, of $\hat{A}$.

(b) Find the probability $P_\psi(a_i)$ that a measurement of $\hat{A}$ on the particle gives each of the eigenvalues a_i.

(c) Find the expectation value of the measurement, that is

$$\langle A\rangle = \sum_{i=1}^{3} a_i P_\psi(a_i),$$

and check that it is equal to $\langle\psi|\,\hat{A}\,|\psi\rangle$.

(d) Find the variance of the measurement

$$\sigma_A^2 = (\Delta A)^2 = \sum_{i=1}^{3} P_\psi(a_i)\,(a_i - \langle A\rangle)^2$$

and show that it is equal to $\langle\psi|\,\hat{A}^2\,|\psi\rangle$.

Question 6

Consider the Hermitian matrices

$$A = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}, \qquad B = \begin{pmatrix} 6 & -4 & -1 \\ -4 & 6 & -1 \\ -1 & -1 & 3 \end{pmatrix}.$$

(a) Show that $[A, B] = 0$.

(b) Find a common set of eigenvectors of A and B and their respective eigenvalues under A and B.

(c) Can you find an eigenvector of A that is *not* also an eigenvector of B?

Question 7

The operators $\hat{A}$ and $\hat{B}$ are, in matrix form:

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$

(a) Show that $\hat{A}$ and $\hat{B}$ do not commute.

(b) Find the eigenvalues and eigenvectors of $[\hat{A}, \hat{B}]$.

(c) Are there any kets for which one can simultaneously find a well-defined value of A and B? If so, find them and give their values for A and B.

Question 8

(Park 4.5) Show that there are no possible solutions for the infinite potential where $E < V < 0$.

Question 9

(Zelevinsky 3.2) A particle in the infinitely deep potential box of width a has an initial wave function $\Psi(x, t = 0) = A \sin^3(\pi x/a)$. Find the wave function at arbitrary time $t > 0$. Does the particle return to the initial state at some moment in time T?

Question 10

(Zelevinsky 3.3) A particle is initially in the ground state of an infinite potential box where the box limits are $x = 0$ and $x = a$. At $t = 0$ the right wall is instantaneously moved from $x = a$ to $x = b > a$. What is the probability thfor the particle to turn out at $t > 0$ in an excited state of the new box. Specifically discuss the case of $b = 2a$

Question 11

(Zelevinsky 3.5) A particle is placed in a potential well of finite depth U_0. The width a of the well is fixed in such a way that the particle has only one bound state with binding energy $\epsilon = U_0/2$. Calculate the probabilities of finding the particle in classically allowed and classically forbidden regions.

Question 12

(past exam) Consider a particle confined inside a three-dimensional potential 'box' with side lengths a, b and c, as shown in Fig. 1.

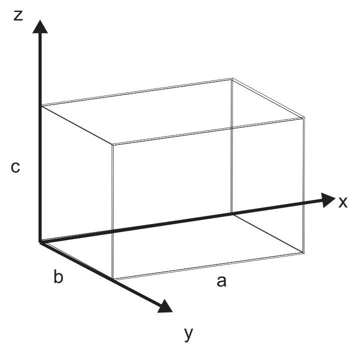

Figure 1:

(a) Starting with the three-dimensional Schrödinger Equation in cartesian coordinates and assuming a separable variable solution, derive an expression for the wave functions and the corresponding energies.

(b) Determine an expression for the normalisation constant for the wave function.

(c) Calculate the probability that the particle will be found on the interval $a/2 \leq x \leq 3a/4$ if the particle is in the ground state.