THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

Practice Mid-term test 1

PHYS2114
Electromagnetism

(1) TIME ALLOWED – 50 minutes

(2) TOTAL NUMBER OF QUESTIONS – 3

(3) ANSWER ALL QUESTIONS

(4) THE QUESTIONS ARE **NOT** OF EQUAL VALUE

(5) TOTAL NUMBER OF MARKS – 20

(6) THIS PAPER MAY BE RETAINED BY THE CANDIDATE

Use a seperate page clearly marked Question 1. This question is worth *6 marks.*

You should answer each part of this question in less than half a page. Include equations and diagrams in your answer where appropriate. Marks will be awarded for logical, succinct reasoning, not just for the correct answer.

1. a) i) Show how Poisson's equation can be derived from Gauss's law.

ii) Consider a uniformly charged sphere with volume charge density ρ and radius R centred on the origin. In this situation explain where Poisson's equation reduces to Laplace's equation.

b) In two or three sentences each explain how polar and non-polar molecules become polarized in an electric field.

c) For the magnetic field $\vec{B} = -z\hat{y} + y\hat{z}$ identify which of these functions (if any) describes the vector potential. Calculate the volume current density $\vec{J}$ for any possible vector potential.

i) $\vec{A} = x^2\hat{x} + xz\hat{y} - 2xz\hat{z}$

ii) $\vec{A} = -x^2\hat{x} + xy\hat{y} + xz\hat{z}$

Use a separate page clearly marked Question 2. This question is worth *8 marks.*

2. An infinite rectangular slot is surrounded by three conducting sides, these are cleverly insulated from each other. There are infinite grounded planes at $x = \pm a$, and an infinite plane at $y = 0$ maintained at a constant potential $V = V_0$. You should assume that there is a separable solution to this problem, that is, there is a solution of the form: $V = X(x)Y(y)Z(z)$.

a) Write down the boundary conditions for this problem.

b) Briefly explain why there can be only one solution for V inside this slot.

c) When writing down the Laplacian for this problem the term $\frac{\partial^2 V}{\partial z^2}$ can be left off. Explain why in one or two sentences.

d) Showing your working, write down Laplace's equation, substitute in $V = X(x)Y(y)$, and seperate it into two terms, one dependent on x and the other on y.

e) In one or two sentencea explain why the term dependent on x must be equal to a constant.

We will assume the term dependent on x is equal to $-k^2$ and the term dependent on y is equal to k^2.

f) Show that:
$X(x) = A\sin(kx) + B\cos(kx)$ and
$Y(y) = Ce^{ky} + De^{-ky}$
are solutions to the equations you wrote down in step d).

g) Simplify the equations in the step above as much as possible making use of the boundary conditions.

h) Recombine your $X(x)$ and $Y(y)$ terms and simplify as much as possible.

i) In two or three sentences explain how you would complete this problem to come up with an expression for V. You do not need to come up with the final solution to V!

Use a separate page clearly marked Question 3. This question is worth *6 marks.* Second longer calculation question

3. A sphere with a radius R centred on the origin carries a polarization described by:

$$\vec{P}(\vec{r}) = kr^2\hat{r}.$$

where $\hat{r}$ is a radial unit vector pointing from the origin.

a) Calculate the bound charge densities: σ_b and ρ_b as a function of r.

b) Calculate the total bound charge inside the sphere and over its surface. Is this what you would expect?

c) What is the electric field, $\vec{E}(\vec{r})$, inside the sphere? Include a diagram in your solution.

d) What is the electric field, $\vec{E}(\vec{r})$, outside the sphere?

e) What is the electric displacement, $\vec{D}(\vec{r})$ inside the sphere?

f) What is the potential, $V(\vec{r})$, inside the sphere? Take $V = 0$ as $r \to \infty$.