1 a) Electric fields do obey the law of superposition. Assume we double the charge distribution, that is replace each charge q with a charge $2q$, then the electric field is doubled.

$$\underset{\sim}{E}_{new} = \underset{\sim}{E} + \underset{\sim}{E} = 2\underset{\sim}{E}$$

However the energy needed to assemble his new charge distribution is

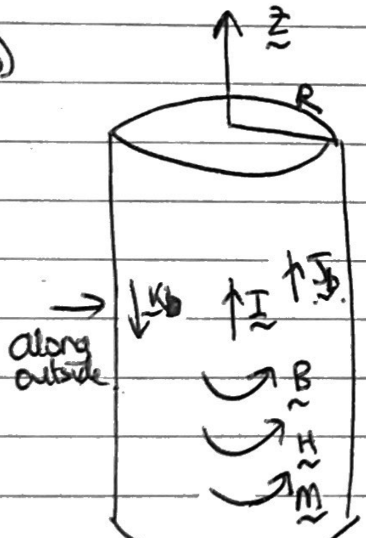
$$W_{new} = \frac{\varepsilon_0}{2}\int (2\underset{\sim}{E})\cdot(2\underset{\sim}{E})\, d\tau = \frac{4\varepsilon_0}{2}\int E^2 d\tau.$$

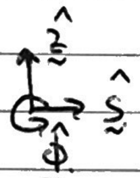
If energy obeyed the law of superposition then

$$W_{new} = W + W = 2W = \frac{2\varepsilon_0}{2}\int E^2 d\tau.$$

As these are not equal superposition is not obeyed.

b)





$\underset{\sim}{I}$ is in $+z$ direction

$\underset{\sim}{B}$ using right-hand screw rule is in $+\phi$ direction.

$\underset{\sim}{H}$ is in same direction as $\underset{\sim}{B}$, so $+\underset{\sim}{\phi}$ direction.

$\underset{\sim}{M}$ is in same direction to $\underset{\sim}{B}$ for paramagnet so $+\underset{\sim}{\phi}$ direction.

$\underset{\sim}{K}_b$ is in $\underset{\sim}{M}\times\hat{\underset{\sim}{n}} = +\hat{\underset{\sim}{\phi}}\times\hat{\underset{\sim}{s}}$ direction $= -\hat{\underset{\sim}{z}}$ along outside of cylinder, no surface current inside!

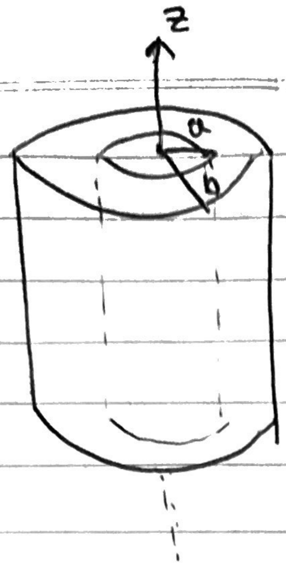
$\underset{\sim}{J}_b$ is in $\nabla\times\underset{\sim}{M}$, ie. $+\hat{\underset{\sim}{z}}$, direction inside cylinder.

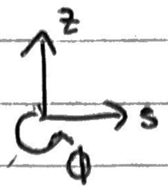
$\Rightarrow$ increases current and hence $\underset{\sim}{B}$.

c) a) Paramagnetism is the alignment of the magnetic dipoles with the magnetic field. This occurs because there is a torque on the magnetic dipole $\underset{\sim}{N} = \underset{\sim}{m}\times\underset{\sim}{B}$ which works to make $\underset{\sim}{m}$ and $\underset{\sim}{B}$ parallel. The magnetic dipole of the atom is a result of the electron spin. In atoms with an odd number of electrons this is the dominant effect.

b) In diamagnetic materials $\underset{\sim}{m}$ aligns to the $-\underset{\sim}{B}$ direction (as opposed to $\underset{\sim}{B}$ direction). This usually happens when there is an even number of electrons as the spins cancel out (Pauli exclusion principle).

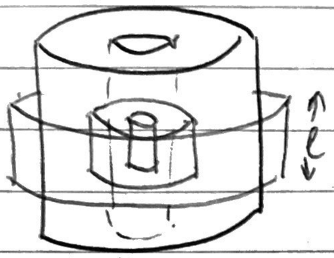
2 a)





cylindrical coordinates.

b) Use Gauss's law for displacement, taking cylindrical Gaussian surfaces



For $s<a$ $\oint \underset{\sim}{D} \cdot d\underset{\sim}{a} = Q_{f,enc} = 0 \Rightarrow \underset{\sim}{D} = 0.$

For $a<s<b$ $\oint \underset{\sim}{D} \cdot d\underset{\sim}{a} = D \cdot 2\pi s l = \sigma \cdot 2\pi a \cdot l.$

$$\Rightarrow D = \frac{\sigma a}{s} \hat{\underset{\sim}{s}}$$

For $s>b$ $\oint \underset{\sim}{D} \cdot d\underset{\sim}{a} = D \cdot 2\pi s l = \sigma \cdot 2\pi a l.$

$$\Rightarrow D = \frac{\sigma a}{s} \hat{\underset{\sim}{s}}$$ (same as $a<s<b$).

c) For $s<a$ $\underset{\sim}{E} = \frac{D}{\varepsilon} = \frac{D}{\varepsilon_0} = 0.$

For $a<s<b$ $\underset{\sim}{E} = \frac{D}{\varepsilon_0(1+\chi_e)} = \frac{\sigma a}{\varepsilon_0(1+\chi_e)s} \hat{\underset{\sim}{s}}$

For $s>b$ $\underset{\sim}{E} = \frac{D}{\varepsilon_0} = \frac{\sigma a}{s\varepsilon_0} \hat{\underset{\sim}{s}}$

d) $\underset{\sim}{P} = \underset{\sim}{D} - \varepsilon_0 \underset{\sim}{E} = \varepsilon_0(1+\chi_e)\underset{\sim}{E} - \varepsilon_0 \underset{\sim}{E} = \varepsilon_0 \chi_e \underset{\sim}{E} = \frac{\sigma a \cancel{\varepsilon_0} \chi_e}{\cancel{\varepsilon_0}(1+\chi_e)s} \hat{\underset{\sim}{s}} = \frac{\sigma a \chi_e}{(1+\chi_e)s} \hat{\underset{\sim}{s}}$

e) $\Delta V = -\int_0^a \underset{\sim}{E} \cdot d\underset{\sim}{s} - \int_a^b \underset{\sim}{E} \cdot d\underset{\sim}{s} - \int_b^s \underset{\sim}{E} \cdot d\underset{\sim}{s}$

$$= 0 - \frac{\sigma a}{\varepsilon_0(1+\chi_e)} \int_a^b \frac{ds}{s} - \frac{\sigma a}{\varepsilon_0} \int_b^s \frac{ds}{s}$$

$$= -\frac{\sigma a}{\varepsilon_0(1+\chi_e)} \ln\left(\frac{b}{a}\right) - \frac{\sigma a}{\varepsilon_0} \ln\left(s/b\right)$$

f) $\sigma_b = \underset{\sim}{P} \cdot \hat{\underset{\sim}{n}} = \cancel{\underset{\sim}{P} \cdot \hat{\underset{\sim}{n}}} = \frac{\sigma a \chi_e}{(1+\chi_e)s} \hat{\underset{\sim}{s}} \cdot \hat{\underset{\sim}{n}}$

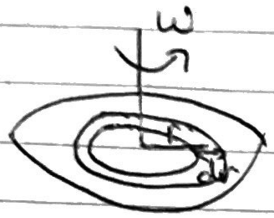
at $s=a$ $\hat{\underset{\sim}{n}} = -\hat{\underset{\sim}{s}}$ (pointing out of surface)

$\Rightarrow \sigma_b = -\frac{\sigma \cancel{a} \chi_e}{(1+\chi_e)\cancel{a}} = -\frac{\sigma \chi_e}{1+\chi_e}$

at $s=b$ $\hat{\underset{\sim}{n}} = \hat{\underset{\sim}{s}}$

$\Rightarrow \sigma_b = \frac{\sigma a \chi_e}{(1+\chi_e)b}$

3a)



Consider a small ~~disk~~ ring at radius r with width dr.
The charge on this ring is:

$$dq = \sigma(r) . 2\pi r . dr . = 2\pi k r^2 dr.$$

To get total charge integrate these rings over the disk

$$Q = \int_0^R 2\pi k r^2 dr = 2\pi k \frac{R^3}{3}.$$

b) $\underset{\sim}{K}(r) = \sigma(r) \underset{\sim}{v} = kr . \omega r . \hat{\underset{\sim}{\phi}} = k\omega r^2 \hat{\underset{\sim}{\phi}}$ $\qquad \left(= \frac{dI}{dr}\right)$

c) $dI = k\omega r^2 dr.$

$\Rightarrow I = \int k\omega r^2 dr = \frac{k\omega r^3}{3}$ is current flowing at radius r.

$$M_{ring} = Ia = \frac{k\omega r^3}{3} . 2\pi r . dr = \frac{2}{3} k\omega r^4 dr$$

d) $$m = \int m_{ring} = \int_0^R \frac{2}{3} k\omega r^4 dr = \frac{2}{3} k\omega \left[\frac{r^5}{5}\right]_0^R = \frac{2}{3} k\omega \frac{R^5}{5}$$

$$= \frac{2k\omega R^5}{15}.$$