TUTORIAL PROBLEMS

WEEK 2: FINISH ELECTROSTATICS, START SPECIAL TECHNIQUES

Textbook problems: You should complete these problems to ensure you are able to solve problems based on the main concepts covered in lectures. These problems are taken from Introduction to Electrodynamics by David J Griffiths (3rd Edition but same in 4th Edition as well).

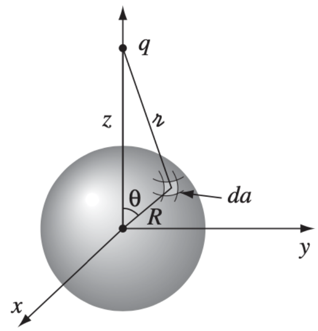
1. In the 19th century, telegraph operators often felt shocks from long wires. Model a telegraph wire as an infinitely long line charge with uniform density λ. Find the potential a distance s from this wire. Compute the gradient of your potential, and check that it yields the correct field. These shocks inspired studies into line charges, aiding the development of safer high-voltage power lines. (Adaptation of Griffiths Problem 2.22).

2. Find the potential on the axis of a uniformly charged solid cylinder, a distance z from the center. The length of the cylinder is L, its radius is R, and the charge density is ρ. Use your result to calculate the electric field at this point. (Assume that $z > L/2$.) (Griffiths Problem 2.27)

3. Consider two concentric spherical shells, of radii a and b. Suppose the inner one carries a charge q, and the outer one a charge $-q$ (both of them uniformly distributed over the surface). Calculate the energy of this configuration,
(a) using $W = \frac{\epsilon_0}{2} \int E^2 d\tau$ (all space), and
(b) using $W_1 + W_2 + \epsilon_0 \int \vec{E}_1 \cdot \vec{E}_2 d\tau$ and that the energy of a uniformly charged sphere with radius R and charge q is $W = \frac{1}{8\pi\epsilon_0} \frac{q^2}{R}$ (Griffiths Problem 2.36).

4. A spherical satellite in geostationary orbit has a radius of $R = 1.0\,\mathrm{m}$. The satellite's two hemispheres are held together by 10 titanium bolts, each with a radius of $r = 1.0\,\mathrm{cm}$. Titanium has a tensile strength of $\sigma = 300\,\mathrm{MPa}$. Calculate the maximum charge Q_{max} the satellite can accumulate before the electrostatic repulsion between the hemispheres exceeds the bolts' mechanical strength. (Adaptation of Griffiths Problem 2.42)

5. Find the average potential over a spherical surface of radius R due to a point charge q located inside (same in image below, only with $z < R$). (In this case, of course, Laplace's equation does not hold within the sphere.) Show that, in general, $V_{avg} = V_{center} + \frac{Q_{enc}}{4\pi\epsilon_0 R}$, where V_{center} is the potential at the center due to all the external charges, and Q_{enc} is the total enclosed charge. (Griffiths Problem 3.1).



Conceptual exam problem:
6. i) Give an example of a case where the electric field is not continuous.
ii) For the case you have given is the potential continuous? Demonstrate this.

Calculated exam problem:
7. The electric potential of some configuration is given by the expression

$$V(\mathbf{r}) = A\frac{e^{-\lambda r}}{r},$$

where A and λ are constants with appropriate units.
You may make use of the following relationships involving Dirac Deltas:

$$f(x)\delta(x) = f(0)\delta(x),$$

$$\nabla \cdot (\frac{\hat{r}}{r^2}) = 4\pi\delta^3(\mathbf{r}),$$

$$\int \delta^3(r)d\tau = 1.$$

And the following standard integral:

$$\int_0^\infty re^{-ar}dr = \frac{1}{a^2}.$$

i) Find an expression for the electric filed $\mathbf{E(r)}$.
ii) Find an expression for the charge density $\rho(r)$.
iii) What is the total charge (through all space)?
Note: An example of a potential with this form is the Yukawa Potential which describes the potential between nucleons (protons and neutrons) in an atomic nucleus. This is a short-range interation dropping off rapidly with distance, it can be thought of as a screened

version of the Coulomb potential.
(Based on Griffiths Problem 2.50)

Answers:

1. $-\frac{1}{4\pi\epsilon_0}2\lambda\ln(\frac{s}{a})$.
2. $\frac{\rho}{2\epsilon_0}[L-\sqrt{R^2+(z+\frac{L}{2})^2}+\sqrt{R^2+(z-\frac{L}{2})^2}]\hat{z}$.
3. $\frac{q^2}{8\pi\epsilon_0}\big(\frac{1}{a}-\frac{1}{b}\big)$
4. 2.9 mC
5. $V_{avg}=\frac{1}{4\pi\epsilon_0}\frac{q}{R}$
7. i) $Ae^{-\lambda r}(1+\lambda r)\frac{\hat{r}}{r^2}$, ii) $\rho=\epsilon_0 A[4\pi\delta^3(\mathbf{r})-\frac{\lambda^2}{r}e^{-\lambda r}]$, iii)0