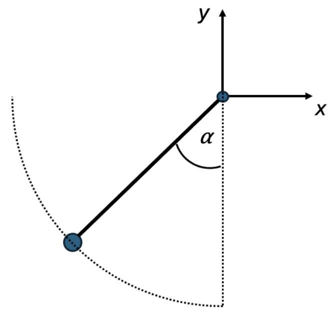
Classical Mechanics and Special Relativity (PHYS2113)

Tutorial Problem Sheet 1

1. Planar pendulum, Newtonian brute-force style

Consider a planar pendulum consisting of a mass m attached to a rigid massless rod of length L that is suspended from the ceiling and can swing freely around its fulcrum.



a) What forces are acting on the mass? Draw a force diagram.

b) Derive expressions for the forces in Cartesian coordinates as a function of the angle α.

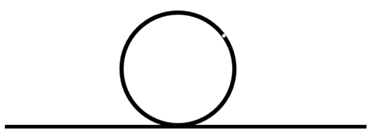
c) Substitute all occurrences of α with the corresponding expressions in terms of x and y and write down the equations of motion in the form

$$m\ddot{x} = F_x(x, y) \tag{1}$$
$$m\ddot{y} = F_y(x, y). \tag{2}$$

d) The result of c) looks messy. Suggest a cleverer way of finding the dynamics of the system and find an approximate analytic solution in the limit $\alpha \ll 1$.

2. Degrees of freedom and holonomy

a) Consider a wheel on a rail. One point on the surface of the wheel is marked with a white dot. The wheel's centre of mass can only move in the direction of the rail, i.e., no jumping allowed.



(i) Assume the wheel is "slidey", i.e., besides rolling, it can also slide or spin in place. How many degrees of freedom does the system have? What is the minimum number of parameters required to describe the state of the system at a given time?

(ii) Now imagine the wheel is "sticky" (e.g., coated with rubber), so it cannot slide and spin in place anymore. How many degrees of freedom does the system have now? What is the minimum number of parameters required to describe the state of the system at a given time?

b) Let's add one dimension to the problem and look at a ball with a dot on it that can move along a two-dimensional planar surface.

(i) A slidey ball. How many degrees of freedom does the system have? What is the minimum number of parameters required to describe the state of the system at a given time?

(ii) A sticky ball. How many degrees of freedom does the system have now? What is the minimum number of parameters required to describe the state of the system at a given time?
Hint: Think about what happens if we roll the ball along a closed path.