THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

Practice Mid-term test 2

PHYS2114
Electromagnetism

(1) TIME ALLOWED – 50 minutes

(2) TOTAL NUMBER OF QUESTIONS – 3

(3) ANSWER ALL QUESTIONS

(4) THE QUESTIONS ARE **NOT** OF EQUAL VALUE

(5) TOTAL NUMBER OF MARKS – 20

(6) THIS PAPER MAY BE RETAINED BY THE CANDIDATE

Use a seperate page clearly marked Question 1. This question is worth *6 marks.*

You should answer each part of this question in less than half a page. Include equations and diagrams in your answer where appropriate. Marks will be awarded for logical, succinct reasoning, not just for the correct answer.

1. a) Consider a spherical shell bounded by $a \leq r \leq b$. The potential in this region is described by $V(r) = \frac{A}{r} + B$, where A and B are positive integers.

i) Show that in this region the potential satisfies Laplace's equation in spherical coordinates.

ii) Hence describe the charge density in the region $a < r < b$.

b) Consider:
$\oint \vec{J} \cdot \vec{da}$.

i) Describe what each symbol in this equation means.

ii) Define what is meant by a steady current.

iii) How would you evaluate this integral for a steady current?

c) Charge is added throughout an initially uncharged dielectric. As a result it becomes polarized. Is it possible to say if in this process the object obtains:

i) a volume bound charge density?

ii) a surface bound charge denisty?

iii) a volume free charge density?

iv) a surface free charge density?

v) Describe the relationship between these properties.

Use a separate page clearly marked Question 2. This question is worth *8 marks.* You may find some of the following standard integrals useful:

$$\int \sin\theta\cos\theta d\theta = -\frac{1}{2}\cos^2\theta + constant$$

2. Consider a solid sphere of charge with radius R, located at the origin, with a variable total charge distribution. The charge distribution is given by $\rho(\theta) = k\cos\theta$, where k is a constant with appropriate units, θ is the angle with the positive z-axis (as is normal in physics in spherical coordinates).

a) Calculate the total charge in the top half of the sphere.

b) In one or two sentences explain why the monopole contribution to the potential is negligible at large distances from the sphere.

c) In order to calculate the dipole moment, $\vec{p}$, of the sphere using $\vec{p} = \int \vec{r'}\rho(\vec{r'})d\tau'$ it is useful to break the sphere into small disks with height dz. Draw a diagram showing this and explain why only the component of $\vec{p}$ in the z-direction needs to be included.

A thin disk at z, with height dz, has a dipole moment given by:

$$\vec{p} = 2\pi k z^2(R - |z|)dz\hat{z}$$

d) What is the average polarization, $\vec{P}$, of the sphere?

e) Calculate the dipole contribution to the potential using $V_{dip}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$.

f) In one or two sentences explain why at large distances $V \approx V_{dip}$.

Use a separate page clearly marked Question 3. This question is worth *6 marks.*

3. A very long, hollow wire with inner radius a, and outer radius b, carries a current with a volume current density described by $\vec{J} = \frac{k}{s}\hat{z}$, where k is a constant with appropriate units.

a) What is the total current, $\vec{I}$, flowing through this wire?

b) Find the magnetic field, $\vec{B}(s)$, in the regions: $s < a$, $a < s < b$ and $s > b$.

c) Show that the vector potential is given by:

$$\vec{A}(s) = \mu_0 k(a - s + a\ln\frac{s}{a})\hat{z}$$

in the region $a < s < b$.

d) Calculate the magnetic force per unit length on this wire if it is placed in a magnetic field, $\vec{B} = B_0\hat{x}$.