THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

Final Exam T2 2024

PHYS2114
Electromagnetism

(1) TIME ALLOWED – 15 minutes reading time + 2 hours

(2) TOTAL NUMBER OF QUESTIONS – 5

(3) ANSWER ALL QUESTIONS

(4) THE QUESTIONS ARE **NOT** OF EQUAL VALUE

(5) TOTAL NUMBER OF MARKS – 100

(6) THIS PAPER MAY BE RETAINED BY THE CANDIDATE

Answer this question in Booklet A. Use a seperate page clearly marked Question 1. This question is worth *18 marks*.

1. a) i) (1 mark) Electric fields obey the superposition principle. Write down an equation to illustrate what this means.

ii) (1 mark) Besides electric fields, list two other quantities that obey the principle of superposition.

iii) (4 marks) Use equations and a written description to show that the electrical energy needed to set up a charge distribution with an electric field $\vec{E}$, given by $W = \frac{\epsilon_0}{2}\int E^2 d\tau$, is not the sum of the energy needed to set up the two charge distributions, $\vec{E}_1$ and $\vec{E}_2$, independently for the case $\vec{E} = \vec{E}_1 + \vec{E}_2$.

b) (6 marks) A steady current flows through a cylindrical paramagnetic material, with radius R, in the positive z-direction. Draw a diagram showing the direction of the current, $\vec{I}$, magnetic field, $\vec{B}$, Auxiliary field, $\vec{H}$, magnetization, $\vec{M}$, volume current density, $\vec{J}_b$ and surface current density, $\vec{K}_b$ inside the material.

c) i) (4 marks) Explain what causes paramagnetism.

ii) (2 mark) Explain how diamagnetism is different to paramagnetism.

Answer this question in Booklet A. Use a separate page clearly marked Question 2. This question is worth *20 marks*.

2. Consider an infinitely long conducting cylinder with radius a and surface charge density σ. The cylinder is surrounded by a neutral dielectric material with susceptibility χ_e out to radius b. Susceptibility is related to relative permittivity (also known as the dielectric constant) by $\epsilon_r = 1 + \chi_e$.

a) (2 mark) Draw a diagram clearly showing this configuration. Include axes showing the coordinate system you are using.

b) (4 marks) Calculate the electric displacement, $\vec{D}(s)$, inside the conducting cylinder, $s < a$, inside the dielectric, $a < s < b$, and outside the dielectric, $s > b$.

c) (4 marks) Calculate the electric field, $\vec{E}(s)$, inside the cylinder, $s < a$, inside the dielectric, $a < s < b$, and outside the dielectric, $s > b$.

d) (2 mark) Calculate the polarization, $\vec{P}(s)$, inside the dielectric, $a < s < b$.

e) (4 marks)Calculate the potential at s, where $s > b$, relative to a point at $s = 0$, ie find $\Delta V = V(s) - V(0)$.
Note: this is not relative to a point an infinite distance away as in this case the potential blows up when taking $V = 0$ at ∞.

f) (4 marks) What is the bound surface charge density, σ_b, at $s = a$ and $s = b$?

Answer this question in Booklet A. Use a separate page clearly marked Question 3. This question is worth *12 marks*.

3. A thin disk with radius R have a variable surface charge density described by $\sigma(r) = kr$ where k is a constant with appropriate units. The disk rotates with an angular velocity ω about the centre of the disk.

a) (4 marks) Calculate the total charge on the disk. Include a diagram showing the disk and coordinate system you are using in your answer.

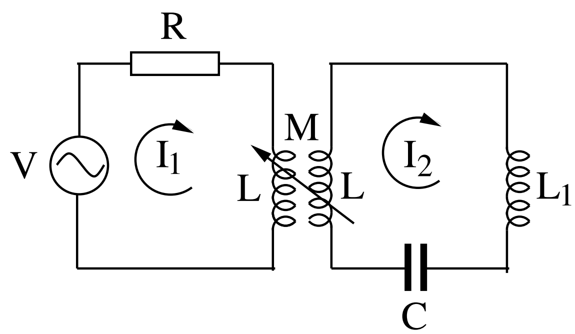
b) (2 mark) Write an expression for the surface current density, $\vec{K}(r)$, of the disk.

c) (3 marks) Write an expression for the magnetic dipole moment, $\vec{m}_{ring}$, of a ring at radius r with width dr on this disk, $r < R$.

d) (3 marks) Find the magnetic dipole moment $\vec{m}$ of the disk.

Answer this question in Booklet B. Use a separate page clearly marked Question 4. This question is worth *25 marks*.

4. Consider the circuit given in the figure that is driven by an AC voltage source, $V = V_0 \cos \omega t$. Each coil of the transformer connecting the two branches has the selfinductance L, the mutual inductance of the coils is M.



Remember that impedances for R, L, C, and M are $Z = R$, $Z = i\omega L$, $Z = \frac{1}{i\omega C}$, $Z = i\omega M$ respectively.

a) (13 marks) Use Kirchhoff's Laws to determine the relationship between the applied voltage in the first branch and the current flowing in each branch.

b) (12 marks) Even if the resistance is nonzero, $R \neq 0$, there is a resonance in the circuit. Find frequency of the resonance.

Answer this question in Booklet B. Use a separate page clearly marked Question 5. This question is worth *25 marks*.

5. For this question you should make use of Maxwell's equations and the Poynting's vector. Here they are (SI units).

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$
$$\vec{\nabla} \cdot \vec{B} = 0$$
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$
$$\vec{S} = \frac{1}{\mu_0} [\vec{E} \times \vec{B}]$$

a) Consider the electromagnetic wave propagating in the x-direction.

$$\vec{E} = E_0 Re \left\{ \left[\sqrt{\frac{2}{3}} \vec{e}_y - i \sqrt{\frac{1}{3}} \vec{e}_z \right] e^{ikx - i\omega t} \right\}$$

Here $\vec{e}_y$ and $\vec{e}_z$ are unit vectors along y- and z-axes respectively. E_0 is the real amplitude.

i) (4 marks) What is the polarisation of the wave? Explain/justify your answer.

ii) (5 marks) Using Maxwell's equations calculate magnetic field in the wave. *There is a useful check of your answer: in a plane wave electric and magnetic fields must be orthogonal.*

iii) (4 marks) Calculate the magnitude and the direction of the energy flux in the wave (Poynting's vector), both instantaneous and averaged over time.

b) Consider the standing wave created by two linearly polarised waves running in opposite directions.

$$\vec{E} = \frac{E_0}{2} Re \left[\vec{e}_z e^{ikx - i\omega t} - \vec{e}_z e^{ikx + i\omega t} \right] = E_0 \vec{e}_z \sin(kx) \sin(\omega t)$$

i) (4 marks) Using Maxwell's equations calculate magnetic field in the wave.

ii) (4 marks) Calculate the energy flux in the wave (Poynting's vector), both instantaneous and averaged over time.

iii) (4 marks) Comment on the physical meaning of your answer to the question **(ii)**.