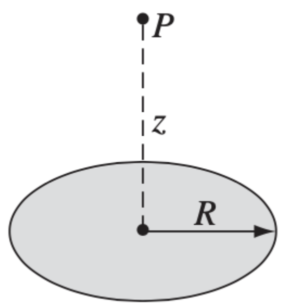
TUTORIAL PROBLEMS

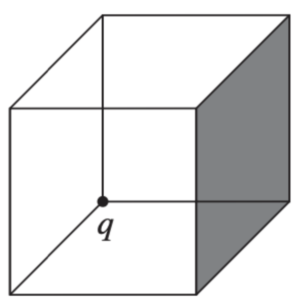
WEEK 1: ELECTROSTATICS 2025

Textbook problems: You should complete these problems to ensure you are able to solve problems based on the main concepts covered in lectures. Because this is the first tutorial this week these problems are based on content you saw in Physics 1B. These problems are taken from Introduction to Electrodynamics by David J Griffiths (3rd Edition but same in 4th Edition as well).

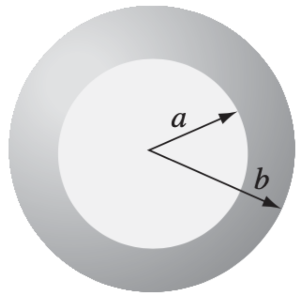
1. Find the electric field a distance z above the center of a flat circular disk of radius R that carries a uniform surface charge σ . What does your formula give in the limit $R \to \infty$? Also check the case $z \gg R$. (Griffiths Problem 2.6).



2. A charge q sits at the back corner of a cube, as shown in the figure below. What is the flux of E through the shaded side? (Griffiths Problem 2.10)



3. A thick spherical shell carries charge density
$\rho = \frac{k}{r^2}(a \leq r \leq b)$.
Find the electric field in the three regions: (i) $r < a$, (ii) $a < r < b$, (iii) $r > b$. Plot $|\vec{E}|$ as a function of r, for the case $b = 2a$. (Griffiths Problem 2.15).



4. Sketch the vector function:

$$\mathbf{v} = \frac{\hat{\mathbf{r}}}{r^2},$$

and compute its divergence and curl in both cartesian and spherical coordinates. Does the answer surprise you? How can you explain this? What is an example from electromagnetism function that has a similar form? (based on Griffiths Problem 1.16).

Conceptual exam problem: There are a few short conceptual questions at the start of the midterm and final. This is an example of a conceptual problem that could be asked on the above content.

5. i) With the use of a diagram and suitable equations explain why the net electric flux through any closed surface in a uniform electric field is zero.
ii) We showed in class that Gauss's law held for a single particle with charge q. Clearly explain why given this Gauss's law must hold for any collection of charges.

Calculated exam problem: The exam and midterm will include two extended calculation questions on electrostatics, covering material from the first five weeks. This example illustrates the type of question you might encounter, based on the content discussed. However, since this material is from Physics 1B, it is unlikely that an exam question will focus solely on this content. These questions are broken into smaller parts than the earlier tutorial questions to help keep you on track during the exam.

6. Consider a sphere of charge with radius r and uniform charge density ρ. You will need to calculate the electric field at a height $z > r$ above the center of this sphere of charge by considering the contribution of a small increment of volume $d\tau$ a distance r' from the center of the sphere and then summing these increments over the entire sphere.
i) Draw a diagram showing this situation, including axes, r, r', a volume increment $d\tau$ and the point where $\vec{E}$ is measured a distance z above the center of the sphere.

ii) Write an expression for $d\vec{E}$ the electric field due to the volume increment you have identified in your diagram. Express your answer in spherical coordinates.
iii) By using a substitution $u = \cos\theta$ and the standard integral:

$$\int \frac{z - r'u}{(r'^2 - 2r'zu + z^2)^{3/2}}\,du = \frac{uz - r'}{z^2\sqrt{r'^2 - 2uzr' + z^2}} + C$$

or otherwise, calculate the electric field at this point due to the entire sphere. Show all your working.
iv) Does this behave as expected in the limit $z \to \infty$?

Answers:
1. $\vec{E}_{disk} = \frac{1}{4\pi\epsilon_0} 2\pi\sigma z[\frac{1}{z} - \frac{1}{\sqrt{R^2+z^2}}]\hat{z}$.
2. $\frac{q}{24\epsilon_0}$.
3. (i) 0; (ii)$\frac{k}{\epsilon_0}(\frac{r-a}{r^2})\hat{r}$; (iii) $\frac{k}{\epsilon_0}(\frac{b-a}{r^2})\hat{r}$
4. $\nabla \cdot \mathbf{v} = 0$; $\nabla \times \mathbf{v} = 0$; clearly a sources so surprising divergence is zero, it is infinite at the origin and zero everywhere else, electric field of a point particle has this form.
5. A suitable equation is $\Phi_E = \oint \vec{E} \cdot d\vec{A}$.
6. ii) $dE_z = \frac{\rho}{4\pi\varepsilon_0}\frac{(z - r'\cos\theta)}{(r'^2+z^2-2r'z\cos\theta)^{3/2}} r'^2 \sin\theta\, dr'\, d\theta\, d\phi$, iii) $\frac{Q}{4\pi\epsilon_0 z^2}$, iv) Yes, behaves like a point charge.