THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

Term 2 Mid-term 2024 Paper

PHYS2114
Electromagnetism

(1) TIME ALLOWED – 50 minutes

(2) TOTAL NUMBER OF QUESTIONS – 3

(3) ANSWER ALL QUESTIONS

(4) THE QUESTIONS ARE **NOT** OF EQUAL VALUE

(5) TOTAL NUMBER OF MARKS – 20

(6) THIS PAPER MAY BE RETAINED BY THE CANDIDATE

(7) YOU MAY REFER TO YOUR 1 A4 PAGE FORMULA SHEET. THIS SHOULD BE HANDED IN WITH YOUR ANSWERS

Use a seperate page clearly marked Question 1. This question is worth *6 marks.*

You should answer each part of this question in less than half a page. Include equations and diagrams in your answer where appropriate. Marks will be awarded for logical, succinct reasoning, not just for the correct answer.

1. a) i) Explain why any net charge in a conductor is found only on its surface.

ii) Explain why electric field lines must be perpendicular to the surface of a conductor.

b) i) Show that $V = A\sin(kx)e^{-ky}$ satisfies Laplace's equation.

ii) In the region $-a < x < a$, $-b < y < b$ where are the maximum and minimum values for V going to be found? Assume that a, b and k are all greater than π.

c) Explain why the magnetic force cannot change the speed of a charged particle.

Use a separate page clearly marked Question 2. This question is worth *8 marks.*

You may find the following standard integrals useful:

$$\int \frac{\sin\theta}{\sqrt{R^2+d^2-2Rd\cos\theta}}d\theta = \frac{\sqrt{R^2-2Rd\cos\theta+d^2}}{Rd} + constant$$

$$\int \frac{\sin\theta\cos\theta d\theta}{\sqrt{R^2+d^2-2Rd\cos\theta}} = \frac{(R^2+dR\cos\theta+d^2)\sqrt{R^2-2dR\cos\theta+d^2}}{3d^2R^2} + constant$$

2. A spherical shell with radius R, is centred on the origin and has a variable charge density, dependent on height, described by $\sigma(z) = k|z|$, where k is a constant with appropriate units.

a) Consider a small increment of the spherical surface with area da, at height $z = h$, $0 < h < R$. Calculate the potential, dV, due to this small increment at the point $z = d$, $d > R$ on the z-axis. Include a diagram of the charge distribution and point $z = d$ in your solution.

b) Making use of this or otherwise, calculate the potential, V due to the sphere at a point $z = d > R$ on the positive z-axis.

Inside the sphere at a point $|z = d| < R$ on the z-axis, the potential is given by:

$$V = \frac{kR}{3\epsilon_0 d^2}((R^2+d^2)^{3/2} - R^3).$$

c) Is the potential continuous along the positive z-axis? Should it be?

d) Calculate E_z along the positive z-axis. Is E_z continuous? Should it be?

Use a separate page clearly marked Question 3. This question is worth *6 marks.*

3. A long, hollow, cylindrical tube with radius R centred and parallel to the z-axis carries a surface current $\vec{K} = k\hat{\phi}$ where k is a constant with appropriate units.

a) Show that outside the tube, $s > R$, the magnetic field is given by $\vec{B} = 0$. Include a diagram showing the current and tube in your solution.

b) Find the magnetic field, $\vec{B}(s)$, inside, $s < R$, the tube.

c) Show that:

$$\oint \vec{A} \cdot d\vec{\ell} = \int \vec{B} \cdot d\vec{a}$$

where $\vec{A}$ is the vector potential and everything has its normal meaning.

d) Using this or otherwise, find the vector potential, $\vec{A}(s)$, inside and outside the tube.

e) Show that outside the tube the curl of the vector potential you have found is equal to $\vec{B}$.