UNSW SCHOOL OF PHYSICS

PHYS2111 – Quantum Mechanics

Tutorial 9

Question 1

(past exam question): The Schödinger Eqn is give by:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \frac{1}{2}m\omega^2 x^2\psi = E\psi \tag{1}$$

and the first eigenfunctions are given by,

$$\begin{aligned}\psi_0(x) &= A_0 e^{-u^2/2} \\ \psi_1(x) &= A_1 u e^{-u^2/2}\end{aligned} \tag{2}$$

where $u = (m\omega/\hbar)^{\frac{1}{2}}\, x$.

(a) Calculate the normalisation constants A_0 and A_1.

(b) For $\psi_0(u)$ and $\psi_1(u)$: calculate the expectation values $\bar{x}$ and $\bar{p}$.

(c) Calculate the energy of the wave function ψ_0.

(d) An atom of mass $m = 4.85 \times 10^{-23}$ g is vibrating. The energy of a vibrational level is $E = 7.2$ meV. What is the displacement amplitude in the classical limit?

Question 2

(Park 4.24) A particle is oscillating in the state $n = 2$ when suddenly the spring loses half its elasticity, $K \to K/2$. Then the particle's energy is measured. What is the probability that

(a) the old ground-state energy is found?

(b) the new ground-state energy is found?

(c) the new energy corresponding to n=1 is found.

Question 3

(a) Using the ladder operator method, calculate the $n = 3$ wave function and verify that it matches that predicted by the power series method.

(b) Does the ladder operator preserve the normalisation condition when generating new eigenstates for the harmonic oscillator.

Question 4

(Park 4.27) Starting with two operators $\hat{a}$ and $\hat{a}^\dagger$ that satisfy the $\left[\hat{a}, \hat{a}^\dagger\right] = 1$, find the eigenvalues and eigenfunctions of $\hat{a}^\dagger\hat{a}$. Now do the same for operators $\hat{b}$ and $\hat{b}^\dagger$ that satisfy $\{\hat{b}, \hat{b}^\dagger\} = 1$, where $\{\hat{b}, \hat{b}^\dagger\}$ is the anti-commutator, $\hat{b}\hat{b}^\dagger + \hat{b}^\dagger\hat{b}$, and $\hat{b}^2 = \hat{b}^{\dagger 2} = 0$

Question 5

Solve Schrödinger's Eqn for the infinitely thin potential barrier $V(x) = +\alpha\delta(x)$ and calculate the corresponding reflectance and transmission.

Question 6

(Griffiths 2.24) Check the uncertainty principle for the wave function described by the bound infinite delta function potential.

Question 7

(Griffiths 2.25) Check that the bound state of the delta function well is orthogonal to the scattering state.

Question 8

(Griffiths 2.27 - harder) Consider the double delta-function potential

$$V(x) = -\alpha\,[\delta(x+a) + \delta(x-a)] \tag{3}$$

where α and a are positive constants.

(a) Sketch the potential

(b) How many bound states does it possess? Find the allowed energies, for $\alpha = \hbar^2/ma$ and for $\alpha = \hbar^2/4ma$ and sketch the wave functions.

(c) What are are the bound state energies in the limiting cases of (i) $a \to 0$ and (ii) $a \to \infty$