UNSW SYDNEY
Australia's Global University

Electron Diffraction

Quantum Physics

Student Notes

SKILLS GAINED

- General lab safety
- Using Vernier Calipers
- How to work safely with high voltages.
- How to work safely with evacuated glass containers

ASSUMED KNOWLEDGE

- Basics of Quantum Mechanics
- Basics of Interference and diffraction
- Knowledge of the Bragg X-ray diffraction experiment and ability to use the Bragg equation.

1 Experimental aim

To demonstrate the wave properties of electrons by observing the diffraction of an electron beam from a polycrystalline layer of graphite. The specific aim of this experiment is to calculate the lattice spacing in graphite from the diameter of the interference rings and the electron accelerating voltage.

2 Background

In 1925, French physicist Louis de Broglie, a graduate student at that time, showed that matter also has a wave nature as well as a particle nature. de Broglie developed his theory of particle-wave duality of matter in 1924, for his PhD dissertation. Matter was previously seen as consisting of particles, such as electrons and protons, characterised by particle properties, energy (E) and momentum (p). Einstein's new paradigm suggested additional wave properties, such as frequency ν and wavelength λ. Louis de Broglie's equation connects the wave and particle aspects:

$$\lambda = \frac{h}{p} \qquad (7.1)$$

where h = 6.625 × 10^{-34} J.s is Planck's constant. There is a short and easy to follow commentary by P. Weinberger revisiting de Broglie's 1924 paper in the *Philosophical Magazine* [1]. It is well worth reading before doing this experiment, if you have time.

This work won de Broglie the Nobel Prize in 1929, after his work was experimentally verified by the US physicists Clinton J. Davisson and Lester H. Germer. In 1927 Davisson and Germer first observed the diffraction and interference of electrons (*particles*) scattered from a nickel target. This discovery happened by accident: the nickel target had crystallised after heating was used to remove the oxide coating and the atomic planes of nickel crystal provided a natural diffraction grating, with the separation of the planes in the crystal being of order of the wavelength of the scattered electrons. The scattered electron intensity was found to be a function of the scattering angle. This phenomenon is analogous to the "Bragg reflections", which occur when X-rays (*waves*) are scattered from the atomic planes of a crystal (see Figure 1).

The Bragg condition for constructive interference gives maxima at:

$$2d\sin\theta = n\lambda \quad (n = 1,\ 2\ ....) \tag{7.2}$$

where *d* is the spacing between the planes of the carbon atoms and θ is the Bragg angle (angle between electron beam and lattice planes), and *n* refers to the order of diffraction (Figure 1).

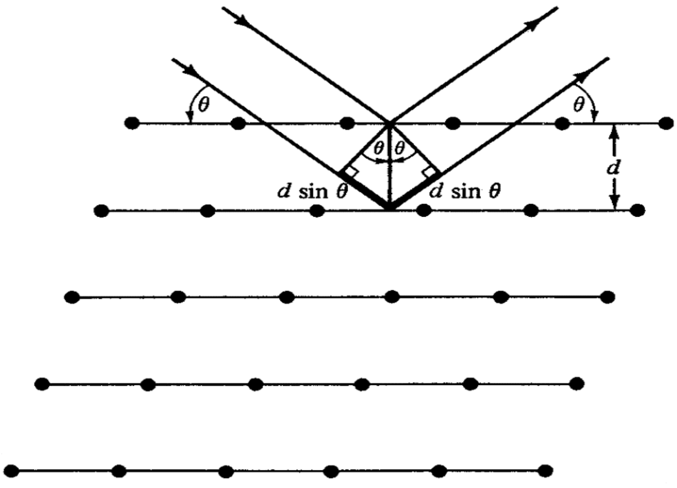

Figure 1: This diagram illustrates how Bragg reflections arise when X-rays scattered off different planes in a crystal interfere to produce an interference pattern. X-rays are waves, but the Davisson-Germer experiment reproduced something similar with an electron beam illuminating a crystal, and the Davisson-Germer experiment also satisfies the Bragg equation (7.2).

An electron beam with de Broglie wavelength λ is reflected in accordance with Bragg equation (7.2), and you will need this equation in your analysis. In the Davisson-Germer experiment, the electron wavelength was calculated from the location of minima and maxima in the diffraction pattern, and it agreed with the de Broglie equation (7.1).

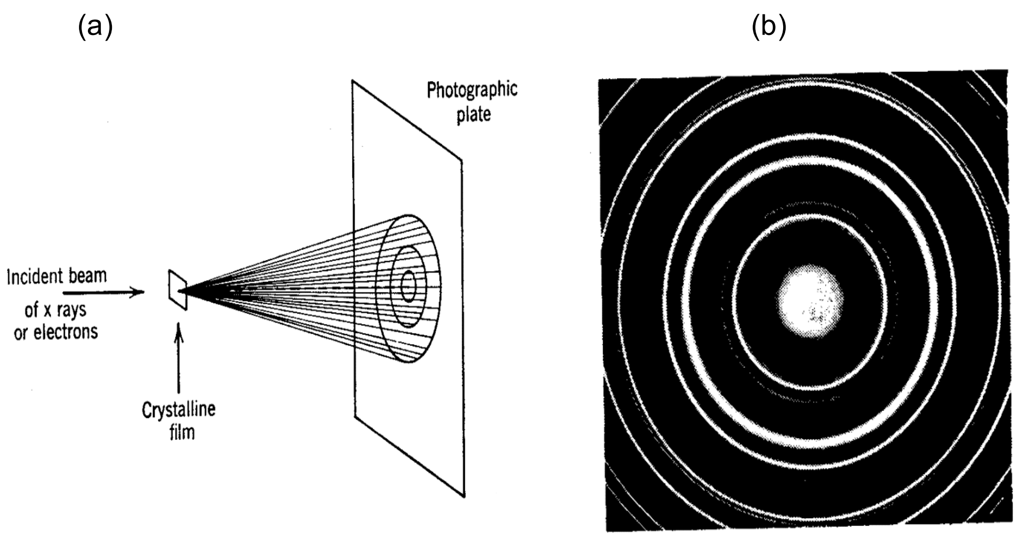

Figure 2: (a) The Debye-Scherrer-Hull technique for diffracting beams of X-rays, electrons or neutrons The crystalline film is consists of powdered crystals with random orientations, equivalent to a two-dimensional diffraction grating, as discussed by Hull [2]. (b) The diffraction pattern from polycrystalline gold foil (from Eisberg & Resnick, Quantum Physics of Atoms, Molecules, etc, Wiley 1985).

In 1928 G.P. Thomson (son of J.J. Thomson, who discovered that electrons are particles) also observed diffraction patterns when an electron beam passed through polycrystalline gold foil. His experiment was similar to the Debye-Scherrer-Hull technique: a polycrystalline powder provides a two-dimensional lattice that diffracts X rays, electrons or neutrons into the form of a cone (Figure 2). This technique is often called *Powder Diffraction*, as opposed to the *Single Crystal Diffraction* of the Davisson-Germer experiment. *Powder Diffraction* is what you will be doing in this experiment. As mentioned above, de Broglie was awarded the Nobel Prize in 1929, while Davisson and Thomson (heads of their groups) shared the Prize in 1937.

Nowadays the electron diffraction technique is used widely in crystallography to study new materials.

2.1 Quantitative information on how the experiment works

The momentum of an electron with kinetic energy KE can be calculated from the velocity *v* that the electrons acquire under the acceleration voltage U_a (Figure 3):

$$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m} = eU_a \qquad (7.3)$$

where *m* is the rest mass of the electron, 9.109×10^{-31} kg, and *e* is the electronic charge 1.6×10^{-19} C. For the accelerating voltages used, the relativistic mass of the electron can be replaced by the rest mass with an error of only 0.5 %.

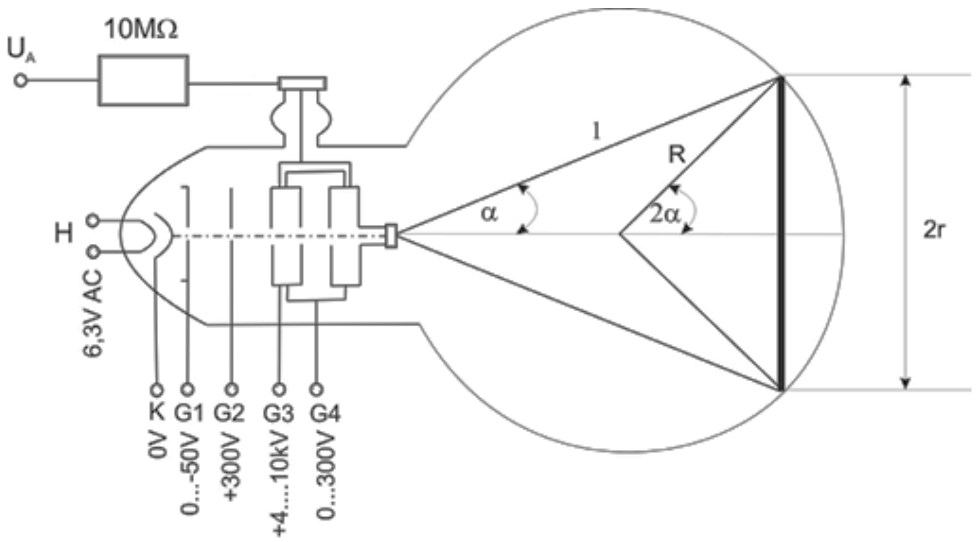

Figure 3: Set-up and beam geometry for the electron beam

Using Equations 7.1 and 7.3 we can calculate the de Broglie wavelength λ:

$$\lambda = \frac{h}{\sqrt{2meU_a}} \qquad (7.4)$$

The electron beam with de Broglie wavelength λ is reflected in accordance with the Bragg condition, shown in Figure 1 and given in Equation 7.2. In this experiment the electrons scatter from a powder of polycrystalline graphite (see Figure 2). It is composed of many small crystals, crystallites, with random orientation to each other. When the electron beam strikes such a polycrystalline target, some of the crystallites will have the atomic planes in the “correct” orientation and reflect the electrons. The electron beam is therefore spread out in the form of a cone, producing interference maxima on the phosphorescent screen in the form of concentric rings around the centre (Figure 2).

The two first order interference rings occur through reflection from the lattice planes of spacing d_1 and d_2 (Figure 4b). Constructive interference results for the beams scattered from the neighboring atomic planes when the rays are in phase (the difference in path length is a whole number of wavelengths).

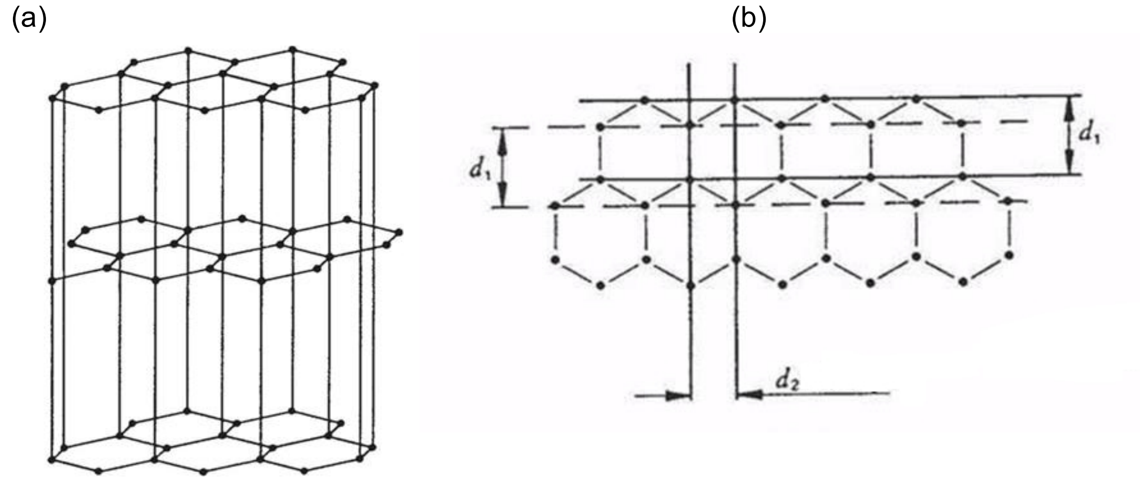

Figure 4: (a) Crystal lattice of graphite, and (b) graphite planes for the first two interference rings.

The Bragg angle θ can be calculated from the radius of the interference ring, but it should be remembered that the angle of deviation α (Figure 3) is twice as great:

$$\alpha = 2\theta \qquad (7.6)$$

From Figure 3:

$$\sin 2\alpha = \frac{r}{R} \qquad (7.7)$$

where $R \approx 65$ mm is the radius of the spherical surface of the glass tube.

Using the trigonometric identity, $\sin 2\alpha = 2\sin\alpha\cos\alpha$ and an approximation for small angle α ($\cos 10^\circ = 0.985$): $\sin 2\alpha \approx 2\sin\alpha$:

$$\sin\alpha = \frac{r}{2R} \qquad (7.8)$$

For the small angles θ we obtain $\sin\alpha = \sin 2\theta = 2\sin\theta$:

$$\sin\theta = \frac{r}{4R} \qquad (7.9)$$

With this approximation and using (7.5) we obtain:

$$r = \frac{2R}{d} n\lambda \qquad (7.10)$$

With n = 1, 2, 3 …

2.2 Useful links:

- Physclips: Diffraction, including electron diffraction.
- The page from some University College London Chemistry notes gives an excellent description of powder diffraction, which is what you will be doing for this experiment, works: http://pd.chem.ucl.ac.uk/pdnn/diff2/kinemat2.htm.

3 Prework

3.1 Theoretical prework

1. Does the particle-wave duality apply to all particles? Calculate λ of a pitched baseball with mass m = 0.15 kg and velocity v = 60m/s. What can you conclude from the result?
2. Calculate λ of 7.0 kV electrons and compare it to the wavelength range of visible light. What is the advantage of an electron microscope compared to a microscope that uses visible light?
3. The operating instructions for this experiment also contain some theory, introduced as necessary while talking about the equipment you will be using, and how to operate it. At this stage, consult both these notes and the operating instructions (Section 2.2) for this experiment, and using Equations 7.1 – 7.10, derive the relationship between the radii of the rings r_1 and r_2 and the accelerating potential U_a. Explain how you can obtain the principal spacings of the graphite lattice, d_1 and d_2, from a plot of r_1, r_2 and U_a.

3.2 Experimental plan

Now you get to undertake this Nobel Prize winning experiment in the Second Year Physics Laboratory. The Operating Instructions for this experiment give you the information you need to undertake the experiment.

What you are required to do is briefly outlined here:

- Go to the Operating Instructions for this experiment, and take careful note of how to keep both yourself and the equipment safe (Section 1) while making your experimental measurements. This experiment involves both high voltages and equipment that is easily damaged.
- Read Section 2.1: The experimental setup carefully, and follow all instructions.
- Work through Section 2.2: Quantitative information on how the experiment works. You will already have done this in Prework question 3, but now do this again with the equipment in front of you.
- Make your measurements and produce appropriate graphs and analysis to extract the key lattice parameters for graphite.

4 Analysis and Discussion

- You may wish to compare your experimental results with known values from the literature (we will let you find them yourself, be sure to cite your source) and discuss any deviations between your measurements and literature values.
- You might like to consider the implications of the 10 Mohm resistor that is in series with the high voltage power supply (see Figure 3). It is possible to graph the data in such a way as to make the effect of this resistor clear.
- You might also compare the range of electron wavelengths with d_1 and d_2. Can you explain why at some values of U_a the electrons behaved as particles, while at others the electrons behaved as waves?
- In your analysis, you should consider the effects of accelerating voltage on electron wavelength, and at the least, aim to obtain d_1 and d_2 for graphite. Would you expect more accurate results for a higher accelerating voltage or a lower accelerating voltage?

5 Conclusions

Write a conclusion to your experimental report. In dot points summarize briefly your main findings, especially with regard to how well the experiment demonstrated the wave nature of electrons.

6 Background References

[1] Weinberger, P. 2006. *Philosophical Magazine Letters*, Vol. 86, No. 7, July 2006, 405-410. You can it find at http://www.cms.tuwien.ac.at/media/pdf/publications/PML-86-405-2006.pdf

[2] In Prince, E. 2004. International Tables for Crystallography, Mathematical, Physical and Chemical Tables, Chapter 2, Section 2.3, p42 (view online at: https://books.google.com.au/books?id=60FoFEGyShIC&pg=PA43&lpg=PA43&dq=Debye-Hull-Scherrer+technique&source=bl&ots=u16kJky_gq&sig=VAOdNaYIf2piTnBq03SIhen2jdM&hl=en&sa=X&ved=0ahUKEwi8o7fBgavLAhUFpZQKHbUVCOcQ6AEIPTAG#v=onepage&q=Debye-Hull-Scherrer%20technique&f=false).