The convention in this thesis will be to say **ZF** when talking about Zermelo-Fraenkel set theory *without* the axoim of choice. When talking about the axoim of choice on its own we will say **AC**, and when talking about Zermelo-Fraenkel set theory together with the axiom of choice use **ZFC**.

We assume that the reader has some familiarity with axiomatic set theory, but for convenience and constistency we restate some of the necessary basics here. For a more thorough review, see [**?**].

0.1 Zermelo-Fraenkel Axioms

Taken (almost) unchanged from [**?**].

0.1.1 Axiom of Extensionality

$$\forall x \forall y \left(x = y \iff \forall z \left(z \in x \iff z \in y\right)\right)$$

Two sets are equal if and only if they contain the same elements.

0.1.2 Empty Set Axiom

$$\exists x \forall y y \notin x$$

There is a set with no elements.

0.1.3 Axiom of Pairs

$$\forall x \forall y \exists z \forall w \left(w \in z \iff \left(w = x \vee w = y\right)\right)$$

For any two sets, there is a set whose elements are precisely these sets.

0.1.4 Axiom of Seperation

$$\forall x \exists y \forall z \left(z \in y \iff \left(z \in x \wedge \phi(z)\right)\right),$$

where $\phi(z)$ is any statement of the formal language with free variable z. For any set x there is a set consiting of all z in x for which $\phi(z)$ holds.

0.1.5 Power Set Axiom

$$\forall x \exists y \forall z \left(z \in y \iff z \subseteq x\right)$$

For any set x there is a set consisting of all subsets of x.

0.1.6 Union Axiom

$$\forall x \exists y \forall z \left(z \in y \iff \exists w \left(z \in w \wedge w \in x\right)\right)$$

For any set x there is a set which is the union of all the elements of x.

0.1.7 Axiom of Infinity

$$\exists x \left(\varnothing \in x \wedge \forall y \left(y \in x \implies y \cup \{y\} \in x\right)\right)$$

There is an inductive set.

0.1.8 Axiom of Replacement

$$\forall x \exists y \forall y' \left(y' \in y \iff \exists x' \left(x' \in x \wedge \phi(x', y')\right)\right),$$

where $phi(s, t)$ is a formula such that

$$\forall s \exists t \left(\phi(s, t) \wedge \forall t' \left(\phi(s, t') \implies t' = t\right)\right).$$

Of $\phi(s, t)$ is a class function, then when its domain is restricted to xm the resulting images form a set y.

0.1.9 Axiom of Foundation

$$\forall x \exists y \left(y \in x \wedge x \cap y = \varnothing\right)$$

Every set is *well-founded*, i.e. contains an $\in$-minimal element.