The convention in this thesis will be to say **ZF** when talking about Zermelo-Fraenkel set theory *without* the axiom of choice. When talking about the axiom of choice on its own we will say **AC**, and when talking about Zermelo-Fraenkel set theory together with the axiom of choice use **ZFC**.

We will use the convention of including 0 at the beginning of the natural numbers $\mathbb{N}$, i.e. $\mathbb{N} = \{0, 1, 2 \ldots\}$. This is a *natural* choice, since we then can use $\mathbb{N}$ to mean the set described by the Axiom of Infinity.

0.1 Zermelo-Fraenkel Axioms of Set Theory

We assume that the reader has some familiarity with axiomatic set theory, but for convenience and consistency we restate some of the necessary basics here. For a more thorough review, see [**?**], from which the formulations below are used as well.

0.1.1 Axiom of Extensionality

$$\forall x \forall y \left(x = y \iff \forall z \left(z \in x \iff z \in y \right) \right)$$

Two sets are equal if and only if they contain the same elements.

0.1.2 Axiom of the Empty Set

$$\exists x \forall y \, y \notin x$$

There is a set with no elements.

0.1.3 Axiom of Pairs

$$\forall x \forall y \exists z \forall w \left(w \in z \iff \left(w = x \lor w = y \right) \right)$$

For any two sets, there is a set whose elements are precisely these sets.

0.1.4 Axiom of Separation

$$\forall x \exists y \forall z \left(z \in y \iff \left(z \in x \land \phi(z) \right) \right),$$

where $\phi(z)$ is any statement of the formal language with free variable z. For any set x there is a set consisting of all z in x for which $\phi(z)$ holds.

0.1.5 Axiom of Power Sets

$$\forall x \exists y \forall z \left(z \in y \iff z \subseteq x \right)$$

For any set x there is a set, denoted by $\mathcal{P}(x)$ and called the power set of x, consisting of all subsets of x.

0.1.6 Union Axiom

$$\forall x \exists y \forall z \left(z \in y \iff \exists w \left(z \in w \land w \in x \right) \right)$$

For any set x there is a set, denoted by $\bigcup x$, which is the union of all the elements of x.

0.1.7 Axiom of Infinity

$$\exists x \left(\varnothing \in x \land \forall y \left(y \in x \implies y \cup \{y\} \in x \right) \right)$$

There is an inductive set.

0.1.8 Axiom of Replacement

$$\forall x \exists y \forall y' \left(y' \in y \iff \exists x' \left(x' \in x \land \phi(x', y') \right) \right),$$

where $\phi(s, t)$ is a formula such that

$$\forall s \exists t \left(\phi(s, t) \land \forall t' \left(\phi(s, t') \implies t' = t \right) \right).$$

If $\phi(s, t)$ is a class function, then when its domain is restricted to a set x, the resulting images form a set y. Since no proper classes will be treated in this text, the relevant statement for us is that the image of any function, of which the domain is a proper set, always is a set.

0.1.9 Axiom of Foundation

$$\forall x \exists y \left(y \in x \land x \cap y = \varnothing \right)$$

Every set is *well-founded*, i.e. contains an $\in$-minimal element.