Unit IV: Complexity

CISC 380 Algorithms

Dr. Miracle

SAT $\rightarrow$ 3SAT

Given f for SAT create a new formula f' as follows:

For each clause C in f,

- If C has ≤ 3 literals, keep it the same.
- If it has > 3 literals replace C by C' as described below:

$$C = (a_1 \vee a_2 \vee \ldots \vee a_k)$$

Add $k-3$ new variables $y_1, \ldots, y_{k-3}$ and replace C by $k-2$ clauses:

$$(a_1 \vee a_2 \vee y_1) \wedge (\overline{y}_1 \vee a_3 \vee y_2) \wedge (\overline{y}_2 \vee a_4 \vee y_3) \wedge \ldots$$

$$\wedge (\overline{y}_{k-4} \vee a_{k-2} \vee y_{k-3}) \wedge (\overline{y}_{k-3} \vee a_{k-1} \vee a_k)$$

- Use f' as input for 3SAT

Claim: f is satisfiable iff f' is satisfiable.

Example: SAT $\rightarrow$ 3SAT

$$(x_1) \wedge (\overline{x_1} \vee \overline{x_2} \vee \overline{x_3} \vee x_4) \wedge (\bar{x_5} \vee x_3 \vee x_2 \vee \bar{x_1} \vee \overline{x_4}) \wedge (x_4 \vee x_5)$$

Procedure for converting input f to SAT to an input to 3SAT

For each clause C in f,

- If C has ≤ 3 literals, keep it the same.
- If it has > 3 literals replace C by C' as described below:

$$C = (a_1 \vee a_2 \vee \ldots \vee a_k)$$

Add $k-3$ variables $y_1, \ldots, y_{k-3}$ and replace C by $k-2$ clauses:

$$(a_1 \vee a_2 \vee y_1) \wedge (\overline{y}_1 \vee a_3 \vee y_2) \wedge (\overline{y}_2 \vee a_4 \vee y_3) \wedge \ldots$$

$$\wedge (\overline{y}_{k-4} \vee a_{k-2} \vee y_{k-3}) \wedge (\overline{y}_{k-3} \vee a_{k-1} \vee a_k)$$

Example: 3SAT $\rightarrow$ Independent Set

$$f = (x) \wedge (\bar{x} \vee y \vee w) \wedge (\bar{y} \vee \bar{w} \vee z) \wedge (x \vee y) \wedge (\bar{y} \vee w \vee \bar{z})$$

1. Create the corresponding input to IS (i.e., a graph G, goal g).
2. Find an IS set and from this create a satisfying assignment.
3. Find a satisfying assignment and from this create an IS in G of size g.

Example 2: 3SAT $\rightarrow$ Subset Sum

$$f = (\overline{x_1} \vee \overline{x_2} \vee \overline{x_3}) \wedge (\bar{x_1} \vee \overline{x_2} \vee x_3) \wedge (x_1 \vee \bar{x_2} \vee x_3) \wedge (x_1 \vee x_2)$$

Procedure for converting input f to 3SAT to an input to SubsetSum

1. In the ith digit of v_i and v_i' put a 1.
2. If x_i appears in c_j put a 1 in v_i in digit $n + j$.
3. If $\bar{x_i}$ appears in c_j put a 1 in v_i' in digit $n + j$.
4. Put a 1 in digit $n + j$ of s_j and s_j' (these function as buffers).
5. All other digits get 0.
6. t contains n 1's followed by m 3's.

Example 1: 3SAT $\rightarrow$ Subset Sum

$$f = (\overline{x_1} \vee \overline{x_2} \vee \overline{x_3}) \wedge (\bar{x_1} \vee \overline{x_2} \vee x_3)$$

$$\wedge (x_1 \vee \bar{x_2} \vee x_3) \wedge (x_1 \vee x_2)$$

Examples:

$$v_1 = 1000011$$

$$v_1' = 1001100$$

$$t = 1113333$$

	x_1	x_2	x_3	c_1	c_2	c_3	c_4
v_1	1	0	0	0	0	1	1
v_1'	1	0	0	1	1	0	0
v_2	0	1	0	0	0	0	1
v_2'	0	1	0	1	1	1	0
v_3	0	0	1	0	1	1	0
v_3'	0	0	1	1	0	0	0
s_1	0	0	0	1	0	0	0
s_1'	0	0	0	1	0	0	0
s_2	0	0	0	0	1	0	0
s_2'	0	0	0	0	1	0	0
s_3	0	0	0	0	0	1	0
s_3'	0	0	0	0	0	1	0
s_4	0	0	0	0	0	0	1
s_4'	0	0	0	0	0	0	1
t	1	1	1	3	3	3	3

Example 2: 3SAT $\rightarrow$ Subset Sum

$$f = (x) \wedge (\bar{x} \vee y \vee w) \wedge (\bar{y} \vee \bar{w} \vee z) \wedge (x \vee y) \wedge (\bar{y} \vee w \vee \bar{z})$$

Procedure for converting input f to 3SAT to an input to SubsetSum

1. In the ith digit of v_i and v_i' put a 1.
2. If x_i appears in c_j put a 1 in v_i in digit $n + j$.
3. If $\bar{x}_i$ appears in c_j put a 1 in v_i' in digit $n + j$.
4. Put a 1 in digit $n + j$ of s_j and s_j' (these function as buffers).
5. All other digits get 0.
6. t contains n 1's followed by m 3's.