Algorithm Analysis

CISC 380 Algorithms

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Bubble Sort

Problem: sort a list of numbers

```
function BubbleSort( List[0,. . . ., n-1])
     i=0
     swapped = true
     while (i <= n-1 and swapped)
          swapped = false
          for j = 0 to n-1
               if List[j] > List[j+1]
                    Swap(List[j], List[j+1])
                    swapped = true
          i = i + 1
     return List
```

Determine worst case input.

Bubble Sort

Problem: sort a list of numbers

Improved Bubble Sort

```
function BubbleSort( List[0,. . . ., n-1])
     i=0
     swapped = true
     while (i <= n-1 and swapped)
          swapped = false
          for j = 0 to n-1-i
               if List[j] > List[j+1]
                    Swap(List[j], List[j+1])
                    swapped = true
          i = i + 1
     return List
```

BOGO Sort

Problem: sort a list of numbers

```
function BOGO( List[0,. . . ., n])
     while not InOrder(List)
          RandomShuffle(List)
     return List
```

Asymptotic Growth Rate Bounds

Let $T(n)$ and $f(n)$ be functions mapping positive integers to positive real numbers.

- $T(n) \in O(f(n))$ if there exist positive constants c and n_0 such that
$$T(n) \leq cf(n) \text{ for all } n > n_0.$$
- $T(n) \in \Omega(f(n))$ if there exist positive constants c and n_0 such that
$$T(n) \geq cf(n) \text{ for all } n > n_0.$$
- $T(n) \in \Theta(f(n))$ iff $T(n) \in O(f(n)$ and $T(n) \in \Omega(f(n))$.
- $T(n) \in o(f(n))$ iff $T(n) \in O(f(n)$ and $T(n) \notin \Theta(f(n))$.
- $T(n) \in \omega(f(n))$ iff $T(n) \in \Omega(f(n)$ and $T(n) \notin \Theta(f(n))$.

Quick Rules

- *Polynomial:* If $T(n)$ is a polynomial of degree k, then $T(n) = \Theta(n^k)$.
- *Addition:* If $T_1(n) = \Theta(f(n))$ and $T_2(n) = \Theta(g(n))$, then $T_1(n) + T_2(n) = \Theta(\max(f(n), g(n)))$
  (Be careful: you can only use the addition rule a constant number of times.)
- *Log:* $log^k n = O(n)$ for any *constant* k.
  (Note that this is not true for $k = n$)
- *Exponential:* n^k is $O(b^n)$ for all constants $b > 1, k \geq 0$.

The Limit Theorem

Consider the following limit:

$$\lim_{n \to \infty} T(n)/f(n)$$

If the limit exists, then the following is true:

1. If the limit is 0 then $T(n) = o(f(n))$ and $T(n) = O(f(n))$.
2. If the limit is ∞ then $T(n) = \omega(f(n))$ and $T(n) = \Omega(f(n))$.
3. If the limit is a constant not equal to 0 then $T(n) = \Theta(f(n))$ (and $T(n) = O(f(n)$ and $T(n) = \Omega(f(n))$.

Asymptotic Growth Rate

Order the following functions by asymptotic growth rate.
Circle functions that grow at the same rate (i.e. $f(n) = \Theta(g(n))$).

$3^n,\ 3+5n,\ 2^{n+1},\ n^2+10n+200,\ 2^{n^2},\ 2^{n/2},\ \log n,\ \ln n, 5\log^2 n,\ 2^n$

Asymptotic Growth Rate

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Answer:

$(\ln n,\ \log n),\ 5\log^2 n,\ 3+5n,\ n^2+10n+200,\ 2^{n/2},\ (2^{n+1},\ 2^n),\ 3^n,\ 2^{n^2}$

Asymptotic Notation

Questions

1. If $T(n) \in O(f(n))$ does this imply that $T(n) \leq f(n)$?

2. If $T(n) < f(n)$ does this imply that $T(n) \in O(f(n))$?

   Does it imply $T(n) \in \Theta(f(n))$?

   Does it imply $T(n) \in o(f(n))$?

Asymptotic Notation

Questions

1. If $T(n) \in O(f(n))$ does this imply that $T(n) \leq f(n)$? No!
   (e.g. $100n \in O(n)$)

2. If $T(n) < f(n)$ does this imply that $T(n) \in O(f(n))$? Yes
   (let $c = 1$ in definition of O)

   Does it imply $T(n) \in \Theta(f(n))$? No

   Does it imply $T(n) \in o(f(n))$? No

Analyzing Pseudo-code

```
for i = 1 to n
     if (i == 5)
          for j = 1 to n
               print "It's finally Friday!"
     else
          print "Have a great weekend."
```